СООБЩЕНИЯ
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ДУБНА

E2 - 9249

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The correlations between the directions of several outgoing mesons of the same sign are discovered in refs. /1,2/. It has been ascertained that like mesons produce, as a rule, a more narrow cone than unlike ones. The aim of this paper is to explain and calculate this effect which, similar to the GGLP-effect /3/, is connected with the Bose statistics in multimeson systems. The conditions, in which the effect can show itself especially strong, are indicated, and the possibility of studying the details of multiple production processes with its help is being analysed. A similar effect from another field of physics, namely, correlations in evaporation of like nucleons from highly excited nuclei, makes it possible to measure the lifetime, dimensions and form of nuclei (see refs. /4-6,9/). The preliminary version of this paper was published earlier /9/.

1. MODEL OF PARTICLE EMISSION AND DETECTION

This paper is a continuation of refs. /5,6/ devoted to the correlations of like particles in multiple production. Let us state briefly the fundamentals of the particle emission model taken in these papers which we'll lean upon.

It is assumed that particles are produced independently in an excited volume V inside which their sources are distributed with a given density. At the instant of excitation (when primary particles collide) all the sources are simultaneously switched on and emit mesons during an average time τ.

Inside the volume V the source current $g_\lambda(r')$ is given, where $r'=\{\vec{r}', t'\}$, $\vec{r}'$ is an arbitrary point inside the source, $t'>0$ is an arbitrary moment and λ are the parameters determining the location and properties of the sources.

For definiteness one can imagine that particles are emitted by heavy oscillators, not necessarily by point ones, which are, for example, the spheres of the radius a

$$g_\lambda(r') = \theta(a - |\vec{r}' - \vec{r}_0|)\,\theta(t')\exp(-iEt' - t'/2\tau). \qquad (1)$$

The entire volume G of each oscillator emits mesons coherently, and the meson production independence means that the phase of each oscillator and its parameters λ, such as the location of the source $\vec{r}_0$ and its frequency E/h, are assumed to be random quantities. As a result of this, the currents $g_\lambda(r')$ are random functions. They form the binary combinations $g_\lambda(r')g_\lambda^*(r'')$ which is averaged over the random parameters λ. The averaged function is referred to as the mutual coherence function of source

$$\beta(r', r'') = \langle g_\lambda(r')\, g_\lambda^*(r'')\rangle .$$

The Fourier transformation of the current g gives the amplitude of detecting the meson with 4-momentum $p = \{\vec{p}, \omega\}$:

$$a_\lambda(p) = \int g_\lambda(r')\exp(ipr')\,dr'.$$

For this reason the Fourier image of the function β yields the mutual coherence function of the meson pairs with 4-momenta p_1, p_2:

$$b(p_1, p_2) = \langle a_\lambda(p_1)\; a_\lambda^*(p_2)\rangle =$$

$$= \int \beta(r', r'')\exp(ip_1 r' - ip_2 r'')\,dr'\,dr''.$$

The correlation effects are expressed in terms of this function if the sources are independent. It is different from zero (mesons are coherent, to a certain extent) even for $\beta(r', r'') = \delta^4(r' - r'')$. Meson production is often expressed in terms of Feynman diagrams, i.e., it is coherent. Our main simplifying assumption is that we consider only mesons produced incoherently (it is reasonable to expect this, e.g., for an increasing number of very gradually complicated Feynman diagrams or simply for statistical production mechanism); as is shown in refs. [5,6], the coherence effects make themselves felt even in this case.

