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THE SCATTERING OF THE ELECTRON ON QUARK AND THE NUCLEON IN THE CASE OF POLARIZED PARTICLES

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Рассеяние электрона на кварке и глюоне
в случае поляризованных частиц

Рассмотрено сечение упругого рассеяния продольно-поляризованного электрона на поляризованном кварке. Вычислены однопетлевые поправки в стандартной модели. Рассмотрено также излучение мягких и жестких фотонов. В случае, когда поляризован только кварк, показано, что появляется асимметрия вверх-вниз относительно плоскости рассеяния, как следствие ненулевой мнимой части амплитуды бокс-диаграмм. Этот эффект также рассмотрен в случае электрон-протонного рассеяния. Приводятся некоторые оценки для асимметрии. Изучение асимметрии, связанной с поляризацией нуклона, может прояснить вопрос о скейлинге в промежуточном состоянии амплитуды комптоновского рассеяния на ненулевой угол. В случае глубоконеупругого рассеяния новая структурная функция, описывающая число партонов и антипартонов в нуклоне с поляризациями поперечными плоскости рассеяния, может быть измерена.

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The Scattering of the Electron on Quark and the Nucleon
in the Case of Polarized Particles

The cross section of elastic scattering of the longitudinally polarized electron on polarized quark is considered. The one loop corrections in the standard model are calculated. The emission of the soft and hard photons also considered. In the case when only quark is polarized the up-down asymmetry relative to scattering plane is shown to appear as a consequence of nonzero imaginary part of the box diagram amplitude. This effect also considered in the case of electron-proton scattering. Some estimates of asymmetry for elastic and inelastic scattering is given. Studying of the one polarized nucleon asymmetry may shed the light in the question of scaling in the intermediate state of the nonforward Compton scattering amplitude. In the case of deep inelastic scattering the new structure function which counts the number of quarks and the antiquarks with the transversal to the scattering plane polarization in the electron may be measured.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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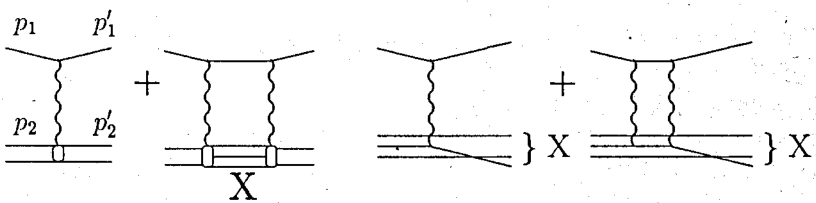
In the quark-parton model the process of elastic electron-quark scattering is the ingredient of the deep inelastic electron-nucleon scattering (DIS) process (see figure).



So we may present the cross section $d\sigma_{DIS}$ of DIS process as a cross section of the elastic electron-quark scattering being averaged on the quark energy fractions

$$d\sigma_{DIS} = \int d^3x\, f(x_1, x_2, x_3)\, \delta(\sum_i x_i - 1) \sum_i d\sigma_{el}^i \tag{1}$$

The distribution function $f(x_1, x_2, x_3)$ of the parton-quarks in the nucleon over the fractions of their energies x in the reference frame where the momentum of the nucleon tends to infinity, satisfy the normalization condition $\int d^3x\ \delta(\sum_i x_i)\ f(x_1, x_2, x_3) = 1$, here $d\sigma_{el}$ is the cross section of the subprocess

$$q^i(p_i) + e(p_1) \longrightarrow q^i(p_i') + e(p_1'), \quad p_i'^2 = p_i^2 = m_i^2, \quad p_1^2 = p_1'^2 = 0.$$

In Born approximation and in the case of unpolarized particles it have the form:

$$d\sigma_{el}^{0i} = dO_1' \,\frac{\alpha^2 (e_i/e)^2}{4\varepsilon_1^2} \frac{\cos^2\theta/2}{\sin^4\theta/2} \left[1 + \frac{Q^2}{2m_i^2} \operatorname{tg}^2\frac{\theta}{2}\right] \left(1 - \frac{Q^2}{2m_i\varepsilon_1}\right), \tag{2}$$

where we suggest the quark of the flavour i is the point-like fermion with charge e_i and constituent mass m_i,

$$Q^2 = -(p_1 - p_1')^2 = 2\varepsilon_1\varepsilon_1'(1 - \cos\theta), \quad \theta = \widehat{\vec{p}_1, \vec{p}_1'}.$$

$\varepsilon_1, \varepsilon_1' = \varepsilon_1(1+\varepsilon_1/m_i(1-\cos\theta))^{-1}$ -are the energies of the initial and scattered electrons in laboratory reference frame, θ -is the electron scattering angle.

Expressing the invariant mass of the hadron M_x in terms of the quark energy fraction

$$M_x^2 = (p+q)^2 - M^2 = Q^2(\frac{1}{x}-1) = ((1-x_i)p+p_i')^2 - M^2 \approx$$
$$\approx \frac{M_p}{m_i} Q_i^2\,(1-x_i)$$

and introducing the standard DIS variables

$$Q^2 = -q^2, \quad x = \frac{Q^2}{2pq}, \quad y = \frac{2pq}{s}, \quad s = 2pp_1 = 2M_p\varepsilon,$$

one may rewrite (1) in the form

$$\frac{d\sigma_{DIS}}{dx\,dy} = \int d^3x\,\delta(\sum_i x_i - 1)\, f(x_1,x_2,x_3) \times$$
$$\times \sum_i \delta\left(x - (1+(1-x_i)\frac{M_p}{m_i})^{-1}\right)\frac{d\sigma_{el}^{0i}}{dy}, \tag{3}$$

where

$$\frac{d\sigma_{el}^{0i}}{dy} = \frac{2\pi}{sy^2}\,\frac{1}{x(1-x_i)}[(1-x)(2-2y+y^2) - 2\frac{m_p^2}{s}\,xy(1-x_i)]. \tag{2a}$$

Below, in part 1, we calculate the cross section $d\sigma_{el}$ with one-loop radiative corrections in the case when the initial particles are polarized:

$$u(p_1)\,\bar{u}(p_1) = \frac{1}{2}\hat{p}_1(1-\gamma_5\xi),$$
$$u(p_2)\,\bar{u}(p_2) = \frac{1}{2}(\hat{p}_2+m)(1-\gamma_5\hat{a}),$$
$$ap_2 = 0, \quad p_2 = (m,0,0,0), \quad |\vec{a}|, |\xi| < 1. \tag{4}$$

Together with the exchange by photons we take into account also Z and $W^{\pm}$ bosons ones. Main attention is paid to box diagrams. In part 2 we consider the elastic scattering of electron on proton in the polarized case for the momentum transferred small compared with Z-boson mass.

Cross section in the Born approximation has a form

$$\begin{aligned}
\frac{d\sigma}{dO_1'} &= \frac{\alpha^2\cos^2\theta/2}{4\varepsilon^2\sin^4\theta/2}\,\mathcal{P}\,(1+\frac{t}{2m\varepsilon}), \quad t = q^2 = (p_1-p_1')^2,\\
\mathcal{P} &= F_1^2 - \frac{t}{4m^2}F_2^2 - \frac{t}{2m^2}(F_1+F_2)^2\,\mathrm{tg}^2\frac{\theta}{2} + \\
&+\xi(F_1+F_2)[F_1(\frac{\varepsilon_1}{m}\,a + \frac{\varepsilon_1'}{m}\,b\,) + F_2\,\frac{\varepsilon_1\varepsilon_1'}{m^2}\,(a-b)]\,\mathrm{tg}^2\frac{\theta}{2},\\
a &= \vec{n}_1\vec{a}, b = \vec{n}_1{}'\vec{a},
\end{aligned} \tag{5}$$

where $\vec{a}$ is the averaged spin vector of the resting proton, $F_{1,2} = F_{1,2}(t)$ are the Dirac and Pauli formfactors of the proton, m-its mass; $\vec{n} = \vec{p}_1/\varepsilon_1$, $\vec{n}' = \vec{p}_1{}'/\varepsilon_1'$, are the orts in the directions of motion of initial and scattered electrons; ε_1, ε_1' their energies in the laboratory frame, $\theta = \widehat{\vec{n}\vec{n}'}$ is the scattering angle. We show that some asymmetry up-down in counting of the scattered electrons events relatively to the scattering plane appears as a consequence of the nonzero imaginary part of the scattering amplitude (originating from box diagrams). Formally it is caused by terms proportional to the quantity I from the cross section:

$$I = \vec{n}\times\vec{n}'\vec{a} = |\vec{a}|\,\sin\theta\,\cos\varphi, \quad \theta = \widehat{\vec{n}\vec{n}'}, \quad \varphi = (\widehat{\vec{n}\times\vec{n}'}),\vec{a}\,. \tag{6}$$

The phenomenon of asymmetry in the case of only one polarized particle in the initial state was first considered in the case of $e-\mu$ scattering in 1960 year paper[1]. The measurement of the recoil protons polarization which arises in the same manner in *ep*-elastic scattering was suggested in [2] as a test of existence of deviation from the one-photon exchange models. The possible manifestation of the currents with the unnatural parity and the T-violating effects in *ep*-collisions with the resonances production was considered in [3, 4]. The experimental investigation of this asymmetry was first done in early seventieth in [5] in scattering of 18 GeV electrons and 12 GeV positrons on proton. The asymmetry was found on the level $< 10\%$ and the statistically reliable indication on T-violating effects was absent. We want to point out on expedient to repeating experiments on elastic as well as inelastic electron polarized proton collisions at the energies and luminosities of the contemporary accelerators. In the case of elastic scattering the main contribution to asymmetry

$$A_{el}(\theta,\varepsilon) = \frac{d\sigma(\vec{a},\theta) - d\sigma(\vec{a},-\theta)}{d\sigma(\vec{a},\theta) + d\sigma(\vec{a},-\theta)} \tag{7}$$

goes from the many particle intermediate state of the nucleon in the box diagram and will measure the imaginary part of the Compton scattering amplitude of the virtual photon on polarized proton on the nonzero angle. It is described by the tensor

$$\frac{e^2}{\pi}\,\mathrm{Im}\,\langle p'|J^\mu(q_1)J^\nu(q_2)|p,\vec{a}\rangle = \frac{e^2}{\pi}\,\mathrm{Im}\,\bar{u}(p')\,T^{\mu\nu}\,u(p,a) =$$
$$= e^2\sum_X \langle p'|J^\mu(q_1)|X\rangle\langle X|J^\nu(q_2)|p,a\rangle\delta((p+q_1)^2-M_X^2),$$
$$p+q_1=p'+q_2. \qquad (8)$$

