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CHANCE TO USE W-DECAY LEPTON CHANNELS FOR WIDTH DETERMINATION

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Возможность использования лептонных мод распада W-бозона для определения его ширины

Рассмотрены процессы $e^+e^- \to W^+W^-$ и $\gamma e \to W\nu$ с последующим распадом W-бозона в лептоны. Благодаря резонансной природе W-бозонного пропагатора в промежуточном состоянии выход лептонов имеет резкий подъем в пороговой области. Вычислено инклюзивное по лептонам сечение. Для случая образования двух лептонов их доли энергии и угол в системе центра инерции определяют некоторое тело в трехмерном пространстве этих параметров. Для случая одиночного рождения W-бозона эти параметры — доля энергии лептона и угол его вылета к оси пучков — образуют плоскую фигуру, ограничивающую разрешенную область. Мы показываем, что измерение «смазывания» границ этих тел за счет резонансной природы W-бозона может дать информацию об отношении ширины W-бозона к его массе. Проанализирован вопрос о влиянии радиационных поправок.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Chance to Use W-Decay Lepton Channels for Width Determination

The process of W^+W^--production at e^+e^--collisions as well as the process of single W-boson production in γe--collision with subsequent W-decay via lepton channel have been considered. Due to resonance nature of W-boson propagator in an intermediate state, lepton output has a sharp rise in the region of W-boson production threshold. Lepton-inclusive cross-sections have been calculated. In the case of two-lepton production their energy fractions and the angle between their momenta in c.m.s. for the majority of events are found to be connected by the condition confining a certain body in 3-dimensional space of these parameters. For the case of single W-production the decay lepton energy fraction and its angle with respect to a beam axis play a role of such parameters; their allowed regions being a plane figure. We show that measurement of «smearing» of lepton distribution due to W-boson resonance nature would give quite reliable information on the ratio of the W-boson total width to its mass. Influence of radiation corrections on this distributions is analysed.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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For the annual integral of luminosity $\int L dt = 500\ \mathrm{pb}^{-1}$, which is planned at LEP-II, having in mind the values of the cross-section $\sigma(e^+e^- \to W^+W^-) \approx 20$ pb and of the probabilities of decay channels [1]

$$e^+e^- \to W^+W^- \to \begin{cases} 4 \text{ jets } 53\% \\ 2 \text{ jets + lepton } 40\%, \\ 2 \text{ leptons } 7\% \end{cases} \tag{1}$$

one may expect $\approx 10^3$ events of two leptons of various kinds per a year.

1. The cross-section of two oppositely charged leptons of various kinds $(\mu^-\tau^+)$ in $e^+(p_+) + e^-(p_-) \to W^+W^- \to \mu^-(r) + \nu_\mu + r^+(\pi) + \bar{\nu}_\tau$ in the energy range from the reaction threshold, $\sqrt{s} \approx 2$ GeV up to $\sqrt{s} \approx M_W$

$$\sigma^{e^+e^- \to \mu^+\tau^-(\nu\bar{\nu})} \approx r_0^2 \left(\frac{m_e m_\tau^3}{m^4}\right)^2 \alpha^2 \approx 10^{-14}\ \mathrm{pb},\ m = M_W \approx 80\mathrm{GeV}, \tag{2}$$

is less than the main background process of two lepton-pair production [2]:

$$\sigma^{e^+e^- \to \mu^+\mu^-\tau^+\tau^-} \approx \frac{\alpha^4}{s} = 10^{-3}\ \mathrm{pb} \left(\frac{200}{\sqrt{s}(\mathrm{GeV})}\right)^2 \tag{3}$$

by several orders of magnitude.

The situation is different at the energy near the threshold of W^+W^--pair production, where the cross-section (2) increases rapidly due to resonance nature of an intermediate state of W-bosons

$$\sigma^{\mu\tau} \approx \left(\frac{m}{\Gamma}\right)^2 \cdot \frac{\alpha^4}{s} \approx 1\ \mathrm{pb} \cdot \left(\frac{200}{\sqrt{s}(\mathrm{GeV})}\right)^2 \tag{4}$$

already exceeding significantly the background cross-section (3).

In the region of the threshold for production of Z-bosons, the analogous increase of the cross-section (3) will take place, and the problems of taking into account of background will arise.