The calculations [5,6], easily performed for the statistical production mechanism, show that the function $b(p_1, p_2)$ is divided into three factors depending separately on p_1, on p_2 and on the difference $p_1 - p_2 \equiv q_{12} = \{\vec{q}_{12}, q^{\circ}_{12}\}$. Thus, for the oscillators, which are distributed in V with a given density, one can write

$$b(p_1, p_2) \sim \tilde{I}(p_1 a)\tilde{I}(p_2 a)\, b_{12}, \tag{2}$$

where, as we see hereafter, the factor

$$b_{12} = b^{*}_{21} \equiv b(q_{12}) = \frac{I(q^{T}_{12} R)}{1 - i\tau q^{\circ}_{12}} \tag{3}$$

is responsible for the discussed correlations. For spherical sources $\tilde{I}(x) = 3(\sin x - x\cos x)x^{-3}$. The form of the function $I(x)$ depends on the distribution $\rho(\vec{r}_0)$ of the centres $\vec{r}_0$ of the spheres which emit particles. So, for $\rho(\vec{r}_0) = \exp(-r_0^2/2R^2)$ we have $I(q_{12}R) = \exp(-\frac{1}{2}\vec{q}_{12}^{\,2}R^2)$. For $\rho(\vec{r}_0) = \theta(R - r_0)$, $I(q_{12}R) = \tilde{I}(|\vec{q}_{12}|R)$. For the sources, which are evenly distributed along the surface of excited spherical volume V_0 we have (only if $\tau \gg R/c$)

$$I(q_{12}R) = 2J_1(q^{T}_{12}R)/(q^{T}_{12}R), \tag{4}$$

where J_1 denotes the Bessel function and
$\vec{q}^{\,T}_{12} = \vec{q}_{12} - \vec{n}(\vec{q}_{12}\cdot\vec{n})$, $\vec{n} = (\vec{p}_1 + \vec{p}_2)|\vec{p}_1 + \vec{p}_2|^{-1}$.
All the forms of $I(x)$ behave similarly: $I(0) = 1$, $I(\infty) = 0$; moreover, they practically coincide in the region $x \lesssim 3$.

The radius R of the volume, in which the centres of the oscillators are placed, and their radii a enter into the arguments of the functions. If $a \ll R$, the maximum of the function $|b_{12}|^2$, i.e., the "interference" maximum, is much narrower than the momenta spectrum (determined, as we'll see below, by the function $\bar{I}(p_i\, a)$). So, for the point oscillators $\bar{I} \equiv 1$, , i.e., the one-particle momentum spectrum turns out to be very wide; it is bounded by phase space only.

2. *MULTIPARTICLE CORRELATIONS OF LIKE MESONS*

Let N particles be produced in a high energy interaction (e.g., in the $\bar{p}p$ annihilation). Among them n are like scalar mesons. Their numbers are 1,2,... n; they are emitted by n independent sources whose parameters are $\lambda_1, \lambda_2, \ldots, \lambda_n$. Other particles emitted by the sources $\lambda_{n+1}, \ldots, \lambda_N$ are different from one another Then the amplitude of simultaneous detection of all particles is equal to the product of the amplitudes $a_{\lambda_j}(p_i)$ of their detection separately, symmetrized over all transpositions of the sources of like particles

$$A = [\sum_{\lambda_1} \sum_{\lambda_2} \ldots \sum_{\lambda_n} a_{\lambda_1}(p_1) a_{\lambda_2}(p_2) \ldots a_{\lambda_n}(p_n)] \prod_{k=n+1}^{N} a_{\lambda_k}(p_k) \qquad (5)$$

(the product Π includes all the amplitudes which should not be symmetrized). The value in the brackets (it is called permutant *) can be obtained from the determinant composed of the amplitudes

$$D_S^{(n)} = D_S^{(n)}(\{a_{\lambda_i}(p_j)\}) = \begin{vmatrix} a_{\lambda_1}(p_1) & a_{\lambda_1}(p_2) & \ldots & a_{\lambda_1}(p_n) \\ a_{\lambda_2}(p_1) & a_{\lambda_2}(p_2) & \ldots & a_{\lambda_2}(p_n) \\ \ldots & \ldots & & \ldots \\ a_{\lambda_n}(p_1) & a_{\lambda_n}(p_2) & \ldots & a_{\lambda_n}(p_n) \end{vmatrix}_S$$

