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THE EFFECT OF THE BOSE-STATISTICS ON AZIMUTHAL CORRELATIONS

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1. *ASSUMPTION FORMULA*

The azimuthal angle distribution for like pions is distinct from that for unlike pions [1-3]. We want to understand this effect, at least qualitatively, taking into account the fact that, first, the like pions obey the Bose-statistics and that, second, between the pion rapidities there exist correlations almost independent of their charge sign.

The influence of the Bose-statistics on the distribution of like pions with small relative momenta has been studied in refs. [4-8]. In particular, it is shown in refs. [6,7] that the pion pairs of the same sign with 4-momenta $p_i = \{\omega_i, \vec{p}_i\}$, $i = 1,2$ are distributed for $p_1 \approx p_2$ with density

$$W(p_1, p_2) = 1 + \frac{\exp[-(p_1 - p_2)^2 R^2]}{1 + (\omega_1 - \omega_2)^2 \tau^2}, \tag{1}$$

where R is the region size in which pions are produced, τ is the characteristic time of the existence of this region. It is assumed that mesons are emitted independently. For $\pi^+\pi^-$ pairs the second term in eq. (1) falls out. When analyzing the formula, it is reasonable to introduce the direction $\vec{n}$ of observation of narrow pion pairs and the projection on the plane $\perp \vec{n}$

$$\vec{p}_i = p_{i||}\vec{n} + \vec{r}_i \qquad (i = 1,2) . \tag{2}$$

In eq. (1) one can separate the factors depending on the difference of energies from those depending on the difference of transverse (to $\vec{n}$) momenta

$$W(p_1,p_2) = 1 + E(\omega_1-\omega_2)\exp[-(\vec{r}_1-\vec{r}_2)^2 R^2], \quad (3)$$

where approximately one can assume either

$$E(\omega_1-\omega_2) = \exp[-(\omega_1-\omega_2)^2 \tilde{\tau}^{-2}] \quad (4')$$

or

$$E(\omega_1-\omega_2) = [1+(\omega_1-\omega_2)^2 \tilde{\tau}^{-2}]^{-1} \quad (4'')$$

and $\tilde{\tau}^2 = \tau^2 + R^2/v^2$

(v is the particle velocity).

We want to see how the distribution (3) affects the azimuthal correlation. For this purpose, in addition to the density in the vicinity of the region $p_1 \approx p_2$, it is necessary to know the distributions of $\vec{p}_1$ and $\vec{p}_2$. In this case it is easier to use phenomenological distributions instead of theoretical ones. Take into account that a larger fraction of pions is emitted in the direction of the interaction axis $\vec{z}$ so that one can assume that $\vec{n} \parallel \vec{z}$. Thus, $\vec{r}_1, \vec{r}_2$ in (3) are considered as the momenta perpendicular not to the variable axis $\vec{n}$ but to the fixed axis - the interaction axis. This is a first simplification. Further let us assume that transverse momenta are distributed independently of longitudinal ones:

$$d\sigma \sim \left(\prod_{i=1}^{n} \exp(-\rho^2 r_i^2)\, d\vec{r}_i\right) \delta\left(\sum_{i=1}^{n} \vec{r}_i\right). \quad (5)$$

Finally let us use experimental data on the joint distribution $\rho(\omega_1,\omega_2)$ of the energies of two pions. It is almost independent of $\vec{r}_1, \vec{r}_2$ and of the pion pair sign (if neither of the pions is leading). It is essential that it is not uniform ($\rho(\omega_1,\omega_2) \neq \text{const}$) and falls to the boundaries of the physical region. These data suffice to obtain the estimate of the azimuthal distributions of interest.