It should be noted that even in the region $2\varepsilon = \sqrt{s} \geq 180$ GeV the background of the process (3) does not exceed 10%, because the cross-section of the process $e^+e^- \to ZZ$ is less by an order than of $e^+e^- \to W^+W^-$.

When studying the process (2) one may contsruct the distribution in the lepton energy fractions

$$x = 2\varepsilon_{\tau^+}/\varepsilon,\ \ y = 2\varepsilon_{\mu^-}/\varepsilon,\ \ \sqrt{s} = 2\varepsilon \tag{5}$$

and the angle $\theta = \widehat{\vec{\pi}^+, \vec{r}^-}$ between the lepton 3-momenta in the c.m.s. of e^+e^--beams, x, y, θ being connected by the condition

$$\begin{gathered} D(x, y, \theta) > 0, \\ D(x, y, \theta) = \sin^2\theta x^2 y^2 \beta^2 - 2(1 + \cos\theta)(1 - \beta^2 - x)(1 - \beta^2 - y)xy - \\ - (1 - \beta^2)^2 (x - y)^2, \end{gathered} \tag{6}$$

where $\beta = (1 - 4m^2/s)^{1/2}$ is the velocity of W-boson in c.m.s., confining a certain body in the kinematically allowed parallelepiped in the space of the parameters x, y, θ:

$$1 - \beta < x, y < 1 + \beta,\ \ 0 < \theta < \pi. \tag{7}$$

The condition (6) may be obtained on integrating over 4-momenta of neutrino and on replacing the Breit—Wigner propagators of intermediate W^+W^--bosons by appropriate δ-functions responsible for the states with real W^+W^--bosons

$$\frac{1}{(q_\pm^2 - m^2)^2 + m^2\Gamma^2} \to \frac{\pi}{m\Gamma}\delta(q_\pm^2 - m^2), \tag{8}$$

in fact, on transforming the phase volume of neutrino final state

$$\begin{aligned} \int d\tilde{\Phi} &= \int \frac{d^3\nu_\mu d^3\nu_\tau}{2\varepsilon_{\nu_\mu} 2\varepsilon_{\nu_\tau}} \delta^4(q_- - \nu_\mu - \pi)\delta^4(q_+ - \nu_\tau - r)\delta(q_-^2 - m^2)\delta(q_+^2 - m^2) = \\ &= \int \frac{d^3q_+ d^3q_-}{2q_{+0}2q_{-0}} \delta((q_- - \pi)^2)\delta((q_+ - r)^2)\delta^4(p_+ + p_- - q_+ - q_-). \end{aligned}$$

as follows:

$$\int d\tilde{\Phi} = \frac{2\beta}{s^2} \int d\varphi dc_- \delta(c_- - a)\delta(c_+ - b)(xy\beta^2)^{-1} = 4\{s^2 xy\beta\,|\,s_- s_0 \sin\varphi\,|\,\}^{-1}, \tag{9}$$

where

$$a = -\frac{1}{x\beta}(1 - \beta^2 - x),\ \ b = -\frac{1}{y\beta}(1 - \beta^2 - y),\ \ c_+ = c_0 c_- - s_0 s_- \sin\varphi,$$
$$c_+ = \cos(\widehat{\vec{q}_+, \vec{\pi}}),\ \ c_- = \cos(\widehat{\vec{q}_+ \vec{r}}),\ \ c_0 = \cos(\widehat{\vec{\pi}, \vec{r}}).$$

Finally one obtains:

$$\int d\tilde{\Phi} = 4\ [s^2\sqrt{D}\,]^{-1},\ s = (p_+ + p_-)^2. \tag{10}$$

It should be noted that the conditions of «reality» for intermediate W-bosons (8) allow one to reconstruct their 4-momenta: for every set of two 4-vectors, π, r associated with real final leptons (μ^-, τ^+) one may find the corresponding two pairs of 4-vectors (q_+, q_-) of $W^\pm$-bosons.

