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G.A.Kozlov

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Козлов Г.А. E2-92-557

Редукционный формализм нуль-относительной энергии для тяжело-легкой кварковой связанной системы

На основе редукционного формализма нуль-относительной энергии в квантовой теории поля и релятивистского ковариантного уравнения предложен полный анализ амплитуд физических тяжело-легких связанных систем (ТЛСС) кварков в положительно-частотном секторе. Получены точные аналитические выражения для константы слабых распадов в зависимости от быстроты кварка в случае псевдоскалярных ТЛСС.

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The Zero-Relative Energy Reduction Formalism for the Heavy-Light Quark Bound System

Based both on the zero-relative energy reduction formalism in quantum field theory and the relativistic covariant equation we perform a complete analysis of physical heavy-light bound system (HLBS) amplitudes for the positive frequency quark states. The calculations in an explicit analytic form of the quark rapidity-dependent leptonic weak decay constants for the pseudoscalar HLBS are presented.

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1. Introduction

Experimenters give hopes in searching for the HLBS $(Q\bar{q})$, where symbols Q and $\bar{q}$ denote a heavy quark and a light anti-quark, respectively. We know the heavy quarkonium can be produced in the relativistic heavy ion collision, so the heavy-light quarkonium will appear in the associated processes. Since in relativistic heavy ion collisions, high temperature is produced by the transformation of great deal of energies, we think the investigation of $(Q\bar{q})$ at a finite temperature will help us to know more about the properties of the HLBS. But in this paper we'll give a brief review of the $(Q\bar{q})$-bound state at zero temperature.

On the other hand the heavy-light quarkonium decays at zeroth temperature are the good current and planned program to search for both the light charged and neutral Higss bosons, h, with masses which are of an order of the quantum chromodynamics (QCD) typical scale $L \sim 0.5 \div 1.0$ Gev. The existence of this type of light h-bosons in the models with several Higgs doublets or with a sufficiently superheavy t-quark ($m_t > m_{Z,W}$, m_t and $m_{Z,W}$

are the masses of t-quark and the gauge bosons, Z and W, respectively), when the Standard Minimal Model (SMM) limit $m_H > 7$ Gev (m_H is the mass of Higgs in the SMM) is not yet relevant, provides the unique possibility of h-production in $(Q\bar{q})$- decays. The following points provide the main field of our interest in this subject:

- the first, in the case of the mass relation $m_h < \hat{m}_Q - \hat{m}_q$ (m_h, $\hat{m}_Q$ and $\hat{m}_q$ are the masses of the h-boson, Q-heavy and q-light quarks, respectively in the constituent quark model) the decays, e.g. $B^-(b\bar{u}) \longrightarrow h^- + D^0(c\bar{u})$ and $B_c^-(b\bar{c}) \longrightarrow h^- + \eta(c\bar{c})$ dominate; in the three generation SMM the Higgs can only be light enough to appear in B-meson decays if $m_t > 80$ Gev, giving the branching ratio for B $\longrightarrow$ Higgs + something of about 20-30 %; the present data eliminate the Higgs masses below 3.7 Gev with the exception of a window between 0,3 Gev and 2.0 Gev; if the Higgs is lighter than the B-meson, it is produced really in the B-meson decays;

-the second, we suppose the dominant is production of $\eta(c\bar{c})$-meson in the decays of B_c^- due to the leading transition b$\longrightarrow$c as compared with b$\longrightarrow$u, where the present status of the data is $|V_{ub}| / |V_{cb}| = 0.09 \div 0.17$ (see the ARGUS and CLEO data in [1]);

-the third, the quark process underlying the decay of the B-meson into a Higgs and charmed hadronic matter is b$\longrightarrow$h+c. The light Higgs field may be considered as the pseudo-Goldstone boson associated with scale invariance of the classical lagrangian;

-the last one, there are no experimental observables of charmless hadron states in the decays of B-meson.