As an intermediate states $|X\rangle$ in (7) can take place the state as well as barionic resonances Δ, N^+ and the many-particles states. The tensor (7) may be build from the tensors and the vectors of the kind

$$(g^{\mu\nu}q_1q_2-q_1^\nu q_2^\mu),\quad A^\mu=\left(\tilde{A}-q_1\frac{\tilde{A}q_1}{q_1^2}\right)^\mu,\quad \tilde{A}\neq q_1,$$
$$B^\nu=(\tilde{B}-q_2\frac{\tilde{B}q_2}{q_2^2})^\nu,\quad \tilde{B}\neq q_2,\quad \varepsilon^{\mu\alpha\beta\gamma}p_\alpha q_{1\beta}a_\gamma,\quad \varepsilon^{\nu\alpha\beta\gamma}p_\alpha q_{2\beta}a_\gamma,$$

which satisfy the current conservation condition: $\langle J^\mu(q_1)\rangle q_1^\mu=\langle J^\nu(q_2)\rangle q_2^\mu=$ $=0$. A lot of structure functions is to be introduced to describe (8) compared to the two ones, g_1, g_2 which describe the DIS of longitudinally polarized electron on polarized proton [6]:

$$\frac{1}{\pi}\,\mathrm{Im}\,\langle p,a|J^\mu(q)J^\nu(q)|p,a\rangle\sim\varepsilon^{\mu\nu\lambda\sigma}q^\lambda(g_1a^\sigma+g_2(a-p\frac{qa}{pa})^\sigma).$$

The contribution to elastic cross section which causes the asymmetry ($\sim I$) has the form

$$\frac{d\sigma_a}{dO}=\frac{2\alpha^3(1-y)^2}{sq^2}\int\frac{d^4q_1\,d^4q_2\,\delta^4((q_1+q_2+q)^2)}{q_1^2q_2^2}\,\delta((p_1-q_1)^2)S_{\mu\nu\lambda}T^{\mu\nu\lambda} \qquad (9)$$

where

$$y=2pq/s,\quad s=2pp_1,$$
$$S_{\mu\nu\lambda}=\frac{1}{4}\,\mathrm{Sp}\;p_1'\gamma_\mu(p_1-q_1)\gamma_\nu p_1\gamma^\lambda$$
$$T^{\mu\nu\lambda}=\mathrm{Im}\,\frac{1}{4}\;\mathrm{Sp}\;(p_2'+m)\hat{T}^{\mu\nu}(\hat{p}_2+m)\gamma^5 a\,\hat{\Gamma}^\lambda(q),$$
$$\Gamma^\lambda(q)=\gamma^\lambda F_1(q^2)-\frac{\gamma^\lambda\hat{q}}{2m}\,F_2(q^2).$$

It may be rewritten in the form:

$$\frac{d\sigma}{dO}=\frac{\alpha^3(1-y)^2}{q^2}\,I\,\left(\varphi_1(y)\,F_1(q^2)+\frac{s}{2m^2}\,\varphi_2(y)\,F_2(q^2)\right)\,\mathrm{tg}^2\frac{\theta}{2} \qquad (10)$$

The asymmetry (7) in terms of form factors φ_1, φ_2 has a form

$$A=\alpha\mathcal{P}^{-1}\,I\,\mathrm{tg}^2\frac{\theta}{2}\,(\varphi_1\,F_1(t)+\frac{s}{2M^2}\,\varphi_2\,F_2(t)),\quad t=q^2. \qquad (11)$$

We had estimated (10, 11) for the case $|X\rangle=p,\Delta$ and some model for tensor $T^{\mu\nu}$ for many-particles state. We find that the proton and Δ-isobara contributions to the asymmetry as a function of scattering angle tends to zero at forward and backward scattering directions $\theta\to 0,\pi$ and has a maximal value for $\theta\sim 20-50°$. The maximal value of A grows from $7\cdot10^{-4}$ to $1\cdot10^{-2}$ when the energies of electron increases from 1.5 to 4.5 GeV. This results are in agreement with the ones obtained in [1, 2]. For the energy 25 GeV the value of A reaches 5% at the scattering angle 4°. The contribution of the same order will, presumably, arise from another resonances. At higher values of momentum transferred $Q=\sqrt{-t}\gg m_p$ the main contribution will be going from the many-particles intermediate states $|X\rangle$. This fact follows from the weak dependence of the structure functions $T^{\mu\nu}$ (8) from Q compared with the rapid falling of elastic form-factors F_1, F_2. Asymmetry will have an order of 10% for $S\sim 50\;GeV^2$, $Q^2\sim 10\;GeV^2$, but elastic cross-section is small.

The same effect of one proton polarized asymmetry will take place also for DIS process $ep\to eX$. The asymmetry will be

$$A_{inel}=\frac{d^2\sigma(\theta,\varepsilon',\vec{a})-d^2\sigma(-\theta,\varepsilon',\vec{a})}{d^2\sigma(\theta,\varepsilon',\vec{a})+d^2\sigma(-\theta,\varepsilon',\vec{a})}=\alpha\,I\,\Pi^{-1}\,g_3\,\mathrm{tg}^2\frac{\theta}{2}\,\frac{\varepsilon'}{2m_p} \qquad (12)$$

where

$$\Pi=(\varepsilon'-\varepsilon)(W_2\cos^2\frac{\theta}{2}+2W_1\sin^2\frac{\theta}{2})=f(x)\,(x\cos^2\frac{\theta}{2}+\frac{\varepsilon-\varepsilon'}{m_p}\,\sin^2\frac{\theta}{2}),$$

$W_{1,2}$-are the structure functions in the nonpolarized case; the structure function g_3 measures the difference of quarks and antiquarks with the polarization transversal to the scattering plane:

$$g_3(x)=\int_0^1 d^3x\,\delta(\sum_i x_i-1)\,f(x_1,x_2,x_3)\,\sum_i\frac{m_i}{m_p}\,(n_{q_i}^{\odot}(x_i)-n_{\bar{q}_i}^{\odot}(x_i)-$$
$$-n_{q_i}^{\oplus}(x_i)+n_{\bar{q}_i}^{\oplus}(x_i))\,(\frac{e_i}{e})^2(1-\frac{m_i^2}{s+xm_i^2})\,\delta(x-(1+\frac{m_i}{m_p}(1-x_i))^{-1}), \qquad (13)$$

where $n_q^{\odot}(x)$, $(n_{\bar{q}}^{\odot}(x))$ are the numbers of quarks (antiquarks) with the averaged spin parallel to the spin of proton, $n_q^{\oplus}$ -antiparallel. The quarks with the spin in the scattering plane will not give the contribution to g_3. Function g_3 is the new one compared with the traditionally considered g_1, g_2. To measure it one need to have the accuracy better then 1%.

The analogous effect take also place in annihilation channel: nucleon-antinucleon annihilation into lepton pair [7] and the nucleon-antinucleon pair creation in the electron-positron collisions due to the nonzero imaginary part of form-factors in the time-like region of momentum transferred. In the latter case the polarization of the created proton in direction $\vec{s}$, $(s^2 = 1)$ $\mathcal{P}_s$ appears:

$$\mathcal{P}_{\vec{s}} = \frac{2\varepsilon}{m}\,(\vec{n}\times\vec{n}_+\vec{s})\,\beta^2\,\cos^2\theta\,\mathrm{Im}(F_1F_2^*)\,[2|F_1-F_2|^2 - \\ -\beta^2(|F_1|^2-\frac{s}{4m}|F_2|^2)\,\sin^2\theta]^{-1}$$

where $\beta^2 = 1-m^2/\varepsilon^2$, $\vec{n} = \vec{p}_-/\varepsilon$, $\vec{n}_+ = \vec{q}_+/|\vec{q}_+|$, $\theta = \widehat{\vec{p}_-\vec{q}_+}$ and p_-, q_+ are the momenta of the initial electron and the proton, m is the mass of the last. (We are grateful to A. Dubničkova for discussion of this question.)

1. The cross section of scattering of a longitudinally polarized electron on the resting quark of mass m and charge $Q_q e$ with the averaged spin $\vec{a}$ in the Born approximation in the frame of SM has a form: [1]

$$\frac{d\sigma_0}{dO_1'} = \frac{\alpha^2}{2s}\,\frac{\varepsilon_1 m}{(m+\varepsilon_1(1-c))^2}\,\mathcal{P}_q, \qquad e(p_1)+q(p_2)\to e(p_1')+q(p_2'),$$
$$s=(p_1+p_2)^2,\quad t=(p_1-p_1')^2,\quad u=(p_1-p_2')^2,$$
$$\begin{aligned}\mathcal{P}_q &= 2s^2[(a_1+a_2)^2+(a_3+a_4)^2]+4s^2\xi(a_1+a_2)(a_3+a_4)+\\ &+4\,a\,[s^2(a_1a_3+a_2a_4)+s(s-u)a_2a_3-sta_1a_4]+\\ &+2\xi\,a\,[2s^2(a_1a_2+a_3a_4)+s(s-u)(a_2^2+a_3^2)-st(a_1^2+a_4^2)]+\\ &+(s\leftrightarrow u,\vec{n}_1\to-\vec{n}_1',a_2\to-a_2,a_3\to-a_3,a_1\to a_1,a_4\to a_4),\end{aligned} \qquad (14)$$

where

$$a_1=\frac{Q_q}{t}+\frac{v_ev_q}{\tau},\ a_2=\frac{a_ea_q}{\tau},\ a_3=\frac{v_ea_q}{\tau},\ a_4=\frac{a_qv_q}{\tau},\ \tau=t-m_Z^2,$$
$$v_e=(1-4s_W^2)(4c_Ws_W)^{-1},\ v_q=(4c_Ws_W)^{-1}[\delta_{qp}(1-\frac{8}{3}s_W^2)-\delta_{qn}(1-\frac{4}{3}s_W^2)],$$

[1] Unpolarized case was considered in [8]

$$a_e=-(4c_Ws_W)^{-1},\ a_q=(4c_Ws_W)^{-1}(\delta_{qp}-\delta_{qn}),$$
$$s_W=\sin\theta_W,\ c_W=\cos\theta_W, \qquad (15)$$

$\vec{n}_1$, $\vec{n}_1'$ are the orts in the laboratory reference frame along the electrons 3-momenta; θ_W, m_Z-are the Weinberg angle and the Z-boson mass and $\theta=\widehat{\vec{p}_1\vec{p}_1}'$, $c=\cos\theta$ is the scattering angle.