The replacement (8) is not strict, so the boundaries of the region (5) are smeared out, i.e., $D > 0$. The ratio of the point number, ΔN, associated with the values of the parameters x, y, θ beyond the region $D > 0$ to the total number of points, N, seems to be the value of an order $\approx \Gamma/M$. Moreover, the quantity

$$\left(\frac{\Delta N}{N}\cdot\frac{M}{\Gamma}\right)_{\Gamma/M<1} = c(s, \theta), \tag{11}$$

for small values of Γ/M is independent of Γ/M, where $c(s, \theta)$ is the smooth function of their arguments. Numerical calculations for $\sqrt{s} = 200$ GeV, $\theta = 120°$ give $c = 5.25$. In our article [3] we calculate the integrated by θ quantity $c(s)$ and find it a smooth decreasing function of s order of one in the threshold region of energy.

We believe that the use of (11) for determination of Γ/M is quite reasonable. Below we shall discuss the effectiveness of various factors which are neglected when calculating $\Delta N/N$. We represent now the concrete formulas for calculation of $N + \Delta N$.

The phase volume of a final state involving Breit-Wigner nature of $W^\pm$ propagator is as follows:

$$\int d\Phi = \left(\frac{m\Gamma}{\pi}\right)^2 \int \frac{d^4q_+ d^4q_- \delta^4(p_+ + p_- - q_+ - q_-)\delta(q_-^2 - m^2)\delta(q_+^2 - m^2)}{((q_+^2 - m^2)^2 + m^2\Gamma^2)((q_-^2 - m^2)^2) + m^2\Gamma^2)}.$$

We transform it by analogy with (10) as follows:($\pi = (\frac{\varepsilon y}{2}, \vec{\pi})$; $r = (\frac{\varepsilon x}{2}, \vec{r})$ are 4-momenta of μ^-, τ^+)

$$\begin{aligned} \int d\Phi = 8\left(\frac{\Gamma(1 - \beta^2)}{m\pi s}\right)^2 \int_{-\infty}^{\infty} d\sigma \int_{-\infty}^{\infty} \rho \tilde{D}^{-1/2}[((2 - \sigma)^2 - \rho^2 - 1 + \beta^2)^2 + \\ + ((1 - \beta^2)\Gamma/M)^2]^{-1}[(\sigma^2 - \rho^2 - 1 + \beta^2)^2 + ((1 - \beta^2)\Gamma/M)^2]^{-1} d\rho, \end{aligned} \tag{12}$$

where

$$\begin{aligned} \tilde{D} = (\sin^2\theta)x^2y^2\rho^2 - 2(1 + \cos\theta)(\sigma^2 - \rho^2 - x\sigma)((2 - \sigma)^2 - \rho^2 - y(2 - \sigma))xy - \\ - ((\sigma^2 - \rho^2)y - ((2 - \sigma)^2 - \rho^2)x + 2xy(1 - \sigma))^2. \end{aligned} \tag{13}$$

The approximation (8) implies: $\sigma = q_{-0}/\varepsilon = 1$, $\rho = |\vec{q}_-|/\varepsilon = \beta$.

The cross-section, inclusive by the 3-momenta of $\mu^-\tau^+$ leptons, is represented as follows:

$$d\sigma^{\mu\tau} = \frac{\alpha^4}{8\pi^4}\cdot\frac{d^3\pi d^3r}{\varepsilon_\pi \varepsilon_r}\int R d\Phi, \tag{14}$$

where $d\Phi$ is given (12) and

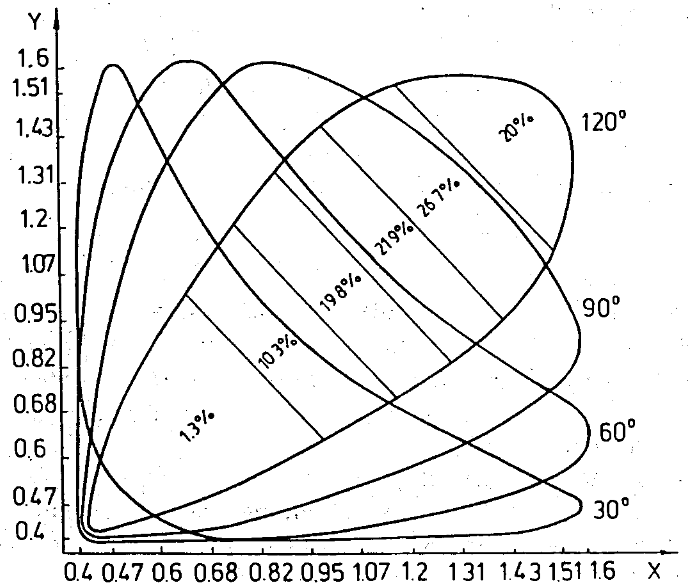

Fig.1. Sections of the body $D = 0$ by planes $\theta = 30°, 60°, 90°, 120°$. Density of pionts in Dalitz-plot for $\theta = 120°$ case are given in per cent.

