

ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ
ДУБНА

E2-92-6

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ANALYSIS OF DISCRETE AMBIGUITIES
FOR $\pi N\Delta$ AND $\rho N\Delta$ VERTEX FUNCTIONS
USING INCLUSIVE CHARGE-EXCHANGE
REACTIONS

Submitted to "Physics Letters"

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Анализ дискретных неоднозначностей
для вершинных функций $\pi N\Delta$ и $\rho N\Delta$ в инклюзивных
зарядово-обменных реакциях

Мы используем формализм диаграмм Фейнмана для описания зарядово-обменных реакций (p,n) , (n,p) и $({}^3\text{He}, t)$ на протонной мишени с учетом спектаторных и распадных мод в $\pi + \rho + g'$ -модели. Характер интерференции между этими модами зависит от используемых параметров вершинных функций. Показано, что существует дискретная неоднозначность для вершинных функций $\pi N\Delta$ и $\rho N\Delta$.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1992

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E2-92-6

Analysis of Discrete Ambiguities for $\pi N\Delta$ and $\rho N\Delta$
Vertex Functions Using Inclusive Charge-Exchange Reactions

We use the formalism of Feynman diagrams to describe the charge-exchange reactions (p,n) , (n,p) and $({}^3\text{He}, t)$ on a free proton target taking into account spectator and decay modes in the $\pi + \rho + g'$ -model. The type of interference between these modes depends on the set of vertex function parameters used. It is shown that discrete ambiguities exist for the $\pi N\Delta$ and $\rho N\Delta$ vertex functions. These are partly dissolved if charge-exchange data for a wide energy range is used.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

1 Introduction

Direct resonant reactions provide a basic source of information on effective interactions, reaction mechanisms and nuclear structure. Their contribution to charge-exchange reactions at intermediate energies $0.6 < T_p < 10$ GeV has attracted much attention recently in on-going studies [1] in the Δ -isobar excitation region.

We explore the coexistence of nucleonic, mesonic and Δ -isobar degrees of freedom in nuclear collisions. Although our approach is classical in the sense that quarks do not appear explicitly, all diagrams of physical importance are included. Being in our approach a resonance in the nucleon-meson system, the Δ -isobar conveys a way to learn about the carriers of the strong nuclear force in the nuclear medium.

Reactions of the type (e, e') , (p, p') , (π, π') , (p, n) , $({}^3\text{He}, t)$, $({}^6\text{Li}, {}^6\text{He})$,... at intermediate energies and at transfer energies ≈ 0.3 GeV involve the Δ -resonance mechanism and allow us to investigate the excitations of nucleon and subnucleon degrees of freedom in nuclei, by strongly interacting probes and by electromagnetic ones as well.

According to estimates [2, 3] the renormalization of the $NN \rightarrow N\Delta$ interaction when taking place inside nuclei, is relatively small ($\approx 5\%$ at $T_p \approx 1$ GeV). The same conclusion was obtained in [4] namely that

all NN and ΔN interactions inside the nucleus are close to free interactions. Therefore the information from the reactions on free nucleons is of special importance. Having this information we can in principle try to understand the mechanism of charge-exchange on a nuclear target and to investigate spin-isospin excitations in a nuclear medium.

In most recent theoretical publications [4]-[9] devoted to the description of the (p, n) reaction and analogous processes, the $\pi + \rho + g'$ -model was used. In this model the phenomenologically determined constants are the Migdal-Landau parameter $g'_{N\Delta}$ and the cut-off parameters Λ_π , Λ_ρ in the monopole meson-baryon formfactors F_B ($B = \pi, \rho$):

$$F_B(t) = \frac{\Lambda_B^2 - m_B^2}{\Lambda_B^2 - t}, \quad (1)$$

where t is the square of the invariant momentum transfer and m_B is the mass of the corresponding meson. The parameters of this model are established from best fits to existing experimental data for a wide class of processes (charge-exchange, π -atoms, photoexcitations, etc.). The situation is completely the same as in the case of elastic scattering of ions on nuclei where the parameters of the optical potential are determined from the best description of the experimental phase set (cross sections).

The principal aspects of this problem can be formulated as follows. Let us consider the elastic scattering at fixed energy from the potential $V(r)$:

$$(T + V)\Psi = E\Psi. \quad (2)$$

Despite the fact that the motion of the particle covers all space the boundary conditions for Ψ are given on both bounds: Ψ has to be regular at $r \rightarrow 0$ and at $r \rightarrow \infty$ Ψ is determined by the experimental set of phase shifts $[\delta_L(E)]_{exp}$. The result is a Sturm-Liouville problem for finding eigen depths, radii and diffusenesses of the potential $V(r)$. Among the "phase equivalent" discrete set of potentials obtained the physical one is selected from additional physical criteria. The energy and L -dependence of the resulting potentials are due to the many-body character of the problem and the field effects.

The aim of this contribution is to discuss discrete ambiguities of the parameters of the formfactors F_B and to search additional criteria for selection of these parameters.

2 Formalism

The cross section of the elementary process $p + p \rightarrow n + X \dots$ can (in the notation of Bjorken and Drell and with $c = \hbar = 1$) be written as

$$d\sigma = \frac{2m^2}{\lambda^{1/2}(s, m^2, m^2)} \frac{m}{E_n} \frac{d\vec{P}_p}{(2\pi)^3} \frac{m}{E_p} \frac{d\vec{P}_n}{(2\pi)^3} \frac{d\vec{P}_\pi}{2E_\pi (2\pi)^3} (2\pi)^4 \delta^{(4)}(P_1 + P_2 - P_n - P_p - P_\pi) S_f \langle |M|^2 \rangle, \quad (3)$$

where P_1 and P_2 are the four momenta of the colliding protons. The indices n , p and π refer to the proton, neutron and pion in the final states, respectively, and S_f is the statistical factor due to the Pauli principle. The symbol $\langle |M|^2 \rangle$ means averaging/summing over the projections of projectile and ejectile spins.