Combining the foregoing, we assume that the cross section is expressed through the formula

$$d\sigma = [\prod_1^n e^{-\rho^2 r_i^2} dr_i\, \delta(\sum_1^n \vec{r}_i)] \times \tag{6}$$

$$\times [d\omega_1 d\omega_2\, \rho(\omega_1, \omega_2)] W(p_1, p_2).$$

The effect of our interest is obtained by integrating (6) over all the variables except the angle ϕ between $\vec{r}_1$ and $\vec{r}_2$. Denote the integral over ω_1, ω_2 normalized to 1 by λ.

$$\lambda = \frac{\int d\omega_1 d\omega_2\, \rho(\omega_1, \omega_2) E(\omega_1 - \omega_2)}{\int d\omega_1 d\omega_2\, \rho(\omega_1, \omega_2)} = \tag{7}$$

$$= \frac{\int d(\omega_1 - \omega_2) \tilde{\rho}(\omega_1 - \omega_2) E(\omega_1 - \omega_2)}{\int d(\omega_1 - \omega_2) \tilde{\rho}(\omega_1 - \omega_2)},$$

Here we introduce a new distribution function $\tilde{\rho}(\omega_1 - \omega_2) = \int \rho(\omega_1, \omega_2) d(\omega_1 + \omega_2)$. Then eq. (6) can be simplified:

$$\delta = \int [1 + \lambda e^{-(\vec{r}_1 - \vec{r}_2)^2 t^2}] \delta(\sum_1^n \vec{r}_i) e^{-\rho^2 \sum_i^n r_i^2} \prod_1^n d\vec{r}_i\,. \tag{8}$$

If we select only events with $\omega_1 = \omega_2$ (more precise, with $(\omega_1 - \omega_2)\bar{\tau} \ll 1$)), then $\lambda = 1$. The estimates of λ in other experimental conditions are presented in sect. 5.

2. AZIMUTHAL CORRELATIONS

Some consequences to be checked experimentally follow from eq. (8). First let us calculate the azimuthal correlations of like pions. The integration in eq. (8) over $\vec{r}_3, \ldots, \vec{r}_n$ gives

$$\sigma \sim \int [1+\lambda e^{-(\vec{r}_1-\vec{r}_2)^2 R^2}] \times \tag{9}$$
$$\times \exp[-\rho^2(r_1^2+r_2^2) - \frac{\rho^2}{n-2}(\vec{r}_1+\vec{r}_2)^2]\, d\vec{r}_1 d\vec{r}_2 \ .$$

We use the formula

$$\int d\vec{r}_1 d\vec{r}_2 \exp[-A(r_1^2+r_2^2) - 2B\, r_1 r_2 \cos\phi] =$$
$$= \frac{\pi}{B^2}\int_0^\pi \frac{d\phi}{[C(\phi)]^2} F\left(\frac{\cos\phi}{C(\phi)}\right), \tag{10}$$

where

$$F(x) = 1 - x\left(\frac{\pi}{2} - \mathrm{arctg}\, x\right),$$
$$C(\phi) = A^2 B^{-2} - \cos^2\phi \ . \tag{11}$$

When integrating the first terms of formula (9), we have

$$A = A_1 = \rho^2 \frac{n-1}{n-2}, \quad B = B_1 = \rho^2/(n-2),$$
$$C(\phi) = C_1 = (n-1)^2 - \cos^2\phi, \tag{12}$$

so that approximately

$$\frac{d\sigma_{+-}(\phi)}{d\phi} = \frac{(n-2)^2}{(n-1)^2\rho^4}\left(1 - \frac{\pi}{2}\frac{\cos\phi}{n-1}\right), \tag{13}$$

This is the known formula [10] which describes the correlations between π^+ and π^-. In order to calculate the correlations between π^+ and π^+, it is necessary to add the integral from the second term. Now

$$A_2 = A_1 + R^2, \quad B_2 = B_1 - R^2. \tag{14}$$

Introduce a dimensionless parameter $\gamma = R^2\rho^{-2}$. For $\pi^+\pi^+(\pi^-\pi^-)$-pairs we have

$$\frac{d\sigma_{++}(\phi)}{d\phi} = F\left(\frac{\cos\phi}{n-1}\right) + \tag{15}$$

$$+ \frac{\lambda}{1+2\gamma+\gamma^2\sin^2\phi} F\left(-\cos\phi \frac{\gamma-(n-2)^{-1}}{\sqrt{1+2\gamma+\gamma^2\sin^2\phi}}\right).$$

When ϕ changes from 0 to π, the first term slowly increases. Its average value is 1 while it increases not more than on $\pi/(n-1)$. The second term decreases due to $-\cos\phi$, and this decrease is rapid due to $\gamma \sim 1$.