$$R = \frac{1}{4}\sum_{\alpha,\beta} M_{\alpha\beta} M^*_{\alpha_1\beta_1} T^{\alpha\alpha_1} S^{\beta\beta_1}. \tag{15}$$

The terms M, T, S represent, respectively, the matrix element of W^+W^--production off mass shell in e^+e^--collisions:

$$M_{\alpha\beta} = \bar{v}(p_+)\left\{\frac{1}{t}\gamma_\beta(p_- - q_-)\gamma_\alpha\omega_- + \left(\frac{1}{s} + \frac{1}{s - M_Z^2}\right)\gamma^\lambda\omega_-\Gamma_{\lambda\alpha\beta} + \right.$$
$$\left. + \left(\frac{1}{s} - \frac{1}{s - M_Z^2}\right)\gamma^\lambda\omega_+\Gamma_{\lambda\alpha\beta}\right\}u(p_-),$$

$$s = (p_+ + p_-)^2,\ t = (p_- - q_-)^2,\ \omega_\pm = \frac{1}{2}(1 \pm \gamma_5), \tag{16}$$

$$\Gamma_{\lambda\alpha\beta} = g_{\lambda\beta}q_{+\alpha} - g_{\lambda\alpha}q_{-\beta} + \frac{1}{2}g_{\beta\alpha}(q_- - q_+)_\lambda, \tag{17}$$

and the current tensors of lepton-neutrino pairs:

$$T^{\alpha\alpha_1} = \frac{q_-^2}{m^2}\left(g^{\alpha\alpha_1} - \frac{1}{m^2}q_-^\alpha q_-^{\alpha_1}\right) + \frac{1}{m^2}(q_- - 2\pi)^\alpha(q_- - 2\pi)^{\alpha_1} - \frac{2i}{m^2}\varepsilon^{\alpha\alpha_1\lambda\sigma}q_{-\lambda}\pi_\sigma,$$

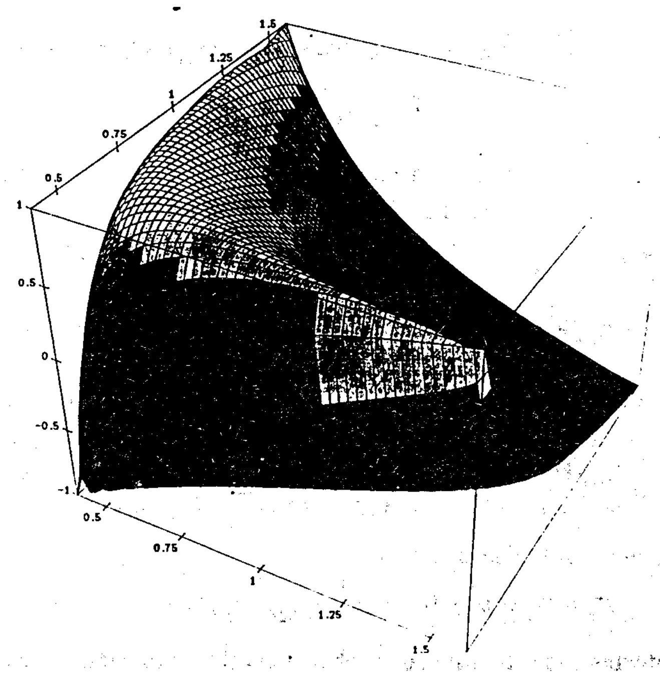

Fig.2. The three-dimensional reconstruction of the body $D = 0$ (see (6)) for $\beta = 0.6$ obtained by means of computer.

