

# объединенный ИНСТитут ядерных исследований 

A.V.Ivkin*, E.A.Kuraev, M.T.Nazirov*

METHOD FOR DETERMINATION OF W-BOSON TOTAL WIDTH

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*Institute of High Energy Physics, Alma-Ata, Kazakhstan

Ивкин А.В., Кураев Э.А., Назиров М.Т.

Предлагается новый метод измерения ширины узкого резонанса по полному сечению его образования. Метод детально излагается для процес$\cos e^{+} e^{-} \rightarrow W^{+} W^{-}, \gamma e \rightarrow W \nu$.

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A new way to measure the total width of a narrow resonance by total cross-section is proposed. The technique is exemplified in detail in the reactions $e^{+} e^{-} \rightarrow W^{+} W^{-}, \gamma e \rightarrow W \nu$.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

The experiments planned at the nearest future in LEP2 seem to be able to solve the problems concerning the nature of bosons which are of great theoretical interest. Theory renormalization and the related character of crosssection behaviour are among these problems. Another important problem is the $W$-boson mass and width measurements, because taking into account of the highest approximation is needed for its theoretical description. The value of the $W$-mass may be defined by rapid growth of the cross-section of the process $e^{+} e^{-} \rightarrow W^{+} W^{-}$near its threshold. For width determination several tricks are proposed. One of them [1], [2] is based on the measuring of sub-threshold cross-section of the $W^{+} W^{-}$-- pair production. Another one [3] is based on the analysis of $W$-decay lepton mode outside the allowed kinematical region.

Herein, we propose the technique based on nonadequacy of the BreitWigner propagator associated with intermediate resonances and the $\delta$-func-tion-like propagators associated with real particles. The difference between the propagators is of an order of $\Gamma / M$. For illustration we consider the expression:

$$
\begin{gather*}
\frac{1}{\pi} \int_{-a}^{b} \frac{\varepsilon d x}{x^{2}+\varepsilon^{2}}=\frac{1}{\pi}\left(\operatorname{arctg} \frac{b}{\varepsilon}+\operatorname{arctg} \frac{a}{\varepsilon}\right)=1-\frac{\varepsilon}{\pi}\left(\frac{1}{a}+\frac{1}{b}\right), \\
a, b>0, \varepsilon \rightarrow 0 . \tag{1}
\end{gather*}
$$

Having in mind the integration in (1) over $x$ from $a$ to $b$, we present the integral in (1) as follows:

$$
\begin{equation*}
\frac{i}{\pi} \frac{\varepsilon}{x^{2}+\varepsilon^{2}}=\delta(x)-\frac{\varepsilon}{\pi a b}, \varepsilon \rightarrow 0,-a<x<b \tag{2}
\end{equation*}
$$

Basing on this fact one may affirm that the difference between the crosssections of narrow-resonance production ( $\varepsilon=\Gamma_{t} / M \ll 1$ ) on and off mass shell is of an order of $\Gamma_{t} / M$ rather than $\left(\Gamma_{t} / M\right)^{2}$.

Now we shall demonstrate for W -boson, as an example, how one can extract it's width analysing the total cross sections of the reactions $e^{+} e^{-} \rightarrow W^{+} W^{-}$and $\gamma e \rightarrow W \nu$.

Thus, in order to measure a resonance width, we propose the following procedure:

$$
\begin{equation*}
\frac{\sigma_{0}^{a b \rightarrow R}(s) \operatorname{Br}(R \rightarrow f)-\sigma_{\exp }^{a b \rightarrow R \rightarrow f}(s)}{\sigma_{\exp }^{a b \rightarrow R \rightarrow f}(s)}=\varphi(s) \Gamma_{t} / M, \tag{3}
\end{equation*}
$$

where $\sigma_{\text {exp }}^{a b \rightarrow R \rightarrow f}(s)$ is a cross-section of the process in which a produced resonance is considered as a free particle with a mass $m_{R}$. It is calculated with
easy (see below). The function $\varphi(s)$ is independent of the R-decay channel and of the total width $\Gamma_{t}$; it is calculable and may be tabulated. It should be noted that this function is different for various processes.

