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We prove the Pomeranchuk-like theorems on the integrated cross sections and on the average spin polarization parameters of different meson-baryon reactions (e.g., $\pi N \to \pi N$, $KN \to KN$, etc.) by using the improvements (see table II) of the usual isospin constraints on the integrated cross sections [1]. All the improvements of isospin constraints are based on the properties [2] of the $I_p[F]$ -integrals defined by

$$I_p[F] = [\int_D |F|^p d\mu]^{1/p} \quad , \quad 1 \le p < +\infty , \tag{1}$$

where F is in general a linear combination of helicity amplitudes, D is a region from the physical domain and μ is a positive measure on the physical domain. Let F_ℓ, $\ell = 1,2,3$, be the functions F for three $(0^- \tfrac{1}{2}^+ \to 0^- \tfrac{1}{2}^+)$ reactions related by isospin invariance ($SU(3)$ - symmetry, quark models, etc.). Then, using the properties of $I_p[F]$-integrals and the isospin sum rules $\sum_{\ell=1}^{3} c_\ell F_\ell = 0$, we obtain

$$|c_i| [\bar{\Sigma}_i^{(n)}]^{\frac{1}{2}} \le |c_j| [\bar{\Sigma}_j^{(n)}]^{\frac{1}{2}} + |c_k| [\bar{\Sigma}_k^{(n)}]^{\frac{1}{2}} \quad , \tag{2a}$$

with

$$\bar{\Sigma}_\ell^{(n)} = \{ \int_D [|F_\ell|^2]^n d\mu \}^{1/n}, \quad \frac{1}{2} \le n < +\infty , \tag{2b}$$

where the three indices $i \neq j \neq k$ represent any permutation of 1,2,3 and each c_ℓ, $\ell = 1,2,3$ is a homogeneous fourth degree polynomial of Clebsch-Gordan coefficients. Next, let $\lambda[x]$ be the function

$$\lambda[x] = (x_1 - x_2 - x_3)^2 - 4 x_2 x_3 = (x_2 - x_3 - x_1)^2 - 4 x_3 x_1 = \tag{3}$$

$$= (x_3 - x_1 - x_2)^2 - 4 x_1 x_2 ,$$

Then it is easy to see that the bounds (2a,b) are equivalent to

$$0 \leq -\lambda\,[\,c_1^2\bar{\Sigma}_1^{(n)},\ c_2^2\bar{\Sigma}_2^{(n)},\ c_3^2\bar{\Sigma}_3^{(n)}\,] \leq 4\cdot\min_{\{i,k\}}\{c_i^2 c_k^2 \bar{\Sigma}_i^{(n)} \bar{\Sigma}_k^{(n)}\}, \qquad (4a)$$

and also to

$$[\,c_i^2\bar{\Sigma}_i^{(n)} - c_j^2\bar{\Sigma}_j^{(n)}\,]^2 \leq 2\,c_k^2\bar{\Sigma}_k^{(n)}\,[\,c_i^2\bar{\Sigma}_i^{(n)} + c_j^2\bar{\Sigma}_j^{(n)} - \frac{1}{2}c_k^2\bar{\Sigma}_k^{(n)}\,]\,. \qquad (4b)$$

Hence the result (4b) improves the theorem 1 from ref. /1/ in the most general form for $\Sigma_\ell^{(n)}$ -integrated (polarized and unpolarized) cross sections (see table I). This result implies that

if

$$\bar{\Sigma}_k^{(n)}[\,c_i^2\bar{\Sigma}_i^{(n)} + c_j^2\bar{\Sigma}_j^{(n)}\,] \underset{s\to+\infty}{\to} 0\,, \qquad (5a)$$

($\sqrt{s}$ is the c.m. energy)

then

$$c_i^2\bar{\Sigma}_i^{(n)} - c_j^2\bar{\Sigma}_j^{(n)} \underset{s\to+\infty}{\to} 0\,, \qquad (5b)$$

or $\bar{\Sigma}_k^{(n)}$ *cannot vanish for* $s \to +\infty$ *if the relation (5b) is violated.*

Next, let $\bar{\sigma}_\ell$ and $\vec{\bar{P}}_\ell = (\bar{A}_\ell, \bar{P}_\ell, \bar{R}_\ell)$, $\ell=1,2,3$, be the usual integrated cross sections and average and spin polarization parameters, respectively. If the functions F_ℓ in eq. (2b) are chosen such that $\bar{\Sigma}_\ell^{(1)} = (1 \pm \vec{\kappa}\cdot\vec{\bar{P}}_\ell)\,\bar{\sigma}_\ell$ (see table I), then eq. (4a) implies the isospin constraints listed in table II. Hence, using the bounds $-\lambda_\kappa^{(\pm)} \geq 0$, $4\bar{H} \leq -\lambda[\bar{\sigma}]$,