The SMM assumes that if the electroweak symmetry breaking vacuum is an absolute minimum of the effective potential for the h-boson field with mass m_h

$$V(h) = - \mu_1 h^2 + \xi h^4 \ln(h^2/ \mu_2^2),$$

where

$$\xi = (4 \pi r)^{-2} [3(m_Z^4 + 2 m_W^4) + m_h^4 - 4 \sum_f m_f^4], \text{ f: all fermions,}$$

one can finds the following mass relation

$$m_h^2 (1 - t\, m_h^2) < t (4 \sum_f m_f^4 - 3 \sum_B m_B^4), \quad t = (4 \pi r)^{-2},$$

f: all fermions and B: Z, W^+, W^-. Here the SMM scale r = 247 Gev, μ_1 and μ_2 are constants. In the presence of the superheavy fermions, e.g. if the top-quark masses $m_t \longrightarrow 100$ Gev, the latter expression would allow for the Higgs particle to be an arbitrarily light.

As is well known, the fundamental role in investigating the pseudoscalar HLBS belongs to the leptonic weak decay $(Q\bar{q}) \longrightarrow l\bar{\nu}$ (l and $\bar{\nu}$ denote the lepton and antineutrino, respectively). The heavy-light quark system can be considered as a simplest one of QCD: one light quark in a static field of a heavy quark source. This is the hydrogen atom-like (with the radius of the kernel$\sim 1/\hat{m}_Q \ll R_c \sim 1/L$) analog as is the positronium-like system for the heavy quark $(Q\bar{Q})$- bound state ($c\bar{c}$, $b\bar{b}$). It is very convenient to study theoreticaly both the perturbative and nonperturbative heavy-light QCD irrespective of consideration of some details of the QCD analysis. The invariant matrix element defining the decay process $(Q\bar{q}) \longrightarrow l\bar{\nu}$ can be represented in the standard form

$$A = \langle 0 | J_\mu^5 | P\rangle G (8 P^0)^{-1/2} \cos\Theta\, \bar{u}_l \gamma_\mu (1 - \gamma_5) v_{\bar{\nu}},$$

where the axial current $J_\mu^5 = \bar{q} \gamma_\mu \gamma_5 Q$ defines the decay constant F_P : $\langle 0 | J_\mu^5 | P\rangle = \sqrt{2} F_P P_\mu$, and P_μ is the 4-momentum of the initial bound state with mass M .

Our aim is to calculate the weak decay constant F_P based on the covariant approach, where the two-quark vertex function (v.f.) Γ is parametrized in the momentum space as follows [2]

$$\Gamma = \Gamma(f_1 , \bar{f}_2 ; P | \tau\lambda) , \qquad (1)$$

where $f_1^\mu + f_2^{-\mu} = P^\mu \pm \tau\lambda^\mu$, f_1^μ and $f_2^{-\mu}$ are momenta of the heavy quark and light antiquark, respectively; $\lambda^\mu = P^\mu / \sqrt{P^2}$ and τ

is a scalar parameter. All the four- momenta are on the respective mass shells: $f_1^2 = \hat{m}_Q^2$, $f_2^2 = \hat{m}_q^2$ and $P^2 = M^2$. The incoming and outgoing four-momenta $\pm \tau\lambda_\mu = (f_1 + f_2)_\mu - P_\mu$ of spurions [2] mean that the v.f. (1) is always off-energy shell (i.e. under the condition $(f_1 + f_2)_\mu = P_\mu$ the v.f. (1) would turn into a constant).

