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As is known, the corrections of the order $O(\frac{v^2}{c^2})$ to the famous Thomas-Reiche-Kuhn sum rule include the retardation term for electric dipole transition /1/. It is defined by the first order of the power-series expansion of $\exp(i\vec{k}\vec{r})$ in transition matrix element between ℓ and $\ell+1$ states. If the photon propagates along the z-axis and is polarized along the y-axis, retardation correction to the oscillator strength f_{on} is

$$f^{R}_{on} = -\frac{2m\omega}{\hbar c^2}(\dot{y}z^2)_{on}(\dot{y})_{on}\,. \tag{1}$$

For the S ground state the summed oscillator strength is equal

$$\sum_n f^{R}_{on} = -\frac{1}{mc^2}\left(z^2\frac{\partial^2 V}{\partial y^2}\right)_{oo} = -\frac{1}{15mc^2}\left(r^2\frac{d^2V}{dr^2} + 4r\frac{dV}{dr}\right)_{oo}, \tag{2}$$

where $V(r)$ is any potential function that commutes with position.

Now consider the matter from another point of view. Evidently, the retardation terms of all the orders for a given ℓ-multipole transition are contained in the electric transverse multipole distribution

$$E_{\ell m}(k^2) = \frac{(2\ell+1)!!}{(-ik)^{\ell}\sqrt{4\pi(2\ell+1)(\ell+1)/\ell}} \int \vec{F}^{*(+)}_{\ell m k}(\vec{r})\vec{J}(\vec{r})d^3r, \quad (3)$$

where $\vec{F}^{*(+)}_{\ell mk}$ is the sum of vector harmonics times appropriate spheric Bessel functions (see, e.g., /2/). Therefore, for finding a certain correction, we have to expand $E_{\ell m}(k^2)$ in series in powers of k^2. The detailed analysis of the structure of $E_{\ell m}(k^2)$ has been performed in /2/. It was shown that $E_{\ell m}(k^2)$ is the sum of the two independent parts:

$$E_{\ell m}(k^2) = \omega\, Q_{\ell m}(0) + k^2 T_{\ell m}(k^2), \quad (4)$$

where $Q_{\ell m}(0)$ is the usual Coulomb moment and $T_{\ell m}(k^2)$ is the toroid multipole distribution, defined as

$$T_{\ell m}(k^2) = \frac{i(2\ell+1)!!}{(-ik)^{\ell+1}\sqrt{4\pi(2\ell+1)(\ell+1)/\ell}} \times$$

$$\times \int \vec{J}(\vec{r})\{\sqrt{\ell}\, f^*_{\ell+1}(kr)\vec{Y}^*_{\ell\ell+1m}(\vec{n}) - \frac{ikr}{2}\sqrt{\ell+1}\,\vec{Y}^*_{\ell\ell-1m}(\vec{n}) f^*_{\ell-1}(kr)\}d^3r. \quad (5)$$

For a given current density the $T_{\ell m}(k^2)$ can be present, no matter whether $Q_{\ell m}$ is present or not. Therefore unlike the standard statements (see, e.g., /3/) the long-range approximation of $E_{\ell m}(k^2)$ is

$$E_{\ell m}(k^2) \underset{k^2\to 0}{\to} \omega\, Q_{\ell m}(0) + k^2 T_{\ell m}(0). \quad (6)$$

From (6) it immediately follows that the retardation effect (1) should be determined by the interference between Q_1 and T_1. The dipole toroid operator in the orthogonal coordinates has the form

$$\vec{T} = \frac{1}{10}[\vec{r}(\vec{r}\vec{J}) - 2r^2\vec{J}]. \qquad (7)$$

Using (7) instead of $\dot{y}z^2$ in (1) the result (2) can be rederived from the straightforward calculations. This demonstrates how to calculate the retardation effects in any $E\ell$-transition strength by formula (6).

Note, that for nuclei just $T_{\ell m}$ in $E_{\ell m}$ depends on the induction current and are neglected in proving the Siegert theorem. If one introduces the magnetization density of a nucleus M, $T_{\ell m}^{ind}$ equals

$$T_{\ell m}^{ind} = -i\sqrt{\frac{4\pi\ell}{(2\ell+1)(\ell+1)}}\int r^{\ell} Y_{\ell\ell m}(\vec{n})\vec{M}(\vec{r})d^3r \qquad (8)$$

and this part of $T_{\ell m}$ is proportional to the usual Blatt-Weiskopf multipole $Q'_{\ell m}$. The total T, taking into account both the convection and the induction currents ($\vec{J}^{ind} = \mathrm{rot}\vec{M}$), has the form

$$\vec{T} = \frac{-i}{5}r^2\vec{V} + \frac{1}{2}[\vec{r}\times(\frac{5}{2}\cdot\vec{L} + \vec{M})]. \qquad (9)$$

Thus $\vec{T}$ can be the reason for the experimental deviation from the TRK sum rule.

In conclusion we stress once more that the toroid moments are defined by the transverse part of the current and are thus independent of the charge density and of the $Q_{\ell m}$ corresponding to it. Usually, if the wave function of a radiated system is not exotic, the value $<T_{\ell m}>$ is compared with the value of $<M_{\ell+1}>$ (compare, for example, the definition (9) and the standard definition of M_{2m}). So, for $1/2^+ \to 3/2^-$ transition we have to evaluate the contributions of both M_2 and T_1 in addition to Q_1 (see, e.g., /4/).

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