$\lambda[\vec{\kappa}\cdot\vec{\bar{P}}\bar{\sigma}] \leq 4\bar{H}$, $2a^{1/2} b^{1/2} \leq a + b$, $|a - b| \leq a + b$, for

$a \equiv -\lambda_\kappa^{(+)}$, $-4\bar{H} - \lambda[\bar{\sigma}]$, $b = -\lambda_\kappa^{(-)}$, $4\bar{H} - \lambda[\vec{\kappa}\cdot\vec{\bar{P}}\bar{\sigma}]$ and the identity

$$\lambda_\kappa^{(+)} + \lambda_\kappa^{(-)} = 2\lambda[\,\bar{\sigma}\,] + 2\lambda[\,\vec{\kappa}\cdot\vec{\bar{P}}\bar{\sigma}\,]\,,$$

we obtain an improvement of the bound (4b) on $\bar{\sigma}$ integrated cross sections to yield the following result:

Table I

The expressions of $|F_\ell|^2$, $\bar{\Sigma}_\ell^{(1)}$ *and* $\bar{\Sigma}_\ell^{(n)}$ *in terms of experimental* observables for some linear combinations of the scattering amplitudes

F_ℓ	[1] Vector $= \{f_\ell^{++}, f_\ell^{+-}\}$	$F_\ell^{(+\kappa)} = \frac{\sqrt{2}}{[1+\|w\|^2]^{1/2}}\{f_\ell^{++} + w f_\ell^{+-}\}$	$F_\ell^{(-\kappa)} = \frac{\sqrt{2}}{[1+\|w\|^2]^{1/2}}\{-w^* f_\ell^{++} + f_\ell^{+-}\}$
$\|F_\ell\|^2$	Differential cross section σ_ℓ	$(1+\vec{\kappa}\cdot\vec{P}_\ell)\,\sigma_\ell$ [2]	$(1-\vec{\kappa}\cdot\vec{P}_\ell)\,\sigma_\ell$ [2]
$\bar{\Sigma}_\ell^{(1)}$	Integrated cross section $\bar{\sigma}_\ell$	$(1+\vec{\kappa}\cdot\bar{\vec{P}}_\ell)\,\bar{\sigma}_\ell$	$(1-\vec{\kappa}\cdot\bar{\vec{P}}_\ell)\,\bar{\sigma}_\ell$
$\bar{\Sigma}_\ell^{(n)}$	$[\int_{\Omega_0} \sigma_\ell^n d\Omega]^{1/n}$	$\{\int_{\Omega_0} [(1+\vec{\kappa}\cdot\vec{P}_\ell)\,\sigma_\ell]^n d\Omega\}^{1/n}$	$\{\int_{\Omega_0} [(1-\vec{\kappa}\cdot\vec{P}_\ell)\,\sigma_\ell]^n d\Omega\}^{1/n}$

1) f_ℓ^{++} and f_ℓ^{+-} are the usual helicity non-flip and helicity flip scattering amplitudes.

2) $P_\ell \equiv (A_\ell, P_\ell, R_\ell)$ are the spin polarization vectors for $\ell = 1,2,3,$ respectively and $\vec{\kappa} \equiv \{\frac{2\mathrm{Re}\,w}{1+|w|^2}, \frac{2\mathrm{Im}\,w}{1+|w|^2}, \frac{1-|w|^2}{1+|w|^2}\}$ is an arbitrary unit vector.

Table II

The isospin constraints derived from eq. (4a) for $n=1$ and $F_\ell \equiv F_\ell^{(+\kappa)}$

