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ELECTROMAGNETIC STRUCTURE OF SPIN - ZERO LIGHT NUCLEI FROM POINT OF VIEW OF ANALYTICITY

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ELECTROMAGNETIC STRUCTURE OF SPIN - ZERO LIGHT NUCLEI FROM POINT OF VIEW OF ANALYTICITY

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1. INTRODUCTION

The usual source of information about the electromagnetic structure of nuclei is the elastic electron scattering experiments. The characteristic feature of measured differential cross sections is the appearance of diffraction minima at definite values of photon momentum transfer which are a reflection of a compound nature of nucleus.

On the other hand, if one confides in quantum electrodynamics than all information about the electromagnetic structure of nucleus is contained in nuclear electromagnetic form factors.

It is important at this point to define clearly what we shall understand by the nuclear electromagnetic form factor.

Generally two definitions of form factors exist in description of electron-nucleus scattering experiments.

The first one [1,2] considers a scalar function of one variable appearing in the general structure (determined only by kinematics) of the electromagnetic current of nucleus. The form factors defined by such a way are the Fourier transform of the nuclear charge distribution. The future dynamical theory of strong interactions of nucleons in nuclei is expected to give their behaviour which of course will clearly exhibit the cause of the appearance of diffraction minima in experimentally measured differential cross sections. As is known, their forms and positions very much depend on the structure of a nucleus [3].

The second type of form factor (defined in a generalized sense) appears in connection with the calculation of electron-nucleus scattering differential cross section in the framework of high energy approximation model [3].

In this case, complex, in general, form factor is a function of two variables (equivalent to the energy and the scattering angle) and plays a role of an effective function in experimental differential cross sections, which compensates the departure from the Mott formula both due to the space extension of nuclei and due to the neglect of two and more photon exchange contributions in the scattering amplitude.

This paper concerns the first type of nuclear electromagnetic form factors only and presents an attempt to analyze them from the point of view of analyticity. The method for model-independent determination of charge radii and charge distributions is proposed.

The paper is organized as follows. In the next section a brief summary of investigations of analytic properties of nuclear electromagnetic form factors in perturbation theory is given. In section 3 we analyze in detail the existing experimental data. Section 4 is devoted to the derivation of explicit parametrization of form factors, the fit of experimental data is carried out, and the correspoding root-mean-squared (r.m.s.) charge radii are calculated. In section 5 we try to get a model-independent information about the charge distribution functions. The final section, 6, presents the conclusions.

2. *ANALYTIC PROPERTIES OF ELECTROMAGNETIC FORM FACTORS*

It is generally believed that the electromagnetic form factors of elementary particles are analytic functions in the cut complex plane of momentum transfer squared $t = q^2$.

For the pion form factor it can be proved exactly starting from the axioms of local quantum field theory /4/. On the other hand, the same results have been obtained by means of the investigation of analytic properties of Feynman diagrams in perturbation theory /5,6/.

This was the reason why the methods of perturbation theory were used with confidence to obtain the information about the analytic properties of electromagnetic form factors of those hadrons (e.g., nucleons, K -mesons, etc.) for which the exact proofs of the analyticity do not exist.

The utilization of the methods of perturbation theory to get the information about the analyticity of electromagnetic form factors of deuteron has been well discussed elsewhere /7-9/. We extend these methods to the spin-zero light nuclei ^4He, ^{12}C and ^{16}O.

Owing to the fact that the spin of ^4He, ^{12}C and ^{16}O is zero, there is only one form factor $F_A(t)$ for each nucleus fully describing its electromagnetic structure, which generally can be represented by the diagram shown in *fig. 1*. The shaded area represents all intermediate

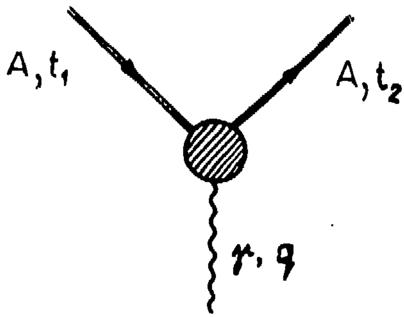

Fig. 1. The electromagentic form factor of nucleus A .

states which conserve quantum numbers and can be understood as a sum of an infinite number of diagrams pictured in *fig. 2*.

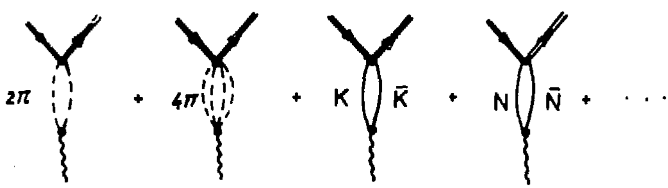

Fig. 2. The decomposition of shaded bubble of fig. 1 into an infinite number of diagrams.