In the framework of the Feynman diagram formalism we take into account the diagrams depicted in Fig.1. Each diagram in Fig.1 has a corresponding matrix element M_j :

$$M_j = (-1)^{j+1} ISF_j G_\Delta(S_\Delta) \frac{f_{\pi N \Delta}}{m_\pi} (\vec{S}^+ \cdot \vec{P}_\pi) [V_C(q_j) (\vec{S} \cdot \vec{\sigma}) + V_{NC}(q_j) S_{12}(\hat{q}_j)], \quad (4)$$

where ISF_j is the isospin factor [5], q_j the impuls of the virtual pion in the Breit system ($q_j^{(0)} = 0, t_j = -\vec{q}_j^2$), $\hat{q} = \vec{q}/q$, \vec{P}_π the impuls of

the real pions in the rest frame of the Δ -isobar, S_Δ the square of the invariant mass of the Δ -isobar and G_Δ its Green's function. We have used the definition of the operators \vec{S} and S_{12} given in [5, 6] and also the notations:

$$V_L(q) = \frac{f_{\pi NN} f_{\pi N \Delta}}{m_\pi^2} F_\pi^2(q) q^2 G_\pi(q), \quad (5)$$

$$V_T(q) = C_p \frac{f_{\pi NN} f_{\pi N \Delta}}{m_\pi^2} F_\rho^2(q) q^2 G_\rho(q), \quad (6)$$

$$V_C(q) = \frac{1}{3} [V_L(q) + 2V_T(q)] + g'_{N\Delta} \frac{f_{\pi NN} f_{\pi N \Delta}}{m_\pi^2} F_\pi^2(q), \quad (7)$$

$$V_{NC}(q) = \frac{1}{3} [V_L(q) - V_T(q)], \quad (8)$$

where G_π and G_ρ are the usual mesonic propagators [10]. The formfactors F_B in eqs.(5) and (6) are given by formula (1) taking into account the fact that $t = -\vec{q}^2$ in the Breit system.

The diagrams included in our calculations dominate in the energy region $0.8 \leq T_p \leq 1.5$ GeV when the neutrons are registered in the angular interval corresponding to the first diffraction peak and the momentum spectra are investigated around the Δ -isobar peak. In principle one might expect that it is necessary to add also the s -wave pion scattering on the nucleon. However, the contribution of this amplitude to the $p+p \rightarrow n + p + \pi^+$ reaction cross section at $T_p \sim 1$ GeV is relatively small (see Fig.4) in the considered region of the angles (in the case of $n(^3He, t)\Delta^+$ this term gives a significant contribution [5]).

It is necessary to stress that the technique used is based not on perturbation theory in the interaction representation, but on decomposition of the total amplitude over the renormalized diagrams. It means that the effects of renormalization, polarization of the vacuum, contributions of the another mesons and resonances and off-shell effects ..., are included in the formfactors F_B . Such treatment can be argued for by comparing with potential models for low energy scattering.

Some additional remarks about the terminology. The sum of diagram 1 and 2 (3 and 4) in Fig.1 is denoted as DET (DEP) and is pictured as one effective diagram in Ref.[5] while we separate out the direct and exchange parts. We have used the terminology of [11] according to which the DET diagram is called the spectator one and DEP the decay one. So we will

call the first diagram the spectator direct diagram, the second as the spectator exchange and so on (SD, SE, DD and DE).

3 Results

The calculations of the cross sections were performed for three very different sets of the vertex function parameters taken from standard literature: OSET ($\Lambda_\pi=1.3$ GeV, $\Lambda_\rho=1.4$ GeV, $C_\rho=3.95$, $g'_{N\Delta}=0.6$) [5]; JAIN ($\Lambda_\pi=1.2$ GeV, $\Lambda_\rho=2.0$ GeV, $C_\rho=2.0$, $g'_{N\Delta}=0.3$) [6]; DMIT ($\Lambda_\pi=0.65$ GeV, $\Lambda_\rho=0.0$ GeV, $C_\rho=0.0$, $g'_{N\Delta}=0.9$) [12]. The coupling constants are standard and equal to $f_{\pi NN}^2/4\pi=0.081$ and $f_{\pi N\Delta}^2/4\pi=0.36$. As is seen from Fig.2 all three sets give nearly the same noncentral potentials $V_{NC}(q)$ while the central potentials $V_C(q)$ have approximately the same shape and are nearly equidistant in the sequence JAIN-OSET-DMIT with JAIN the deepest.

Calculated energy spectra for a number of forward angles are compared with experimental data [13] in Fig.3 for the reaction $p+p \rightarrow n+X$ at $T_p=1$ GeV incident energy. Full detailed agreement should not be expected since the theoretical curves are not folded with the experimental resolution function. The general agreement is still quite satisfactory, but the data for the angular region $\theta_{lab} \in [0^\circ, 15^\circ]$ are not able to discriminate between the three transition potentials JAIN, OSET and DMIT. Thus we are left with a potential ambiguity involving apparently widely different potentials (see Fig.2). Neither the momentum spectrum nor the angular distribution help to distinguish between the three potential families (Within each family some minor variation can be obtained due to the continuous ambiguities in the definitions of the parameters Λ_π , Λ_ρ , C_ρ and $g'_{N\Delta}$. The theoretical estimations [15, 16] of the Λ_π value are also close to the set [5] and [6] and are not in drastic disagreement with the set [12]). The same conclusions were obtained in [14] at $T_p=1$ GeV.

The purpose of this paper is to point out to and investigate these ambiguities further. We will discuss both the $p+p \rightarrow n+X$ and $n+p \rightarrow p+X$ process.

