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1. Wess, Zumino, Ferrara [1,2] and Salam and Strathdee [3,4] have proposed a gauge supersymmetric theory. The internal symmetry transformation parameters used there are scalar superfunctions of a special type - the so called chiral superfunctions [4] $\lambda_i(x, \theta)$, where θ are the spinor anticommuting coordinates. These theories involve in an essentially nonlinear manner a gauge real scalar superfield containing a gauge vector Yang-Mills field and a gauge spinor field. One can call them chiral gauge supersymmetric theories (CGST)*.

In this paper we construct a *general* gauge supersymmetric theory (GGST) in which internal symmetry is realized locally in superspace (x, θ) in the most general way. The chiral parameters $\lambda_i(x, \theta)$ of CGST are replaced by general real scalar superfunctions. In this case the gauge superfield is a Majorana *spinor* superfield which includes a vector Yang-Mills field and a spin 3/2 field. The gauge superfield enters into the Lagrangian polynomially and the coupling constant is dimensionless

2. Consider a set of superfields $V_m(x, \theta)$ belonging to some internal symmetry group representation with generators $(T_i)_{mn}$

$$V'_m(x, \theta) = [\exp(ig\lambda_i T_i)]_{mn} V_n(x, \theta). \tag{1}$$

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* The CGST gave raise to a large number of papers which treat the renormalization of these theories [5-7] and their spontaneous breaking [7]. A deep analogy of the CGST with "quasi" Yang-Mills theories [9] has been pointed out in [10].

An invariant Lagrangian for these superfields contains V_m and some differential operators constructed out of spinor derivatives /4,9/ $D_\alpha = \frac{\partial}{\partial \bar{\theta}^\alpha} - \frac{i}{2}(\not{\partial}\theta)_\alpha$ *.

By analogy with the Yang-Mills approach we replace the constant parameters λ_i in (1) by arbitrary scalar superfunctions $\Lambda_i(x,\theta)$

$$V'_m(x,\theta) = [\exp(ig\Lambda_i(x,\theta)T_i)]_{mn} V_n(x,\theta) . \qquad (2)$$

The product of scalar superfunctions is again a scalar superfunction, therefore transformations (2) form a group. Thus the internal symmetry group is localized in superspace in the most general way**.

3. As in the Yang-Mills theories a lengthened covariant spinor derivative can be defined

$$\Delta_\alpha V_m(x,\theta) = (D_\alpha + ig\Psi_{\alpha i} T_i)_{mn} V_n(x,\theta) . \qquad (3)$$

Here a new superfield - the gauge spinor superfield $\Psi_\alpha(x,\theta) = \Psi_{\alpha i}(x,\theta)\, t_i$ is introduced (t_i are the adjoint representation generators). It is easy to verify that $\Delta_\alpha V_m$ transforms according to the same rule (2) as the superfield V_m itself provided $\Psi_\alpha(x,\theta)$ transforms as follows

$$\Psi'_\alpha(x,\theta) = e^{ig\Lambda(x,\theta)}\Psi_\alpha(x,\theta)\, e^{-ig\Lambda(x,\theta)} - \frac{i}{g} e^{ig\Lambda(x,\theta)} D_\alpha e^{-ig\Lambda(x,\theta)} . \qquad (4)$$

The group property is evident.

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* The following notations are used: $\not{\partial} = \gamma^\mu \partial_\mu$, $(\gamma_\mu)^+ = g_{\mu\mu}\gamma_\mu$, $g_{\mu\nu} = \mathrm{diag}(+---)$, $\gamma_5 = \gamma_0\gamma_1\gamma_2\gamma_3$.

** In CGST only chiral superfunctions $\Lambda_i(x,\theta)$ and consequently only chiral material superfields are considered. In contrast, in GGST the material superfields V_m , as well as Λ_i, have to be general ones.

4. Now it is easy to construct a Lagrangian for the superfields V_m which is invariant under gauge transformations (2). For this purpose one has to lengthen all the spinor derivatives D_α in the original Lagrangian according to (3). In this way an invariant interaction of V_m with the gauge superfield is switched on.