The electromagnetic form factor $F_A(t)$ depends on one variable usually chosen in the form of squared momentum transfer $t=-\vec{q}^{\,2}$ ($\vec{q}$ is 3-dimensional space-vector of momentum transfer $q=(t_2-t_1)$ in the Breit system), the physical region for the scattering process being $t<0$.

Further we shall be interested in the analytic properties of $F_A(t)$ in the complex t-plane. For our purpose it will be sufficient to know the lowest branch point t_L.

Applying the well known rules (see, e.g., ref. /10/) to the diagrams shown in *fig. 2* one can see immediately that the lowest normal threshold in all three cases is that coming from the first diagram at $t_0=4m_\pi^2$ (m_π is the pion mass).

The lowest anomalous thresholds for $F_{^4He}(t)$, $F_{^{12}C}(t)$ and $F_{^{16}O}(t)$ are given by triangle diagrams shown in *figs. 3(a)-3(c)* respectively and their positions (the masses of corresponding particles are denoted by m_A)

$$t_{^4He}=4m_p^2-\frac{[m^2_{^4He}-m^2_{^3H}-m_p^2]^2}{m^2_{^3H}}\approx 9.63m_\pi^2$$

$$t_{^{12}C}=4m_p^2-\frac{[m^2_{^{12}C}-m^2_{^{11}B}-m_p^2]^2}{m^2_{^{11}B}}\approx 6.72m_\pi^2 \qquad (1)$$

$$t_{^{16}O}=4m_p^2-\frac{[m^2_{^{16}O}-m^2_{^{15}N}-m_p^2]^2}{m^2_{^{15}N}}\approx 4.92m_\pi^2$$

are still higher (unlike the deuteron) than the lowest normal threshold t_0.

So, the consideration of perturbation theory shows that each of $F_A(t)$ ($A={}^4He, {}^{12}C, {}^{16}O$) is an analytic function of t in a plane cut along the real axis from $4m_\pi^2$ to ∞.

Further, it follows from the unitarity condition

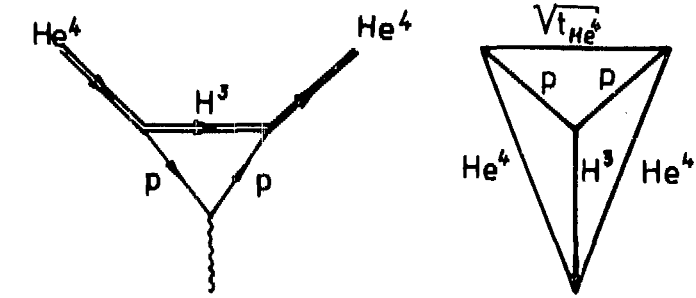

Fig. 3a. The triangle diagram and its dual diagram giving the lowest anomalous threshold of $F_{^4He}(t)$.

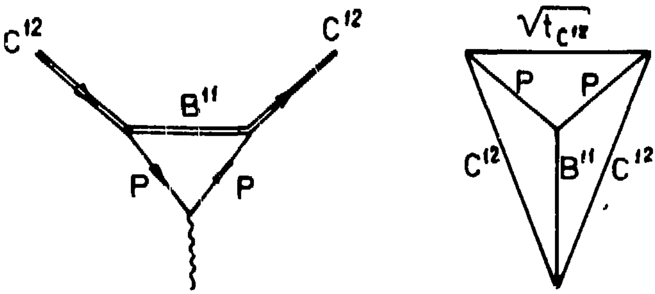

Fig. 3b. The triangle diagram and its dual diagram giving the lowest anomalous threshold of $F_{^{12}C}(t)$.

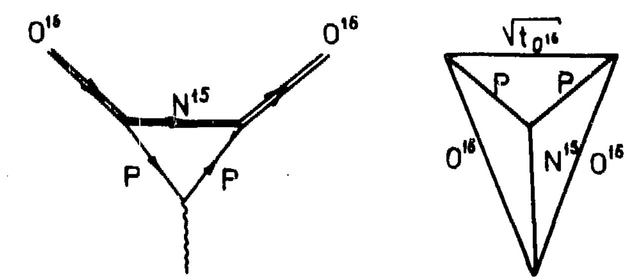

Fig. 3c. The triangle diagram and its dual diagram giving the lowest anomalous threshold of $F_{^{16}O}(t)$.

$$T_{fi} - T^*_{if} = i(2\pi)^4 \sum_n T_{fn} T^*_{in} \delta^{(4)}(P_f - P_i) \qquad (2)$$

that the vertex amplitude corresponding to *fig. 1* (the photon is off the mass shell) is hermitian for space-like region values of momentum transfer squared. This results in a hermiticity of electromagnetic current of nucleus from which the reality of corresponding form factors follows.