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* According to J.Schwinger, "symmetrant".

if all subtractions in it are replaced by additions. The symmetrical sum of the products, permutant, is distinguished from the usual determinant by the subscript S. So we have

$$A = D_S^{(n)}(\{a_{\lambda_j}(p_i)\}) \prod_{k=n+1}^{N} a_{\lambda_k}(p_k) \quad , \quad (i,j = 1,2,\ldots, n).$$

The density of the momentum distribution of N particles is proportional to $|A|^2$

$$W(p_1, \ldots, p_N) \sim \langle |A|^2 \rangle , \tag{6}$$

where the sign < > denotes the averaging over all unobserved random parameters of the sources. Let all these parameters λ_1, $\lambda_2, \ldots, \lambda_n$ be evenly distributed and be independent of one another so that the result of averaging the products

$$a_{\lambda_k}(p_i)\, a^*_{\lambda_k}(p_j)$$

over λ_k (mutual coherence function of any particle pair) for $\lambda_1, \lambda_2, \ldots, \lambda_n$ is the same

$$\langle a_{\lambda_1}(p_i)\, a^*_{\lambda_1}(p_j)\rangle = \langle a_{\lambda_2}(p_i)\, a^*_{\lambda_2}(p_j)\rangle = \ldots = b(p_i, p_j). \tag{7}$$

Then it is not difficult to establish the identity which is valid both for permutants and for determinants

$$\begin{vmatrix} a_{\lambda_1}(p_1) & a_{\lambda_1}(p_2) & \cdots & a_{\lambda_1}(p_n) \\ a_{\lambda_2}(p_1) & a_{\lambda_2}(p_2) & \cdots & a_{\lambda_2}(p_n) \\ \cdots & \cdots & & \cdots \\ a_{\lambda_n}(p_1) & a_{\lambda_n}(p_2) & \cdots & a_{\lambda_n}(p_n) \end{vmatrix}^2 = \tag{8}$$

$$= n! \begin{vmatrix} b(p_1,p_2) & b(p_1,p_2) & \cdots & b(p_1,p_n) \\ b(p_2,p_1) & b(p_2,p_2) & \cdots & b(p_2,p_n) \\ \cdots & \cdots & & \cdots \\ b(p_n,p_1) & b(p_n,p_2) & \cdots & b(p_n,p_n) \end{vmatrix} .$$

One should substitute this in (6) and remember eq. (2). We are thus led to the following expression for momentum distribution density

$$W(p_1,\dots,p_N) \sim \begin{vmatrix} 1 & b_{12} & \dots & b_{1n} \\ b_{21} & 1 & \dots & b_{2n} \\ \dots & \dots & & \dots \\ b_{n1} & b_{n2} & \dots & 1 \end{vmatrix}_S \prod_{i=1}^{N} \tilde{I}^2(p_i a). \qquad (9)$$

We see that the probability of detecting n like particles in the vicinity of the region $p_1 = \dots = p_n$ (we denote it by T) is $n!$ times as large as the same probability for unlike particles. Far from T these probabilities coincide and there remains the momentum spectrum only

$$W(\vec{p}_1,\dots,\vec{p}_n) \sim \prod_{i=1}^{N} \tilde{I}^2(p_i a), \qquad (10)$$

The formula (9) is written on the assumption that all particles are emitted independently, in particular, that their momenta are mutually independent. But, in fact, the momenta are connected due to the conservation laws. To take this into account, we must put the probability density (9) into the usual expression for the differential cross section of N particle production:

$$d\sigma(\vec{p}_1,\dots,\vec{p}_N) \sim \prod_i^N I^2(p_i a) P^{(n)} \delta^4\left(\sum_i^N p_i - P\right) \prod_i^N \frac{d\vec{p}_i}{2\omega_i}, \qquad (11)$$

where

$$P^{(n)} = D_S^{(n)}(\{b_{ij}\}). \qquad (12)$$

Hereafter talking about the interference of the states of like particles in multiple production, we'll mean the permutant in the general formula (11). If there are several kinds of like particles in the final state, we must put in (11) the product of several permutants $P^{(n)}$. Let us write them explicitly for some n.