However, a small coefficient in the second term makes the latter a little addition to the first term (*fig. 1*). *Figure 2* shows $d\sigma/d\phi$ versus n for constant γ and versus γ for constant n. One can see from *fig. 1* that eq. (15) gives a satisfactory qualitative description of the experimental data which are not very reliable for the present to search a quantitative coincidence.

One usually characterized the azimuthal distributions by their asymmetry coefficients β:

$$\beta = \frac{\sigma(\phi > \pi/2) - \sigma(\phi < \pi/2)}{\sigma(0 \le \phi \le \pi)}.$$

The integrals over $d\phi$ in eq. (10) and therefore β can be calculated exactly. If we denote $(n-2)^{-1} = a$, then

$$\beta = \frac{a}{1+a} - \frac{\lambda\gamma}{(1+\gamma)(1+\lambda+2\gamma)} + \frac{\lambda a\gamma(a-\gamma)}{(1+a)(1+\gamma)(1+\lambda+2\gamma)(1+a+\gamma)}, \tag{16}$$

or approximately if $n \gg 1$

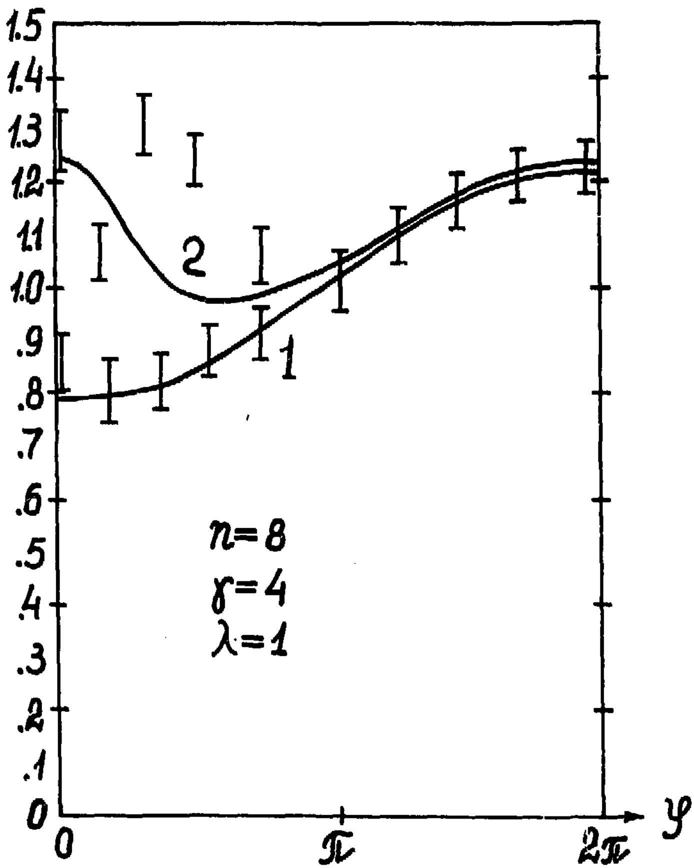

Fig. 1. The distribution of the azimuthal angles between $\pi^+\pi^-$ (curve 1) and $\pi^+\pi^-$ (curve 2) pairs. The calculation by formula (15) is given for $n = 8$, $\gamma = 4$, $\lambda = 1$. *The experimental data are taken from paper* [1] *: the sample with* $n = 8$, $|\Delta y| < 0.4$. *The ratio of the area between curves 2 and 1 to the area under curve 1 is the quantity* η *, formula (18)).*