Now, since in this case the Schwartz reflection principle holds one can write for nuclear electromagnetic form factors the following Schwartz condition

$$F^*_A(t) = F_A(t^*) \qquad (3)$$

which must be fulfilled in the framework of the future form factor dynamical theory.

3. *ANALYSIS OF EXISTING EXPERIMENTAL DATA*

The electromagnetic form factors of nuclei are not directly measurable quantities. The main experimental source of information about them is measurements of the elastic electron scattering differential cross section * at different energies. An essential ingredient in the practical analysis of such measurements is the assumption that the process can be approximately described in one photon exchange approximation (see the first diagram of series pictured in *fig. 4*) i.e., that the matrix element is proportional to a photon propagator times a form factor which depends only on the structure of the nucleus. Then the differential cross section in an invariant form looks as follows

$$\frac{d\sigma}{d\Omega} = \frac{1}{16\pi^2 s} \frac{e^2 Z^2}{t^2} |F_A(t)|^2 |(s - m_e^2 - M^2)(s + t - m_e^2 - M^2) + M^2 t|, \quad (4)$$

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* In future the same information may be obtained from the positron and muon scattering experiments.

where $F_A(t)$, M and Z are form factors, mass and charge number of corresponding nucleus respectively, m_e mass of the electron and s, t are common invariant variables defined through four-momenta of electron and nucleus (see *fig. 4*) by the following expressions

$$s = (p_1 + t_1)^2,$$
$$t = (p_2 - p_1)^2, \qquad (5)$$

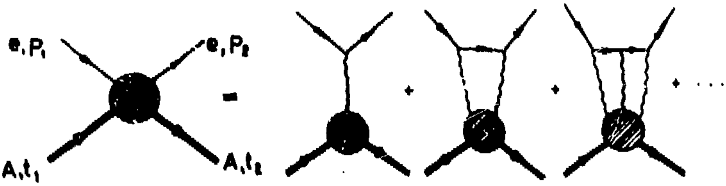

Fig. 4. The elastic electron-nucleus scattering amplitude and its power series expansion in coupling parameter $(Z\alpha)$.

The formula (4) requires some comments. Although it is now becoming clear [2] that even for very light nuclei the one photon exchange approximation of the amplitude is no longer sufficient to describe the electron scattering process it is however still used with advantage as a qualitative guide of what to expect in experiments.

Now using the experimental data on the differential cross sections of $e\,^4He \to e\,^4He$ from ref. [11], $e^{12}C \to e^{12}C$ from refs. [12-14], $e^{16}O \to e\,^{16}O$ from refs. [12,13] and formula (4) one can calculate the absolute values of the electromagnetic form factors $|F_A(t)|$ in finite interval of momentum transfer t. If further one supposes that the form factors for $t < 0$ take the real and positive values only we get their approximate behaviour around the diffraction minima as is shown in *figs. 5(a)-5(c)* (see also refs. [11-14] where they are drawn in a logarithmic scale).

Here we would like to analyze the afore-mentioned two assumptions in more detail.

It follows from the analyticity and unitarity (see the discussion in the previous section) that the form factors for $t < t_L$ (t_L is the lowest branch point) are real quantities. However, there are no fundamental principles forbidding the form factors to acquire the values zero or even negative values in this region of t. Just opposite, if one believes in the analytic properties obtained in section 2 it is too hard to understand the nature of the sharp change in the behaviour of $F_A(t)$ (see *figs. 5(a)-5(c))* in the vicinity of the diffraction minimum. There is well known fact in mathematics that function suffers intense changes in behaviour in the vicinity of some singularity only and there are no near singular points on the first Riemann sheet of the t-plane following from the perturbation theory which should be responsible for such sharp changes.

It seems for us to be natural to explain this phenomenon as an occurence of a zero (see the next section) of the form factor at the value of t which corresponds roughly to the diffraction minimum of elastic electron-nucleus scattering.

So, the application of the analyticity of form factors to their experimental data leads to the results, which have been obtained in nuclear physics simply by Fourier transform of various charge distribution functions.

Further we shall try to clarify that the introduction of form factors zeros seems not to be in contradiction with the present-day experimental data about electron scattering differential cross sections.