In the $p+p \rightarrow n+X$ process the decay amplitudes have lower isospin weight factor with the consequence that the decay mode is negligibly small as is demonstrated in Fig.4 for $T_p=1$ GeV and $\theta = 0^\circ$ and the JAIN

potential. To be able to assess the relative importance of various components of the full amplitude, Fig.5 displays their partial (hypothetical) cross sections as functions of exit neutron momentum. These components interfere to give the full physical result. The $p+p \rightarrow n+X$ process is essentially determined by the spectator term alone, a result which also holds for larger angles (see Fig.5) and all three potentials. The relative role of the SD and SE parts as a function of potential family, is as for the richer $n+p \rightarrow p+X$ process, to which we now turn.

The decay amplitudes play a more noticeable role for the reaction $n+p \rightarrow p+X$, the total cross section nevertheless does not allow us to give preference to any of the tree potentials (Fig.6). The decay and spectator amplitudes are nearly orthogonal at $T_p=1$ GeV and $\theta = 0^\circ$, i.e. the full cross section is close to the sum of the partial hypothetical spectator and decay cross section. Notice that this result applies to all three potentials. The separation between the S-mode and D-mode maxima is equal to $\Delta E \approx 140 \text{ MeV} \approx m_\pi$, thus has a kinematical origin.

Finally we investigate in detail the SD, SE, DD, and DE amplitudes and their interplay for all three potentials. Figs.7A-7C correspond to the JAIN, OSET and DMIT potentials respectively, for the same five forward angles. As we have alluded to above partial cross sections for the three potential choices have some common features; all spectator terms are peaked and at nearly the same momentum while all decay terms are rather flat with a shoulder and drop off at essentially the same hard momentum. The total S contribution and total D contribution is nearly the same for all three potentials (see Fig.6) with S the dominant part in the central momentum region.

If we now, however, address the SD and SE constitution of the spectator amplitude using partial hypothetical cross sections, it varies (Figs.7A-7C) dramatically with potential set. For the JAIN potential the SE term is substantially larger than the SD for all angles investigated and the two amplitudes are nearly orthogonal (i.e. $\sigma(S) \approx \sigma(SE) + \sigma(SD)$). For OSET the relative magnitude is the opposite at 0° , but reverses to the JAIN case beyond $\sim 10^\circ$. The peak heights are also smaller than for JAIN. Thus there is a rather complicated and angular dependent interference in this case, since the total S contribution is the same as for JAIN. In the case of the DMIT potential we have the opposite extreme of the JAIN case, the SE term is negligible and the full S contribution coincides with SD for all angles.

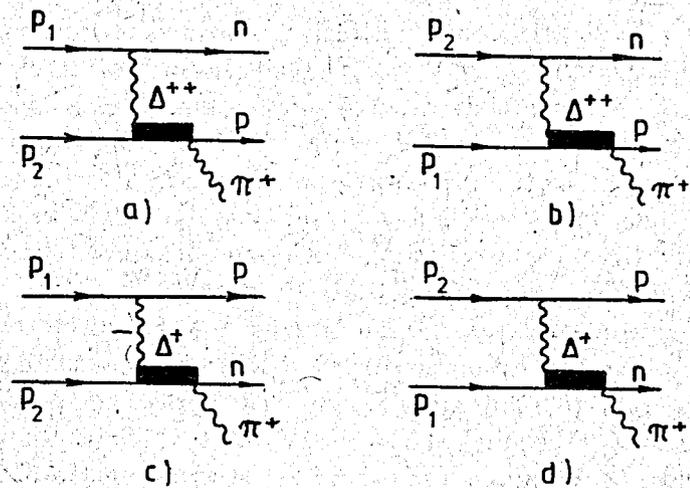


Fig.1 Diagrams contributing to the NN-NA cross sections. a) SD - spectator direct, b) SE - spectator exchange, c) DD - decay direct, d) DE - decay exchange.

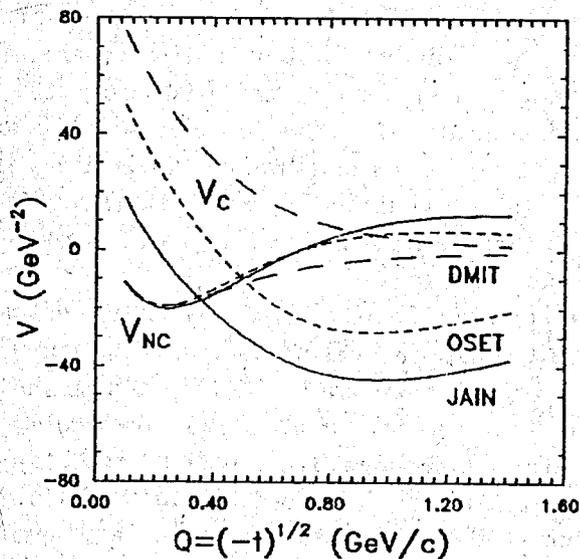


Fig.2 Effective NN-NA transition potential calculated with parameter sets from Refs./5,6,12/. Notice the similarity in shape and almost equidistant spectrum for the central part V_c of the potential and the weak sensitivity on the noncentral part V_{nc} to the chosen set. Lines: full - JAIN, short dash - OSET, long dash - DMIT.