2. A Higgs-Quarkonium Interplay

The local interaction lagrangian for the couplings of the pseudoscalar mesons to a light Higgs-boson h in the low momentum limit $\vec{p}_h = 0$ ($m_h < \hat{m}_Q$, M) linear with the Higgs-field term can be written in the standard form (keeping both the heavy and light flavour numbers, N_H and N_L , respectively, since $m_h > \hat{m}_q$)

$$L_{int.} \longrightarrow L_{eff.} + L_{Z,W} \qquad \text{as } \vec{p}_h \longrightarrow 0 ,$$

where

$$L_{eff.} = -(1+\frac{h}{r}) \sum_l m_l \bar{l} l + \frac{N}{12\pi}[\alpha_s G^a_{\mu\nu} G^{\mu\nu a} + \frac{13}{2} \alpha F_{\mu\nu} F^{\mu\nu}) \frac{h}{r} , \quad (2)$$

$N = N_H + N_L$, l: all light fermions with the masses m_l and

$$L_{Z,W} = -(1+ \frac{h}{r})^2 (m_Z^2 Z^\mu Z_\mu + 2 m_W^2 W^{+\mu} W^-_\mu). \qquad (3)$$

The first term in (2) represents the direct contact interaction of the h-boson with all light fermions, but the second term and the third one provide the effective gluon-Higgs and gamma-Higgs interaction, respectively, induced by the constituent light- and heavy-quark loops. Here we take into account the mass reduction mass $\longrightarrow$ mass (1+ h/r) of all massive particles involving the (2) and (3); $G^a_{\mu\nu}$ and $F^{\mu\nu}$ are the field strengths for gluons and photons, respectively.

The expression for the vertex of interaction of h-boson field, {higgs}, with the pseudoscalar unequal mass- and an equal one-

quarkonia is defined through the couplings of Higgs to the gluons

$$\frac{g_{eff.}}{r} A'_{Q\bar{q}} \{higgs\} A_{q\bar{q}} =$$

$$= < Q\bar{q} \,|\, \frac{\alpha_s}{8\pi} G^a_{\mu\nu} G^{\mu\nu a} - \frac{3}{2N} \sum_{f:u,d,s} \hat{m}_f \bar{f} f \,|\, q\bar{q}> \frac{2N}{3r} \{higgs\}, \quad (4)$$

where $A_{Q\bar{q}}$ and $A_{q\bar{q}}$ are the $(Q\bar{q})$- and $(q\bar{q})$-bound system amplitudes, respectively, $\hat{m}_f$ and f are the mass and field of any light flavours. In the case of the three light flavours the product operator $G^a_{\mu\nu} G^{\mu\nu a}$ is related to the trace of the energy momentum tensor Θ^ν_μ by means of [3].

$$\Theta^\mu_\mu = - \frac{b\,\alpha_s}{8\pi} G^a_{\mu\nu} G^{\mu\nu a} + \sum_{f:\,u,d,s} \hat{m}_f \bar{f} f ,$$

where $b = 11 - (2/3)\, N_l = 9$ is the first coefficient of the β - function

$$\beta(\alpha_s) = - b\, (\frac{\alpha_s}{2\pi}) + O(\alpha_s^2)$$

in the QCD without superheavy quarks. Therefore, the Higgs-gluon couplings (4) can be rewritten in the following useful formula

$$g_{eff.} \bar{A}_{Q\bar{q}} \{higgs\} A_{q\bar{q}} = - <Q\bar{q}| \frac{2N}{27} \Theta^\mu_\mu +$$

$$+ (1- \frac{2N}{27}) \sum_{f:\,u,d,s} \hat{m}_f \bar{f} f \,|\, q\bar{q} > \{higgs\},$$

which is valid for an arbitrary number of light and heavy flavours.

To obtain the expression for the effective constant $g_{eff.}$ it therefore suffices to renormalize the fermion amplitude $f \longrightarrow (\sqrt{2} / 3) f$ in (4). One gets

$$g_{eff.} \bar{A}_{Q\bar{q}} \{higgs\} A_{q\bar{q}} = - \frac{2}{9} <Q\bar{q} \,|\, \Theta^\mu_\mu \,|\, q\bar{q}> \{higgs\}$$

and $g_{eff.} = - (2 / 9)\, \vec{m} = - 0.90$ Gev, where the effective mass $\vec{m}$ is defined through the gluon contribution

$$\tilde m\; A_{Q\bar q}\; \dot A_{q\bar q} = \langle Q\bar q \mid \Theta^{\mu}_{\mu} \mid q\bar q \rangle .$$

Here the heavy quarks have been integrated out of the theory.