The one-loop radiative corrections to the vertex functions as well as to photon and Z-boson Green functions in the SM was in most complete form obtained in [9, 10] by groups of W. Hollik and D. Bardin. We use their results here. Cross section (14) will be modified in such a way:

$$\frac{d\sigma}{dO_1'} = \frac{\alpha^2}{2s}\frac{\varepsilon_1 m}{(m+\varepsilon_1(1-c))^2}\,\tilde{\mathcal{P}}_q\,(1+\frac{\alpha}{2\pi}\Lambda_{1e}+\frac{\alpha}{2\pi}Q_q^2\Lambda_{1q}), \qquad (16)$$

where Λ_{1e} is the known QED-vertex correction factor:

$$\begin{aligned}\Lambda_{1e} &= -2\ln\frac{s}{\lambda^2}(\ln\frac{s}{m_e^2}-1)+\ln\frac{s}{m_e^2}+\ln^2\frac{s}{m_e^2}+\frac{\pi^2}{3}-4,\\ \Lambda_{1q} &= \Lambda_{1e}(m_e\to m_q),\end{aligned}$$

and the quantity $\tilde{\mathcal{P}}_q$ may be obtained from $\mathcal{P}_q$ by replacement $a_i\to\tilde{a}_i$,

$$\begin{aligned}\tilde{a}_1 &= \frac{Q_q}{t}(1-\Pi_\gamma)\{1+\frac{\alpha}{4\pi}[\Lambda_{2Z}(a_e^2+v_e^2+a_q^2+v_q^2)+G_L^e+Q_q^{-1}G_L^q]\}+\\ &+\frac{v_ev_q}{\tau}(1-\Pi_Z)\{1+\frac{\alpha}{4\pi}[\Lambda_{2Z}(3a_e^2+v_e^2+3a_q^2+v_q^2)+v_e^{-1}F_L^e+v_q^{-1}F_L^q]\}+\\ &+\frac{v_q+Q_qv_e}{\tau}\,\Pi_{\gamma Z},\\ \tilde{a}_2 &= \frac{a_ea_q}{\tau}(1-\Pi_Z)\{1+\frac{\alpha}{4\pi}[\Lambda_{2Z}(a_e^2+3v_e^2+a_q^2+3v_q^2)+a_e^{-1}F_L^e+a_q^{-1}F_L^q]\},\\ \tilde{a}_3 &= \frac{v_ea_q}{\tau}(1-\Pi_Z)\{1+\frac{\alpha}{4\pi}[\Lambda_{2Z}(3a_e^2+v_e^2+a_q^2+3v_q^2)+v_e^{-1}F_L^e+a_q^{-1}F_L^q]\}+\\ &+\frac{\alpha}{4\pi}[2Q_qv_qa_q\Lambda_{2Z}+G_L^q]-\frac{1}{\tau}a_q\Pi_{\gamma Z},\\ \tilde{a}_4 &= \frac{v_qa_e}{\tau}(1-\Pi_Z)\{1+\frac{\alpha}{4\pi}[\Lambda_{2Z}(a_e^2+3v_e^2+3a_q^2+v_q^2)+a_e^{-1}F_L^e+v_q^{-1}F_L^q]\}+\\ &+\frac{\alpha}{4\pi}[2Q_qv_ea_q\Lambda_{2Z}+Q_qG_L^e]-\frac{1}{\tau}Q_qa_e\,\Pi_{\gamma Z}.\end{aligned} \qquad (17)$$

The functions λ_{2Z}, $F_L^{e,q}$, $G_L^{e,q}$, Π_Z, Π_γ, $\Pi_{\gamma Z}$ are given in Appendix 1.

Consider now the box diagrams. The amplitudes of the diagrams with noncrossed boson lines (straight boxes) have nonzero imaginary part which will caused the contribution to the cross section proportional to the quantity I (6). It has a form:

$$\frac{d\sigma}{dO_1'} = \frac{d\sigma_0}{dO_1'}\,(1 + \alpha\frac{\varepsilon_1'}{2m}\mathrm{tg}^2\frac{\theta}{2}\,\mathcal{P}^{-1}\,I\,R), \tag{18}$$

where

$$\begin{aligned} R = {} & Q_q^3\Phi_1 + Q_q^2(a_e a_q \Phi_{aa} + a_e v_q \Phi_{av}) + Q_q(a_e^2 a_q^2 \Phi_{a2a2} + a_e^2 v_q^2 \Phi_{a2v2} + \\ & + a_e^2 a_q v_q \Phi a2av) + a_e^3 a_q^3 \Phi_{a3a3} + a_e^3 v_q^3 \Phi_{a3v3} + a_e^3 a_q^2 v_a \Phi_{a3a2v} + \\ & + a_e^3 a_q^2 v_q^2 \Phi_{a3av} + (4\sin^4\theta_W)^{-1}\Phi_W \delta_{pq}. \end{aligned} \tag{19}$$

In calculations we use the relation:

$$\begin{aligned} & \sum_{l,s,n} \frac{1}{4}\ \mathrm{Sp}\ \gamma^l \hat{a}_1 \gamma^s \hat{b}_1 \gamma^n \hat{c}_1(\alpha_1 + \beta_1\gamma_5)\ *\ \frac{1}{4}\ \mathrm{Sp}\ \gamma_l \hat{a}_2 \gamma_s \hat{b}_2 \gamma_n \hat{c}_2(\alpha_2 + \beta_2\gamma_5) = \\ & = 8(\alpha_1\alpha_2 + \beta_1\beta_2)\cdot a_1 a_2 \cdot b_1 b_2 \cdot c_1 c_2 + \frac{1}{2}\ \mathrm{Sp}\ \hat{c}_1 \hat{a}_2 \hat{b}_1 \hat{c}_2 \hat{a}_1 \hat{b}_2(\alpha_1 - \beta_1\gamma_5) \times \\ & \times(\alpha_2 + \beta_2\gamma_5). \end{aligned}$$

The first term in r.h.s. (18) corresponds to pure QED contribution, the last present only in the case when quark is a p-quark and arises from W^+, W^- exchange by fermions. The quantities Φ_i free from infrared divergences. They are put down in Appendix 3. Numerically main contribution to R go from Φ_1, Φ_W. In deriving (18) we put $a_v = 0$.

Consider now the contribution of the real part of the amplitude of the box diagrams. Extracting the terms which contain the "photon mass" λ, we may present it in the form:

$$d\sigma_0 + d\sigma_{box} = d\sigma_0\left(1 + \frac{2\alpha}{\pi}Q_q \ln(\frac{-s}{u})\ln(\frac{-t}{\lambda^2})\right)(1 + \frac{\alpha}{\pi}\mathcal{P}_q^{-1}\mathcal{K}), \tag{20}$$

$$\mathcal{K} = Q_q F_1 + a_e a_q F_{aa} + a_e v_q F_{av} + F_W + \Delta F_W + O(a_e^2 a_q^2), \tag{21}$$

where

$$\begin{aligned} a &= \vec{n}_1\vec{a},\ b = \vec{n}_1{}'\vec{a}, \\ F_W &= Q_q^{-1}(1-\xi)(4\sin^4\theta_W)^{-1}(A\delta_{pq} + B\delta_{qu}), \\ F_1 &= \frac{1}{2}(-\varphi(t,s)\ell_s + \varphi(t,u)\ell_u) + \pi^2\frac{s-u}{t} + \frac{1}{2}\xi(\ell_u - \ell_s)(\frac{u}{t}b - \frac{s}{t}a), \\ F_{aa} &= \frac{s-u}{2\tau}(\ell_s - \ell_u) + \frac{\xi}{2}\frac{u}{\tau}((2\frac{s}{t} - \frac{t}{u})\ell_s + 2\frac{u}{t}\ell_u)b - \end{aligned}$$

$$\begin{aligned} & - \frac{\xi}{2}\frac{s}{\tau}((2\frac{u}{t} - \frac{t}{s})\ell_u + 2\frac{s}{t}\ell_s)a + \frac{\pi^2(a+b)(u-s)}{2\tau}, \\ F_{av} &= \xi\frac{t}{2\tau}(-\varphi(t,s)\ell_s + \varphi(t,u)\ell_u) + \frac{\pi^2(s-u)}{\tau} + \frac{1}{2}(\ell_u - \ell_s)\frac{ub - sa}{\tau}, \\ & \ell_s = \ln^2(\frac{s}{t}),\ \ell_u = \ln^2(\frac{u}{t}),\ \varphi(t,s) = 1 + 2\frac{s}{t} + 4(\frac{s}{t})^2, \\ A &= (\frac{1}{2}sI_{WW} - I_{QWW})[(\frac{Q_q}{t} - \frac{a_e v_q}{\tau})(s^2 + \frac{1}{2}sub - \frac{1}{2}s(s-t)a) + \\ & + \frac{a_e a_q}{\tau}(s^2 - \frac{1}{2}sub - \frac{1}{2}s(2s-u)a)], \end{aligned} \tag{22}$$

$$B = \frac{1}{4}\ln(\frac{-u}{M_W^2})[\frac{s}{t}Q_q + \frac{s}{\tau}a_e(a_q - v_q)](-2 + a + b). \tag{23}$$

The quantity ΔF_W is given in Appendix 3; I_{WW}, I_{QWW} in Appendix 2. We omit in (20) the contributions of higher then 2 powers in a_q, v_q, v_e as well as the terms which don't contain doblelogarithms in F_1, F_{aa}, F_{av}. They have very cumbersome form and give small ($< 10\%$) contribution compared with ones putted in (20).