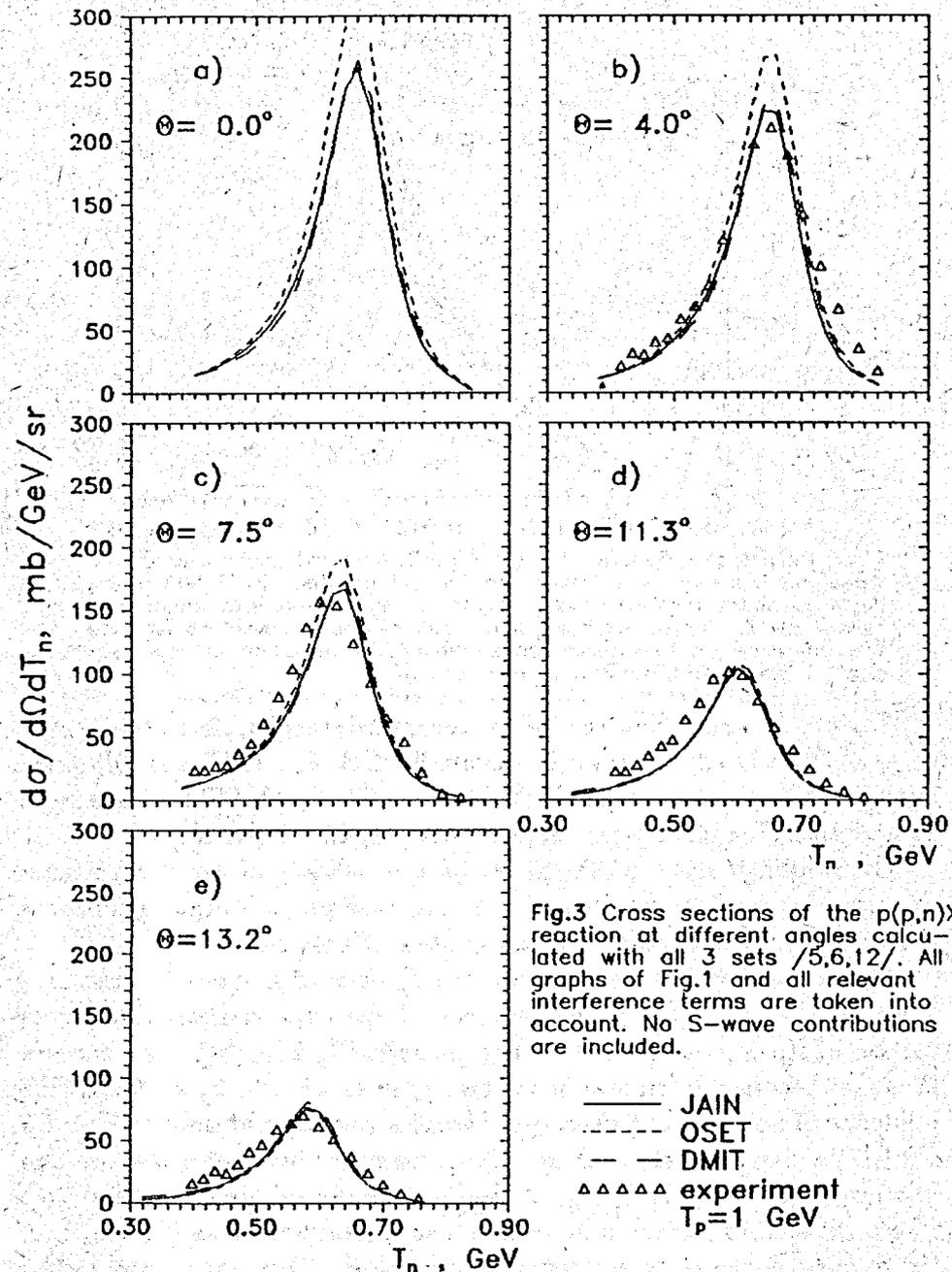


Fig.3 Cross sections of the $p(p,n)X$ reaction at different angles calculated with all 3 sets /5,6,12/. All graphs of Fig.1 and all relevant interference terms are taken into account. No S-wave contributions are included.

— JAIN
 - - - OSET
 - · - DMIT
 ΔΔΔΔΔ experiment
 $T_p = 1 \text{ GeV}$

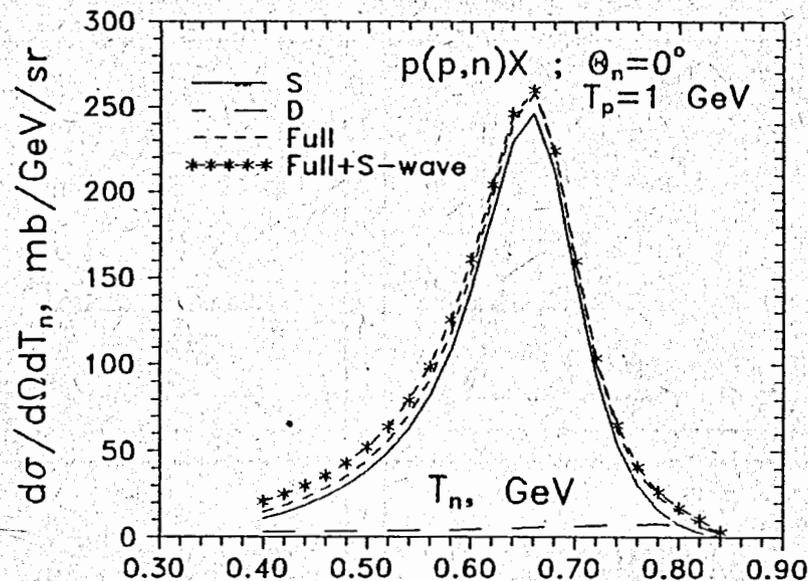


Fig. 4 Partial contributions to the $p(p,n)X$ forward cross section from the summed 'spectator' and 'decay' graphs. The interference between direct and exchange graphs is also taken into account giving the full cross section. Jain's set of the parameters is used. The influence on the full cross section from inclusion of nonresonant S-wave contributions is also shown.

For the D terms now the DD is completely negligible in the DMIT case, i.e. the D contribution is essentially DE. For JAIN the DD term again contributes more than the DE term, while for OSET they are more comparable. For both cases there is some angular dependence.