	$\vec{\kappa}$	The isospin bounds (4a) imply:	The λ-functions and $\bar{H}_{ij}$ are defined by: [1]
I	$\pm\vec{\kappa}$	$0 \leq -\lambda_{\bar{\kappa}}^{(\pm)} \leq 4 \min_{\{ij\}} \{c_i^2 c_j^2 \bar{\sigma}_i (1 \pm \vec{\kappa}\cdot\vec{\vec{P}}_i)\bar{\sigma}_j (1 \pm \vec{\kappa}\cdot\vec{\vec{P}}_j)\}$	$\lambda_{\bar{\kappa}}^{(\pm)} \equiv \lambda[x],\ x_\ell \equiv (1 \pm \vec{\kappa}\cdot\vec{\vec{P}}_\ell)\bar{\sigma}_\ell c_\ell^2$
II	$-\vec{\vec{P}}_\ell$	$\lambda_{\bar{P}_\ell}^{(-)} = 0 \rightarrow \bar{H} \equiv c_1^2 c_2^2 \bar{H}_{12} = c_2^2 c_3^2 \bar{H}_{23} = c_3^2 c_1^2 \bar{H}_{31}$ [2]	$\bar{H}_{ij} \equiv \frac{1}{2}[1 - \vec{\vec{P}}_i \cdot \vec{\vec{P}}_j]\bar{\sigma}_i \bar{\sigma}_j$
III	$\vec{\vec{P}}_\ell$	$4\bar{H} \leq -\lambda[\bar{\sigma}] \leq 4 \min_{\{ij\}} \{c_i^2 c_j^2 \bar{\sigma}_i \bar{\sigma}_j\}$ [2]	$\lambda[\bar{\sigma}] \equiv \lambda[x],\ x_\ell \equiv c_\ell^2 \bar{\sigma}_\ell$
IV	$2\vec{\kappa}\cdot\vec{\vec{P}}_\ell \vec{\kappa} - \vec{\vec{P}}_\ell$	$4 \max\{-c_i^2 c_j^2 \kappa\cdot\vec{\vec{P}}_i\ \vec{\kappa}\cdot\vec{\vec{P}}_j\ \bar{\sigma}_i \bar{\sigma}_j\} \leq \lambda[\vec{\kappa}\cdot\vec{\vec{P}}\bar{\sigma}] \leq 4\bar{H}$ [2]	$\lambda[\vec{\kappa}\cdot\vec{\vec{P}}\bar{\sigma}] \equiv \lambda[x],\ x_\ell \equiv c_\ell^2 \vec{\kappa}\cdot\vec{\vec{P}}_\ell \bar{\sigma}_\ell$

1) For the definition of $\lambda[x]$ see eq. (3).
2) These results are valid if and only if $\bar{A}_\ell^2 + \bar{P}_\ell^2 + \bar{R}_\ell^2 = 1$ and are the integrated analogs of the corresponding results on differential observables derived in ref. [3,4].

$$[c_i^2\bar{\sigma}_i - c_j^2\bar{\sigma}_j]^2 + [c_i^2\bar{\sigma}_i \bar{X}_i^{(\kappa)} - c_j^2\bar{\sigma}_j \bar{X}_j^{(\kappa)}]^2 + \bar{A}^{(\kappa)} \leq \tag{6a}$$

$$\leq 2c_k^2\bar{\sigma}_k [c_i^2\bar{\sigma}_i + c_j^2\bar{\sigma}_j - \frac{1}{2}c_k^2\bar{\sigma}_k][1 + \bar{X}_k^{(\kappa)}\bar{X}_{ijk}^{(\kappa)}],$$

where

$$\bar{X}_\ell^{(\kappa)} \equiv \vec{\kappa}\cdot\vec{\bar{P}}_\ell, \quad \ell = i, j, k, \tag{6b}$$

$$\bar{A}^{(\kappa)} \equiv \max\{[-\lambda_{\bar{\kappa}}^{(+)}]^{1/2}[-\lambda_{\bar{\kappa}}^{(-)}]^{1/2}, 2[-4\bar{H}-\lambda[\bar{\sigma}]]^{1/2}[4\bar{H}-\lambda[\vec{\kappa}\cdot\vec{\bar{P}}\sigma]]^{1/2},$$

$$\frac{1}{2}|\lambda_{\bar{\kappa}}^{(+)} - \lambda_{\bar{\kappa}}^{(-)}|, |8\bar{H} + \lambda[\bar{\sigma}] - \lambda[\bar{X}^{(\kappa)}\bar{\sigma}]|\}, \tag{6c}$$

$$\bar{X}_{ijk}^{(\kappa)} = \frac{2c_i^2\bar{\sigma}_i \bar{X}_i^{(\kappa)} + 2c_j^2\bar{\sigma}_j \bar{X}_j^{(\kappa)} - c_k^2\bar{\sigma}_k \bar{X}_k^{(\kappa)}}{2c_i^2\bar{\sigma}_i + 2c_j^2\bar{\sigma}_j - c_k^2\bar{\sigma}_k}, \tag{6d}$$

valid at any energy for any unit vector $\vec{\kappa}$ *in any spin reference frame.*

It is important to note that, if $\bar{\sigma}_\ell^{(D)}$ and $\vec{\bar{P}}_\ell^{(D)}$ are defined by integration on a region D from the physical domain, where the experimental data on differential cross sections and on polarization parameters are available, then the results presented in table II and eq. (6a) remain valid when we substitute $\bar{\sigma}_\ell$ and $\vec{\bar{P}}_\ell$ by $\bar{\sigma}_\ell^{(D)}$ and $\vec{\bar{P}}_\ell^{(D)}$, $\ell = 1,2,3$, respectively, for any region D.

