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SMALL ANGLES BHABHA SCATTERING: TWO LOOP APPROXIMATION

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Процесс рассеяния Баба: двухпетлевое приближение

Рассмотрены сечения упругого и неупругого рассеяния электрона и позитрона на малые углы при высоких энергиях. Доказано, что диаграммы с двумя и более фотонными линиями в канале рассеяния могут быть опущены при вычислении радиационных поправок с точностью до 0,1%. Мы рассматриваем также процессы одно- и двухкратного тормозного излучения и процессы образования пар. Основываясь на этих расчетах, мы получаем комбинированную формулу для инклюзивной постановки эксперимента в терминах структурных функций.

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Small Angles Bhabha Scattering: Two Loop Approximation

The elastic and inelastic cross sections for small angles e^+e^- scattering at high energies are considered. We prove that all the diagrams with two or more virtual photons in scattering channel may be omitted when calculating the radiative corrections with accuracy of the order 0.1%. It is the consequence of the generalized eikonal representation for elastic and inelastic amplitudes. We take into account the processes of single and double bremsstrahlung in the same and opposite directions and the pair production processes. Basing on this calculations we construct the combined formula for the inclusive scattering electron and positron cross section in terms of the structure functions.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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The small angles Bhabha scattering, process (SBS), planned, is to use to measure the luminosity at LEP-11. The high accuracy of it's determination $\sim 0.1\%$ is necessary to extract the standard model (SM). A lot of attention was devoted to SBS (see [1] and the references therein). In this paper we suggested another variant of calculations, based on detailed analysis of elastic as well as inelastic $2 \to 3$, $2 \to 4$ small angles processes which was fulfilled in 70-80 years, and the ideas of renormalization group. The need of this calculation also followed from the discrepancy at $0.5 - 1\%$ level of results of several calculations of the last years.

Bhabha scattering process with possible inelastic channels

$$e^+(p_+)+e^-(p_-) \to e^+(p'_+)+e^-(p^{-'})+\gamma(k), \gamma(k_1)+\gamma(k_2)+\gamma(k_2), e_+(p''_+)+e_-(p''_-)+.. \quad (1)$$

is the statistically main one in the e^+e^- colliders.Mainly it caused by electromagnetic interactions. So the "pollution" caused by weak interactions for the scattering angles $3^0 < \theta < 6^0$ less 1% [1].We put the result of calculation in SM of elastic cross sections in Born approximation[2]

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{8s} 4B_1 + (1-c)^2 B_2 + (1+c)^2 B_3$$

$$B_1 = (\frac{s}{t})^2|1+(g_v^2-g_a^2)\xi|^2, \qquad B_2 = |1+(g_v^2-g_a^2)|^2$$

$$B_3 = \frac{1}{2}|1+\frac{s}{t}+(g_v+g_a)^2(\frac{s}{t}\xi+\chi)|^2,$$

$$\chi = \frac{\Lambda s}{s-m_z^2+im_z\Gamma_z}, \qquad \xi = \frac{\Lambda t}{t-m_z^2} \qquad \Lambda = \frac{G_F m_z^2}{2\sqrt{2}\pi\alpha}$$

$$s = (p_+ + p_-)^2 = 4\varepsilon^2, \qquad t = (p_- - p'_-)^2 = -\frac{1}{2}s(1-c),$$

$$u = (p_- - p'_+) = \frac{1}{2}s(1+c), \qquad c = cos\theta, \qquad \theta = \widehat{\vec{p}_-\vec{p}'_-}$$

Supposing the angles of the detected scattered e^+e^- are the range

$$2^0 < \theta < 5^0, 4Gev/c < \theta\varepsilon < 10Gev/c \tag{2}$$

(it corresponds to moment transferred of order $Q \approx \sqrt{-t} \sim \varepsilon\theta \sim 10Gev/c$) and the accuracy of measurement cross section of order

$$\frac{\delta\sigma}{\sigma} \sim 10^{-3} \tag{3}$$

We see that one may neglect the quantity of order 10^{-5}

$$\frac{\alpha}{\pi}|\frac{t}{s}|, \quad (\frac{\alpha}{\pi})^2, \quad \frac{m_e^2}{s}, \quad \frac{m_e^2}{|t|}, \quad (\frac{\alpha L}{\pi})^n, n > 3, \quad L = ln(\frac{Q^2}{m_e^2}) \tag{4}$$

and to take into account the terms of order 10^{-3} (see fig.1)

$$\gamma = \frac{\alpha}{\pi}, \qquad (\gamma L)^{1,2,3}, \qquad \gamma^2 L \tag{5}$$

The renormalization group's technique give a possibility to take into account all terms of kind $(\gamma L)^n$ (first column in fig.1). But the terms situated in diagonal are of the same order due to the fact $\gamma L^2 \sim 1$.Our efforts are devoted to take into account the terms (5).

Paper constructed in such a way.In the part I we consider the diagrams up to two level approximation describing the elastic e^+e^- scattering. Here we prove that in frames of accuracy (3,4) one may consider diagrams with only one photon state in t-channel.Our proof essentially coincide with the derivation of the generalized eikonal representation of the small-angle scattering amplitude [3].The result have a form

$$A(s,t)|_{s\gg|t|\gg m^2} = A_0(t)(\Gamma(t))^2(1-\Pi(t))^{-1}exp\{-i\alpha(ln(\frac{-t}{\lambda^2}+\frac{\alpha}{6\pi}ln^2(\frac{-t}{m^2}))\}, \tag{6}$$

where $A_0(t)$ is the amplitude in Born approximation, $\Gamma(t)$ is the Dirac's form factor of the electron, λ is the "mass" of the photon, $\Pi(t) = \frac{1}{3}\gamma ln\frac{-t}{m^2} + ...$ is the vacuum polarization operator. Representation (7) is violated in three-loop level [3a]. However we may consider (7) as exact one due to accuracy frames (3) adopted. From (7) one may deduce that the knowledge of electron formfactor and polarization operator up two loop level accuracy is necessary. The relevant expressions present in the literature. Contribution of Pauli's formfactor of electron is a quantity of order $\gamma|\frac{t}{s}|$ and may be omitted (4). Total expression of the elastic scattering cross section there is in the part 2. We present here also calculation of contributions from inelastic process of single bremsstrahlung, as well as soft photon's emission. In the same way as in part I we may prove that only diagrams with one virtual photon in y-chanel are relevant. Really the two-photons t-channel exchange diagrams contribution to the amplitude is pure imaginary compared with real ones from one-photon exchange diagrams. So the interference between them is absent whereas the taking into account of square of module of the to-photon exchange diagrams leads to exceeding of required accuracy (3). When calculating the interference of Born's amplitude of single bremsstrahlung with r.c. to it we use the known expressions for the vertex functions F and Π [4,5] and essentially by the results of our previous calculations of r.c to the single bremsstrahlung cross section [6].

In part 3 we present the differential cross section of processes $2 \to 4$ which contribute to the inclusive electron and positron cross sections. We use some modification of the infinite momentum frame method to obtain totally differential cross sections as a function transversal to the beam axes components of momenta of final particles and their energy fractions. Further integration of such kind differential cross sections is convenient because of the integrands are free from singularities.

In conclusion we construct the combined formulas based on structure function method and the Drell-Yan picture of the cross section which accumulate the results.