Our findings raise questions about the validity of specific pictures (inspired by diagrams) of what goes on, and provide arguments for a search for ways to dissolve the ambiguities, if possible.

One possibility is to investigate the T_p -dependence of the cross sections for the sets. The T_p -dependence of the experimental total cross section $\sigma_{tot}(p + p \rightarrow n + p + \pi^+)$ is given in Fig. 3 in [14]. One can see that the Δ -isobar dominates in the energy interval $0.8 < T_p < 3$ GeV. The influence of nonresonant processes becomes pronounced only at $T_p > 1.5$ GeV. The approximate scaling of the reduced invariant cross sections $\sigma_r = [\sigma_{tot}(T_{p0})]/[\sigma_{tot}(T_p)] \frac{d\sigma}{pdQd\Omega} \equiv B \frac{d\sigma}{pdQd\Omega}$ holds in this energy region [14]. These reduced experimental cross sections σ_r are shown in Figs. 8a, b, c, the calculational procedure is described in [14]. They are normalized to B containing a "background" from the s-wave πN interaction and other resonances, but the calculated cross sections contain only the resonance

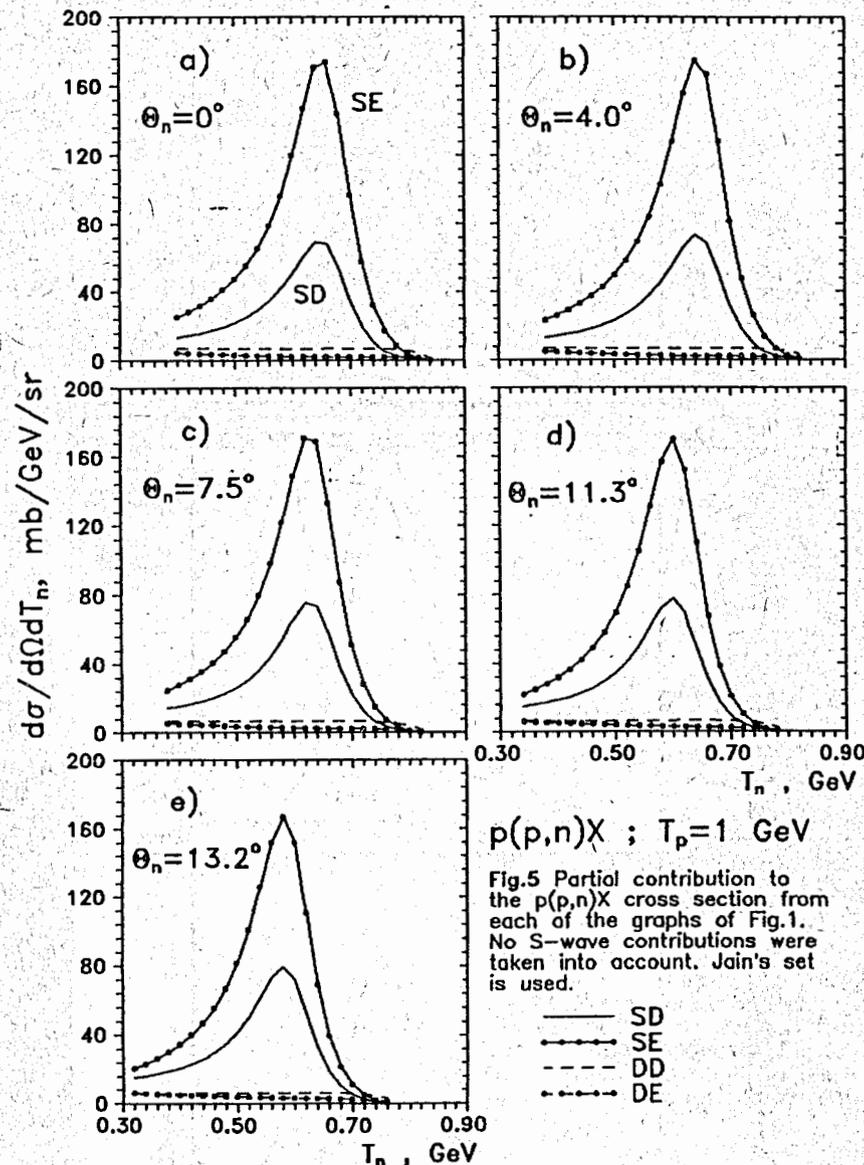


Fig. 5 Partial contribution to the $p(p,n)X$ cross section from each of the graphs of Fig. 1. No S-wave contributions were taken into account. Jain's set is used.

terms. It means that the calculated σ_r has to be not greater than the experimental inclusive one.

The results of the calculations are presented in Fig. 8. It is evident that the approximate scaling holds for the calculated reduced cross sections, when Jain's set is used. This scaling is not so good in the case of Oset's set and is strongly violated with the parameter set of Dmitriev. Thus