3. Bound State

The v.f. (1) obeys the unlocal bound state eq. [4]

$$(P+\tau\lambda)^2\,\Gamma(p;P|\tau\lambda) = 8\int d\Omega_{p'}\,\tilde w(p,p';P|\tau\lambda)\,\frac{\Gamma(p';P|\tau\lambda)}{2(\lambda p')[\lambda(2p'-P)-i0]}\,(p'P)\left[\frac{(p'P)}{M^2}(P+\tau\lambda)^2-\bar\mu^2\left(1+\frac{\tau}{\sqrt{P^2}}\right)\right] \qquad (5)$$

($d\Omega_p = d^3p/[(2\pi)^3\, 2\sqrt{p^2+\bar\mu^2}\,]$, $\tilde w(..)$ is the interaction kernel) in the case of an arbitrary total momentum $P = \{E, \vec P\}$; where $\bar\mu$ is the relativistic reduced flavour mass, that in the asymptotic limit $\hat m_Q \to \infty$ is defined only by the mass of a light antiquark and p^μ is the 4-vector of relative momentum: $f_1^\mu = \varepsilon P^\mu + p^\mu$, $f_2^\mu = (1-\varepsilon)P^\mu - p^\mu$ and the mass coefficient $\varepsilon = 0.5[1+(\hat m_Q^2 - \hat m_q^2)/M^2]$, which leads to 1 as $\hat m_Q \gg \hat m_q$, and to 0.5 as $\hat m_Q \sim \hat m_q$. Note that $(p+f_2) \overline{=} 0$

as $\hat m_Q \gg \hat m_q$, which means for the $(Q\bar q)$-bound systems the typical internal momenta are determined mostly by the light quark with momentum f_2^μ. The v.f. Γ is the nonlocal function that depends on the spectral parameter M taking the values of the total energy of the two quark bound system. It is clear, this fact is true due to direct consideration of the quark-antiquark interaction problem in the framework of the local quantum field theory. A way leading to a local approximation for the dynamical function Γ is to restrict the structure of interacting particles: $(p^2/\bar\mu^2) < 1$.

In the one-loop approximation the QCD characteristic scale, i.e. the scale on which the nonperturbative fluctuation dominates, $R_c = \Lambda^{-1}\exp(-2\pi/b)$, $b = (11-2n_f/n_c)/3$, is $R_c \sim (0.12 \div 0.24)$ fm at $n_c = n_f$ and $R_c \sim (0.07 \div 0.13)$ fm at $n_c = n_f / 2$ and at decreasing of the nonperturbative QCD dimensional parameter Λ from 200 Mev up to 100 Mev, respectively (n_c and n_f are numbers of

colours and flavours). In the nonperturbative region at large distances one can find the following general representation for the interaction kernel $w_{np}(R > R_c)$ in R^D in the scalar form [5]

$$w_{np}(R > R_c) \sim R^{\alpha}\,[\,a + b\,R^{\beta}\ln(R/l)\,] , \qquad (6)$$

where α, β and a, b are arbitrary numbers and functions, respectively; l^{-1} is a mass parameter. The Fourier transformation of (6)

$$\tilde w_{np}(k^2) = \left[\frac{4}{(l\cdot k)^2}\right]^{\frac{\alpha+\beta+D}{2}} \frac{\pi^{D/2}}{2}\left\{\frac{a(k^2)^{\beta/2}}{2^{\beta-1}}\, l^{\alpha+\beta+D}\,\frac{\Gamma(\frac{\alpha+D}{2})}{\Gamma(-\alpha/2)} + b\,\frac{\Gamma(\frac{\alpha+\beta+D}{2})}{\Gamma(-\frac{\alpha+\beta}{2})}\left[\ln\left(\frac{4}{l^2k^2}\right) + \Psi\left(\frac{\alpha+\beta+D}{2}\right) + \Psi\left(-\frac{\alpha+\beta}{2}\right)\right]\right\} , \qquad (7)$$