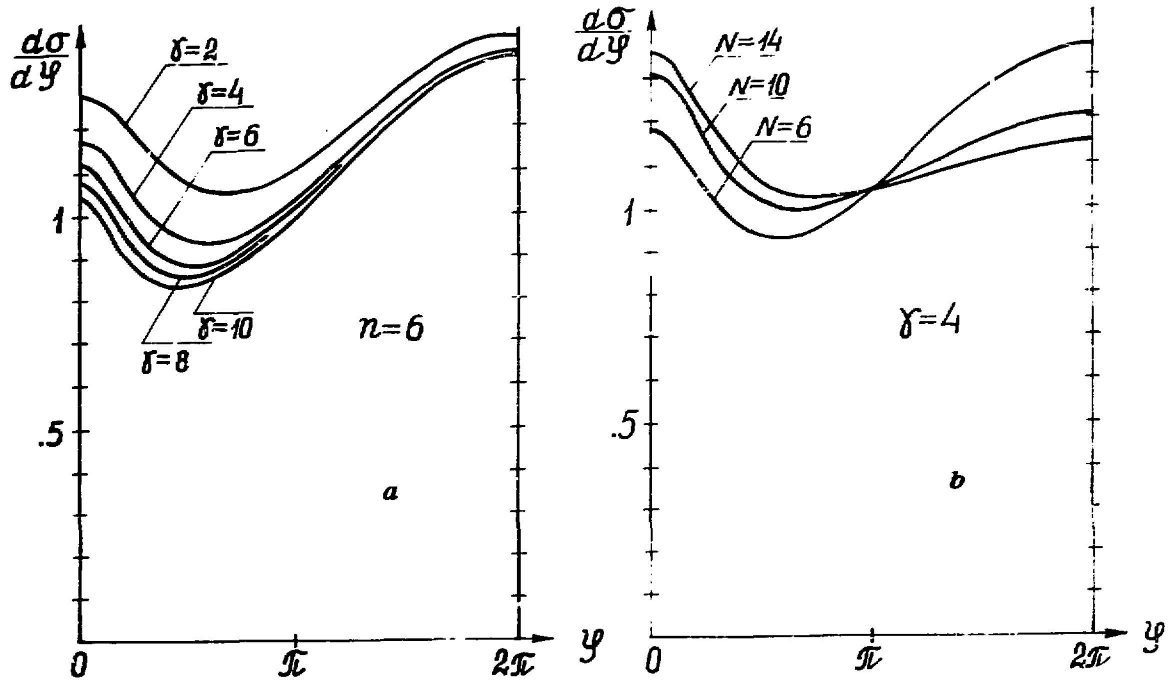

Fig. 2. a) $\frac{d\sigma_{++}(\phi)}{d\phi}$ *versus* γ *, b) versus* n .

$$\beta = \frac{1}{n-1} - \frac{\lambda\gamma}{(1+\gamma)(1+\lambda+2\gamma)} - \frac{\lambda}{n-1}\,\frac{\gamma^2}{(1+\gamma)^2(1+\lambda+2\gamma)}. \qquad (17)$$

For $n \gg 1$, $\gamma \gg 1$ the last term can be neglected. From eq. (17) it follows that $\beta_{+-} > \beta_{++}$, in qualitative agreement with experiment /1,3/.

It is of interest to calculate one more quantity which characterizes the difference between σ_{++} and σ_{+-}. This is a relative increase of the $\pi^+\pi^+$ production cross section due to their identity in comparison with $\pi^+\pi^-$. We normalize $d\sigma(\phi)/d\phi$ for $\pi^+\pi^+$ and $\pi^+\pi^-$ pairs to $d\sigma(\pi)/d\phi$. We could normalize them to $\sigma(\phi > \pi/2)$ (*fig. 1*). The result would be the same. Then integrating the first and second terms of eq. (9) over r_1, r_2, we obtain

$$\eta = \frac{\sigma_{++} - \sigma_{+-}}{\sigma_{+-}} = \frac{\lambda}{2\gamma+1}. \qquad (18)$$

Now η is independent of multiplicity (in contrast to β). An experimental confirmation of this fact could be an argument in favour of the chosen model (9). Measuring experimentally β and η for known n, one can determine the parameters λ and γ. They are indirectly connected with the parameters R and τ of multiple production.