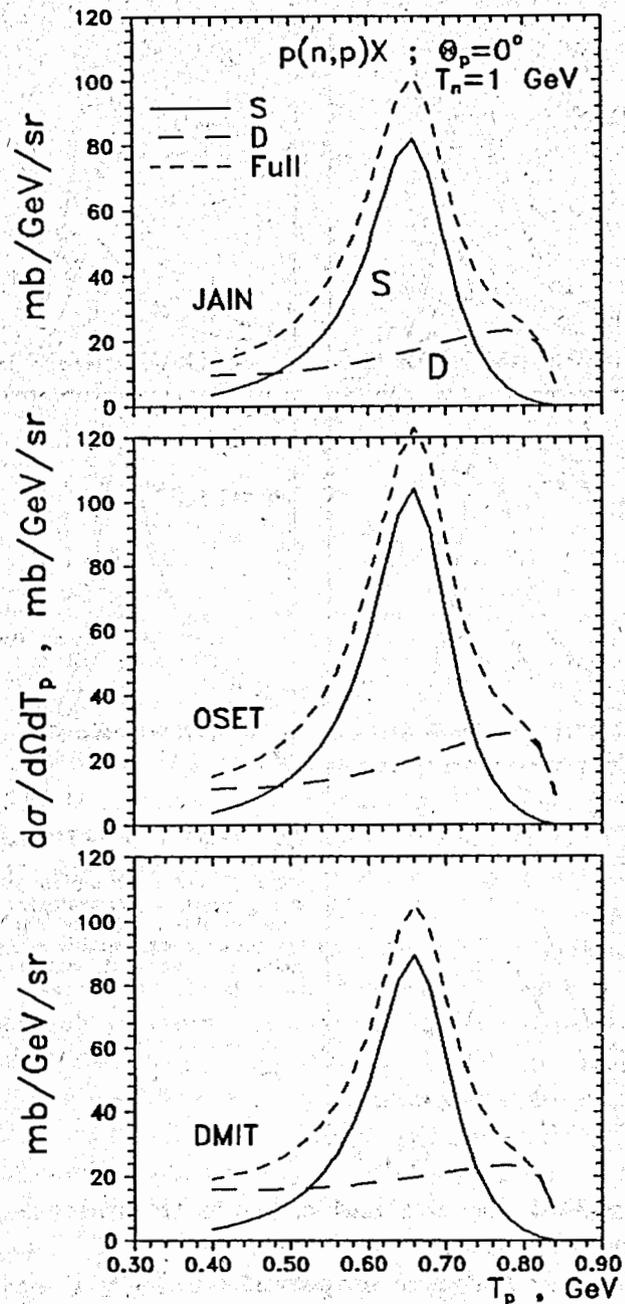


Fig.6 The same as for Fig.4 but for $p(n,p)X$ reaction. S-wave contributions are omitted.