$$S^{\beta\beta_1} = \frac{q_+^2}{m^2}\left(g^{\beta\beta_1} - \frac{1}{m^2}q_+^\beta q_+^{\beta_1}\right) + \frac{1}{m^2}(q_+ - 2r)^\beta(q_+ - 2r)^{\beta_1} - \frac{2i}{m^2}\varepsilon^{\beta\beta_1\lambda\sigma}q_{+\lambda}r_\sigma. \tag{18}$$

The distribution in the parameters x, y, θ has the following form:

$$\frac{d\sigma^{\mu\tau}}{dx\,dy\,d\cos\theta} = \frac{xy\alpha^4}{8\pi^4 s}\int d\Omega_\mu d\Omega_\tau R\, d\Phi\,\delta(\cos\theta - \cos(\widehat{\vec{r},\vec{\pi}})). \tag{19}$$

In fig.1 the sections of the body $D = 0$ by the planes $\theta = 30°, 60°, \ldots, 150°$ are depicted. Distribution of points calculated by means of (15) is given in fig.1 for $\theta = 120°$ section of body $D = 0$. The picture of this body for $\beta = 0.6$ is drawn in fig.2. It should be noted that the distribution in the fraction of single-lepton energy coincides with that given by R.Kleiss 1, although the tensors T, S, used there, differ from (18).

2. Now we consider the process $\gamma e \to W\nu_e \to l\nu_e\bar{\nu}_e$. When integrating over neutrino phase volume

$$\int d\Phi = \int \frac{d^3 v_1 d^3 v_2}{2\varepsilon_1 2\varepsilon_2}\delta^4(p + k - l - v_1 - v_2)\delta((l + v_2)^2 - m^2) = \int d^4 v_2 \delta(v_1^2)\delta(v_2^2)\delta((l + v_2)^2 - m^2), \tag{20}$$

it is convenient to use parametrization by Sudakov in order to write down lepton 4-momenta,

$$v_i = \alpha_i p + \beta_i k + v_i^{\perp},\ l + \alpha p + \beta k + l^{\perp},\ v_i^{\perp} p = v_i^{\perp} k = l^{\perp} p = l^{\perp} k = 0,$$
$$v^{\perp 2} = -\vec{v}^2 < 0,\ l^{\perp 2} = -\vec{l}_{\perp}^2 < 0,\ d^4 v_i = \frac{s}{2} d\alpha_i d\beta_i d^2 v_i^{\perp},\ s = (p + k)^2 = 4\varepsilon\omega. \tag{21}$$

So, as experimentally the energy ε_l and the 3-momentum of a produced lepton are measured, we consider the parameters α, β, $l^{\perp}$ as known ones

$$\alpha = \frac{\varepsilon_l}{\omega}\left(\frac{1 - c}{2}\right),\ \beta = \frac{\varepsilon_l}{\varepsilon}\left(\frac{1 + c}{2}\right),\ c = \cos(\widehat{\vec{l}, \vec{k}}),\ s\alpha\beta = \vec{l}_{\perp}^2 \tag{22}$$

as well as the neutrino invariant mass

$$M^2 = (k + p - l)^2 = (v_1 + v_2)^2 = s(1 - \alpha - \beta). \tag{23}$$

In these terms the integral (20) over phase volume takes the form:

$$\int d\Phi = \frac{1}{2}s\int d\alpha_2 d\beta_2 d\vec{v}_2^2 \frac{1}{2} d\varphi \delta(s\alpha_2 \beta_2 - \vec{v}_2^2)\delta(s(\alpha + \alpha_2)(\beta + \beta_2) - (\vec{l} + \vec{v}_2)^2)\delta(M^2 + m^2 - s(\alpha + \beta)) = \frac{1}{2s}\int d\beta_2 d^{1/2}\theta(d)\theta(\beta^0 - \beta_2), \tag{24}$$

where

$$\beta^0 = \frac{1}{s}(m^2 + M^2) = 1 - \alpha - \beta - \mu^2,\ \mu^2 = m^2/s,$$
$$d = (\alpha + \beta)^2(\beta_2 - \beta_-)(\beta_+ - \beta_2), \tag{25}$$

$$\beta_{\pm} = (\alpha + \beta)^{-2}\{\beta(\alpha + \beta)\beta^0 + \mu^2(\alpha - \beta) \pm [4\alpha\beta\mu^2(\beta^0 - \mu^2)(1 - \beta^0)]^{1/2}\}.$$