3. *CRITICAL COMMENTS*

In ref. /1/ the formula is derived having approximately such a structure

$$\frac{d\sigma_{++}(\phi)}{d\phi} = \frac{d\sigma_{+-}(\phi)}{d\phi}\, I^{(s)}, \qquad (13')$$

where

$$I^{(s)} = 1 + \exp[-4(1-\cos\phi)] \qquad (15')$$

is an approximate estimate of the type (9) integral. This formula does not seem satisfactory to us. When deriving (13'), a logic inaccuracy is committed: twice one integrates over the same variables $\vec{r}_3,\ldots,\vec{r}_n$, $|\vec{r}_1|$, $|\vec{r}_2|$: for the first time when calculating $d\sigma_{+-}(\phi)/d\phi$, another time when calculating $I^{(s)}$. As we see, the correction term should be added to (not multiplied by) $d\sigma_{+-}(\phi)/d\phi$. Further from (15') it follows that the peak in the (++) spectrum is always twice higher than in (+-):

$$\frac{d\sigma_{++}(0)}{d\phi} = 2\,\frac{d\sigma_{+-}(0)}{d\phi}\,.$$

This is incorrect. The integration over ω_1, ω_2, $|\vec{r}_1|$, $|\vec{r}_2|$ strongly decreases the effect; the peak height should depend on λ and γ (see (18)) tending to zero for $\lambda \to 0$ or for $\gamma \to \infty$.

4. *DISTRIBUTIONS AT THE FIXED TRANSVERSE MOMENTUM*

The dependence of the asymmetry coefficient β on the transverse momentum r_1 of one of the pions has been measured in paper /3/. It is found that β is linearly dependent on r_1. We'll show that the same property of β follows from formula (9). Rewrite it in the form

$$\frac{d^2\sigma}{d\phi\, dr_1} = r_1 e^{-A_1 r_1^2}\,\Psi(\phi, A_1, B_1) + \lambda r_1 e^{-A_2 r_1^2}\,\Psi(\phi, A_2, B_2)\,, \tag{19}$$

where

$$\Psi(\phi, A_i, B_i) = \int \exp[-A_i r_2^2 - 2B_i r_1 r_2 \cos\phi]\, r_2\, dr_2\,. \tag{20}$$

Using eq. (14), divide (19) by $r_1 \exp(-A_1 r_1^2)$:

$$\frac{d^2\sigma}{d\phi\, dr_1} - \Psi(\phi, A_1, B_1) + \lambda e^{-R^2 r_1^2}\,\Psi(\phi, A_2, B_2)\,. \tag{21}$$

From here it follows that for sufficiently large r_1 the second term (which expresses the Bose effect) can be neglected. As $R > \rho$ ($R \sim m_\pi^{-1}$, $\rho \sim (2m_\pi)^{-1}$), this takes place already for $r_1 \sim 3m_\pi$. Therefore the ϕ-distributions of $\pi^\pm\pi^\pm$ and $\pi^+\pi^-$ pairs practically coincide for large r_1 ($\geq 3m_\pi$).

Let us calculate this distribution for $\pi^+\pi^-$ pairs. To do this, expand $\exp(-B\cos\phi)$ in a series and obtain ($r \sim r_1$)

$$\frac{d^2\sigma}{d\phi\, dr} \sim \frac{1}{2A_1}\left[\sum_0^\infty \frac{(-2B_1 r\cos\phi)^{2k}}{(2k)!}\cdot\frac{k!}{A_1^k} + \right. \tag{22}$$

$$\left. + \sum^\infty \frac{(-2B_1 r\cos\phi)^{2k-1}}{(2k-1)!}\left(\frac{\pi}{A_1}\right)^{1/2}\frac{(2k-1)!!}{2^k A_1^{k-1}}\right].$$

Leaving only the terms linear in $\cos\phi$,

$$\frac{d^2\sigma}{d\phi\, dr} \sim 1 - \tilde\beta\cos\phi, \tag{23}$$

we see that the asymmetry $\tilde\beta$ indeed increases proportional to r

$$\tilde\beta = \frac{2}{\pi}B_1\left(\frac{\pi}{A}\right)^{1/2} r \equiv \frac{2}{\pi^{1/2}}\frac{\rho}{[(n-1)(n-2)]^{1/2}}\, r\,. \tag{24}$$

From eq. (22) it is easy to obtain an exact formula for the asymmetry coefficient. It is clear the asymmetry arises only due to odd powers of $\cos\phi$, i.e., due to the second term in brackets. The asymmetry coefficient is (with minus sign) the ratio of the second and the first sum integrated over ϕ from 0 to $\pi/2$. The calculation gives

$$\beta = \tilde\beta\left(\sum_0^\infty \frac{(\tilde\beta\sqrt{\pi})^{2k}}{(2k+1)!!\,2^k}\right) \Big/ \left(\sum_0^\infty \frac{(\tilde\beta\sqrt{\pi})^{2k}}{(2k)!!\,2^{2k}}\right). \tag{25}$$