The result (6a) requires that, if

$$\bar{\sigma}_k [c_i^2\bar{\sigma}_i + c_j^2\bar{\sigma}_j] \underset{s\to+\infty}{\to} 0, \tag{7a}$$

then

$$c_i^2\bar{\sigma}_i - c_j^2\bar{\sigma}_j \underset{s\to+\infty}{\to} 0, \tag{7b}$$

$$[\bar{A}_i, \bar{P}_i, \bar{R}_i] - [\bar{A}_j, \bar{P}_j, \bar{R}_j] \underset{s\to+\infty}{\to} 0, \tag{7c}$$

and

$$\bar{A}^{(\kappa)} \underset{s\to+\infty}{\to} 0, \qquad \textit{for all } \vec{\kappa}, \tag{7d}$$

and, conversely $\bar{\sigma}_k$ *integrated cross sections cannot* *vanish for* $s \to +\infty$ *if one of the Pomeranchuk-type theorems (7b,c) is violated or if* $\bar{A}^{(\kappa)} \neq 0$ *at high energies.*

We note of course that the consequence (7b,c) can also be obtained directly from the bound $\bar{H} \leq -\lambda[\bar{\sigma}]$, rewritten in the equivalent form:

$$|c_i^2\bar{\sigma}_i - c_j^2\bar{\sigma}_j|^2 + \bar{H} \leq 2c_k^2\bar{\sigma}_k [c_i^2\bar{\sigma}_i + c_j^2\bar{\sigma}_j - \frac{1}{2}c_k^2\bar{\sigma}_k], \quad (8)$$

if the condition (7a) holds for $s \to +\infty$.

Therefore, if the indices i, j, k are chosen as in table III, then the bound (6a) enables us to understand the small elastic cross section differences: $|\bar{\sigma}_{\pi^+p} - \bar{\sigma}_{\pi^-p}|$, $|\bar{\sigma}_{K^+p} - \bar{\sigma}_{K^0p}|$, $|\bar{\sigma}_{K^-p} - \bar{\sigma}_{\bar{K}^0p}|$, $|\bar{\sigma}_{K^+n} - \bar{\sigma}_{K^-n}|$, $|\bar{\sigma}_{K^+p} - \bar{\sigma}_{K^-p}|$ at high energies in terms of the small charge exchange cross sections: $\bar{\sigma}_{\pi^-p \to \pi^0 n}$, $\bar{\sigma}_{K^+n \to K^0 p}$, $\bar{\sigma}_{K^-p \to \bar{K}^0 n}$ and $\bar{\sigma}_{K_L^0 p \to K_S^0 p}$, $\bar{\sigma}_{K_L^0 n \to K_S^0 n}$, respectively, and to predict the validity of Pomeranchuk-like theorems (7b,c,d) for the elastic integrated cross sections and average spin polarization parameters for $s \to +\infty$. The actual experimental data on the $\bar{\sigma}_k$ of the reactions listed in table III, seem to satisfy the condition (7a) since a fit of the energy dependence, according to formula $\bar{\sigma}_k = AP_{LAB}^{-n}$ implies the n-values presented in table IV. Moreover, we see that the values of $\bar{\sigma}_{\pi^+p}$ (43 GeV/c) = (3.22±0.05)mb, and $\bar{\sigma}_{\pi^-p}$ (40 GeV/c) = (3.32 ± 0.06) mb, $\bar{\sigma}_{\pi^-p}$ (42 GeV/c) = (3.36 ± 0.07) mb, as well as $\bar{\sigma}_{K^+p}$(43 GeV/c) = (2.31 ± 0.05) mb and $\bar{\sigma}_{K^-p}$(43 GeV/c) = (2.33 ± 0.05)mb (see ref. /5/), respectively, are in good agreement with the predictions (7a,b). Therefore, it is of great interest to obtain the accurate experimental data on the integrated parameters and to test in more detail the isospin predictions (7a,b,c,d) (and also the predictions listed in table II) for the above reactions at high energies. On the other hand, it would be interesting to obtain the experimental data for average polarization parameters $(\bar{A}_\rho, \bar{P}_\rho, \bar{R}_\rho)$ in the pion-nucleon scattering at energies below the one-pion production threshold where the