The "photon mass" λ entering in (20), $\Lambda_{1,e}$, $\Lambda_{1,q}$ will disappear in the sum with the soft photon emission cross section.

The cross section of emission of photon of energy (in laboratory frame) $\omega < \Delta\varepsilon$, $\Delta\varepsilon \ll \varepsilon_1$, has a form:

$$\begin{aligned} d\sigma^{soft} &= d\sigma_0\left(-\frac{\alpha}{4\pi^2}\right)\int_{k<\Delta\varepsilon}\frac{d^3k}{(k^2+\lambda^2)^{1/2}}\left(-\frac{p_1}{p_1k} - \frac{p_2}{p_2k} + \right. \\ & \left. + Q_q(\frac{p_1'}{p_1'k} + \frac{p_2'}{p_2'k})\right)^2. \end{aligned} \tag{24}$$

This cross section may be naturally separated into two parts: odd end even relative to change of leptons momenta $p_1 \leftrightarrow -p_1'$:

$$d\sigma^{soft} = d\sigma^{soft}_{odd} + d\sigma^{soft}_{even}. \tag{25}$$

The standard methods of calculations (see Appendix 2) give:

$$\begin{aligned} d\sigma^{soft}_{odd} &= d\sigma_0\frac{4\alpha Q_q}{\pi}\ln(-\frac{u}{s})\left(\ln(\frac{2\Delta\varepsilon}{\lambda}) - \frac{1}{2}\ln(\frac{-t}{m^2})\right), \\ d\sigma^{soft}_{even} &= d\sigma_0\frac{2\alpha}{\pi}\left\{(\ln(\frac{-t}{m_e^2}) - 1)\ln(\frac{m_e\Delta\varepsilon}{\lambda\sqrt{\varepsilon_1\varepsilon_1'}}) + \frac{1}{4}\ln^2(\frac{-t}{m_e^2}) - \frac{1}{4}\ln^2(\frac{\varepsilon_1'}{\varepsilon_1}) - \frac{\pi^2}{12} + \right. \\ & \left. + Q_q^2\left[(\ln(\frac{2\varepsilon_2'}{m}) - 1)\ln(\frac{m\Delta\varepsilon}{\lambda\varepsilon_2'}) + \frac{1}{2}\ln^2(\frac{2\varepsilon_2'}{m}) - \frac{1}{2}\ln(\frac{2\varepsilon_2'}{m}) + \frac{1}{2} - \frac{\pi^2}{12}\right]\right\}, \end{aligned}$$

The quantity λ disappears in the sum of (20), (16), (25). The auxiliary parameter $\Delta\varepsilon$ will also disappear when one add the hard photons emission process cross section:

$$e(p_1)+q(p_2)\to e(p_1')+q(p_2')+\gamma(k), \qquad k_0>\Delta\epsilon. \tag{26}$$

We define first the kinematical invariants:

$$\begin{gathered}
\chi_i=2p_ik,\quad \chi_i'=2p_i'k,\quad s=(p_1+p_2)^2,\quad s_1=(p_1'+p_2')^2,\\
t=(p_1-p_1')^2,\quad t_1=(p_2-p_2')^2,\quad u=(p_2-p_1')^2,\quad u_1=(p_1-p_2')^2,\\
p_i^2=p_i'^2=k^2=0,\quad s+s_1+t+t_1+u+u_1=0
\end{gathered} \tag{27}$$

and the chiral states of the photon, leptons and the quarks:

$$\begin{aligned}
\hat{e}^\lambda &= \begin{cases} 2N_1(\hat{p}_1'\hat{p}_1\hat{k}\omega_{-\lambda}-\hat{k}\hat{p}_1'\hat{p}_1\omega_\lambda)=e_1^\lambda \\ 2N_2(\hat{p}_2'\hat{p}_2\hat{k}\omega_{-\lambda}-\hat{k}\hat{p}_2'\hat{p}_2\omega_\lambda)=e_2^\lambda\ ; \end{cases}\\
u_\lambda &= \omega_\lambda u, \qquad \omega_\lambda=\frac{1}{2}(1+\lambda\gamma_5),\\
e_1^\lambda &= B_\lambda e_2^\lambda,\\
4N_1 &= ((p_1k)(p_1'k)(p_1p_1'))^{-\frac{1}{2}},\quad 4N_2=((p_2k)(p_2'k)(p_2p_2'))^{-\frac{1}{2}},\\
B_+ &= (B_-)^*,\quad |B_\lambda|=1,\quad B_+=2N_1N_2\ \mathrm{Sp}\ \hat{p}_1\hat{p}_1'\hat{k}\hat{p}_2'\hat{p}_2\hat{k}\omega_-.
\end{aligned}$$

To make the contraction over the vector indices we use the projection operators of kind

$$P=-(st_1)^{-1}\bar{u}(p_1)\hat{p}_2\omega_+u(p_2')\bar{u}(p_2')\hat{p}_2\omega_+u(p_1).$$

Chiral amplitudes $M^{\lambda_1\lambda_2'\lambda}$ have a form:

$$\begin{aligned}
M^{\pm\pm\pm} &= 4\bar{u}(p_2')\hat{p}_2\omega_\pm u(p_1)\bar{u}(p_1')\hat{p}_1\omega_\pm u(p_2)\rho_{\pm\pm},\\
M^{\pm\pm\mp} &= -4\bar{u}(p_2')\hat{p}_1'\omega_+u(p_1)\bar{u}(p_1')\hat{p}_2'\omega_+u(p_2)\rho_{\pm\pm},\\
M^{\pm\mp\pm} &= 4\bar{u}(p_2')\omega_\pm u(p_1)\bar{u}(p_1')\hat{p}_1\hat{p}_2'\omega_\mp u(p_2)\rho_{\pm\mp},\\
M^{\pm\mp\mp} &= -4\bar{u}(p_2')\hat{p}_2\hat{p}_1'\omega_\pm u(p_1)\bar{u}(p_1')\omega_\mp u(p_2)\rho_{\pm\mp}.
\end{aligned}$$

Using the completeness of the spinor chiral states and the density of polarization matrix of the initial particles (4) one obtains:

$$\begin{aligned}
|\bar{u}(p_2')\hat{p}_2\omega_\pm u(p_1)|^2 &= -st_1;\\
|\bar{u}(p_2')\hat{p}_1'\omega_\pm u(p_1)|^2 &= -s_1t;
\end{aligned}$$

$$\begin{aligned}
|\bar{u}(p_2')\omega_\pm u(p_1)|^2 &= -u_1;\\
|\bar{u}(p_2')\hat{p}_2\hat{p}_1'\omega_\pm u(p_1)|^2 &= -utt_1;\\
|\bar{u}(p_1')\hat{p}_1\omega_\pm u(p_2)|^2 &= -\frac{1}{2}st(1\mp\vec{n}_1\vec{a});\\
|\bar{u}(p_1')\hat{p}_2'\omega_\pm u(p_2)|^2 &= -\frac{1}{2}s_1t_1(1\mp\vec{n}_2'\vec{a});\\
|\bar{u}(p_1')\hat{p}_1\hat{p}_2'\omega_\pm u(p_2)|^2 &= -\frac{1}{2}u_1t_1t(1\pm\vec{n}_2'\vec{a});\\
|\bar{u}(p_1')\omega_\pm u(p_2)|^2 &= -\frac{1}{2}u(1\pm\vec{n}_1'\vec{a}).
\end{aligned}$$

For the summed on polarization states of the final particles of module square of the matrix element we obtain:

$$\begin{aligned}
\sum|M^{(\lambda)}|^2 \;=\;& 4tt_1(1+\xi)\{\rho_{++}(s^2(1-\vec{n}_1\vec{a})+s_1^2(1-\vec{n}_2'\vec{a}))+\\
&+\rho_{+-}(u_1^2(1+\vec{n}_2'\vec{a})+u^2(1+\vec{n}_1\vec{a}))\}+\\
&+4tt_1(1-\xi)\{\rho_{--}(s^2(1+\vec{n}_1\vec{a})+s_1^2(1+\vec{n}_2'\vec{a}))+\\
&+\rho_{-+}(u_1^2(1-\vec{n}_2'\vec{a})+u^2(1-\vec{n}_1\vec{a}))\},
\end{aligned} \tag{28}$$

where

$$\begin{gathered}
\rho_{\lambda_1\lambda_2}=\left|N_1B_{\lambda_1}\left(\frac{Q_q}{t_1}+\frac{C^e_{\lambda_1}C^q_{\lambda_2}}{\tau_1}\right)+N_2\left(\frac{Q_q}{t}+\frac{C^e_{\lambda_1}C^q_{\lambda_2}}{\tau}\right)\right|^2,\\
C^{e,q}_\lambda=v_{e,q}-\lambda a_{e,q},\quad n_2'=\frac{\vec{p}_2'}{\epsilon_2'},\quad \tau_1=t_1-M_Z^2.
\end{gathered} \tag{29}$$

The hard photons emission cross section has the form:

$$d\sigma=\frac{\alpha^3}{8\pi^2s}(\sum|M^{(\lambda)}|^2)\frac{d^3kd^3p_1'd^3p_2'}{\omega\ \epsilon_1'\epsilon_2'}\delta^4(p_1+p_2-p_1'-p_2'-k). \tag{30}$$