1. Consider first the set of 9 diagrams with one, two and three photons in t-channel. We parametrize their momenta k_1, $k_2 - k_1$, $q - k_2$ using the Sudakov representation

$$k_i = \alpha_i\tilde{p}_+ + \beta_i\tilde{p}_- + k_{\perp i}, \; q = p_- - p'_- = \alpha_q\tilde{p}_+ + \beta_q\tilde{p}_- + q_\perp,$$
$$\tilde{p}_\pm = p_\pm - \frac{m^2}{s}p_\mp, \; k_\perp p_\pm = q_\perp p_\pm = 0, \; d^4k = \frac{s}{2}d\alpha d\beta d^2k_\perp. \tag{7}$$

The four-vectors $\tilde{p}_{\pm}$ are almost light-like, $\tilde{p}_{\pm}^2 = O(\frac{m^6}{s^2})$. The parameters $\alpha_i, \beta_i, \alpha_q, \beta_q$ are small in the region of integration, which give the main contribution to the amplitude:

$$|\alpha_q| \sim |\beta_q| \sim |\alpha| \sim |\beta| \sim \frac{-q_{\perp}^2}{s} = \frac{-t}{s} << 1. \qquad (8)$$

The characteristic transversal momenta of photons are of order transverse momentum $|k_{i\perp}| \sim \sqrt{-t}$. This fact follows from the ultraviolet convergence of box-diagrams amplitudes. The smallness of $|\alpha_q|, |\beta_q|$ follows from the mass-shell conditions of the scattered leptons: $p_-'^2 - m^2 = (p_- - q)^2 - m^2 = s\alpha_q\beta_q - (m^2+s)\alpha_q + t = 0, |\alpha_q| \sim \frac{-t}{s}$. We will see later that the four vectors of the virtual fermions are near mass shell:

$$(p_- - k)^2 - m^2 = -s\alpha - \vec{k}^2 + i0, \; (p_+ + k)^2 - m^2 = s\beta - \vec{k}^2 + i0,$$
$$(p_-' - k)^2 - m^2 = -s\alpha - (\vec{q} - \vec{k})^2 + i0. \qquad (9)$$

The relations (9) follow from this fact. The same conclusion ensue from The analysis of the situation of the poles in α, β planes. The nonzero contribution, which corresponds to the case when the poles in α, β planes are situated on the different sides of the real axes, corresponds $|\alpha, \beta| < |\alpha_q, \beta_q|$.

It is conveniet to present the relevant diagrams in the symmetrized form:

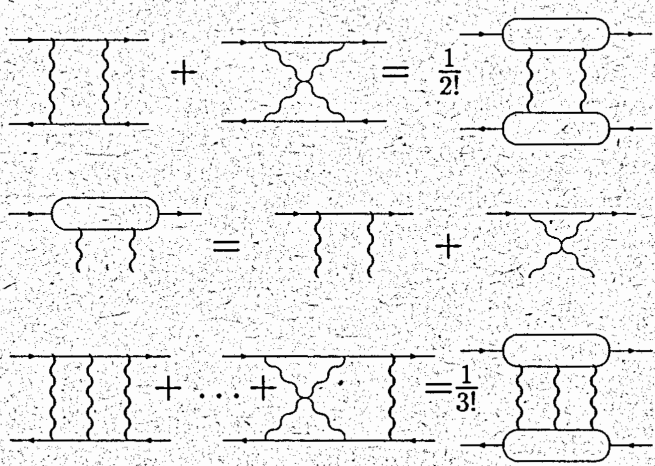


Fig.1

The next step is the simplification of the nominators of the integrands. Using the smallnes of α, β parameters and the Dirac equation for the spinors of the external particles $(\hat{p}_+ + m)v(p_+) = (\hat{p}_- - m)u(p_-) = 0$ one obtains:

$$\bar{u}(p_-')\gamma^{\mu}(p_- - k + m)\gamma^{\nu}u(p_-)\bar{v}(p_+)\gamma^{\nu}(-p_+ - k + m)\gamma^{\mu}v(p_+') = -2sB,$$
$$\bar{u}(p_-')\gamma^{\mu_1}(p_- - k_1 + m)\gamma^{\mu_2}(p_- - k_2 + m)\gamma^{\mu_3}u(p_-)\bar{v}(p_+)\gamma^{\mu_3}(p_+ - k_1 + m)\gamma^{\mu_2}$$
$$(-p_+ - k_2 + m)\gamma^{\mu_1}v(p_+') = (-2s)^2B,$$
$$B = \bar{u}(p_-')\gamma\mu u(p_-)\bar{v}(p_+)\gamma^{\mu}v(p_+') \approx \frac{2}{s}\bar{u}(p_-')\hat{p}_+u(p_-)\bar{v}(p_+)\hat{p}_-v(p_+') \qquad (10)$$

Taking into account (9) and (10), we may rewritten the contribution of all the 9 diagrams as

$$M = A_0(t)\{1 + i\frac{\alpha}{4\pi^2}\frac{2s}{2!}\int \frac{s}{2}d\alpha d\beta \phi^{(1)}(\alpha)\phi^{(1)}(\beta)I_2((\vec{q})^2)$$
$$+(-2s)^2(\frac{\alpha}{4\pi^2})^2\frac{1}{3!}\int (\frac{s}{2})^2 d^2\alpha d^2\beta \phi^{(2)}(\alpha_1,\alpha_2)\phi^{(2)}(\beta_1,\beta_2)I_3((\vec{q})^2)\}. \qquad (11)$$

where $A_0(t) = \frac{4\pi\alpha i}{t}B$ is the Born amplitude and

$$\phi^{(1)}(x) = \frac{1}{-sx + i0} + \frac{1}{sx + i0} = -2\pi i\delta(sx),$$
$$\phi^{(2)}(x_1, x_2) = \frac{1}{(sx_1 + i0)(sx_2 + i0)} + \frac{1}{(s(x_2 - x_1) + i0)(sx_2 + i0)} +$$
$$\frac{1}{(s(x_2 - x_1) + i0)(-sx_2 + i0)} + \frac{1}{(sx_1 + i0)(s(x_2 - x_1) + i0)} +$$
$$\frac{1}{(-sx_1 + i0)(-sx_2 + i0)} + \frac{1}{(-sx_2 + i0)(-s(x_2 - x_1) + i0)}$$
$$= (-2\pi i)^2\delta(sx_1)\delta(sx_2) \qquad (12)$$

The quantities $I_{2,3}$ entering (12) are defined as follows:

$$I_2((\vec{q})^2) = \frac{-t}{\pi}\int \frac{d^2k}{(\vec{k}^2 + \lambda^2)((\vec{k} - \vec{q})^2 + \lambda^2)};$$
$$I_3((\vec{q})^2) = \frac{-t}{\pi^2}\int \frac{d^2k_1 d^2k_2}{(\vec{k}_1^2 + \lambda^2)((\vec{k}_1 - \vec{k}_2)^2 + \lambda^2)((\vec{q} - \vec{k}_2)^2 + \lambda^2)}$$

Carring the α, β integration by means of δ-functions and using

$$I_2 = 2ln^2(\frac{-t}{\lambda^2}), \qquad I_3 = 3ln^2(\frac{-t}{\lambda^2}),$$

we obtain:

$$M = A_0(t)\{1 - i\alpha ln\frac{-t}{\lambda^2} - \frac{1}{2}(\alpha ln\frac{-t}{\lambda^2})^2\} = A_0(t)exp\{-i\alpha ln(\frac{-t}{\lambda^2})\}. \qquad (13)$$