$\lim 1/[\Gamma(-\alpha/2)] \to 0$ as $(\alpha/2) \to n$, $n = 0,1,\ldots$ ($\Gamma(\ldots)$ and $\Psi(\ldots)$ are the gamma- and psi-functions, respectively) allows one to get the following momentum distribution of the heavy-light quarks inside the hadron ($\bar\psi(p)(E_Q + E_q - M) = \Gamma(p)$, $E_j = \sqrt{m_j^2 + p^2}$, j: Q, q)

$$\bar\psi(\chi_p) = C_p \exp[-\bar\mu^2 \sinh^2\chi_p/(2\nu^2)] , \qquad (8)$$

$$C_p = \sqrt{M}/(\pi^{3/4}\,\nu^{3/2}) , \qquad (9)$$

obtained from the solution of eq.(5) in D = 3-space with the interaction kernel (7) in the leading order at $\alpha = 2$, $\beta = 0$, predicted by the calculation of the transition amplitudes [4] and many spectroscopic data. Here χ_p is the quark rapidity and ν is the universal spring constant accumulating the full flavour independent QCD-oriented three-dimensional structure of the two-body Bethe-Salpeter picture. This model is characterized by the reduced constant $\tilde\nu = [\alpha_s(M)]^{-1}\nu$ common to all the heavy-light flavour sector, α_s is the QCD spring constant.

4. Weak Decay Constant

Using the general principles of constructing the pseudoscalar bound state amplitude on the base of the zero-relative energy reduction formalism in quantum field theory (or the equal-time spurion's technique) [2], it is easy to find for the weak decay constant

$$F_P = F^{(0)} + F^{(1)}, \qquad (10)$$

where

$$F^{(0)} = 4\sqrt{2\pi} \cdot \int_0^\infty \frac{dp\, p^2}{\sqrt{1 + p^2/\hat{m}_Q^2}} \; \frac{\Gamma(p^2)}{4E_q^2 - M^2 - i0},$$

but the second term in (10) arises due to the unequal-mass contribution

$$F^{(1)} = 2\sqrt{2\pi} \cdot \Delta\, M(1-x) \int_0^\infty \frac{dp\, p^2}{E_Q\, E_q} \; \frac{\Gamma(p^2)}{M^2 - 4E_q^2 + i0}.$$

Here $\Delta = \hat{m}_Q - \hat{m}_q$ and by "x" we denote the fraction of the anti-quark momentum, f_2 in the total momentum P^μ : $f_2^\mu = x P^\mu$. It is clear that in the $\hat{m}_Q \sim M \longrightarrow \infty$ limit x is going to zero.

The effective mass of a heavy quark, $\hat{m}_Q^{eff}$, is defined through the energy-momentum constraint: $(\hat{m}_Q^{eff})^2 = f_2^2 + M(M - 2 f_2^0)$. The heavy quark energy E_Q differs from its physical value of the mass $\hat{m}_Q$ by some finite small amount e ($E_Q = \hat{m}_Q + e$) that can be interpreted as:

- a binding energy;
- an average energy of the light quark $< E_q >$;
- a second term $q^2/(2\,\hat{m}_Q)$ in the expression of the energy of a free quark with the small momentum q^μ.