As a check one may put $Q_q=1, a=\xi=0$ in (30) and use the relation [11]:

$$\begin{aligned}
|\frac{N_1B_\pm}{t_1}+\frac{N_2}{t}|^2 &= -\frac{1}{8tt_1}\left(-\frac{p_1}{(p_1k)}-\frac{p_2}{(p_2k)}+\frac{p_1'}{(p_1'k)}+\frac{p_2'}{(p_2'k)}\right)^2=\\
&= -\frac{1}{8tt_1}(V_e-V_q)^2
\end{aligned} \tag{31}$$

to obtain the known result for the differential cross section of the process $e\mu\to e\mu\gamma$:

$$d\sigma=\frac{\alpha^3}{8s}(-(V_e-V_q)^2)\frac{s^2+s_1^2+u^2+u_1^2}{tt_1}\frac{d^3kd^3p_1'd^3p_2'}{\omega\ \epsilon_1'\epsilon_2'}\times$$

$$\times\delta^{(4)}(p_1+p_2-p_1'-p_2'-k). \qquad (32)$$

In conclusion of this section we discuss the possibility to improve the accuracy of the differential cross section cited above to take into account the terms of kind $(\frac{\alpha}{\pi}\ln\frac{-t}{m^2})^n$ by means of the renormalization group technique. We may present the cross section in the form:

$$d\sigma = \int_y^{1-\kappa}\frac{dx_1}{x_1}D(\frac{y}{x_1},\beta_e)D(x_1,\beta_e)\times$$
$$\times\int_0^{1-\kappa}dx_2\,D(x_2,\beta_q)d\hat{\sigma}_0\,(x_1p_1,\;x_2p_2;\;\frac{y}{x_1}p_1',\;x_2p_2')(1+\frac{\alpha}{\pi}K) \qquad (33)$$

where

$$D(x,\beta)=\frac{\beta}{2}(1+\frac{3}{8}\beta)(1-x)^{\frac{\beta}{2}-1}-\frac{1}{4}\beta(1+x),$$
$$\kappa=\Delta\epsilon/\epsilon,\quad \beta_e=\frac{2\alpha}{\pi}(\ln(\frac{-t}{m_e^2})-1),\quad \beta_q=\frac{2\alpha Q^2}{\pi}(\ln(\frac{-t}{m_q^2})-1)$$

and the cross section $d\hat{\sigma}_0=|1-\Pi(t)|^{-2}d\sigma_0$, $d\sigma_0$ is given in (14), where the substitutions of momenta according to (30) is to be done. The quantity K is a sum of the finite in m_e, m_q tend to zero contributions of $\Lambda_{1e,1q}$, K_W, $K_{soft,hard}$, $K_{\gamma,\gamma}$, $K_{\gamma,z}$. It may be reconstructed using the formulae cited above.

2. Return now to the question of asymmetry in electron-polarized proton elastic scattering. We consider now the contribution of the proton state as an intermediate state in box diagrams. Some closed expression may be obtained if we consider the form-factors entering the box amplitudes as constants evaluating the asymmetry. This approximation is rather good for energies $\epsilon_1=2-3\ GeV$ and give the result which exceeds about 10-20% the exact numerical calculation for higher energies. Adopting this approximation, one obtain for asymmetry:

$$A_{el}^{(p)}=\alpha\frac{\epsilon'}{2m}\mathrm{tg}^2\frac{\theta}{2}(\mathcal{P}^{-1})_{\xi=0}IR,\quad m=m_p, \qquad (34)$$

where $\mathcal{P}$ is given above (5) and the quantity R has a form:

$$R=\frac{1}{\pi}\mathrm{Im}\int\frac{d^4k}{i\pi^2\,\Delta Q}\left\{A+B(\frac{-ts}{q_1^2q_2^2}+\frac{s}{q_1^2}+\frac{s}{q_1^2})+\frac{s-u}{t}(D\frac{s}{q_2^2}-C\frac{s}{q_1^2})\right\},(35)$$

where

$$A=(F_1-\frac{s}{2m^2}F_2)(F_{11}F_{12}+F_{11}F_{22}+F_{21}F_{12})+\frac{s}{2m^2}(\frac{m^2}{s}-1)\times$$
$$\times(F_1+\frac{s}{2m^2}F_2)F_{21}F_{22};$$
$$B=\frac{s}{4m^2}(su-tm^2)^{-1}\{-(s^2-su-um^2)F_1F_{22}F_{11}-sm^2F_1F_{12}F_{21}+$$
$$+(s+m^2)(s-u)F_2F_{11}F_{12}+\frac{1}{2}(s(s-u)+tm^2)(F_1+\frac{s}{2m^2}F_2)\times$$
$$\times F_{21}F_{22}-\frac{t}{2}(s+m^2)F_2(F_{21}F_{12}-F_{11}F_{22})\}-\frac{s}{t}D-\frac{u}{t}C;$$
$$C=\frac{s}{2m^2}F_{11}(F_2F_{12}-F_1F_{22});\qquad D=\frac{s}{2m^2}F_{12}(F_1F_{21}-F_2F_{11}), \qquad (36)$$

and we have denoted:

$$F_i=F_i(t),F_{ij}=F_i(q_j^2);\;i,j=1,2;$$
$$\Delta=(p_1-q_1)^2-m_e^2+i0;\quad Q=(p_2+q_1)^2-m^2+i0;$$
$$s=2p_1p_2;\quad t=-2p_1p_1';\quad u=-2p_2p_1';\quad q_1+q_2+q=0. \qquad (37)$$

We use for the exact calculation the form-factors in the dipole model:

$$F_1(t)=(1-\frac{2.79\,t}{4m^2})(1-\frac{t}{4m^2})^{-1}Z,\quad F_2(t)=(1-\frac{t}{4m^2})^{-1}Z,$$
$$Z=(1-\frac{t}{0.71\,(GeV/c)^2})^{-2}. \qquad (38)$$

The result of them, discussed above, qualitatively similar to one obtained in [4] but the model different from (38) was used in [4] is drawn in figure 1.

Consider further the $\Delta(1232)$ isobar $I(Jp)=\frac{3}{2}(\frac{3}{2}^+)$ state in the intermediate state of the "box" diagram. We will use the following expression for $p\gamma\Delta$ vertex function $\Gamma^{\mu}_{p\gamma\Delta}$ [12]:

$$\Gamma^{\mu}_{p\gamma\Delta}=\frac{2i}{3}\bar{\Psi}_\nu(p_\Delta)\Gamma^{\nu\mu}u(p),\quad q=p_\Delta-p,$$
$$\Gamma_{\mu\nu}=G_M(q^2)K^{(M)}_{\mu\nu}+G_E(q^2)K^{E}_{\mu\nu}+G_C(q^2)K^{C}_{\mu\nu}=G_M(q^2)K^{(M)}_{\mu\nu},$$
$$K^{(M)}_{\mu\nu}=-\frac{3}{2}\frac{m_\Delta+m}{m}((m_\Delta+m)^2-q^2)^{-1}\epsilon_{\mu\nu\alpha\beta}p^\alpha_\Delta q^\beta,$$
$$G_\alpha(q^2)=G_\alpha(0)Z[0.83+0.17\exp(\frac{q^2}{0.192(GeV/c)^2})],\quad \alpha=M,E,C;$$
$$G_M(0)=2.28,\quad G_E(0)=0.07,\quad G_C(0)=0.0\pm0.34 \qquad (39)$$

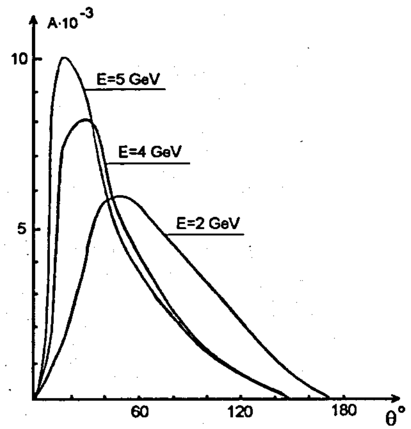


Figure 1: Contribution to $A_{el}(\theta)$ (see (34)) from proton's intermediate state.

and the known expression for the polarization density matrix for the isobar [13]:

$$\sum \Psi_\alpha(p)\bar{\Psi}_\beta(p) = (\hat{p} + m_\Delta)\left[g_{\alpha\beta} - \frac{1}{3}\gamma_\alpha\gamma_\beta - \frac{2}{3m_\Delta^2}p_\alpha p_\beta + \right.$$
$$\left. + \frac{1}{3m_\Delta}(p_\beta\gamma_\alpha - p_\alpha\gamma_\beta)\right]. \qquad (40)$$

Contribution to the asymmetry has the form:

$$\begin{aligned} A_{el}^{(\Delta)} &= \frac{\alpha}{12\pi}P^{-1}\mathrm{tg}^2(\frac{\theta}{2})\mathrm{Im}\int \frac{d^4q_1}{i\pi^2}\frac{N}{q_1^2q_2^2}\frac{G_M(q_1^2)G_M(q_2^2)}{\Delta Q}\frac{(m_\Delta + m)^2}{m^2} \times \\ &\quad \times((m_\Delta + m)^2 - q_1^2)((m_\Delta + m)^2 - q_2^2))^{-1}; \\ N &= S_{\sigma\mu\lambda}\mathrm{Re}\frac{i}{4}\ \mathrm{Sp}\ (\hat{p}_2' + m)(\hat{p}_2 + \hat{q}_1 + m)(A^\sigma B^\mu + 3B^\sigma A^\mu) \times \\ &\quad \times(\hat{p}_2 + m)\gamma_5\hat{a}\hat{\Gamma}_\lambda(q), \\ A^\sigma &= \gamma^\sigma\hat{p}_2'\hat{q}_2 - \hat{q}_2\hat{p}_2'\gamma^\sigma, \quad B^\mu = \gamma^\mu\hat{p}_2\hat{q}_1 - \hat{q}_1\hat{p}_2\gamma^\mu. \qquad (41) \end{aligned}$$

The plot of $A_{el}^{(\Delta)}$ is presented as a function of θ for the different energies in fig. 2.