The expression (14) is known one of eikonal amplitude of the small angles scattering of charged particles [3]. To generalize (14) consider first the diagrams with the one-loop corrections to the electron lines and with two photons exchange in t-channel (there are 8 two loop diagrams of such a kind, another 8 described the corrections to the positron lines; remaining diagrams with additional photon line connecting the electron and positron ones will describe the pure eikonal in the next order of perturbative theory). We will prove now that in the frames of the using accurasy it take place the generalized eikonal representation [3]

$$M = A_0(t)(\Gamma_1(t))^2 exp\{-i\alpha ln(\frac{-t}{\lambda^2})\}, \qquad (14)$$

where $\Gamma_1(t)$ is the Dirac form factor part of the electron vertex function

$$\Gamma^\mu(t) = \Gamma_1(t)\gamma^\mu + \frac{\sigma^{\mu\nu}q^\nu}{2m}\Gamma_2(t), \qquad \Gamma_1(t) = 1 + \gamma\Gamma_1^{(2)}(t) + \ldots .$$

In order to obtain (15) express first the phase volume of the exchanged photons in terms of invariant masses of electron and positron blocks of the diagram

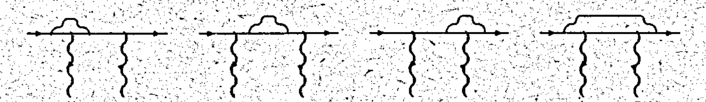
$$d^4k = \frac{s}{2}d\alpha d\beta d^2k_\perp = \frac{1}{2s}ds_1 ds_2 d^2\vec{k}, \qquad s_1 = (k-p_-)^2 = -s\alpha - \vec{k}^2,$$

$$s_2 = (k+p_+)^2 = s\beta - \vec{k}^2.$$

Consider for definitness the diagram with the corrections to the electron line. Performing the integration over s_2 by means of $\phi^{(1)}(\beta)$ we obtain instead of the second term in (12) the expression

$$\frac{8i\alpha^2}{s^2(\vec{q})^2}\int\frac{d^2k(\vec{q})^2}{((\vec{k}-\vec{q})^2+\lambda^2)(\vec{k}^2+\lambda^2)}\bar{v}(p_+)\hat{p}_- v(p'_+)\int_C ds_1 p_+^\mu p_-^\nu \bar{u}(p'_-)A^{\mu\nu}u(p_-), \qquad (15)$$

where $\bar{u}(p'_-)A^{\mu\nu}u(p_-)$ is the part of the Compton amplitude with the virtual photons described by the set of the diagrams



We note that it is not complete set of Feynman diagrams' so the quantity $\bar{u}(p'_-)A^{\mu\nu}u(p_-)$ is not gauge invariant. (But $\int_C p_+^\mu p_+^\nu \bar{u}(p'_-)A^{\mu\nu}u(p_-)ds_1$ is sertainly gauge invariant).

The contour C of the integration in the s_1 plane is the causal Feynman's one. It drawn in the figure 2:

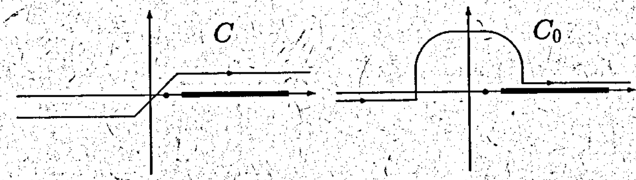


Fig. 2

The singularities of Compton amplitude are the pole at $s_1 = m^2$ which corresponds to the one electron intermediate state and the right cut starting from $s_1 = (m+\lambda)^2$ which corresponds to the one electron one photon intermediate state. Using the Sudakov's representation for the photon momentum k we may represent the p_+^μ in (16) in the form:

$$p_+^\mu = \frac{1}{\alpha}(k - k_\perp - \beta p_-)^\mu \approx \frac{-s}{s_1+\vec{k}^2}(k-k_\perp)^\mu. \qquad (16)$$

Let us cosider the action of two terms in (16) on the Compton amplitude. The contribution of the second term $\sim k_\perp^\mu$ in (17) to the (16) is zero:

$$|\vec{k}_\perp| s p_+^\nu \int_C \frac{ds_1 k_\perp^\mu}{(s_1+\vec{k}^2)|k_\perp|}\bar{u}(p'_-)A^{\mu\nu}(k, k-q, s_1)u(p_-) = 0. \qquad (17)$$

The statement (18) follows from the two reasons: the convergence of the integral on "large circle" in the plane s_1 plane and the absence of the left cut. The absence of the left cut is the known property of the planar diagrams. The convergence follows from the fact that only the physical polarizations for the photon with momentum k take place so the quantity $e^\mu p_+^\nu A^{\mu\nu}$ behave at large s_1 as $\frac{m^2}{s_1}$, $e^\mu = k_\perp^\mu \frac{1}{|k_\perp|}$.

Applying the Ward identity to the first term in (17) we immediately obtain:

$$p_+^\mu p_+^\nu \bar{u}(p'_-)A^{\mu\nu}(s_1)u(p_-) = -\frac{se^2}{s_1}p_+^\mu \bar{u}(p'_-)\Gamma^{\mu(2)}(q)u(p_-), \qquad s_1 >> m^2. \quad (18)$$

Both singularities in s_1 plane are situated under the integration path. Therefore we can calculate the integral over s_1 by deforming the contour into semicircle of large radius (see fig.1). The result differs from the second term in r.h.s. (14) by a factor $\Gamma(t)$. The same arguments may be used to the diagrams with three photons in the t-channel and the one loop corrections to the electron line, as well as with the corrections to the positron line. So appear the result (15).

It may include in this scheme also the diagrams with the self-energy insertions to the photon lines. This results according to J.Schwinger [4] in the replacement:

$$\frac{1}{\vec{k}^2+\lambda^2} \to D(\vec{k}^2) = \frac{1}{\vec{k}^2+\lambda^2}(1+\gamma\int_0^1 \frac{dv\phi(v)\vec{k}^2}{4m^2+(1-v^2)\vec{k}^2}),$$
$$\frac{1}{(\vec{q})^2} \to \frac{1}{(\vec{q})^2}(1+\frac{1}{3}\gamma ln(\frac{(\vec{q})^2}{m^2})),$$
$$\phi(v) = \frac{2}{3}-\frac{1}{3}(1-v^2)(2-v^2). \qquad (19)$$

Calculations with the logarithmical accuracy give:

$$\int\frac{d^2k}{\pi}D((\vec{q})^2)D((\vec{k}-\vec{q})^2) = 2L_1+\frac{\gamma}{3}L_2(L_2+2L_1),$$
$$\int\frac{d^2k_1}{\pi}\int\frac{d^2k_2}{\pi}D((\vec{k}_1)^2)D((\vec{k}_1-\vec{k}_2)^2)D((\vec{k}_2-\vec{q})^2) =$$
$$3(L_1)^2+\gamma L_1L_2(L_1+L_2),$$
$$L_1 = ln\frac{\vec{q}^2}{\lambda^2}, \qquad L_2 = ln\frac{\vec{q}^2}{m^2}. \qquad (20)$$

Using (21),(15) one obtains (7).