There is a question why these zeros of form factors are unobserved as zeros in elastic electron scattering experimental data.

The whole problem resides in the rate of convergence of expansion of scattering amplitude into the series represented by means of Feynman diagrams in *fig. 4* *.

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* We have not shown in *fig. 4* the diagrams corresponding to the radiative corrections, because they are not important in the understanding of the idea which we would like to clarify here. But in practical calculations they must be taken into account.

Let us denote this series by

$$M(s,t) = M_1 + M_2 + M_3 + \dots \tag{6}$$

where indices 1,2,3... mean the number of exchanged photons. Then the elastic scattering differential cross section exactly looks as follows

$$\frac{d\sigma}{d\Omega} = \frac{1}{8\pi^2 s} \sum_{s_i, s_f = -1/2}^{+1/2} \{ |M_1|^2 + |M_2|^2 + |M_3|^2 + \dots + M_1 M_2^* + M_1 M_3^* + \dots \tag{7}$$

$$+ M_2 M_3^* + \dots \},$$

and the form factor squared is connected with experimentally measurable differential cross section $\frac{d\sigma}{d\Omega}_{exp}$ in the following way

$$F_A^2 = \frac{\frac{d\sigma}{d\Omega}_{exp} - \{\text{contributions from two and more photon exchanges} + \text{interference terms}\}}{\frac{d\sigma}{d\Omega}_{Mott}}. \tag{8}$$

One can see from eq. (8) that the zero value of form factor does not cause the zero in $d\sigma/d\Omega_{exp}$ inevitably. If elastic electron scattering experiments give for corresponding value of t the nonzero value of $d\sigma/d\Omega_{exp}$ then it insinuates that the contributions in brackets of eq. (8) are noticeable and they must be taken into account in determination of experimental behaviour of nuclear-electromagnetic form factors. Moreover the series (6) is a power series expansion in coupling parameter $Z\alpha$ (Z is the charge number of nucleus and $\alpha \approx \frac{1}{137}$) and its rate of convergence will be changed from nucleus to nucleus. As a consequence, the contributions in brackets of eq. (8) will increase with an enlargement of the charge number Z. If one uses the one photon exchange approximation for extraction of form factors data for all nuclei under consideration uniformly then this effect must appear in minima of obtained behaviours of form factors as is shown in *figs. 5(a)-5(c)*.

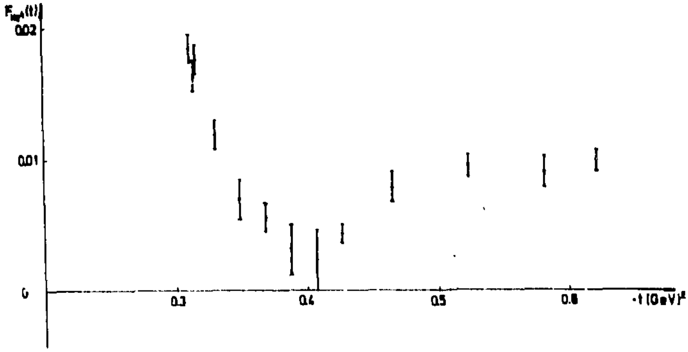

Fig. 5a. The data on $F_{^4He}(t)$ *obtained in one-photon exchange approximation of the electron scattering amplitude.*

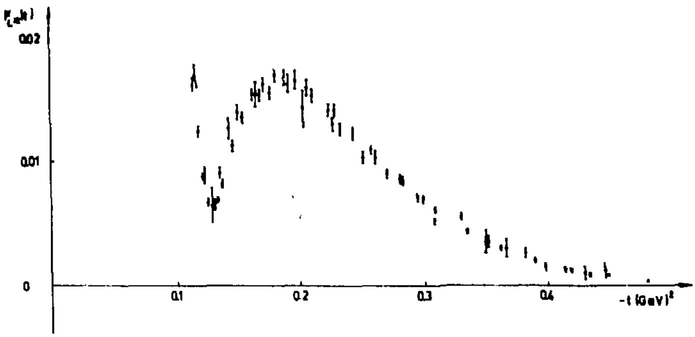

Fig. 5b. The data on $F_{^{12}C}(t)$ *obtained in one-photon exchange approximation of the electron scattering amplitude.*