The interference of two-meson states $\pi^+\pi^+$ (or $\pi^-\pi^-$) must be given by the factor

$$P^{(2)} = 1 + |b(q_{12})|^2 = 1 + \frac{I^2(q_{12}R)}{1+(\tau q^o_{12})^2}. \tag{13}$$

already studied in detail in refs. [5,6]. To observe this interference, it is necessary to compare, for the pairs of like mesons, the distribution on the plane (q^{T2}_{12}, q^o_{12}) in the vicinity of the origin with the background. The first one is twice as high as the latter. The plane of the variables $\Delta\omega = \omega_1 - \omega_2 = q^o_{12}$ and $\Delta\vec{p}^{\,2}_T = (\vec{q}^{\,T}_{12})^2$ is called the $\Lambda\Lambda$ -plot [7]. It is easy to show that the two-particle distribution ($P^{(2)}$) actually depends on q^o_{12}, q^T_{12} even when the function $I(q_{12}R)$, which enters into $P^{(2)}$, depends on the whole vector Δp the components other than q^T_{12} enter into q^o_{12} because $\vec{q}_{12}\vec{n} = q^o_{12}/v$, v being the velocity of $\pi\pi$ pair. The background can be obtained, for example, as in [8]

The three-particle-state interference gives the following expression for the correlation of the momenta of three particles

$$P^{(3)} = 1 + |b_{12}|^2 + |b_{23}|^2 + |b_{31}|^2 + $$
$$+ 2\mathrm{Re}(b_{12} b_{23} b_{31}). \tag{14}$$

This effect in the vicinity of the point $q = 0$, where

$$q = \{ q^T_{12}, q^T_{23}, q^T_{31}, q^o_{12}, q^o_{23}, q^o_{31} \}, \tag{15}$$

is 6 times as large as the background. But it is very difficult to use this many-dimensional space. It will be shown in § 3 that the effect reveals itself on the $\Lambda\Lambda$ -plot but in a weak, integrated form.

Finally let us write the expected interference between the quadruples of like mesons. In the region $p_1 = \ldots = p_4$ it must exceed the unlike particle density by a factor of 24:

$$P^{(4)} = 1 + (|b_{12}|^2 + |b_{13}|^2 + |b_{14}|^2 + |b_{23}|^2 + |b_{24}|^2 + |b_{34}|^2) +$$

$$+ 2\mathrm{Re}\,(b_{12}b_{23}b_{31} + b_{12}b_{24}b_{41} + b_{23}b_{34}b_{42} + \tag{16}$$

$$+ b_{13}b_{34}b_{41}) + (|b_{12}b_{34}|^2 + |b_{13}b_{24}|^2 + |b_{14}b_{23}|^2) +$$

$$+ 2\mathrm{Re}(b_{12}b_{23}b_{34}b_{41} + b_{13}b_{34}b_{42}b_{21} + b_{14}b_{42}b_{23}b_{31}).$$

3. THE POSSIBILITY OF OBSERVATION OF THE INTERFERENCE OF THREE-PARTICLE STATES

1. Of course, it is practically impossible to observe the triple pion interference in the 6-dimensional manifold (15). But the three-pion-state interference can affect the correlation between pion pairs. The density of pion pairs near the origin of the $\Delta\Delta$ -plot will increase not by a factor of 2 (as in (13)) but somewhat higher.