It is necessary to note that in the two-loop corrections to electron line the left cut in s_1 -plane appears and the simple relation as (19) will not have a place. Fortunately the accounting this fact leads to exceeding the taken accuracy.

In the same way we may prove that only diagrams with one virtual photon in the t-channel are relevant in the case when SBS is acompanied by photons emission.Really the two-photons t-channel exchange diagrams amplitudes are pure imaginary as compared with ones from one-photon exchange diagrams.So the interference between them is absent,whereas the accounting of the square of modul of the amplitude of two-photon exchange diagrams lead to the exceeding accuracy.

2. Consider now the Feynman diagrams which describe the one and two loop corrections to the elastic e^+e^- scattering with one photon in t-channel.They are drawn in fig.3.

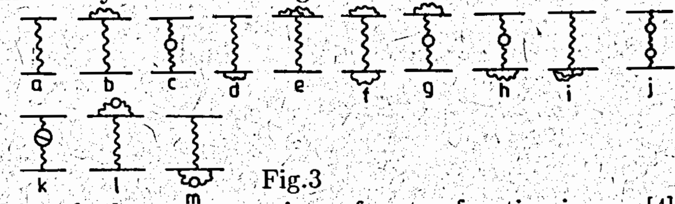


Fig.3

Using the known expressions of vertex function in one [4] and two loops [5] as well as the polarization operator [7] one obtain their contribution to the cross section:

$$d\sigma = d\sigma_0\{1+2m_b+m_c+2m_e+(m_b)^2+2m_cm_b+(m_c)^2+m_k+2m_l\}^2,$$
$$m_b = \gamma[(L_2-1)(L_1+1)-\frac{1}{4}L_2-\frac{1}{4}(L_2)^2+\frac{\pi^2}{12}],$$
$$m_c = \gamma[\frac{1}{3}L_2-\frac{5}{9}+\Pi_\tau+\Pi_\mu+\Pi_h],$$
$$m_k = \frac{1}{4}(\gamma)^2L_2,$$
$$m_e+m_l = (\gamma)^2[\frac{1}{32}(L_2)^4-\frac{31}{144}(L_2)^3+(\frac{229}{288}$$
$$-\frac{\pi^2}{48}(L_2)^2+(-\frac{1627}{864}+\frac{3}{4}\zeta(3)-\frac{13}{144}\pi^2)L_2$$
$$+\frac{1}{2}(L_2-1)^2(L_3)^2+(L_2-1)(-\frac{1}{4}(L_2)^2+\frac{3}{4}-1+\frac{\pi^2}{12})L_3], \qquad (21)$$

where $\gamma=\frac{\alpha}{\pi}$, $L_3 = ln(\frac{\lambda}{m})$, $L_2 = ln(\frac{-t}{m^2}$, λ is the "photons mass", $\Pi_\mu, \Pi_{tau}, \Pi_h$ the contributions to the polarization operator from muons,τ-leptons and hadrons:

$$\Pi_\mu = \frac{1}{3}ln(\frac{-t}{m_\mu^2})-\frac{5}{9},$$
$$\Pi_\tau = -\frac{1}{3y}[-\frac{22}{3}+\frac{5}{3}(x+\frac{1}{x})+(x+\frac{1}{x}-4)\frac{1+x}{1-x}lnx],$$
$$\Pi_h = \frac{-t}{4\pi(\alpha)^2}\int_{4m_\pi^2}^{\infty}dx\frac{1}{x-t}\sigma^{e^+e^-\to h}(x),$$
$$x = 1+\frac{y}{2}+\frac{1}{2}\sqrt{y(y+4)}, \qquad y = \frac{-t}{m_\tau^2}$$

where $\sigma^{e^+e^-\to h}(s)$ is the cross section of e^+e^- annihilation to the hadrons.

Note that due to the fact that the momentum transferred is large compared to the mass of the light leptons we may use asymptotic expressions for the functions Γ, Π.

consider now the single bremsstrahlung process (fig.4)

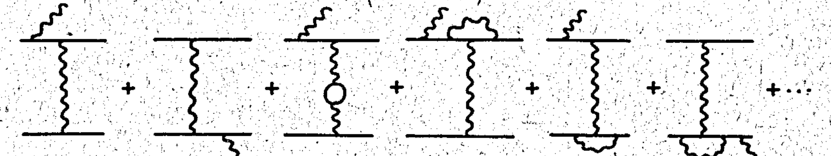


Fig.4

In the case of the soft photon one may use the result of the work[8] of P.I.Fomin: neglecting the terms of order $\frac{-t}{s}\gamma$ one obtains:

$$d\sigma^{\gamma}_{SV} = d\sigma_0 2\gamma[(L_2-1)ln(\frac{m\Delta\epsilon}{\lambda\epsilon}) + \frac{1}{4}(L_2)^2 - \frac{\pi^2}{12}]\{1+\gamma[(L_2-1)(L_3+1) -\frac{1}{4}L_2 - \frac{1}{4}(L_2)^2 + \frac{\pi^2}{12}] + \gamma(\frac{1}{3}L_2 - \frac{5}{9})\}^2 \quad (22)$$

In (23) the $\Delta\epsilon$ is the auxiliary parameter. We will suppose for it $m << \Delta\epsilon << \epsilon$. Final expressions will not depend on it when one consider the emission of hard photons with energy $\omega > \Delta\epsilon$. We present here also the cross section of process of emission of two soft photons with energies $\omega_1, \omega_2 < \Delta\epsilon$ (see fig.5)

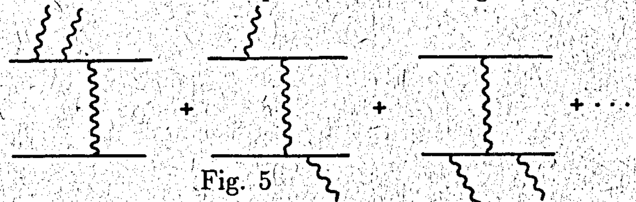


Fig. 5

$$d\sigma^{\gamma\gamma}_{SS} = 2d\sigma_0\{\frac{1}{2}(2\gamma)^2\left[(L_2-1)L_4 + \frac{1}{4}(L_2)^2 - \frac{\pi^2}{12}\right]^2 - (\gamma)^2(L_2-1)^2\frac{\pi^2}{3}\} +d\sigma_0(2\gamma)^2[(L_2-1)L_4 + \frac{1}{4}(L_2)^2 - \frac{\pi^2}{12}]^2, \qquad L_4 = ln(\frac{m\Delta\epsilon}{\lambda\epsilon}). \quad (23)$$

The first term in r.h.s.(24) describes the double bremsstrahlung by electrons only and by positrons only (initial and final). The second one corresponds to the simultaneous emission by electron and positron. Energy of each photon's in the c.m.s. do not exceed $\Delta\epsilon$.