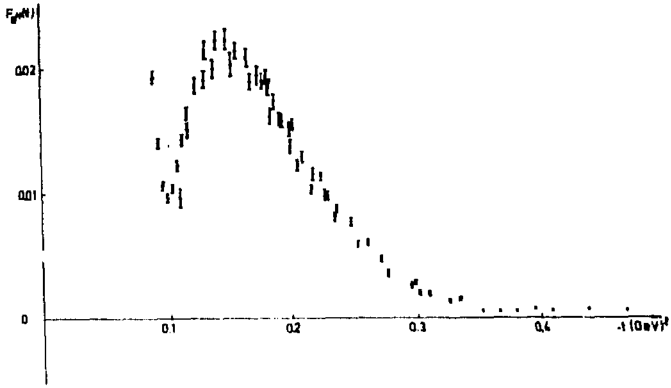

Fig. 5c. The data on $F_{^{16}O}(t)$ *obtained in one-photon exchange approximation of the electron scattering amplitude.*

Really while ^4He is the case where the contributions in brackets of eq. (9) are tiny (for $t \simeq -0.407\ GeV^2$ $F_{^4He}(t)$ is equal to zero in the limit of experimental error) the values of $F_{^{12}C}(t)$ and $F_{^{16}O}(t)$ at the minimum are different from zeros and are increasing with an enlargement of the charge number Z.

This is the unrefutable evidence for our conjecture that *figs. 5(a)-5(c)* (see also the *figs.* in ref. /11-14/) are no real behaviours of corresponding nuclear electromagnetic form factors and can be considered at most as a behaviour of an effective function F(t) compensating the neglect of two and more photon exchange contributions (here we include also dispersion effects and radiative corrections) in the one photon exchange approximation of elastic electron scattering amplitude.

Another convincing evidence supporting our hypothesis about zeros is a result obtained in /15/ where in the framework of the multiscattering theory the contribution of two photon exchange is evaluated. It is clearly demonstrated that already in the case of ^4He the two-photon exchange contribution takes around 30-40% of the form factor values in the vicinity of diffraction minimum.

The model-independent support of our hypothesis is expected in elastic positron-nucleus scattering in which the interference terms of one- and two-photon exchange diagrams have opposite sign to the corresponding terms in elastic electron-nucleus scattering. If form factors really acquire the value zero at diffraction minima and just higher photon exchange contributions are responsible for the nonzero value of differential cross section in diffraction minima, the data obtained in elastic positron scattering experiments must differ noticeably from the electron scattering data.

4. *PARAMETRIZATION OF FORM FACTORS, RESULTS OF THE FIT AND CHARGE RADII*

If we consider the results obtained in sect. 2 (i.e., the electromagnetic form factors of ^{4}He, ^{12}C and ^{16}O are analytic functions in the whole complex plane t besides the cut from $4m_\pi^2$ to ∞) with the aim of optimal use of analyticity one can do the conformal mapping into the unifocal ellipse in the z-plane by the following way. The point $t=0$ and the most distant experimental point in space-like region are mapped into $+1$ and -1, respectively, and the cut for $4m_\pi^2 < t < \infty$ is mapped onto the ellipse.

By such a procedure we get the analytic function $F_A(z)$ in the ellipse and one can write the following maximally convergent series in the whole t-plane (see /16/ and reference therein) composed of the Tschebyscheff polynomials

$$F_A[z(t)] = 1 + \sum_{n=1}^{M} A_n B_n [T_n(z) - 1] \tag{9}$$

in which the normalization condition is automatically taken into account. Here $B_n = (R^{2(n-1)} + R^{-2(n-1)} + 2\delta_{n-1,0})$, R is the sum of the semiaxes of the ellipse and A_n are coefficients to be found from a fit.

Now there is a question to what experimental data the parametrization (9) will be applied through the least squares method procedure to find the minimal number of unknown coefficients A_n.

We would like to note that besides the question with zeros discussed in the previous section and the fact, that also after the introduction of zeros the data are approximate only (they have been calculated in one photon exchange approximation using (4)) there are inconsistent data obtained in different measurements (see, e.g., the data on $F_{4_{He}}(t)$ from ref. /11/ and ref. /17/). It is transparent that through fitting of such data one can never get the χ^2 inside its normal region $\chi^2_{NORMAL}=(N-1)\pm\sqrt{2(N-1)}$ (N is the number of degrees of freedom (n.d.f.)) also in the best possible agreement of existing data with our parametrization.

So, before fitting we have reelaborated the data in the following sense.

All inconsistent data nearly for the same values of momentum transfer squared were replaced with the effective value obtained by method of a calculation of the centre of gravity where the reversed value of an experimental error is playing the role of the mass.

Further, by our arguments discussed in the previous section we have desired to alter the sign in diffraction minimum of an experimental form factor and moreover we have removed some points with large partial value of χ^2 around those form factor zeros.