So, we consider the process $\gamma e \rightarrow W \nu \rightarrow f$, then $\varphi(s)$ in the expression (3) is as follows:

$$
\begin{equation*}
\varphi(s)=\lim _{(\Gamma / M)->0}\left(\frac{M}{\Gamma} \frac{\sigma_{0}(s)-\tilde{\sigma}(s)}{\tilde{\sigma}(s)}\right), \tag{4}
\end{equation*}
$$

where $\sigma_{0}(s)$ is the cross-section of the process with mass-shell $W$-boson:

$$
\begin{equation*}
\sigma_{0}\left(s, M^{2}\right)=\frac{4 \pi \alpha^{2}}{M^{2}}\left\{\left(1-x^{-1}\right)\left(1+5 x^{-1}+\frac{7}{4} x^{-2}\right)-x^{-1}\left(2+x^{-1}+x^{-2}\right) \ln x\right\} \tag{5}
\end{equation*}
$$

and $\widetilde{\sigma}(s)$ is the appropriate cross-section with virtual $W$-boson:

$$
\begin{equation*}
\tilde{\sigma}(s)=\int_{0}^{s} d s_{1} \rho\left(s_{1}\right) \sigma_{0}\left(s, s_{1}\right) \tag{6}
\end{equation*}
$$

and

$$
\begin{equation*}
\rho(x)=\frac{x\left(\Gamma_{t} / M\right) \pi^{-1}}{\left(x-M^{2}\right)^{2}+M^{2} \Gamma_{t}^{2}} \tag{7}
\end{equation*}
$$

The function $\varphi(s)$ is a decreasing one and this value is of an order of 0.7 in the energy $\sqrt{s}=100 \mathrm{GeV}$. It can be evaluated beforehand.

For the process $e^{+} e^{-} \rightarrow W^{+} W^{-} \rightarrow f$ one may incorporate the analogous procedure:

$$
\begin{equation*}
\frac{\sigma_{0}(s) B r^{2}(W \rightarrow f)-\sigma_{\exp }(s)}{\sigma_{\exp }(s)}=f(s) \frac{\Gamma_{t}}{M} \tag{8}
\end{equation*}
$$

The function $f(s)$, independent of the $W$-decay channel mode, can be calculated by the following relation:

$$
\begin{equation*}
f(s)=\lim _{(\Gamma / M)->0}\left(\frac{\sigma_{0}(s)-\tilde{\sigma}(s)}{\tilde{\sigma}(s)} \frac{M}{\Gamma}\right) \tag{9}
\end{equation*}
$$

$\sigma_{0}(s)$ is the cross-section of on-mass-shell $W^{+} W^{-}$production in $e^{+} e^{-}$-collisions:

$$
\begin{gather*}
\sigma_{0}(s)=\frac{16 \pi \alpha^{2}}{s}\left\{\left(1+\frac{1}{2} x^{-1}+\frac{1}{8} x^{-2}+\frac{8 x+1}{16 x^{2}(3 x-1)^{2}}\right) \ln \frac{1+\beta}{1-\beta}-\right. \\
\left.-\frac{5}{4} \beta-\beta \frac{\left(4 x^{2}+20 x+3\right)}{12 x^{2}(3 x-1)^{2}}\right\}, x=s / 4 M^{2}, \beta=\left(1-x^{-1}\right)^{1 / 2} \tag{10}
\end{gather*}
$$

and $\tilde{\sigma}(s)$ has the form:

$$
\begin{equation*}
\tilde{\sigma}(s)=\int_{0}^{s} d s_{1} \int_{0}^{\left(\sqrt{s}-\sqrt{s_{1}}\right)^{2}} d s_{2} \rho\left(s_{1}\right) \rho\left(s_{2}\right) \sigma\left(s ; s_{1}, s_{2}\right) \tag{11}
\end{equation*}
$$

where $\sigma\left(s ; s_{1}, s_{2}\right)$ is the cross-section of production of two virtual $W$-bosons with the masses squared $s_{1}$ and $s_{2}$ in $e^{+} e^{-}$-collisions. It was calculated primary in [2]. For our case the approximation $\sin ^{2} \theta_{w}=1 / 4$ is relevant. Then

$$
\begin{equation*}
\sigma\left(s ; s_{1}, s_{2}\right)=\frac{\pi \alpha^{2}}{2 s^{2} s_{1} s_{2}}\left\{-\left(\frac{1}{s^{2}}+\frac{1}{\left(s-M_{z}^{2}\right)^{2}}\right) G_{1}-2 G_{2}+2\left(\frac{1}{s}+\frac{1}{s-M_{z}^{2}}\right) G_{3}\right\} \tag{12}
\end{equation*}
$$