The sum of the cross sections (22,23,24) do not depend on parameter λ and have the form:

$$d\sigma_V + d\sigma_{VS} + d\sigma_{SS} = d\sigma_0\{1+\gamma[4(L_2-1)L_k + \frac{11}{3}L_2 - \frac{46}{9} + 2(\Pi_\mu + \Pi_\tau + \Pi_h)] +(\gamma)^2[-\frac{1}{9}(L_2)^3 + (8(L_k)^2 + \frac{40}{3}L_k + \frac{62}{9} - \frac{2}{3}\pi^2)(L_2)^2 + (-16(L_k)^2 - \frac{284}{9}L_k + 6\zeta(3) + \frac{13}{18}\pi^2 - \frac{4351}{216})L_2] + \gamma^2(\Pi_\mu + \Pi_\tau + \Pi_h)[3(\Pi_\mu + \Pi_\tau + \Pi_h) +4(L_2-1)L_k + \frac{11}{3}L_2 - \frac{46}{9}]\}, \qquad L_k = ln\frac{\Delta\epsilon}{\epsilon} \quad (24)$$

The term in (25) of order $(\gamma)^2(L_2)^3$ arise from the diagrams l,m fig.3.It disappears if one takes into account the emission of the soft e^+e_- pairs (see fig.6). the total energy of the pairs component don't exceed $\Delta\epsilon >> m$ in the c.m.s.

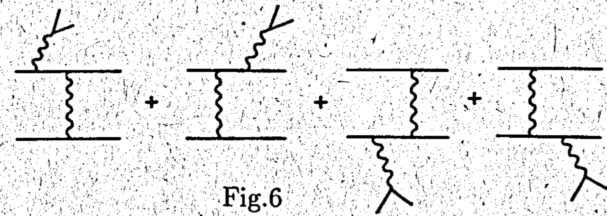


Fig.6

Taking into account both kinematic configurations when the soft pairs are emitted by electrons and positrons we obtain [15]

$$d\sigma^{softpairs} = d\sigma_0\gamma^2\{\frac{8}{9}L_m^3 + \frac{8}{3}(ln\theta - \frac{5}{6})L_m^2 + \frac{8}{3}(ln^2 2 - \frac{5}{3}ln2 + \frac{14}{9} - \frac{\pi^2}{6})L_m\},$$
$$L_m = ln\frac{\Delta\epsilon}{m} \quad (25)$$

where θ is the scattering angle $\theta = \widehat{\vec{p}_-, \vec{p}'_-} = \widehat{\vec{p}_+, \vec{P}'_=}$.

The corrections (25), (26) regard the elastic kinematic of the scattered e'^+, e'_-. Now let us consider the hard photon emission process and the corrections to it's cross section due to virtual and soft photon emission [9b].

Cross section of the process hard bremsstrahlung accompanied by soft photon emission described by diagrams figure 7. is given by the expression:

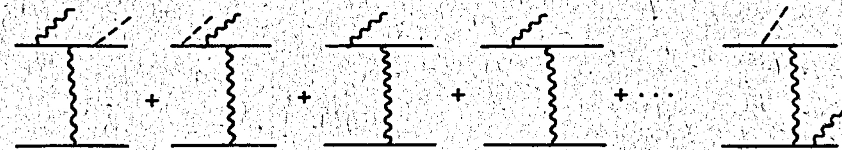


Fig.7

$$d\sigma^{Hs} = (d\sigma^{e^+e^-\to(e^+\gamma)e^-} + d\sigma^{e^+e^-\to e^+(e^-\gamma)})2\gamma\{(\rho_+ - 1)^2(L_k - L_3) + \frac{1}{4}\rho_+^2 - \frac{\pi^2}{12} + (\rho_- - 1)^2(L_k - L_3) + \frac{1}{4}\rho_-^2 - \frac{\pi^2}{12}\}, \quad (26)$$

where $\rho_\pm = ln\frac{q_\pm^2}{m^2}, q_\pm^2 = 2\epsilon_\pm\epsilon'_\pm(1 - cos(\theta_\pm)) \approx \epsilon\epsilon'_\pm\theta_\pm^2$, and $\Delta\epsilon$ is the maximal energy of the soft photon.

Basing on the results of the papers [6] for the case of small scattering angles we obtain for the cross section of emission of the hard photon by electron with r.c. in the one loop level:

$$d\sigma_{Hv}^{e^+e^-\to e^+(e^-\gamma)} = d\sigma_0^{e^+e^-\to e^+(e^-\gamma)}\{1 + \frac{\gamma}{2}[4L_3(\rho_- - 1) - \rho_-^2 + 3\rho_+ + \frac{\pi^2}{3} - \frac{9}{2}]\} (27)$$

The sum of (27) and (28) gives for the bremsstrahlung of the e^- with the virtual and real soft photon corrections

$$d\sigma_{Hs+Hv}^{e^+e^-\to e^+(e^-\gamma)} = d\sigma_0^{e^+e^-\to e^+(e^-\gamma)}\{1 + 2\gamma[(\rho_- 1)ln\frac{\Delta\epsilon_-}{\epsilon} + \frac{3}{4}\rho_- + (\rho_+ - 1)ln\frac{\Delta\epsilon_+}{\epsilon} + \frac{3}{4}\rho_+ O(1)]\}(1 - \Pi(q_+^2))^{-1},$$

$$\Pi(q^2) = \frac{1}{3}\gamma(\rho - \frac{5}{3}) + \Pi_\mu(q^2) + \Pi_\tau(q^2) + \Pi_h(q^2). \quad (28)$$

The cross section of this process in Born approximation have a form:

$$d\sigma_0^{e^+e^-\to e^+(e^-\gamma)} = 2\frac{(\alpha)^3}{(\pi)^2}\frac{1-x}{((\vec{p}_+)^2)^2}d^2p'_- d^2p'_+\{\frac{1+x^2}{dd_1} - 2x\frac{m^2}{(\vec{p}_+)^2}(\frac{1}{d^2} + \frac{1}{d_1^2})\},$$

$$d = m^2(1-x)^2 + (\vec{p}'_- - x\vec{p}'_+)^2, \qquad d_1 = m^2(1-x)^2 + (\vec{p}'_- - \vec{p}'_+)^2, \quad (29)$$

where $x, \vec{p}'_-$ is the energy fraction of the scattered electron and transversal to the beam axes component of it's momentum; $1, \vec{p}'_+$ -the same quantities for the scattered positron.

The bremsstrahlung of the positron may be taken into account by means of replacements from the (29,30).

3.We denote as $d\sigma_{\gamma\gamma}$, $d\sigma^{\gamma\gamma}$, $d\sigma_\gamma^\gamma$ the cross sections of double bremsstrahlung of hard photons by positron,electron and both of them (see fig.$8_{a,b,c}$).

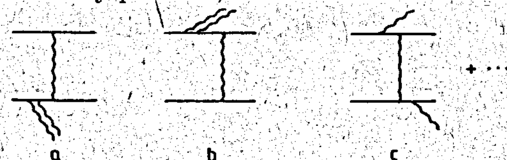


Fig.8

There are two different mechanisms of hard pair ($\epsilon"_+ + \epsilon"_- > \Delta\epsilon >> m$) of the processes $e^+e^- \to 2e^+2e^-$ creation: bremsstrahlung's and two - photon's ones.The amplitudes is described by the Feynman diagrams in the fig.9, where the signs also depicted: the Fermi statistics of leptons is to be taken into account.