To prove this, we have simulated the reaction $\pi^+ p \to p 4\pi^+ 3\pi^-$. We generated the Monte-Carlo $p7\pi$ phase space events. From each event we chose randomly three pions imitating a $3\pi^-$ -triple. Events were taken with weights W_3 calculated from eq. (11) for $n=3$, $a=0$:

$$W = W_3 = (\text{Phase Space})\,P^{(3)}.$$

The function $I(q_{ij}R)$ was taken from eq. (4) on the assumption that $R = r = 1/m_\pi$, $a=0$ (point-like pion source). For a randomly chosen pair belonging to the chosen triple, one built the distribution on the $\Delta\Delta$ -plot restricted by the values $|q^0_{12}| < 1$ *GeV*, $|q^T_{12}|^2 < 0.05$ *GeV*. So, the

$(q^{o}_{12}, q^{T2}_{12})$ distribution was obtained assuming that all $\pi^-\pi^-$ -correlations were the result of the $3\pi^-$ -state interference given by eq. (15).

For comparison one can plot the $(q^{o}_{12}, q^{T2}_{12})$ -distribution giving to each event the weight $W = W_2 = (P.S.)P^{(2)}$. This means that the triple-pion interference is switched off and the two-pion-state interference remains.

The results of the numerical experiments are as follows. In the last case, when the pair correlations are originated by the two-pion interference only, we have $W/(P.S.) \approx 2$ at the origin of the $\Delta\Delta$ -plot (as could be expected). But in the first case, when the pair correlations were due to the three-pion interference, the ratio $W/(P.S.)$ at the origin of the $\Delta\Delta$ -plot was higher than 2. This ratio turns out to be more higher, the less the interaction energy (phase space available) is. It equals not 2 but 2.40 for 5 *GeV/c*, 2.10 for 20 *GeV/c*, 2.06 for 40 *GeV/c*. For the reaction $\pi^+ p \to p\,3\pi^+ 2\pi^-$ (5 *GeV/c*) it equals 2.38.

We conclude that the three-pion-state interference considerably changes the distribution on the $\Delta\Delta$ -plot for the pion pairs taken from the triples of like pions if the phase space available is not very high. The same result can be obtained from a simple model which assumes that all components of particle moments are Gauss-like distributed in the phase space.

2. The reaction in which the three-pion interference is expected has been investigated experimentally in ref. /1/. But it has been stated there that many-pion correlations are the result of two-pion ones. This contradicts, e.g., the interference formula (14): except for "two-pion" terms $|b_{ij}|^2$ it contains a "three-pion" term $2\mathrm{Re}(b_{12} b_{23} b_{31})$. We think that the question of the existence of properly three-pion correlations hasn't yet a final solution. It is not excluded that the distributions investigated are not sensitive in the question of interest.

To analyze this point, one can again simulate the reaction $\pi^+ p \to p\,4\pi^+ 3\pi^-$ at the energy investigated in ref. /1/. In each event we have calculated the solid angle Ω between three momenta π^-, π^-, π^- in the CMS of secondary pions. We built these Ω-distributions introduced in ref. /1/ making some assumptions. Four hypothesis about the correlation properties have been considered:

1) Matrix element is constant so there are no correlations. The weight of events is given by the phase factor

$$W_1 - (P.S.)$$

2) There exists only the two-pion interference (this may happen, e.g., in triples (– – +)

$$W_2 = W_1 P^{(2)} .$$

3) There exists the three-pion interference

$$W_3 = W_1 P^{(3)} = W_1[1 + |b_{12}|^2 + |b_{23}|^2 + |b_{31}|^2 +$$

$$+ 2Re(b_{12} b_{23} b_{31})] .$$

4) There is a correlation between the triples but it can be explained by the pair correlations only. We could not repeat the proof procedure chosen in ref. /1/ because it is not described there completely enough. But we tried to exclude from the correlations $P^{(3)}$ the "properly triple" term. Thus two additional hypothesis have been considered

$$W_4 = W_1(1 + |b_{12}|^2 + |b_{23}|^2 + |b_{31}|^2)$$

$$W_5 = W_1(1 + |b_{12}|^2)(1 + |b_{23}|^2)(1 + |b_{31}|^2).$$

The parameter $\beta = n(\Omega > \pi/2)/n(\Omega < \frac{\pi}{2})$ was calculated for each hypothesis; its values were compared with the experimental results obtained in ref. /1/. The comparison (see *Table 1*) shows that they (i.e., W_1, W_2, W_3) qualitatively coincide with the data; therefore within the framework of these hypothesis one can try to test the conclusions of ref. /1/. (But in no case the considered model can explain the dependence of β on the pion sign).