The reason for the latter is our conjecture that one photon exchange approximation of an electron scattering amplitude totally fails around the diffraction minimum. The practical performance supports it because after the removing of corresponding experimental points the values of coefficients A_n are nearly the same.

The truncation point of the expansion (9) in the fitting procedure is found through minimization of quantity /16/

$$X = \chi^2 + \phi, \tag{10}$$

where ϕ is the so-called Cutkosky convergence test function [18] which controls the goodness of convergence of our parametrization.

The results of the fit are (see *figs. 6(a)-6(c)*) as follows:

$$\begin{array}{llll} \text{for } {}^4\text{He}: & M=5 & \chi^2=15.1 & \text{n.d.f.}=19 \\ \text{for } {}^{12}\text{C}: & M=7 & \chi^2=53.3 & \text{n.d.f.}=45 \\ \text{for } {}^{16}\text{O}: & M=7 & \chi^2=59.9 & \text{n.d.f.}=43 \end{array} \quad . \tag{11}$$

At the beginning we carried out the fits without introducing zeros of form factors at the diffraction minima and we have no criterion where to stop in the fitting. The last knowledge supports the indications that the data of nuclear electromagnetic form factors shown in *fig. 5(a)-5(c)* are in contradiction with the analytic properties obtained in section 2.

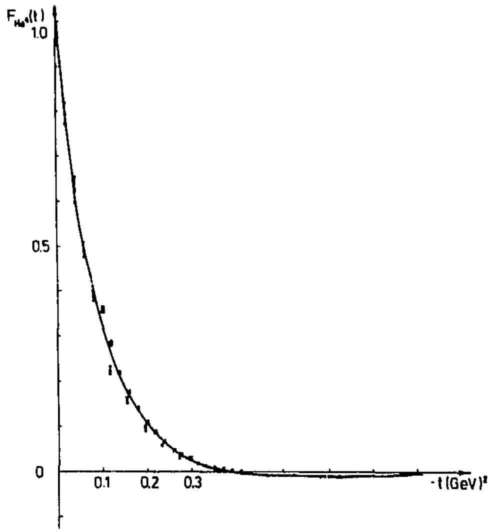

Fig. 6a. The result of the fit of (9) to the experimental data of $F_{^4He}(t)$ with a zero in the diffraction minimum.

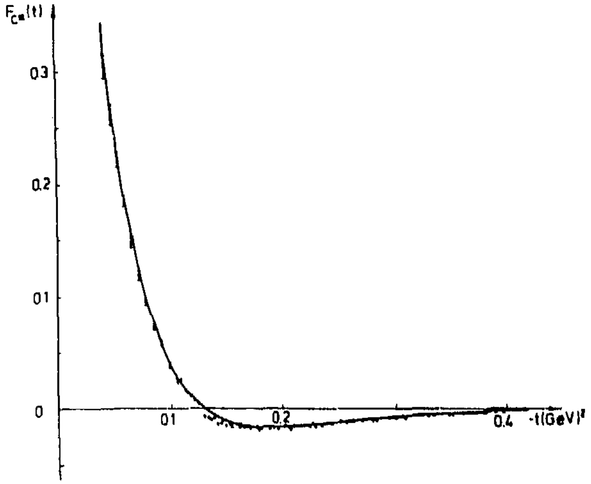

Fig. 6b. The result of the fit of (9) to the experimental data of $F_{^{12}C}(t)$ with a zero in the diffraction minimum.

Fig. 6c. The result of the fit of (9) to the experimental data of $F_{^{16}O}(t)$ with a zero in the diffraction minimum.

Finally we shall discuss here a model-independent calculation of nuclear electromagnetic charge radii (for more detail see ref. /19/).

If somebody speaks about the model-independent method of determination of r.m.s. charge radius one has in mind the use of the well known expansion

$$F(t) = 1 + \frac{1}{6}<r^2> t + \dots \tag{12}$$

from which the formula for the calculation of $<r^2>^{1/2}$

$$<r^2> = 6 \left.\frac{dF(t)}{dt}\right|_{t=0} \tag{13}$$

follows.

However the expansion (12) is nothing but the Taylor series which can be written provided that F(t) is an analytic function inside the circle around the point $t=0$. Its radius of convergence equals $R = t_L$ (t_L is the position of the lowest branch point) and so it is clear that one may use (12) to fit the data only in the t -region where $|t| < R$.