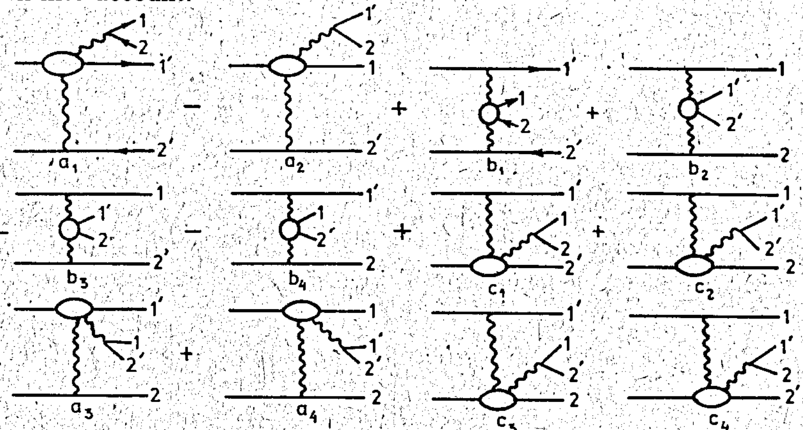


Fig.9

We are to consider the processes $e^+e^- \to e^+e^-\mu^+\mu^-$ which is described by diagrams a_1, b_1, c_1 of the fig.9 and, in principle the processes $e^+e^- \to e^+e^-\pi^+\pi^-, e^+e^-\tau^+\tau^-$.But the first have too small cross section and it's contribution don't exceed 0.1%.The second's one have the same order of magnitude in the scattering angles range (3).

It is convenient to draw the diagrams for cross sections for the processes of fig.8,9:

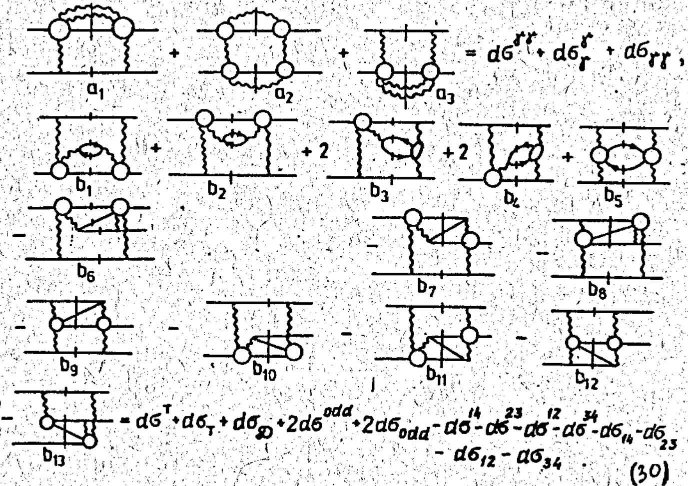


Fig.10

For the process $e^+e^- \to e^+e^-\mu^+\mu^-$ the contributions $b_1 - b_5$ only remained.

$$d\sigma^{e^+e^-\to e^+e^-\mu^+\mu^-} = d\sigma^T + d\sigma_T + d\sigma_D + 2d\sigma_{odd} + 2d\sigma^{odd}. \quad (31)$$

All the contribution which are not mentioned here including the diagrams of the annihilation type give the contribution of order $|\frac{t}{s}|$ compared with the ones listed above.

All the processes of fig.8,9 was considered in [9] where the totally differential cross sections where calculated in terms of energy fractions and the transversal momenta of the final particles. In [10] some analytical results for the case ep - scattering are given.

The Sudakov's parameter modification of the infinite momentum frame method [9] is very convenient in this case.

The most simplest expression may be obtained for the double bremsstrahlung in the opposite sides cross section (see fig.10 a_3):

$$\frac{d\sigma_\gamma^\gamma}{d^2q_1d^2q_2dxdy} = \frac{\alpha^4(1-x)(1-y)}{\pi^3(\vec{k}^2)^2}\int\frac{d^2k}{\pi}\{(\vec{k})^2d_1d_2(1+x^2) - 2xm^2(d_1-d_2)^2\}$$
$$\{d_1, d_2 \to \tilde{d}_1, \tilde{d}_2, \ x \to y\}[d_1^2d_2^2\tilde{d}_1^{\,2}\tilde{d}_2^{\,2}]^{-1} \quad (32)$$

where

$$d_1 = m^2(1-x)^2 + (\vec{q}_1 - x\vec{k})^2, \qquad d_2 = m^2(1-x)^2 + (\vec{q}_1 - \vec{k})^2,$$
$$\tilde{d}_1 = m^2(1-y)^2 + (\vec{q}_2 - y\vec{k})^2, \qquad \tilde{d}_2 = m^2(1-y)^2 + (\vec{q}_2 - \vec{k})^2, \quad (33)$$

and $x, -\vec{q}_1(y, -\vec{q}_2)$ are the energy fraction and the transversal momentum of the scattered electron (positron); $\theta_1 = \frac{|\vec{q}_1|}{x\epsilon}, \theta_2 = \frac{|\vec{q}_2|}{y\epsilon}$ are the corresponding scattered angles.

The expression (33) is convenient for analytical as well as for numerical further integration. It is necessary to integrate (33) over whole 2 - dimensional plane $\vec{k}$ and by $\vec{q}_1, \vec{q}_2, x, y$ in the frame aperture of the contours and the energy cuts.

We present here the result of analytical integration (33) by $\vec{k}$ with the logarithmical accuracy, using the method of the quasireal electrons [9a]. It

may consider four regions where 3 - momenta of photons are parallel to one of the charged particles:

$$\frac{d\sigma_\gamma^\gamma}{dO} = \frac{d\sigma_1}{dO}(\vec{k}_1 \parallel \vec{p}_-, \vec{k}_2 \parallel \vec{p}_+) + \frac{d\sigma_2}{dO}(\vec{k}_1 \parallel \vec{p}_-, \vec{k}_2 \parallel \vec{p}_+)$$
$$+\frac{d\sigma_3}{dO}(\vec{k}_1 \parallel \vec{p}_-, \vec{k}_2 \parallel \vec{p}_+)\frac{d\sigma_4}{dO}(\vec{k}_1 \parallel \vec{p}_-, \vec{k}_2 \parallel \vec{p}_+) \quad (34)$$

where

$$a) \quad \frac{d\sigma_1}{dO} = G\int_{\kappa_1}^{1-\eta_1} dx_1P(x_1)\int_{\kappa_2}^{1-\eta_2} dx_2P(x_2)((1-x_1)(1-x_2))^{-1},$$
$$\theta_- = \theta; \theta_+ = \frac{1-x_2}{1-x_1}\theta;$$
$$b) \quad \frac{d\sigma_2}{dO} = G\int_{\kappa_1}^{1-\eta_1} dx_1P(x_1)\int_{\kappa_2}^{1-\eta_2} dx_2P(x_2)(1-x_1)^{-1},$$
$$\theta_- = \theta, \theta_+ = (1-x_1)\theta;$$
$$c) \quad \frac{d\sigma_3}{dO} = G\int_{\kappa_1}^{1} dx_1P(x_1)\int_{\kappa_2}^{1-\eta_2} dx_2P(x_2)(1-x_2))^{-1},$$
$$\theta_- = \theta, \theta_+ = \frac{1}{1-x_2}\theta;$$
$$d) \quad \frac{d\sigma_4}{dO} = G\int_{\kappa_1}^{1} dx_1P(x_1)\int_{\kappa_2}^{1} dx_2P(x_2),$$
$$\theta_- = \theta, \theta_+ = \frac{1-x_2}{1-x_1}\theta;$$
$$G = (\frac{\alpha^2L_2}{\epsilon\theta)})^2, \qquad \kappa_{1,2} = \frac{\Delta\epsilon_{1,2}}{\epsilon},$$
$$P(z) = \frac{1}{z}[1+(1-z)^2 - 2(1-z)L_2^{-1}]. \quad (35)$$