The asymmetry in the $\pi^{\pm}\pi^{\pm}$ azimuthal angles for fixed r can be obtained similarly; we write it in the approximation (24) only for $n \gg 1$

$$\tilde{\beta} = \frac{2\rho r}{\pi^{1/2}}\left[\frac{1}{n} - \frac{\lambda \exp(-R^2 r^2)}{1+\gamma}\,\frac{\gamma}{(1+\gamma)^{1/2}}\right]. \qquad (26)$$

We see that the difference of the asymmetry coefficient for $\pi^{\pm}\pi^{\pm}$ pairs from $\pi^{+}\pi^{-}$ can be observable only for small r. The parameter R could be estimated from this difference.

5. *THE PARAMETER* λ

All Bose effects in the azimuthal correlations would be offset if λ vanishes. The maximum effect is achieved when we select the events with $|\omega_1 - \omega_2|\tilde{r} \ll 1$. The remarkable effect is seen in paper /1/ where the ϕ-distribution for the events with $|\Delta y| < 0.4$ has been obtained. In contrast, this effect vanishes for the events with $|\Delta y| > 1.5$. But the experiment shows that the effect does not vanish if one takes all the events (with any y). This means that the parameter $1/\tilde{r}$ and the correlation length Ω in the energy distribution do not differ considerably.

Indeed, as the experiment proves, the distribution $\tilde{\rho}(\omega_1 - \omega_2)$ where ω_1, ω_2 is the C.M.S. energy, for the reaction $\pi^- p \to \pi^+\pi^- X$ (40 *GeV/c*) can be very good approximated by

$$\tilde{\rho}(\omega_1 - \omega_2) \sim \exp(-|\omega_1 - \omega_2|/\Omega), \qquad (27)$$

where Ω is comparatively small, $\Omega = .465$ *GeV* $= 3.33 m_\pi$. Therefore, even if integration over $|\omega_1 - \omega_2|$ in (7) goes up to ∞, the parameter λ remains comparable with 1. This means that the azimuthal Bose-effect can be observed even if one does not reject the events with large $|y_1 - y_2|$ or $|\omega_1 - \omega_2|$. The dependence of λ on $\tilde{r}$ is shown on *fig. 3* where two approximations (4') and (4'') have been taken ((4') lies somewhat higher). The very dependence on $\tilde{r}$ lies between these curves. So *fig. 3* permits to calculate $\tilde{r}$, if one knows λ, within the 20-25% limit. The analytic form of lower curve is

$$\lambda = \sqrt{\pi}\ \kappa \exp(\kappa^2)[1-\Phi(\kappa)], \quad \kappa = 1/2\Omega\tilde{r}, \tag{28}$$

where $\Phi(\kappa)$ denotes the probability integral.

But, naturally, the azimuthal corrections for cut $|\omega_1-\omega_2|\ \Omega$ are more pronounced: the value of λ grows and $\tilde{r}$ can be determined better. We emphasize that we prefer to cut the events with $|\omega_1-\omega_2|<\Omega$ to the cut $|y_1-y_2|<Y$, because the effect, as the formula (1) shows, depends directly on $(\omega_1-\omega_2)$.