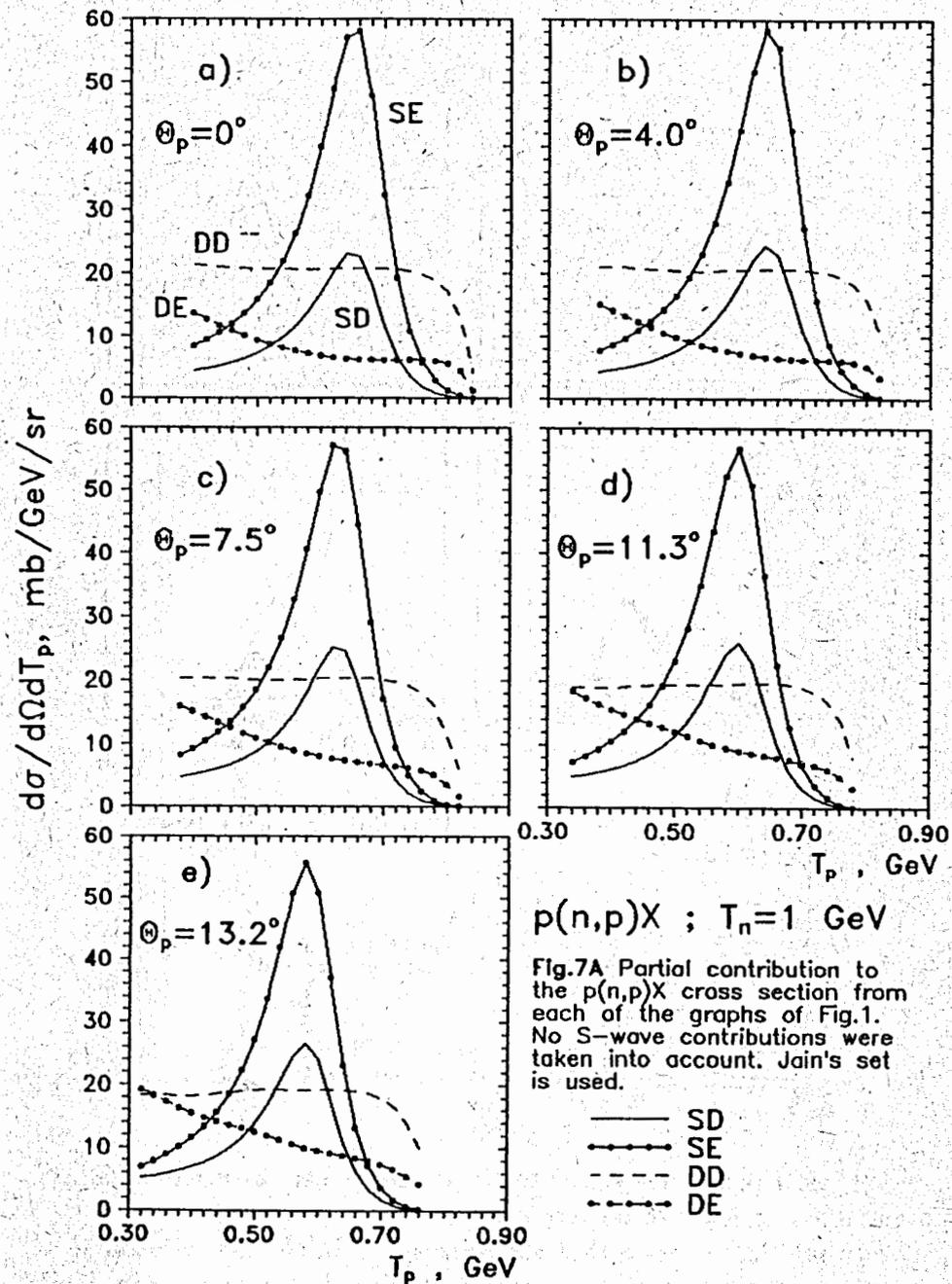
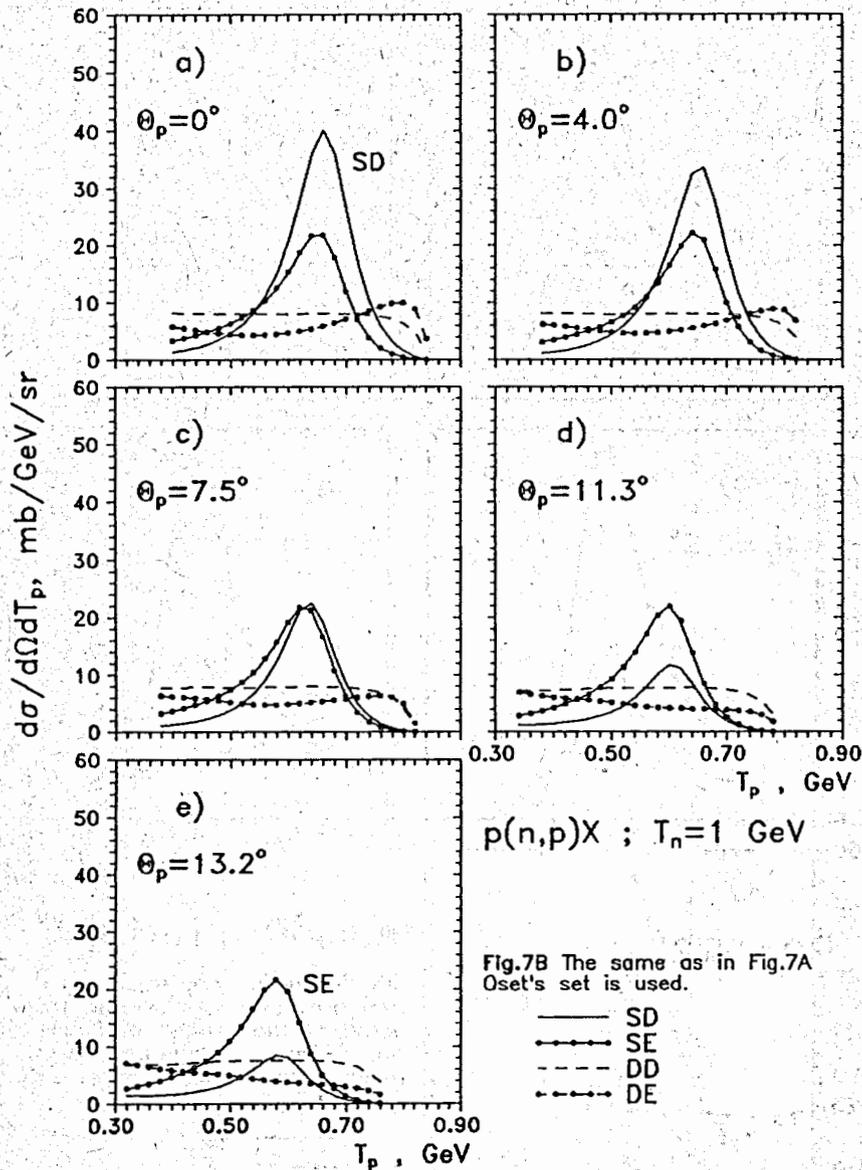
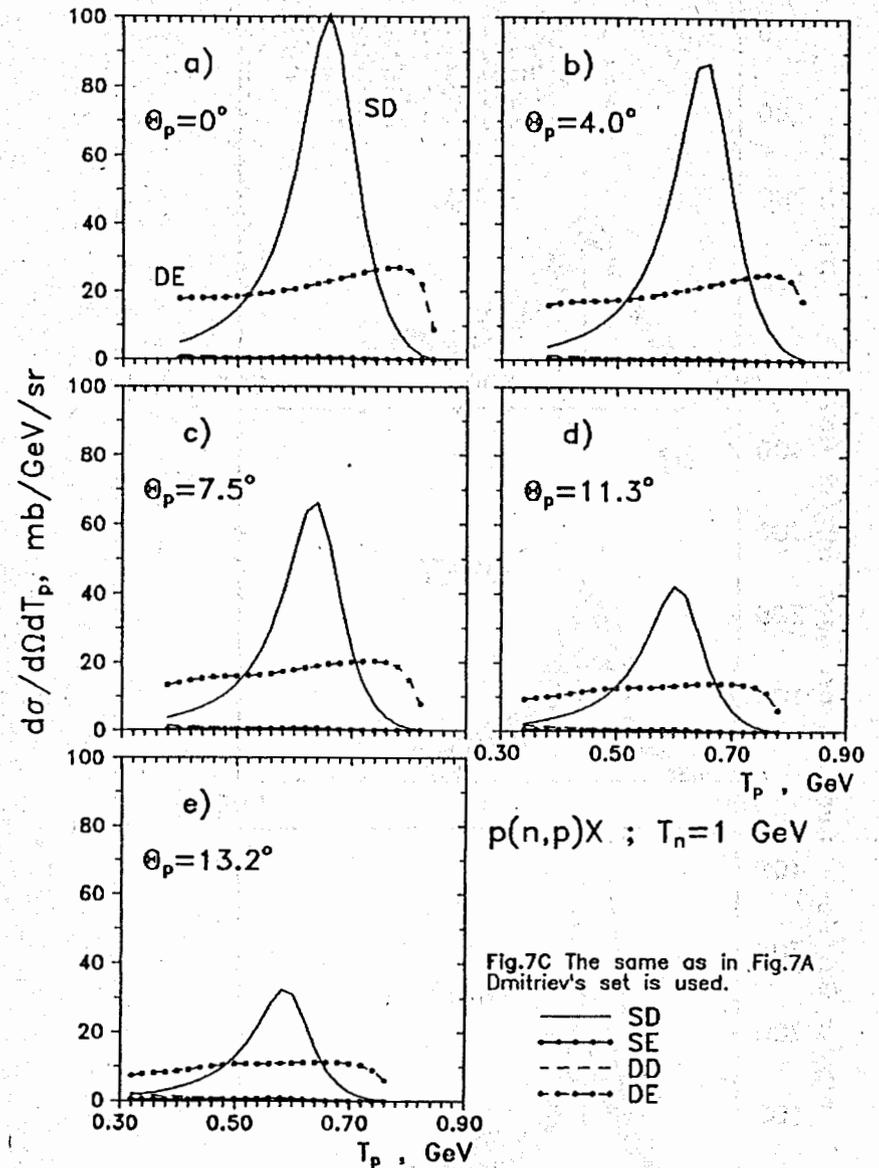