For the double bremsstrahlung in the same direction process totally differential cross section have more complicated form [9b, 10, 11]. We will present here the result of calculation within the quasireal electron approximation. For definiteness we consider the emission of two photons by electrons. Again it is convenient to consider four regions

$$\frac{d\sigma^{\gamma\gamma}}{dO} = \frac{d\sigma_1}{dO}(\vec{k}_1, \vec{k}_2 \parallel \vec{p}_-) + \frac{d\sigma_2}{dO}(\vec{k}_1 \parallel \vec{p}_-, \vec{k}_2 \parallel \vec{p}'_-) + \frac{d\sigma_3}{dO}(\vec{k}_1 \parallel \vec{p}'_-, \vec{k}_2 \parallel \vec{p}_-) + \frac{d\sigma_4}{dO}(\vec{k}_1, \vec{k}_2 \parallel \vec{p}'_-) \quad (36)$$

where

$$\begin{aligned}
\frac{d\sigma_1}{dO} &= \frac{1}{2}G\int_{\kappa}^{1-\eta} dx_1 \int_{\kappa}^{1-\eta-x_1} dx_2\left[\frac{P(x_1)}{1-x_1}P\left(\frac{x_2}{1-x_1}\right) + \frac{P(x_2)}{1-x_2}P\left(\frac{x_1}{1-x_2}\right)\right], \\
\theta_- &= \theta, \quad \theta_+ = \theta(1-x_1-x_2); \\
\frac{d\sigma_4}{dO} &= \frac{1}{2}G\int_{\kappa}^{1} dx_2 \int_{\kappa}^{1-x_2} dx_1\left[\frac{P(x_2)}{1-x_2}P\left(\frac{x_1}{1-x_2}\right) + \frac{P(x_1)}{1-x_2}P\left(\frac{x_2}{1-x_1}\right)\right], \\
\theta_- &= \theta, \quad \theta_+ = \theta; \\
\frac{d\sigma_2}{dO} &= \frac{1}{2}G\int_{\kappa}^{1-\eta} dx_1 \int_{\kappa}^{1-x_1} dx_2\frac{P(x_1)}{1-x_1}P\left(\frac{x_2}{1-x_1}\right), \\
\theta_- &= \theta, \quad \theta_+ = \theta(1-x_1); \\
\frac{d\sigma_3}{dO} &= \frac{1}{2}G\int_{\kappa}^{1-\eta} dx_2 \int_{\kappa}^{1-x_2} dx_1\frac{P(x_2)}{1-x_2}P\left(\frac{x_1}{1-x_2}\right), \\
\theta_- &= \theta, \quad \theta_+ = \theta(1-x_2).
\end{aligned} \quad (37)$$

The parameter η entering (34,35) is the minimal energy fraction of the jet hitting the detectors.The case when the scattered lepton is accompanied by photons which move in the close direction one cannot distinguish from the case of the bare electron of the same total energy. So we put $\eta = 0$ in (34b,c,d) and 35c). We give in (34,35) the expressions for the scattering angles of positron θ_+ in terms of θ_- and energy fractions.It must be taken into account when integrating on detectors aperture.

Consider now the pairs creation processes.Basing on the estimation of the total cross section [9a,12,13] in the charge-blind experimental set up we see that the main contribution arises from the process of light pair creation by two-photon mechanism (see above).

The totally differential cross section have the most simplest form for the

case when the created pair have the energies much less than the initial $e^{\pm}$:

$$\begin{aligned}
d\sigma^{e^+e^-\to e'^+e'^-q_+q_-} &= 2\frac{\alpha^4}{\pi^4}\frac{d^2q_1d^2q_2}{((\vec{q}_1)^2)^2}\frac{d^2k}{((\vec{k})^2)^2}\frac{d\beta_1}{\beta_1}dx\Big\{\frac{(\vec{k})^2(\vec{q}_1)^2}{c\ c_1} - \frac{x(1-x)}{c^2c_1^2}[((\vec{q}_1)^2 \\
&+2(\vec{q}_1\vec{q}_2))((\vec{k})^2 - 2(\vec{k}\vec{q}_2)) + 2(\vec{k}\vec{q}_1)((m^2 + (\vec{q}_2)^2)]^2\Big\}, \quad x = \frac{\beta_2}{\beta_1}, \quad \beta_1 << 1, \\
c &= m^2 + (\vec{q}_2)^2 + x(\vec{q}_1)^2 + 2x(\vec{q}_1\vec{q}_2), \\
c_1 &= m^2 + (\vec{q}_2 - \vec{k})^2 + x(\vec{q}_1)^2 + 2x(\vec{q}_1(\vec{q}_2 - \vec{k})),
\end{aligned} \quad (38)$$

where $x_1 = 1, -\vec{q}_1; 1, \vec{k}$ are the energy fractions and the transversal to the beam axis component of the momenta of scattered e'^- and e'^+ .Mass of created particle is m.The energy fractions and the transversal momentum of the created negative and positive charged particles are respectively: $x\beta_1 + \frac{m^2+(\vec{q}_2)^2}{sx\beta_1}, -\vec{q}_2; (1-x)\beta_1 + \frac{m^2+(\vec{q}_2)^2}{sx\beta_1(1-x)}, \vec{q} = \vec{q}_1 + \vec{q}_2 + \vec{k}$. The created pair moves into the electron's direction if the conditions are fulfilled:

$$\left(\frac{m^2 + (\vec{q}_2)^2}{s}\right)^{\frac{1}{2}} < x\beta_1, \qquad \frac{m^2 + (\vec{q}_2)^2}{s})^{\frac{1}{2}} < (1-x)\beta_1; \quad (39)$$

and it moves along positron's direction if:

$$\frac{m^2}{s} < x\beta_1 < \left(\frac{m^2 + (\vec{q}_2)^2}{s}\right)^{\frac{1}{2}}, \qquad \frac{m^2}{s} < (1-x)\beta_1 < \left(\frac{m^2 + (\vec{q}_2)^2}{s}\right)^{\frac{1}{2}} \quad (40)$$

The more complicated form have the cross section in the case when the pair,being hard moves in the direction close to the initial electrons one:

$$\begin{aligned}
d\sigma^{e^+e^-\to e'^+e'^-\mu_+\mu_-} &= \frac{\alpha^4}{\pi^4}\frac{d^2q_1d^2q_2}{((\vec{q}_1)^2)^2}\frac{d^2k}{((\vec{k})^2)^2}\frac{x_1^3dx_1dx_2}{d_2'^2d_2^2}\Big\{\frac{(\vec{q}_1)^2}{x_1}[(d_2' - d_2)^2(4m^2x_2\Delta \\
&+m^2(x_2^2 + (\Delta)^2) + (\vec{r})^2) + 2(d_2' - d_2)(x_2d_2' + \Delta d_2)(\vec{k}\vec{r}) + (\vec{k})^2(x_2^2d_2'^2 \\
&+(\Delta d_2)^2)] - 2(d_2' - d_2)^2[2m^2x_2\Delta - (x_2^2 + m^2 + (\vec{q}_2)^2)(m^2 \\
&+(\Delta)^2 + (\vec{q})^2) + m^2(x_2^2 + (\Delta)^2) + (\vec{r})^2] - 4(\vec{k}\vec{r})(d_2' - d_2) \\
&(x_2d_2' + \Delta d_2) + 2(\vec{k})^2d_2'^2(m^2 + (\vec{q}_2)^2) + 2(\vec{k})^2d_2^2(m^2 + (\vec{q})^2) \\
&+4(d_2' - d_2)[d_2'(\vec{k}\vec{q})(x_2^2 + m^2 + (\vec{q}_2)^2) + d_2(\vec{k}\vec{q}_2)((\Delta)^2 + m^2 + (\vec{q})^2)] \\
&+4d_2'd_2[-m^2(\vec{k})^2 + 2(\vec{k}\vec{q})(\vec{k}\vec{q}_2) - (\vec{k})^2(\vec{q}\vec{q}_2)]\Big\}.
\end{aligned} \quad (41)$$