Table I

The values of parameter β for some three-pion states (+ - 0 means $\pi^+\pi^-\pi^\circ$). In column 1 - experimental data, in column W_i - data calculated with the weight W_i.

State	1	W_1	W_2	W_3	W_4	W_5
+ - 0	1.ol	1.02				
+ + -	1.1		.94			
- - +	.98		.94			
+ + +	.78			.82	.84	.81
- - -	.57			.82	.84	.81

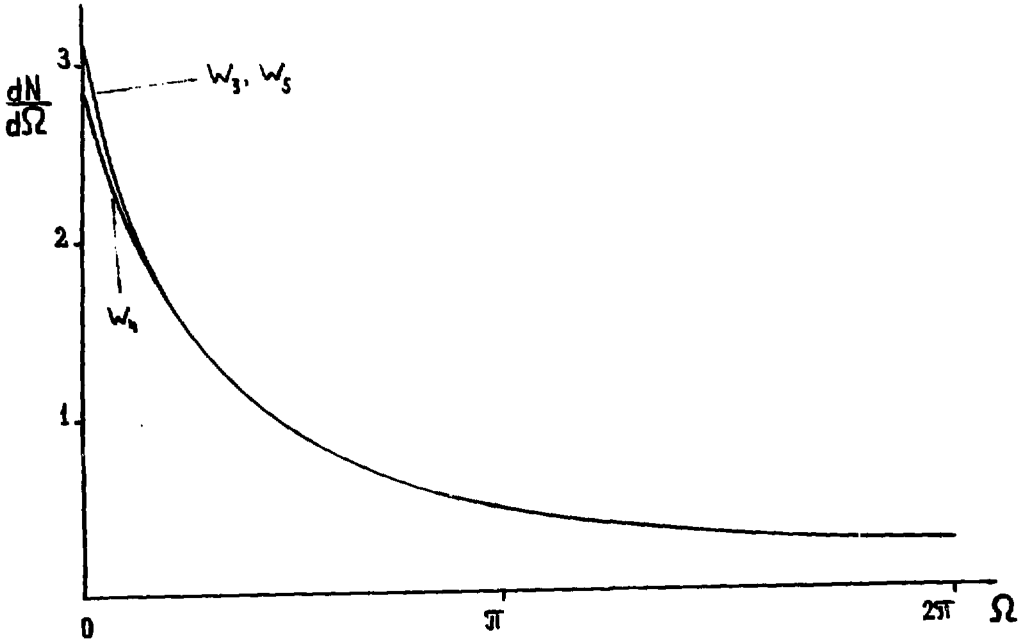

Fig. 1. Distribution of the solid angles Ω calculated for various hypotheses.

So we have plotted the Ω-distributions for the hypothesis W_3, W_4, W_5 after normalizing all of them to unity (*fig. 1*). They practically coincide in all three cases. Because there exists the triple term in W_3 (and not in W_4, W_5), this proves that the Ω-spectrum cannot be used to conclude what hypothesis (W_3, W_4 or W_5) is valid. As the Ω-variable is not sensitive to the "proper" triple interference term, the statement that the triple correlations can be explained by the pair ones cannot be considered as proved. Further investigations are necessary, for example, with the $\Delta\Delta$-plot not at very high energies.

We are indebted to V.I.Luboshitz for valuable discussions.

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Received by Publishing Department on October 27, 1975.