On the other hand, in our expansion (9) we have no such restriction. If one believes in analytic properties obtained in the framework of perturbation theory (see sect. 2), the series (9) is convergent in the whole region $-\infty < t < 4m_\pi^2$ and always it can be used for the model-independent determination of $<r^2>^{1/2}$. For that purpose one must take the limit

$$\lim_{t\to 0} 6 \frac{dF[z(t)]}{dz} \frac{dz(t)}{dt} = <r^2>, \tag{14}$$

where $F[z(t)]$ is given by (9).

Now in concrete cases (taking the coefficients A_n determined by fitting the data with zeros) from (14) one gets the following values of the charge radii.

$$<r^2>^{1/2}_{^4He} = 1.51\,F, \quad <r^2>^{1/2}_{^{12}C} = 1.86\,F, \quad <r^2>^{1/2}_{^{16}O} = 2.08F, \tag{15}$$

which are smaller than the averaged values

$$\langle r^2 \rangle^{1/2}_{^4He} = 1.67\,F, \quad \langle r^2 \rangle^{1/2}_{^{12}C} = 2.44F, \quad \langle r^2 \rangle^{1/2}_{^{16}O} = 2.69\,F \qquad (16)$$

of those now accepted in literature.

The last effect is again an immediate consequence of the dependence of the rate of convergence of series (6) on Z and the use of the data about $F_A(t)$, obtained in one photon exchange approximation.

While for ^{4}He the difference between the values (15) and (16) is only 0.16 F, for ^{12}C it is already 0.58 F and in the case of ^{16}O the latter acquires the value 0.61 F. So, the contributions in brackets of (8) are considerably larger for ^{12}C and ^{16}O than for ^{4}He, what again confirms our conjecture that for nonzero values of $d\sigma/d\Omega_{exp}$ in diffraction minima two and more photon exchange contributions are responsible.

If, at least two photon exchange contributions might be evaluated, one could expect an improvement in behaviour of $F_A(t)$.

Its slope will be steeper and as a consequence the values of charge radii obtained by our method will approach the averaged values (16).

We urge experimenters to measure the data on $F_{^4He}(t)$, $F_{^{12}C}(t)$ and $F_{^{16}O}(t)$ also for lower values of momentum transfer $|t|$ than those shown in *figs. 6(a)-6(c)* because these can be crucial in the finite decision between the future model-independent values obtained by our method and those now scattered in wide intervals ($1.63F \le \langle r^2 \rangle^{1/2} \le 1.71F$ ref./11/ $2.35F \le \langle r^2 \rangle^{1/2}_{^{12}C} \le 2.53\,F$ refs. /12–14/ and $2.65\,F \le \langle r^2 \rangle^{1/2}_{^{16}O} \le 2.73\,F$ refs. /12,13/).

5. *DETERMINATION OF CHARGE DISTRIBUTIONS*

The procedure is commonly to test various model-dependent forms of charge distributions $\rho(r)$ containing a number of free parameters. By χ^2 minimization technique these are chosen to be such as to optimize the agreement between the calculated and experimentally measured values of cross sections.

Due to the fact that there are a few charge distributions well describing the experimental data it is impossible also within models, to determine the charge distribution unambiguously. For a particular charge distribution it may be possible to determine at most the parameters necessary to fit the experimental data. However, in this manner no information is obtained about possible variations in the shape of the charge distribution if other parametrizations are taken into account. As a consequence, we do not know how the results are restricted by the assumed form of the model charge distribution and thus it is impossible to decide whether certain properties of $\rho(r)$ follow from the experimental data or from the concrete parametrization.

One of the possibilities to solve this problem is the utilization of inverted Fourier transform

$$\rho(r) = \frac{1}{2\pi^2 r} \int_0^\infty F(q) \sin(qr)\, q\, dq \tag{17}$$

and the calculation of $\rho(r)$ in a model-independent way, which however is not also without defects. The experimental behaviour of $F(q)$ is always known only in a restricted region of q and there is a question to what extent one can get the objective information about $\rho(r)$ from the approximate equation

$$\rho(r) = \frac{1}{2\pi^2 r} \int_0^{q_{max}} F(q) \sin(qr)\, q\, dq \tag{18}$$

where q_{max} is the position of the last experimental point of $F(q)$ in space-like region.

We shall not discuss here the reliability of expression (18) and we confine ourselves only to quoting that such an approximation is justified to some extent by present-day experimental data. One can see in *figs. 6(a)-(c)* that form factors for higher values of q are relatively small. From here we expect that the contribution from the integral $\int_{q_{max}}^{\infty} F(q) \sin(qr)\, q\, dq$ will be negligible.