Where we use the notations:

$$\begin{aligned}
\vec{r} &= x_2(\vec{q}_1 - \vec{k}) + (1-x_1)\vec{q}_2, \qquad \Delta = 1 - x_1 - x_2, \\
d_2 &= m^2x_1(1-x_1) + x_2(1-x_1)\Delta + x_2(1-x_2)(\vec{q}_1)^2 + x_1(1-x_1)(\vec{q}_2)^2 + 2x_1x_2(\vec{q}_1\vec{q}_2) \\
d_2' &= d_2 - 2x_1x_2(\vec{k}\vec{q}_1) + x_1(1-x_1)((\vec{k})^2 - 2(\vec{k}\vec{q}_2)).
\end{aligned} \quad (42)$$

We put $m_e = 1$ in (37).The energy fractions and the transversal momenta of the scattered e'_-, e'_+, μ_+, μ_- are respectively: $x_1, -\vec{q}_1; 1, \vec{k}; \Delta, \vec{q}; x_2, -\vec{q}_2$.

We give here also the expression for the pair production process cross section in the collinear cinematic.It means that the components of the pair move in the direction close to one of the charged particles e^+, e^-in final or initial state. So, naturally:

$$d\sigma^{e^+e^-\to e'^+e'^-q_+q_-}\frac{d\sigma^{\gamma\gamma}}{dO} = \frac{d\sigma_1}{dO}(\vec{q}_+, \vec{q}_- \parallel \vec{p}_-) + \frac{d\sigma_2}{dO}(\vec{q}_+, \vec{q}_- \parallel \vec{p}'_-)$$
$$+\frac{d\sigma_3}{dO}(\vec{q}_+, \vec{q}_- \parallel \vec{p}_+) + \frac{d\sigma_4}{dO}(\vec{q}_+, \vec{q}_- \parallel \vec{p}'_+) \quad (43)$$

Using the structure function method[15] one obtains:

$$a) \quad \frac{d\sigma_1}{dO} = G\int_{\eta}^{1-\kappa} dx F^{e^+e^-}(x),$$
$$\theta_- = \theta, \theta_+ = x\theta,$$
$$b) \quad \frac{d\sigma_2}{dO} = G\int_{\kappa}^{1-\kappa} dx F^{e^+e^-}(x),$$
$$\theta_- = \theta_+ = \theta,$$
$$c) \quad \frac{d\sigma_3}{dO} = G\int_{\eta}^{1-\kappa} dx F^{e^+e^-}(x),$$
$$\theta_- = \theta, \theta_+ = x\theta,$$
$$d) \quad \frac{d\sigma_4}{dO} = G\int_{\kappa}^{1-\kappa} dx F^{e^+e^-}(x),$$
$$\theta_- = \theta_+ = \theta, \kappa = \frac{\Delta\epsilon}{\epsilon}, \quad (44)$$

where

$$F^{e^+e^-} = \frac{1}{12}\left[\frac{1+x^2}{1-x} + 2\frac{1-x^3}{x} + \frac{3}{2}(1-x) + 3(1+x)lnx\right]. \quad (45)$$

Both bremsstrahlung and two-photon mechanisms are considered in deriving (40),$2m << \Delta\epsilon << \epsilon$ is the auxiliary parameter of the energy of the soft unobserved pair.

It is necessary to note that (40) don't takes into account such effects as charge-odd interference of the bremsstrahlung's and two-photon's amplitudes $d\sigma^{odd}$,as well as the identity of the fermions effects.They considered in details

in [9b].In [14] the spectral distributions in the fragmentation region also considered which may be used for the numerical estimates.

In conclusion we present here the Drell-Yan's type representation of the Bhabha process cross section (see fig. 11)

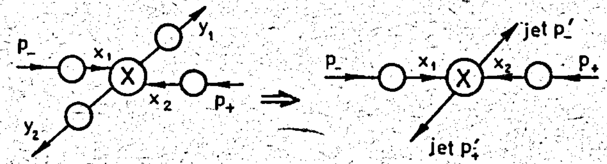


Fig.11

Remembering that if the detectors are constructed such that they registered the general energy of the jet which hit them, then one may omit the structure functions on the scattered particles lines.So we :have

$$d\sigma^{e^+e^-\to e'^+,jet;e'^-,jet} = \int_0^1 dx_1 D(x_2,\beta_q)\int_0^1 dx_2 D(x_2,\beta_q) d\hat{\sigma}(x_1x_2Q^2)(1+\gamma K(x_1,x_2)) \quad (46)$$

where $\beta_q = 2\Gamma(ln(\frac{Q^2x_1x_2}{m^2}) - 1), Q^2 = (\epsilon\theta)^2$; the structure functions are known [15]:

$$D = D^\gamma + D^{e^+e^-}, \qquad D^{e^+e^-} = \frac{(\beta_q)^2}{4}F^{e^+e^-}(x),$$
$$D^\gamma = \frac{\beta_q}{2}(1-x)^{-1+\frac{\beta_q}{2}}\left[1 + \frac{3}{8}\beta_q - \frac{1}{48}\beta_q^2\left(\frac{1}{3}ln\left(\frac{Q^2x_1x_2}{m^2}\right) - \frac{47}{8} + (\pi)^2\right)\right]$$
$$-\frac{(1+x)\beta_q}{4} + \frac{(\beta_q)^2}{32}\left[4(1+x)ln\left(\frac{1}{1-x}\right) + \frac{1+3x^2}{1-x}ln\left(\frac{1}{x} - 5 - x\right]. \quad (47)$$

The quantity $K(x_1, x_2)$ have a contributions from nandrolone terms in corrections of two-loop virtual,one-loop corrected single bremsstrahlung, and from the processes $2 \to 4$:

$$K = K_v + K_S^\gamma + K_h^\gamma + K_{VV} + K_{Sh}^{\gamma\gamma} + K_{VS}^\gamma + K_{SS}^{\gamma\gamma} + K_{hh}^{\gamma\gamma} + K_{SS}^{e^+e^-} + K_{hh}^{e^+e^-}. \quad (48)$$

All the contributions to (43) except K_h^γ, $K_{hh}^{\gamma\gamma}$, $K_{hh}^{e^+e^-}$ may be established from the calculations given above. The contributions K_h^γ, $K_{hh}^{\gamma\gamma}$, $K_{hh}^{e^+e^-}$ may be found by numerical integration taking into account the experimental conditions.

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