The calculation of the imaginary part of the loop momenta integrals

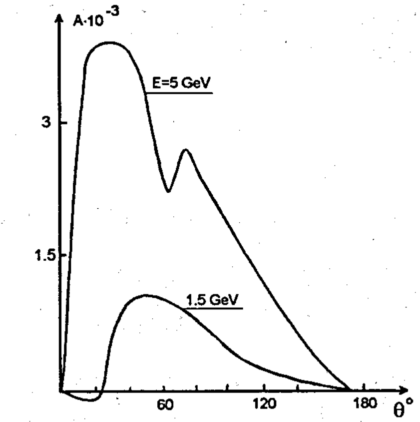


Figure 2: Contribution to $A_{el}(\theta)$ (see (41)) from isobara's intermediate state.

(41) may be transformed to the two fold integrals:

$$\begin{aligned} &\frac{1}{\pi}\mathrm{Im}\int\frac{d^4q}{i\pi^2}(Q\Delta)^{-1}F\left(\frac{-q_1^2}{m^2}, \frac{-q_2^2}{m^2}\right) = \\ &= \frac{s}{4\pi}\int_{-1}^{1}dc\int_0^{2\pi}d\varphi\, m^4\rho^{-2}F\left(\frac{s\epsilon(1-c)}{\rho}, \frac{s\epsilon'(1-c')}{\rho}\right), \\ &s = 2m\epsilon, \quad \epsilon' = \epsilon(1 + \frac{\epsilon}{m}(1 - \cos\theta))^{-1}, \quad \rho = m^2(m + \epsilon(1-c)), \\ &c' = c\cos\theta - \sin\theta(1-c^2)^{\frac{1}{2}}\cos\varphi. \end{aligned}$$

Discuss now the contribution to asymmetry from the many particles intermediate state. As we have noted a lot of tensor structures may be used to describe the nonforward Compton amplitude (8). We will use here such a model of it:

$$\begin{aligned} \hat{T}^{\mu\nu} &= e^{-2}\sum |J^\mu|X\rangle\langle X|J^\nu|\delta((p_2+q_1)^2 - M_X^2) = \int_{\Delta_{min}}^{1}\frac{d\Delta\ \rho(\Delta)}{(p_2+q_1)^2} \times \\ &\quad \times(\hat{p}_2 + \hat{q}_1 + M_X)\ (q_1q_2\cdot g^{\mu\nu} - q_1^\nu q_2^\mu)\ \delta((p_2+q_1)^2 - M_X^2). \qquad (42) \end{aligned}$$

Really one may consider the expression (42) as a scaling hypothesis. The function ρ which describe the density of many-particles states with mas M_X, $\Delta = M_X^2/s$, we suggest to be normalized

$$\int_{\Delta_{th}}^{1} dx\, \rho(x) = 1. \tag{43}$$

The estimation of asymmetry with $T^{\mu\nu}$ from (42) give the result:

$$A = \frac{\alpha}{4} y(1-y) \ln(\frac{s}{m_e^2})(F_2)^{-1}\left(1 + c\left(\frac{\alpha \ln(\frac{s}{m_e^2})}{4\pi F_2}\right)^2\right)^{-1} \vec{n}' \times \vec{n}\vec{a},\ y = \frac{Q^2}{s}. \tag{44}$$

In deriving (44) we neglect the contribution of Dirac form-factor F_1 and the contributions which don't contain large logarithmic enhancement. The estimation (44) is rather rough. The second term in the denominator (44) represent the contribution of the loop diagram. It provide the fact that the asymmetry don't exceed the unity.

The reason for scaling (42) is the fact that many channels at high electron energies are open. Contribution to asymmetry of each one is small, nevertheless the question about the total contribution is open. Naive quark counting rule is to be modified at higher orders of perturbation theory. The large values of asymmetry (of orders of 10-20%) in the future precise experiments will be the arguments in favor of validity the scaling hypothesis (42).

We note also the connection of the double Compton forward scattering

$$\langle p, a | J^\mu(q_1) J^\nu(q_2) J^\lambda(-q_1 - q_2) | p, a \rangle \tag{45}$$

which describes the unpolarized electron-polarized proton inelastic scattering asymmetries with the odderons problem [14]: in both cases we have C-odd state of three vector particles in the scattering channel.

In conclusion we say some words about the one particle polarization induced asymmetry in the case of deep inelastic scattering. The order of it is 1%, as follows from (12). So to measure the g_3 structure function the accuracy required is to be better then 1%. The result (13) follows from (36) if one put $F_1 = 1$, $F_2 = 0$.

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Appendix 1

We use the expressions for vertex functions and self-energy functions of boson and fermions which was obtained in [10]. The t'Hooft-Feynman gauge and the on-mass-shell regularization scheme was used in [10]. The renormalized vertex functions have a form:

$$\begin{aligned}
\Gamma_\mu^{Zff} &= ie\gamma_\mu(v_f - a_f\gamma_5)(1 + \frac{\alpha}{4\pi}\Lambda_{1f})\{1 + \frac{\alpha}{4\pi}(v_f^2 + a_f^2 - 2v_f a_f \gamma_5)\Lambda_{2Z} + \\
&+ (v_f + a_f)^{-1} F_L^f (1 - \gamma_5)\} \\
\Gamma_\mu^{\gamma ff} &= -ie\gamma_\mu Q_f (1 + \frac{\alpha}{4\pi} Q_f^2 \Lambda_{1f})\{1 + \frac{\alpha}{4\pi}(v_f^2 + a_f^2 - 2v_f a_f \gamma_5)\Lambda_{2Z} + \\
&+ Q_f^{-1} G_L^f (1 - \gamma_5)]\},
\end{aligned}$$

where Q_f is the fermion charge in units of an electron charge. Schematically representing the sum of Born amplitude with loop correction to the vertex functions and Green function of boson (except of "box"-diagrams) in the form (spinors are missed)

$$\begin{aligned}
m^{Born} + m^{virt} &= 4\pi\alpha i\{\frac{Q_q}{t}(1 - \Pi_\gamma)\Gamma_\mu^{\gamma ee} * \Gamma_\mu^{\gamma qq} + \frac{1}{T}(1 - \Pi_Z)\, \Gamma_\mu^{Zee} * \Gamma_\mu^{Zqq} - \\
&- \frac{1}{T}\Pi_{Z\gamma}(\Gamma_\mu^{\gamma ee} * \Gamma_\mu^{Zqq} + \Gamma_\mu^{Zee} * \Gamma_\mu^{\gamma qq})\} = 4\pi\alpha i(1 + \frac{\alpha}{4\pi}\Lambda_{1e})(1 + \frac{\alpha}{4\pi}Q_q^2\Lambda_{1q}) \times \\
&\times\{a_1\gamma_\mu * \gamma_\mu + a_2\gamma_\mu\gamma_5 * \gamma_\mu\gamma_5 + a_3\gamma_\mu * \gamma_\mu\gamma_5 + a_4\gamma_\mu\gamma_5 * \gamma_\mu\}
\end{aligned}$$

the quantities a_i are listed in text in terms of Λ, F, G, Π.

$$\begin{aligned}
\Lambda_{1f} &= -2\ln|\frac{t}{\lambda^2}|(\ln|\frac{t}{m_f^2}| - 1) + \ln|\frac{t}{m_f^2}| + \ln^2|\frac{t}{m_f^2}| + \frac{\pi^2}{3} - 4, \\
F_L^e &= (8s^3c)^{-1}\Lambda_{2W} - \frac{3}{4}cs^{-3}\Lambda_{3W}, \\
F_L^u &= -(1 - \frac{2}{3}s^2)(8s^3c)^{-1}\Lambda_{2W} + \frac{3}{4}cs^{-3}\Lambda_{3W}, \\
F_L^d &= (1 - \frac{4}{3}s^2)(8s^3c)^{-1}\Lambda_{2W} \mp \frac{3}{4}cs^{-3}\Lambda_{3W}, \quad G_L^e = -\frac{3}{4}s^{-2}\Lambda_{3W}, \\
G_L^u &= -\frac{1}{12}s^{-2}\Lambda_{2W} + \frac{3}{4}s^{-2}\Lambda_{3W}, \quad G_L^d = \frac{1}{6}s^{-2}\Lambda_{2W} - \frac{3}{4}s^{-2}\Lambda_{3W}.
\end{aligned}$$

In expressions for F_L^i, G_L^i we have used notations $s = \sin\theta_W$, $c = \cos\theta_W$. The functions Λ_{2W}, Λ_{3W} are dependent on $x = m_W^2/t$:

$$\Lambda_2(x) = -\frac{7}{2} - 2x - (2x + 3)\ln(-x) + 2(1 + x)^2\,(\frac{1}{2}\ln^2(-x) +$$

$$+\int_{1+z}^{1}\frac{dz}{t}\ln(1-z)),$$

$$\Lambda_3(x)=\frac{5}{6}-\frac{2}{3}x+\frac{1}{3}(2x+1)L+\frac{2}{3}x(x+2)L^2,$$
$$L=\ln((\sqrt{1-4x}+1)/(\sqrt{1-4x}+1)).$$