The next step is to write an invariant under (4) Lagrangian for Ψ_α itself. This Lagrangian is found to have the form

$$\mathcal{L} = \frac{1}{16}\mathrm{Tr}\{\bar{\Psi} i\partial\!\!\!/ \Psi - \frac{1}{2}[(\bar{D} + ig\bar{\Psi})\gamma_\mu \Psi]^2 + \frac{1}{12}[(\bar{D} + ig\bar{\Psi})\sigma_{\mu\nu}\Psi]^2\} . \quad (5)$$

where Tr means trace over the internal symmetry indices and the invariant action is $S = \int d^4x\, d^4\theta\, \mathcal{L}(x,\theta)$ /10,11/. The derivation which is based on the projection operators /12/ will be described elsewhere. Let us stress that Ψ_α enters into (5) polynomially and that the coupling constant g is dimensionless.

It is necessary to note that there is one peculiarity which could produce some difficulties. In the free case at $g = 0$, the Lagrangian is invariant under the transformation $\Psi'_\alpha = \Psi_\alpha - D_\alpha \Lambda$ (see (4)). However, in this case it becomes also invariant under a new transformation

$$\Psi'_\alpha(x,\theta) = \Psi_\alpha(x,\theta) + (\partial\!\!\!/ \gamma_5 D)_\alpha \Omega(x,\theta) , \quad (6)$$

where Ω is an arbitrary scalar superfield. We do not know whether it is possible to modify this invariance for $g \neq 0$.

5. To make the results more transparent let us discuss them in terms of ordinary fields. The superfield $\Psi_\alpha(x,\theta)$ has the decomposition

$$\Psi_\alpha(x,\theta) = \psi^{(1)}_\alpha(x) + \bar{\theta}^\beta \phi^{(1)}_{\beta\alpha}(x) + \frac{1}{4}\bar{\theta}\theta\,\psi^{(2)}_\alpha(x) + \frac{1}{4}\bar{\theta}\gamma_5\theta\,\psi^{(3)}_\alpha(x) +$$

$$+ \frac{1}{4}\bar{\theta} i\gamma^\mu\gamma_5\theta\,\psi_{\alpha\mu}(x) + \frac{1}{4}\bar{\theta}\theta\bar{\theta}^\beta \phi^{(2)}_{\beta\alpha}(x) + \frac{1}{32}(\bar{\theta}\theta)^2\,\psi^{(4)}_\alpha(x) . \quad (7)$$

It contains four Majorana spinor fields $\psi^{(i)}_{a}(x)$, Majorana spinvector field $\psi_{a\mu}(x)$ and four real Bose fields $\phi_{\beta a}(x)$:

$$\phi_{\beta a}(x) = (v(x)\,1 + a(x)\gamma_5 + iv^{\mu}(x)\gamma_{\mu} + iA^{\mu}(x)\gamma_{\mu}\gamma_5 + ie^{\mu\nu}(x)\sigma_{\mu\nu})_{\beta a} \quad (8)$$

The calculations in terms of fields are very cumbersome. Here we restrict ourselves to the free case (g=0) only. The gauge invariances (4) and (6) make some degrees of freedom in decomposition (7), (8) arbitrary and they do not appear in the equations of motion. Some other components in (7), (8) vanish due to the equations of motion. Finally, in the free case the Lagrangian (5) describes a massless vector field $V_{\mu}(x)$ and a massless spinvector field $\chi_{\mu a}(x)$ which obey the Proca and Rarita-Schwinger equations, respectively

$$\Box V_{\mu} - \partial_{\mu}\partial^{\nu}V_{\nu} = 0;\quad \hat{\partial}\chi_{\mu} - \partial_{\mu}\gamma^{\nu}\chi_{\nu} - \gamma_{\mu}\partial^{\nu}\chi_{\nu} + \gamma_{\mu}\hat{\partial}\gamma^{\nu}\chi_{\nu} = 0.$$

Note that these equations are invariant under the transformations $V'_{\mu} = V_{\mu} + \partial_{\mu} b$; $\chi'_{\mu} = \chi_{\mu} + \partial_{\mu}\lambda$ connected with (4) and (5) correspondingly.

We hope to give a detailed analysis of the theory proposed in further publications.

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