The restriction imposed on the parameters

$$c = \cos(\widehat{\vec{l}, \vec{k}}),\ x = \frac{2\varepsilon_l}{\sqrt{s}}, \tag{26}$$

follows from the reality of $\beta^{\pm}$ and has the form

$$\frac{m^2}{2\omega^2\sqrt{\lambda}\,[1 + \lambda - c(1 - \lambda)]} < x < \frac{2\sqrt{\lambda}}{1 + \lambda - c(1 - \lambda)},\ \lambda = \frac{\varepsilon}{\omega}. \tag{27}$$

Herein we don't give a lepton-inclusive cross-sections. By analogy with the process $e^+e^- \to WW \to l l' v \bar{v}$, it is constructed from the matrix element of the

$e^-\gamma \to W(q_-)v$ process and the current tensor of a lepton-neutrino pair. The construction of the 4-vector of an intermediate W-boson is significant. We give the scheme of constructions of $q_- = l + v_2$, $v_2 = \alpha_2 p_- + \beta_2 k + v_2^{\perp}$. Specifying β_2 from the region of integration, we construct the values $-v_2^{\perp 2} = \vec{v}^2 = s\beta_2(\beta^0 - \beta_2)$ and $\alpha_2 = \vec{v}^2/s\beta_2$. The azimuthal angle φ between the vectors $v_2^{\perp}$ and $l^{\perp}$ is reconstructed by the relation

$$\sin\varphi = \frac{s(\alpha + \beta)}{2|\vec{l}^{\perp}|\ |\vec{v}|}[(\beta_+ - \beta_2)(\beta_2 - \beta_-)]^{1/2}. \tag{28}$$

giving a pair of values. Thus, for even values of β_2 we construct two 4-vectors q_- differing by the value of the azimuthal angle $\varphi_1 - \varphi_2 = \pi$. Then we calculate the squared modula of the process matrix element (ME) and sum their values, the sum being used as the ME modulus squared in the inclusive cross-section.

3. Evaluating the quantities $\Delta N/N$ we considered a produced leptons as massless one, admitting the error in $\Delta N/N$ of an order $(m_\tau/m_W)^2 \approx 10^{-4}$. It is significantly less than $\delta\Gamma/M$, where $\delta\Gamma = 0.6$ GeV is a characteristic error in measurement of the W-boson width with usage of hadron collisions. Substantial corrections occur as a result of changes of $2\varepsilon = \sqrt{s}$, entering the equation $D = 0$ as a parameter. The corrections $\approx \Delta\varepsilon/\varepsilon$ originate from nonmonochromatic beams. They are the most significant in the case of the process $\gamma\gamma \to WW$, when photon is produced in scattering of a laser beam by an electron one. Decreasing the total energy produces the decreasing of the region of variation of the fraction of lepton energy, $1 - \beta < x,y < 1 + \beta, \beta \to 0$ as well as the volume of a body enclosed in the parallelepiped (7) of the process of two-lepton production. Bremsstrahlung along the beam axis effectively leads to decrease of the c.m.s. total energy of W^+W^- [4] and, according to the mentioned above, it does not change the value of ΔN. Taking into account of radiation corrections leads to the change of the event total number $N \to N\left(1 + 0\left(\frac{\alpha}{\pi}\ln\frac{s}{m_e^2}\right)\right)$, resulting in error of Γ/M measurement of the form:

$$\frac{\delta\Gamma}{M} = \left(\frac{\Gamma}{M}\right)\cdot\frac{\alpha}{\pi}\ln\frac{s}{m_e^2} \tag{29}$$

comprising, for the parameter $\frac{\alpha}{\pi}\ln\frac{s}{m_e^2} = 0.1$, the value of an order of $\approx 10\%$.

Thus, specifying an accuracy 10%, one may neglect the effect of radiation corrections in Γ/M — measurement.

In conclusion, we would like to note the influence of final-state rescattering on the cross-section of two-lepton production [5]. The scattering cross-section of produced lepton is large in the case of a small angle of scattering, $\theta = 0$, however, as it follows from definition of an allowed region $D > 0$, its phase volume tends to zero. So this effect is negligible.

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