The quantities $\Pi_{\gamma\gamma}(t)$, $\Pi_{\gamma Z}(t)$, $\Pi_{ZZ}(t)$ have the form

$$\Pi_{\gamma\gamma}(t)=\frac{\alpha}{4\pi t}(f_{\gamma\gamma}(t)-tf'_{\gamma\gamma}(0)),$$
$$f_{\gamma\gamma}(t)=\frac{4}{3}\sum_f Q_f^2(t+2m_f^2)\,F(t,m_f,m_f)-(3t+4m_W^2)\,F(t,m_W,m_W),$$
$$\Pi_{\gamma Z}(t)=\frac{\alpha}{4\pi}(\frac{1}{t}\,f_{\gamma Z}(t)+a_{\gamma Z}),\quad \Pi_{ZZ}(t)=\frac{\alpha}{4\pi}\left(\frac{f_{ZZ}(t)-f_{ZZ}(M_Z^2)}{t-M_Z^2}+a_{ZZ}\right),$$
$$f_{\gamma Z}(t)=-\frac{4}{3}\sum Q_f v_f(t+2m_f^2)\,F(t,m_f,m_f)+\frac{1}{cs}[(3s^2+\frac{1}{6})t+$$
$$+4(c^2+\frac{1}{3})m_W^2]\,F(t,m_W,m_W),$$
$$f_{ZZ}(t)=\frac{4}{3}\sum_{l=e,\mu,\tau}2a_l^2t(\frac{5}{3}-\ln\frac{-t}{m_l^2})+\frac{4}{3}\sum_{f\neq\nu}[(v_f^2+a_f^2)(t+2m_f^2)-$$
$$-\frac{3}{8c^2s^2}m_f^2]\,F(t,m_f,m_f)+\frac{1}{12c^2s^2}\{[-40c^4(t+2m_W^2)+$$
$$+(c^2-s^2)(8m_W^2+t)+12m_W^2]\,F(s,m_W,m_W)+$$
$$+[10M_Z^2-2m_H^2+t+\frac{m_H^2-m_t^2}{t}]\,F(t,M_Z,m_H)\}$$

where

$$F(t,m_1,m_2)=-1+\frac{m_1^2+m_2^2}{m_1^2-m_2^2}\ln\frac{m_1}{m_2}-$$
$$-\int_0^1 dx\,\ln\frac{-tx(1-x)+x(m_2^2-m_1^2)+m_1^2}{m_1m_2},\qquad F(0,m_1,m_2)=0,$$

and finite renormalization constants ($a_{\gamma Z}$, a_{ZZ}) are listed in [9, 10].

Appendix 2

We use the Wick rotation for loop momenta k:

$$k^2=-z,\quad d^4k=i\pi^2z\,dz,\quad 0<z<\Lambda^2.$$

Here we restrict ourselves to integrals concerned to "box" diagrams and containing from 2 to 4 propagators of kind that are contained in "straight boxes"

$$(\Delta)=(-k+p_1)^2-m_e^2,\quad (Q)=(k+p_2)^2-m^2,$$
$$(1)=k^2-\lambda^2,\qquad (2)=(q+k)^2-\lambda^2,$$
$$(1Z)=k^2-M^2,\qquad (2Z)=(q+k)^2-M^2,$$

$$M^2=M_Z^2,\quad p_2'^2=p_2^2=m^2,\quad p_1'^2=p_1^2=m_e^2,\quad q=p_1'-p_1=p_2-p_2'.$$

Integrals corresponding to the "crossing boxes" contain propagator $(Q')=(p_2'-k)^2-M^2$ instead of (Q). They may be drawn from the integrals listed above by the change $p_2\leftarrow -p_2'$. Let us define the kinematical invariants

$$s=(p_1+p_2)^2,\quad t=(p_1-p_1')^2,\quad u=(p_2-p_1')^2,\quad s+t+u=2m^2$$

and we shall assume $s\sim|-t|\sim|u|\sim M_Z^2$. For the case of denominators (ΔQ) we have

$$((\Delta)(Q))^{-1}=\int_0^1 dx\,[k^2-2kp_x]^{-2}=\int_0^1 dx\,[(k-p_x)^2-p_x^2]^{-2}$$

and after integration over the k

$$\int_0^1\{\ln\frac{\Lambda^2}{p_x^2}-1\},p_x^2=m_e^2x+m^2(1-x)-sx(1-x).$$

As a result we have

$$I_{(\Delta Q)}=L_e+1-\ln(\tfrac{-s+i0}{m_e^2}),\quad I_{(\Delta 1)}=I_{(\Delta 2)}=L_e+1,$$
$$I_{(1Q)}=I_{(2Q)}=L_m+1,\qquad I_{(12)}=L_e-\ln(\tfrac{-t}{m_e^2})+1,$$
$$I_{(2,2Z)}=I_{(1,1Z)}=I_{(\Delta,1Z)}=I_{(\Delta,2Z)}=I_{(Q,1Z)}=I_{(Q,2Z)}=L_M,$$
$$I_{(1,2Z)}=I_{(2,1Z)}=L_M-\int_0^1 dx\ln(1-x\tfrac{t}{M^2}),$$
$$I_{(1Z,2Z)}=L_M-\int_0^1 dx\ln(1-x(1-x)\tfrac{t}{M^2})-1,$$

where $L_e = \ln\frac{\Lambda^2}{m_e^2}$, $L_m = \ln\frac{\Lambda^2}{m^2}$, $L_M = \ln\frac{\Lambda^2}{M^2}$. It is possible to use these integrals for checking of vector and tensor integrals with large number of denominators. Here and below we use notations

$$\left(I_{(ij)},\ I_{(ijk)},\ I_{(ijkl)}\right) = \int \frac{d^4k}{i\pi^2}\left(\frac{1}{(i)(j)}, \frac{1}{(i)(j)(k)}, \frac{1}{(i)(j)(k)(l)}\right).$$

Scalar integrals with three denominators. As an example let us consider $1/(\Delta Q1)$. Combining first denominators (Δ) and (Q) with the aid of parameter x and then the result with denominator (1) with the aid of z, integrating over 4-momentum in loop we reduce it to the form

$$I_{(\Delta Q1)} = \int \frac{d^4k}{i\pi^2}\frac{1}{((p_1-k)^2-m_e^2)\,((k+p_2)^2-m^2)\,(k^2-\lambda^2)} =$$

$$= -\int\limits_0^1 \frac{z\,dz\,dx}{z^2p_x^2+\lambda^2(1-z)} = -\int\limits_0^\varepsilon \frac{z\,dz\,dx}{z^2p_x^2+\lambda^2} - \int\limits_\varepsilon^1 \frac{dz}{zp_x^2}\,dx = -\frac{1}{2}\int\limits_0^1 \frac{dx}{p_x^2}\ln\frac{p_x^2}{\lambda^2}.$$

Further integration over x may be done with the aid of relations

$$\int\limits_0^1 \frac{dx}{p_x^2} = -\frac{2}{s}\ln\frac{-s-i0}{m_e m}$$

$$\int\limits_0^1 \frac{dx}{p_x^2}\ln\frac{p_x^2}{m^2} = -\frac{1}{s}\left\{-\frac{\pi^2}{3} + \frac{1}{2}\ln^2\left(\frac{-s}{m_e^2}\right) + \frac{1}{2}\ln^2\left(\frac{-s}{m^2}\right) + \ln\left(\frac{-s}{m_e^2}\right)\ln\frac{m_e^2}{m^2}\right\}.$$

Similar computations yield the following expressions:

$$I_1 \equiv I_{(\Delta Q1)} = I_{(\Delta Q2)} = \tfrac{1}{2s}\left[\ln^2\left(\tfrac{-s}{m_e m}\right) - 2\ln\tfrac{\lambda^2}{m_e m}\ln\left(\tfrac{-s}{m_e m}\right) - \tfrac{\pi^2}{3} - \ln^2\left(\tfrac{m}{m_e}\right)\right],$$

$$I_{(\Delta 12)} = I_{(Q12)} = \tfrac{1}{t}\left(\tfrac{1}{2}\ln^2\left(\tfrac{-t}{m_e^2}\right) + \tfrac{2}{3}\pi^2\right),$$

$$I_{(\Delta Q1Z)} = I_{(\Delta Q2Z)} = \tfrac{1}{s}\left(\tfrac{\pi^2}{6} - F\left(\tfrac{s+M^2}{M^2}\right)\right),$$

$$I_{(Q1Z2)} = I_{(Q12Z)} = \tfrac{1}{t}\left[F\left(\tfrac{t}{M^2}\right) + 2F\left(\tfrac{t}{t-M^2}\right) + \ln\tfrac{M^2-t}{M^2}\ln\tfrac{M^2}{m^2} + \ln^2\left(\tfrac{M^2-t}{M^2}\right)\right],$$

$$I_{(\Delta 1Z2)} = I_{(\Delta 12Z)} = \tfrac{1}{t}\left[F\left(\tfrac{t}{M^2}\right) + 2F\left(\tfrac{t}{t-M^2}\right) + \ln\tfrac{M^2-t}{M^2}\ln\tfrac{M^2}{m_e^2} + \ln^2\left(\tfrac{M^2-t}{M^2}\right)\right],$$

$$I_{\Delta 1Z2Z} = I_{Q1Z2Z} = \tfrac{2}{t}\int\limits_0^1 \tfrac{dy}{y}\ln\left(1 - \tfrac{t}{M^2}y(1-y)\right), \quad F(x) = -\int\limits_0^1 \tfrac{dz}{z}\ln(1-zx).$$

Scalar integrals with 4 denominators and various mass relations are considered in [9] and recently were analyzed in [15] with the dispersion relation. The method we apply is similar to ones used in [16]. So for $I_{(\Delta Q12)} \equiv I$ we obtain

$$I_{(\Delta Q12)} = \int vx^2\,dv\,dx\,dy\,[x^2v^2p_y^2 + x^2(1-v)q^2 - x(1-v)(q^2-\lambda^2) + \\ +\lambda^2(1-x)]^{-2}.$$

The sequence of operations [16] gives

$$I \equiv I(\Delta Q12) = -\frac{1}{t}\ln\left|\frac{-t}{\lambda^2}\right|\int\frac{dx}{p_x^2} = \frac{2}{ts}\ln\left(\frac{-t}{\lambda^2}\right)\ln\left(\frac{-s}{mm_e}\right).$$