Taking, instead of F(q) in (18), our parametrization (9) obtained by combination of analyticity and experimental data with zeros in diffraction minima and calculating the corresponding integral we get the behaviour of $\rho(r)$ for ^4He, ^{12}C and ^{16}O as they are shown in *figs. 7(a)-(c),* respectively. Also the comparison with the most popular Fermi distribution is presented there.

By using of (9) with coefficients A_n obtained by fitting of the data without zeros in diffraction minima, the general feature of $\rho(r)$ is an absolute inconsistency with the Fermi charge distribution. In that case $\rho(r)$ approaches rapidly to zero already for lower values of momentum transfer q.

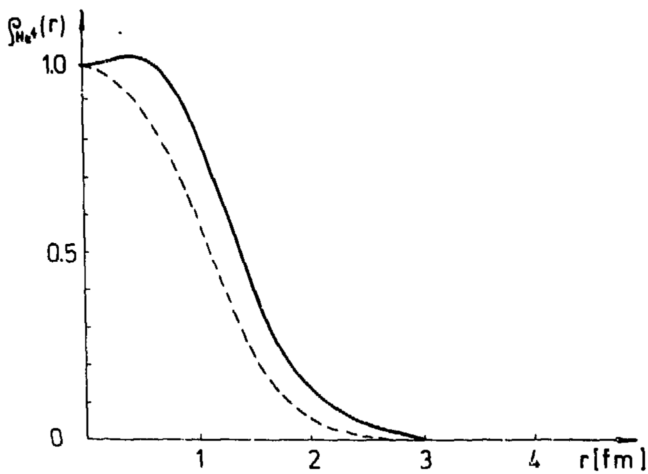

Fig. 7a. The charge distribution $\rho_{^4He}(r)$ obtained from (18) and parametrization (9). The dashed line corresponds to the two-parameter Fermi distribution.

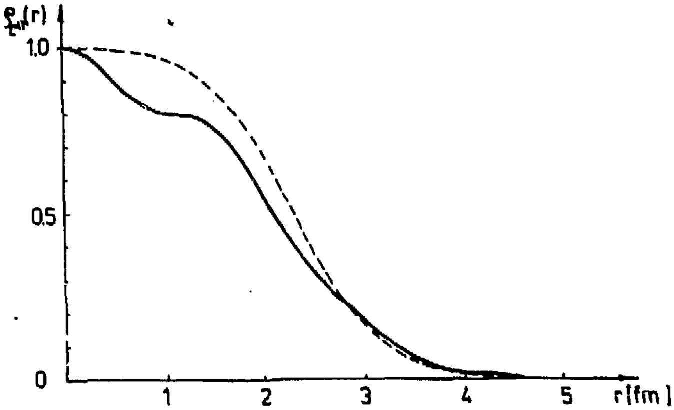

Fig. 7b. The charge distribution $\rho_{^{12}C}(r)$ obtained from (18) and parametrization (9). The dashed line corresponds to the two-parameter Fermi distribution.

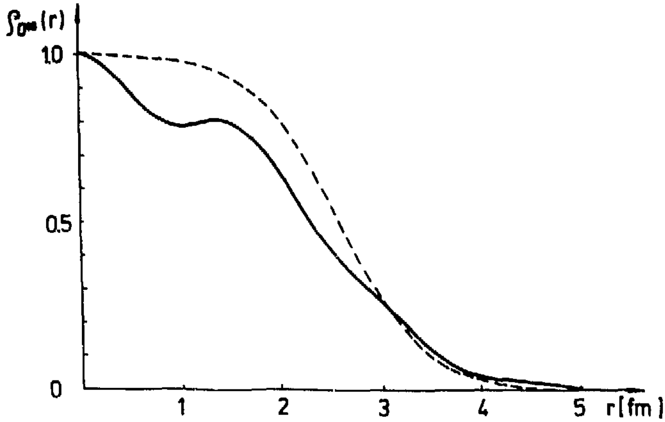

Fig. 7c. The charge distribution $\rho_{^{16}O}(r)$ obtained from (18) and parametrization (9). The dashed line corresponds to the two-parameter Fermi distribution.

6. CONCLUSIONS

In this section we summarize what has been learnt by the application of analyticity to the spin-zero light nucleus form factors.

The diffraction minima in elastic electron scattering differential cross section seem to be real zeros of form factors. The arguments supporting such interpretation are discussed in detail.

The maximally convergent parametrization for nuclear electromagnetic form factor is presented by which the model-independent values of charge radii and charge distributions can be found, provided that reliable experimental data do exist.

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