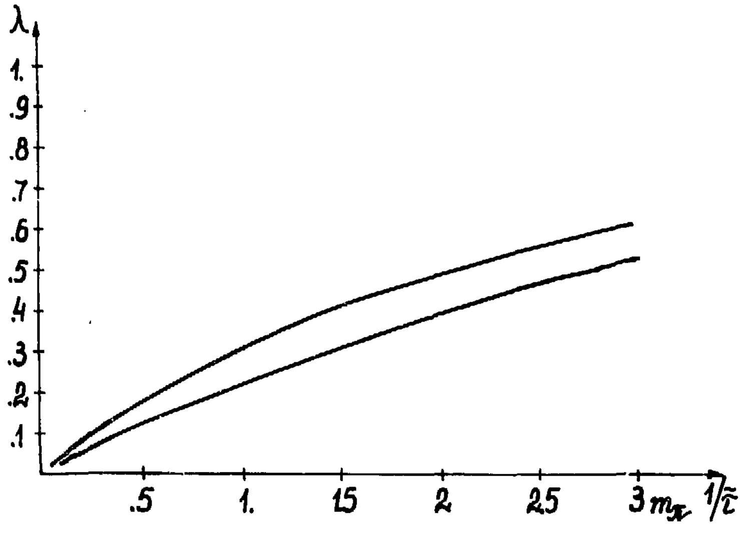

Fig. 3

We understand that the simplicity of formulae of this paper arises mainly due to the assumption that $\tilde{\rho}(\omega_1-\omega_2)$ is the same for any $\vec{r}_1,\vec{r}_2$. This hypothesis seems to be reasonable but needs to be experimentally checked.

We wonder why it was easier to observe the azimuthal correlation of like pions than the direct Podgoretsky's effect - the peak on the $\Delta\Delta$ -plot [9] (The $\Delta\Delta$ -plot is two-dimensional plot with the axis $\Delta\omega = \omega_1 - \omega_2$ and $\Delta \vec{r}^{\,2} = (\vec{r}_1 - \vec{r}_2)^2$, where $\vec{r}_1, \vec{r}_2$ are momenta perpendicular to the variable axis $\vec{n} - \vec{p}_1 + \vec{p}_2$). It is strange because we assume that the azimuthal effect is the reflexion of that of Podgoretsky's and we might expect that the inequality holds

$$\sigma_{az}(\phi_{++} \to 0) \leq \sigma_{Podg}(\Delta p \to 0). \tag{29}$$

Here σ_{az} is the cross section of excess of events on the ϕ-curve lying in the peak over the background of unlike pairs, and $\sigma_{Podg.}$ is the excess of events on the $\Delta\Delta$ -plot lying in the peak region. If this inequality did not hold it would mean that there is an additional source of azimuthal correlations or that the $\Delta\Delta$ -plot is not a proper way to represent the effect of interference. May be it is worth to check the plot $(\Delta\omega, (\vec{r}_1 - \vec{r}_2)^2)$ where $\vec{r}_1, \vec{r}_2$ are perpendicular to the z -axis, instead of usual $\Delta\Delta$-plot. We suggest to experimenters to clear up this point.

6. CONCLUSIONS

1. The opinion exists which connects the azimuthal correlations with clusters. It is too early to insist on this only explanation. It is shown here that simple"protostatistical" [10] model can explain qualitatively all the data available if we include in it the interference between the sources of like pions. We cannot differ experimentally two moving clusters from long excited volume, the data available now are too poor for this.

2. Equally with a direct method of measurement of R and r (by means of the $\Delta\Delta$-plot [9]) we may use another one: to compare the azimuthal correlations in $\pi^{\pm}\pi^{\pm}$ and $\pi^{+}\pi^{-}$ pairs. Both methods should give independently the same values of R and r. In order to

increase a precision of the second method, it is necessary, first, to measure the ϕ -distribution as a function of $\omega_1-\omega_2$ (and not of Δy) and, second, to measure the $\bar{\rho}(\omega_1-\omega_2)$ distribution experimentally. This would enable us to know an accurate value of the integral λ (7) instead of its approximate estimates.

2. The simple model (6) used in this paper, qualitatively correctly explains: 1) the $d\sigma(\phi)/d\phi$ dependence for $\pi^+\pi^+$ and $\pi^+\pi^-$ pairs; 2) the $d\sigma(\phi)/d\phi$ dependence on the multiplicity n ; 3) the dependence of the asymmetry coefficient β on n and on the pair charge; 4) the $d\sigma(\phi)/d\phi$ dependence on $|\Delta y|$; 5) the dependence of the $d^2\sigma/d\phi\, dr$ asymmetry distribution on r.

   Only the fact of decreasing β for large $|\Delta y|$ found in ref. [3] cannot be explained by this model although we can understand other properties of the curve $\beta(\Delta y)$, i.e., $\beta_{++}(0) < \beta_{+-}(0)$, $\max \beta_{++}(\Delta y) \approx \max \beta_{+-}(\Delta y)$.

Both the theoretical model of the phenomenon and the experiments require further improvement.

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