Therefore, the heavy quark virtuality $v_Q = (f_1^2/\hat{m}_Q^2 - 1) = 2e/\hat{m}_Q$ is much smaller than unity. In the limit $m_Q \longrightarrow \infty$ the virtuality $v_Q \longrightarrow 0$ and thus, the heavy quark is almost on-shell. The exact relativistic expression for F_P in terms of the bound state wave function (w.f.) $\bar{\psi}(p)$ becomes:

$$F_P = \frac{1}{\sqrt{2\pi}} \int \frac{d^3p}{E_Q\, E_q}\; \psi(\bar{p}) \{ \frac{E_Q + E_q - M}{M^2 - 4\, E_q^2 + i0} \} [\, \Delta\, M(1-x) - 2\hat{m}_Q E_q] \qquad (11)$$

In the limit $\hat{m}_Q \longrightarrow \infty$ one can find from (11)

$$F_P^{AS} \longrightarrow 2\sqrt{2}\;\pi\; M^{-1} \int_0^d dp\, p^2\, \bar{\psi}(p) \cdot (1 - 2p/ M), \; d < M \qquad (12)$$

with taking into account the normalization constant (9) proportional to $M^{1/2}$. The asymptotic expression (12) based on the w.f.(8) provides the $M^{-1/2}$ asymptotic scaling law (ASL) and becomes

$$F_P^{AS} \longrightarrow F\, (1 + \delta F), \qquad (13)$$

where

$$F = 2\, \pi^{1/4}\, \nu^{3/2} / \sqrt{M}$$

and

$$\delta F = - 4\sqrt{2}\;\; \nu/(\sqrt{\pi}\;\; M). \qquad (14)$$

The numerical calculations of the variations (with increasing x up to 0.3) of F_P (11) and the asymptotic one F_P^{AS} (13) both with using the w.f. (8) for D, D_S, B, B_S– mesons and the exotic heavy top-quark state T(t=100 Gev, $\bar{q}$=0.5 Gev) are given in Table 1. The values of the universal constant ν in (8) and (9) are determined for the hadron mass spectra and have been suggested by Mitra et al. in [6]. In going both from D- to B-mesons and from D_S- to B_S-mesons the ratios of perturbative gluon interaction factors, calculated in [7,8] and which should be multiplied by F_P-constants, are

$$[\, \alpha_S(M_D) / \alpha_S(M_B)]^{-2/b} = 0.80$$

and

$$[\, \alpha_S(M_{D_S}) / \alpha_S(M_B)]^{-2/b} = 0.82,$$

where $b = 11-(2/3)\, n_f$. Therefore, we shall not take it into account in our numerical data presented in Table 1.

Table 1

Meson	ν, Gev	F_P, Mev fraction x 0.0	0.1	0.2	0.3	F_P^{AS}, Mev	$\vert \delta F \vert$, %
D(1.869)	.268	138.7	107.9	77.0	46.2	146.4	45.8
D_s(1.971)	.299	150.5	113.8	77.1	40.3	160.4	48.4
B(5.271)	.262	131.2	116.0	100.9	85.7	132.0	15.8
B_s(5.373)	.298	142.9	124.5	106.5	88.5	153.9	17.7
T(100.0)	.039	2.0	1.9	1.6	1.4	2.0	0.12

The resulting calculations for the T-meson indicate the $M^{-1/2}$-ASL for M > 5 Gev as was shown by Simonov [9]. The asymptotic weak decay constants F_D^{AS} and F_{Ds}^{AS} have very large corrections $| \delta F |$ (listed in Table 1) which suggest that these decay constants break the ASL. Small values of F_{Ds} at x > 0.1 could be explained by the quark content inside the D_s-meson. In fact, we have used the constituent strange (~ 0.419 Gev) and the charmed content (~ 1.50 Gev) flavour masses [10,11]. Therefore, the expected value of F_{Ds} is strongly dipped by the term in the last brackets in the r.h.s. of (11). A possible discrepancy of F_B (F_D) in recent potential models is due to different values of the b (c)-flavour mass used.

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