Table III

The coefficients c_ℓ from the sum rule $\sum_{\ell=1}^{3} c_\ell F_\ell = 0$ for some usual meson-baryon reactions related by isospin invariance via two channels

i	j	k
$\pi^+P \to \pi^+P$ $[c_i = +1]$	$\pi^-P \to \pi^-P$ $[c_j = -1]$	$\pi^-P \to \pi^o n$ $[c_k = -\sqrt{2}]$
$K^+P \to K^+P$ $[c_i = +1]$	$K^oP \to K^oP$ $[c_j = -1]$	$K^+n \to K^oP$ $[c_k = -1]$
$K^-P \to K^-P$ $[c_i = +1]$	$\bar{K}^oP \to \bar{K}^oP$ $[c_j = -1]$	$K^-P \to \bar{K}^o n$ $[c_k = -1]$
$K^+n(P) \to K^+n(P)$ $[c_i = +1$	$K^-n(P) \to K^-n(P)$ $[c_j = -1]$	$K_L^oP(n) \to K_S^oP(n)$ $[c_k = +2]$

Table IV

The values of n from a fit of energy dependence according to formula $\bar{\sigma}_k = A P_{LAB}^{-n}$

Reaction	Momentum range (GeV/c)	n	Ref.
$\pi^- P \to \pi^0 n$	6 – 100	1.15 ± 0.07	/5/
$K^- P \to K^0 n$	4 – 40	1.57 ± 0.05	/5/
$K^+ n \to K^0 P$		1.3	/6/
$K^0_L P \to K^0_S P$	1.0 – 12	2.18 ± 0.05	/7/

bound (4b) for $\bar{\Sigma}^{(1)}_{\rho} \geq \bar{\sigma}_{\rho}$ is saturated (see ref. /1/) in order to see if the equalities $\bar{X}^{(\kappa)}_i = \bar{X}^{(\kappa)}_j = \bar{X}^{(\kappa)}_k$ are verified. These equalities are direct consequences of the inequalities: $0 \leq |H| \leq -\lambda[\bar{\sigma}]$ (see table II).

Next, we remark that, if SU(3)-symmetry is assumed to be valid at high energies, then the results (6a) and (7a,b,c,d) can be applied to the following reactions: (i) $\pi^+P \to \pi^+P\ [c_i = +1]$, (j) $K^+P \to K^+P\ [c_j = -1]$, (k) $\pi^+P \to K^+\Sigma^+\ [c_k = +1]$, and also to: (i) $\pi^-P \to \pi^-P\ [c_i = +1]$, (j) $K^-P \to K^-P\ [c_j = -1]$, (k) $K^-P \to \pi^-\Sigma^+\ [c_k = +1]$, since:

$$\sum_{\rho=1}^{3} c_\rho F_\rho = 0, \quad \bar{\sigma}_{\pi^+P \to K^+\Sigma^+} \approx P_{LAB}^{-2.19 \pm 0.34} \quad \text{and}$$

$$\bar{\sigma}_{K^-P \to \pi^-\Sigma^+} \approx P_{LAB}^{-2.0 \pm 0.07},$$

(see ref. /8/).

From the values of elastic integrated cross sections at $P_{LAB} = 43$ GeV/c we obtain $\bar{\sigma}_{\pi^+P} - \bar{\sigma}_{K^+P} = -(0.91 \pm 0.07)$ mb and $\bar{\sigma}_{\pi^-P} - \bar{\sigma}_{K^-P} = (1.03 \pm 0.08)$ mb, which are in disagreement with the predictions (7b).

Finally, we note that the results (4a,b), (6a,b,c,d) and table II are of great interest for a systematic study of the possible breaking effects of the isospin invariance (SU(3) -symmetry, quark models, etc.) when complete and accurate experimental data will be available. In particular the bounds (4a,b) will be very useful for a test of the isospin invariance directly from the coefficients obtained by polynomial fits of the differential (polarized or unpolarized) cross sections, since $\bar{\Sigma}^{(n)}$ -integrated cross sections can be expressed as functions of these coefficients.

We remark that all the results (4a,b), table II and (6a) can be extended to the cases when 0 -spin particles are replaced by unpolarized J -spin particles and also to the three-body final state $(0^- \tfrac{1}{2}^+ \to 0^- 0^- \tfrac{1}{2}^+)$ reactions.

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