$$
\begin{align*}
& \text { where } \begin{array}{c}
G_{1}=-\lambda^{3 / 2}\left[\frac{1}{6} \lambda+2 s\left(s_{1}+s_{2}\right)+2 s_{1} s_{2}\right] \\
G_{3}=-\lambda^{1 / 2}\left[\frac{1}{6} \lambda\left(s+11\left(s_{1}+s_{2}\right)\right)+2 s\left(s_{1}^{2}+s_{2}^{2}+3 s_{1} s_{2}\right)-2\left(s_{1}^{3}+s_{2}^{3}\right)\right] \\
-4 s_{1} s_{2}\left[s\left(s_{1}+s_{2}\right)+s_{1} s_{2}\right] \ln L, \\
G_{2}=-\lambda^{3 / 2}\left[\frac{1}{6} \lambda+2 s\left(s_{1}+s_{2}\right)-8 s_{1} s_{2}\right]+4 s_{1} s_{2}\left(s-s_{1}-s_{2}\right) \ln L \\
\lambda=s^{2}+s_{1}^{2}+s_{2}^{2}-2\left(s s_{1}+s s_{2}+s_{1} s_{2}\right), \quad L=\frac{s-s_{1}-s_{2}-\sqrt{\lambda}}{s-s_{1}-s_{2}+\sqrt{\lambda}}
\end{array}
\end{align*}
$$

The function $f(s)$ can be tabulated. It's graph is shown in the figure. One may convence that it is a decreasing function (it is of an order of 1.0 in the region $\sqrt{s} \approx 200 \mathrm{GeV}$ ). In the table the values of $f(s)$ for various values of $\Gamma_{t}$ are shown.

Table. $f(s)$ for different values of $\Gamma_{t}$

| GeV | 1.5 GeV | 2.25 GeV | 3.0 GeV |
| :---: | :---: | :---: | :---: |
| 150 | $2.27 \pm 0.03$ | $2.35 \pm 0.02$ | $2.39 \pm 0.02$ |
| 200 | $1.49 \pm 0.01$ | $1.53 \pm 0.01$ | $1.55 \pm 0.01$ |
| 220 | $1.11 \pm 0.01$ | $1.14 \pm 0.01$ | $1.16 \pm 0.01$ |
| 240 | $0.87 \pm 0.01$ | $0.74 \pm 0.03$ | $0.76 \pm 0.02$ |
| 260 | $1.71 \pm 0.04$ | $0.68 \pm 0.01$ | $0.68 \pm 0.02$ |

Now we shall discuss influence of various factors on the given technique accuracy. If a width is defined by lepton modes, e.g., of the reaction $e^{+} e^{-} \rightarrow W^{+} W^{-} \rightarrow \mu \tau \nu_{\mu} \bar{\nu}_{\tau}$, then the basic background at $\sqrt{s} \geq 180 \mathrm{GeV}$ will be the process $e^{+} e^{-} \rightarrow Z Z \rightarrow \mu^{+} \mu^{-} \tau^{+} \tau^{-}$in the case when a single pair of opposite-


Dependence of $f(s)$ for the different values $\Gamma_{t} / M$.
charge leptons is not detected. However, likely, the $e^{+} e^{-} \rightarrow Z Z$ crosssection is almost by an order of magnitude less than for $e^{+} e^{-} \rightarrow W^{+} W^{-}$[4] and, taking into account the branching ratios of the extra pair, we obtain the discrepancy of an order of more than $10^{-1}$.

A significant contribution of an order of $\Delta \varepsilon / \varepsilon$ may occur due to uncertainty, $\Delta \varepsilon$, of the initial energy, $2 \varepsilon=\sqrt{s}$, if colliding beams (or one of beams) are not monochromatic. Then photon beam usage can be substantially restricted for the technique proposed.

Radiative corrections should strongly change the value of an extracted width, because contribute significantly to total cross-sections [5]. However, one can reveal with the case that the value of this contribution is of an order of $\delta \Gamma=\Gamma(\alpha / \pi) \ln \left(s / m_{e}^{2}\right)$ and it is small for LEP2 energies $(\sqrt{s}=200 \mathrm{GeV}$ $\left.(\alpha / \pi) \ln \left(s / m_{e}^{2}\right)=0,1\right)$,

Thus, we propose a new technique for determination of W-boson total width which is effective only for the energy of LEP2, because the difference of total cross-section, is small for $W$ being on the mass shell and off $i t$, and it is maximal (of an order of 2 pb ) in the energy region of LEP2. Outside this energy region the $|\sigma-\widetilde{\sigma}|$ value is of the order of the background effects which we have just discussed, so we can't extract $\Gamma$ with needed accuracy.

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