Fig.7A Partial contribution to the $p(n,p)X$ cross section from each of the graphs of Fig.1. No S-wave contributions were taken into account. Jain's set is used.



some physical preference seems to be present, where at least the DMIT potential is restricted to a specific energy.

We would like to note that the analysis [2] of the π -mesoatom data gives $g'_{N\Delta} = 0.4 \pm 0.2$. It is in agreement with the value given in Jain's and Oset's parameter sets. One might treat this fact as an indication of a possible weak dependence of the $g'_{N\Delta}$ on the excitation energy.



4 Conclusions

We have shown that momentum spectra and angular dependence of the charge-exchange cross sections in the projectile kinetic energy interval

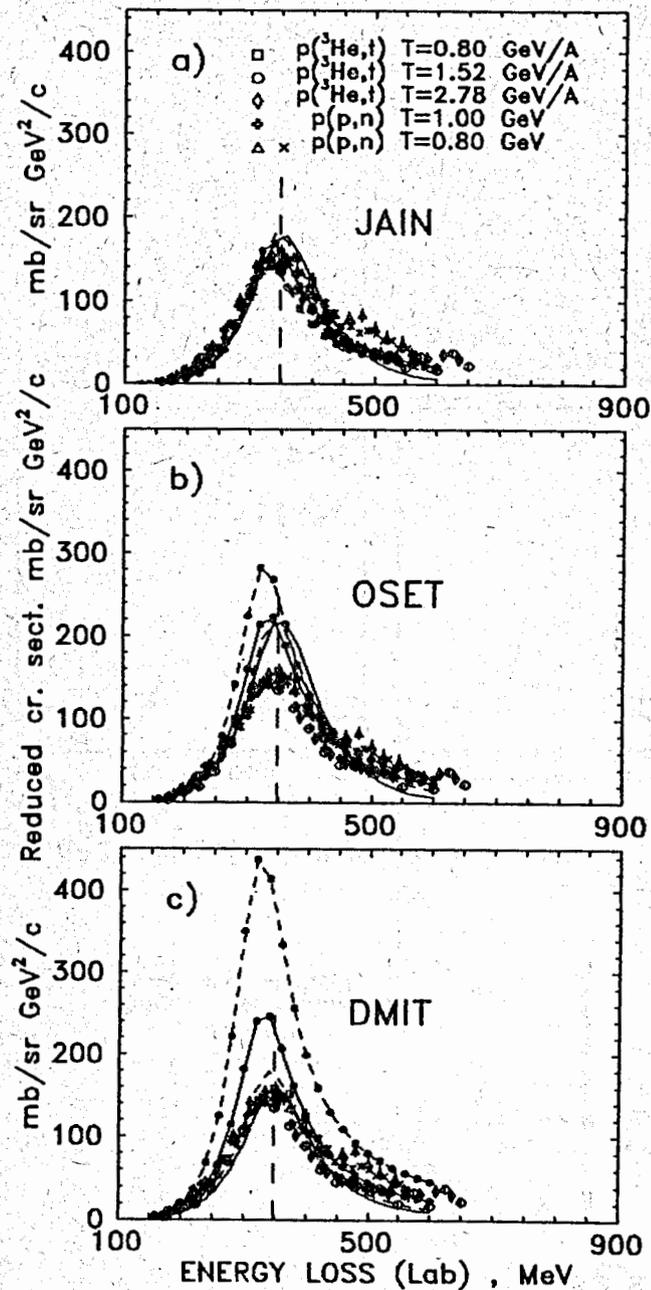


Fig.8 Reduced invariant cross sections for the $p(^3\text{He},t)$ and $p(p,n)$ reactions at 0° . Data are taken from compilation /14,17/. The lines represent our calculations with parameters from /5,6,12/. Full - $T=0.8$; dashed - $T=1.0$; full with dots - $T=1.52$ and dashed with dots - $T=2.78$ GeV/A.

$0.8 < T_p < 1.5$ GeV can be well described with three parameter sets Λ_π , Λ_ρ , C_ρ and $g'_{N\Delta}$ given in refs.[5, 6, 12].

These sets of parameters give nearly shape-equivalent central $NN \rightarrow N\Delta$ transition potentials, which however differ significantly in depth, with an almost equidistant depth spectrum. This gives a strong indication of a far-going analogy with the Sturm-Liouville problem and shows that a discrete ambiguity exists in the determination of the values of the $\pi N\Delta$ and $\rho N\Delta$ vertex formfactors. We have discussed various aspects of this ambiguity in detail.

Using the approximate scaling of the reduced inclusive invariant charge-exchange cross sections in the Δ -peak region for energies $0.8 < T_p < 3$ GeV, we are able to give preference to the parameter sets [6] and [5], thus partially solving the ambiguity.

Taking into account these results, it is interesting to look for discrete ambiguities in the case of $\pi N\Delta$ and $\rho N\Delta$ vertex formfactors and to confront parameters, characterizing these formfactors, obtained from charge-exchange data with other reactions such as (π^-, A) , (e, e') (π^+, π^0) etc. Such work is in progress.

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