In complete analogy with [16] we get

$$I_Z \equiv I_{(\Delta Q1,2Z)} = I_{(\Delta Q,1Z,2)} = \int vx^2\,dv\,dx\,dy\,[x^2v^2p_y^2 + x^2(1-v)q^2 -$$

$$-x(1-v)(q^2-\lambda^2) + M^2(1-x)]^{-2} = \frac{1}{2(M^2-q^2)}\int\limits_0^1 \frac{dy}{p_y^2}\left\{\ln\frac{p_y^2}{\lambda^2} + \right.$$

$$\left. +2\int\limits_0^1 \frac{dv}{a\Delta}[b + vcp_y^2 - 2M^2a(M^2-q^2)vp_y^2\Delta^{-1/2}L]\right\},$$

where

$$\begin{aligned}
p_y^2 &= ym^2 + (1-y)m_e^2 - sy(1-y),\\
a &= q^2(1-v) + vp_y^2, \quad b = -q^4(1-v)(M^2 + q^2(1-v)),\\
c &= 4M^2q^2 + v(q^4 - 5q^2M^2) + v^2(4M^2p_y^2 - q^4),\\
\Delta &= (q^2(1-v) - M^2)^2 - 4M^2v^2p_y^2,\\
L &= \ln[4M^2p_y^2v^2(M^2 - q^2(1-v) - \sqrt{\Delta})^{-2}].
\end{aligned}$$

An integral with two $Z-$ or $W^\pm-$ boson propagators may be reduced by the described routine to the single-fold integral. It is free of infrared divergencies and has the form

$$I_{(\Delta QZZ)} = \int vx^2\,dv\,dx\,dy\,[x^2(p_y^2v^2 + (1-v)s) + xv(M^2-m^2) - \\ -x(1-v)(s-m^2)]^{-2}$$

It is possible to put $m^2 = 0$ here. Direct integration yields

$$I_{ZZ} \equiv I_{\Delta QZZ} = \int\limits_0^1 \frac{dy}{tsy(1-y) - M^2(M^2+s)} \ln\frac{(-s-i0)(M^2 - ty(1-y))}{M^4}$$

$$\mathrm{Im}\, I_{\Delta QZZ} = -\pi \int\limits_0^1 \frac{dy}{sty(1-y) - M^2(M^2+s)}.$$

Let us to note that the combinations which are free from infrared divergencies are

$$\Phi = tI_{\Delta Q12} - 2I_{\Delta Q1} = \frac{1}{s}\left[-\ln^2\left(\frac{-s-i0}{-t}\right) + \ln^2\left(\frac{-t}{m_e m}\right) + \frac{\pi^2}{3} + \ln^2\frac{m}{m_e}\right],$$

$$\Phi_Z = \tau I_Z - I_1 = -\int \frac{dx}{p_x^2} \int \frac{dv}{\Delta A}\{b + vcp_x^2 + 2M^2 av\tau p_x^2 \Delta^{-1/2} L\},$$

$$\tau = t - M^2, \quad \mathrm{Im}\, \Phi = \frac{2\pi}{s}\ln\frac{s}{t}.$$

Taking into account the symmetry properties of expressions under integrals the vector integrals of the form

$$\int \frac{d^4k}{i\pi^2} k^\mu \left(\frac{1}{(i)(j)(k)}, \frac{1}{(i)(j)(k)(l)}\right) \equiv \left(\frac{k}{(i)(j)(k)}, \frac{k}{(i)(j)(k)(l)}\right)$$

can be reduced to the following expressions

$$\frac{k}{\Delta Q12} = \alpha(p_1 + p_1') + \gamma p, \quad \frac{k}{\Delta 12} = \alpha_\Delta (p_1 + p_1'),$$

$$\frac{k}{Q12} = \alpha_Q(p_1 + p_1') + \gamma_Q p, \quad \frac{k}{\Delta Q1} = \beta_1 p_1' + \gamma_1 p, \quad \frac{k}{\Delta Q2} = \beta_1 p_1 + \gamma_1 p,$$

where

$$\alpha = \frac{1}{t}I_{\Delta Q1} + \frac{1}{2u}I_{\Delta 12} + \frac{1}{2u}I_{Q12} - \frac{s}{2ut}\Phi,$$

$$\gamma = \frac{t}{2us}I_{\Delta 12} + \frac{u-s}{2us}I_{Q12} - \frac{1}{2u}\Phi,$$

$$\alpha_Q = \frac{1}{t}\ell_q^t, \quad \gamma_Q = I_{Q12} - \frac{2}{t}\ell_q^t, \quad \beta_1 = I_{\Delta Q1} - \frac{1}{s}(\ell_q^s + \ell_e^s),$$

$$\gamma_1 = \frac{1}{s}\ell_q^s, \quad \alpha_\Delta = \frac{1}{t}\ell_e^t, \quad p = p_1 + p_2$$

In the same way:

$$\frac{k}{\Delta Q, 1Z, 2} = \alpha_{Z\gamma} p_1 + \beta_{Z\gamma} p_1' + \gamma_{Z\gamma} p,$$

$$\frac{k}{\Delta Q, 1, 2Z} = \alpha_{Z\gamma} p_1' + \beta_{Z\gamma} p_1 + \gamma_{Z\gamma} p,$$

$$\alpha_{Z\gamma} = I_{\Delta Q, 1Z, 2} - (2stu)^{-1}[s(u-t)\Phi_Z - stI_{QZ\gamma} - s^2 I_{\Delta QZ} - stI_{\Delta Z\gamma}],$$

$$\beta_{Z\gamma} = -(2stu)^{-1}[s^2\Phi_Z - tsI_{QZ\gamma} + s(t-u)I_{\Delta QZ} - stI_{\Delta Z\gamma}],$$

$$\gamma_{Z\gamma} = -(2stu)^{-1}[ts\Phi_Z + t(s-u)I_{QZ\gamma} - stI_{\Delta QZ} - t^2 I_{\Delta Z\gamma}],$$

$$\frac{k}{\Delta QZ} = \beta_Z p_1' + \gamma_Z p, \quad \beta_Z = \frac{s + 2M^2}{s} I_{\Delta QZ} - \frac{2}{s}(\ell_M^s - 1),$$

$$\gamma_Z = -\frac{M^2}{s} I_{\Delta QZ} + \frac{1}{s}(\ell_M^s - 1),$$

$$\frac{k}{\Delta Z2} = \alpha_{\Delta Z} p_1 + \beta_{\Delta Z} p_1',$$

$$\alpha_{\Delta Z} = \frac{1}{t}[M^2 I_{\Delta Z1} + 1 + \ln\frac{M^2}{m_e^2} + i_M^t], \quad \beta_{\Delta Z} = \frac{1}{t} i_M^t,$$

$$\frac{k}{\Delta QZZ} = \alpha_{ZZ}(p_1 + p_1') + \gamma_{ZZ} p,$$

$$\alpha_{ZZ} = -\frac{s + 2M^2}{2u} I_{ZZ} + \frac{1}{u} I_{QZZ} - \frac{1}{u} I_{\Delta QZ}$$

$$\gamma_{ZZ} = -\frac{t - 2M^2}{2u} I_{ZZ} - \frac{1}{u} I_{QZZ} + \frac{1}{u} I_{\Delta QZ},$$

$$\frac{k}{QZZ} = \alpha_{QZZ}(p_1 + p_1') + \gamma_{QZZ} p, \quad \frac{k}{\Delta ZZ} = \alpha_{QZZ}(p_1 + p_1'),$$

$$\alpha_{QZZ} = \frac{1}{t}[1 + j + m^2 I_{QZZ}], \quad \gamma_{QZZ} = \frac{t - 2M^2}{t} I_{QZZ} - \frac{2}{t}(1 + j)$$

Besides of integrals defined we use notations

$$\ell_M^s = \ln\frac{s}{M^2}, \quad \ell_e^s = \ln\frac{s}{m_e^2}, \quad \ell_q^s = \ln\frac{s}{m^2},$$

$$i_M^t = \int\limits_0^1 dx \ln(1 - x\frac{t}{M^2}), \quad j = \int\limits_0^1 dx \ln(1 - x(1-x)\frac{t}{M^2}).$$

This method of computation [2] requires no additional integrals.

We put here the scalar integrals of "box" diagram with excited state of the intermediate nucleon: $I_1^\star = I_{\Delta Q^\star 1}$, $I_0^\star = I_{\Delta Q^\star 12}$, which may be obtained

[2]one of the authors (EAK) is grateful to D. Yu. Bardin for useful discussion on this problem

from the cited above by replacement $(Q) \to (Q^\star) = (p_2+k)^2 - M_X^2$, $M_X^2 - m^2 > 0$. In the similar manner one obtains

$$I_1^\star = -\int_0^1 dx \int_0^1 dy\,[p_x^2 y + xM^2 - i0]^{-1},$$

$$I_0^\star = \int_0^1 dz \int_0^1 dx \int_0^1 dy\,\{z[p_x^2y^2 + (1-y)q^2] - q^2(1-y) + xyM^2 - i0\}^{-2},$$

Using the known relation Im $(x-i0)^{-1} = \pi\delta(x)$ one obtains:

$$\mathrm{Im}\, I_1^\star = \frac{\pi}{s}\ln(\frac{s}{m_e^2}),\quad \mathrm{Im}\, I_0^\star = \frac{2\pi}{q^2(s-M^2)}\ln(\frac{s}{m_e^2}).$$

The cross section of emission of soft (in laboratory frame) photon in ep-scattering has the form

$$d\sigma^{soft} = d\sigma_0(-\frac{4\pi\alpha}{16\pi^3})\int_0^{\Delta\varepsilon}\frac{d^3k}{\omega}\left(-\frac{p_1}{p_1k} - \frac{p_2}{p_2k} + \frac{p_1'}{p_1'k} + \frac{p_2'}{p_2'k}\right)^2$$

$$p_1^2 = p_1'^2 = m_e^2,\ p_2^2 = p_2'^2 = m^2,\ p_2=(m,0,0,0),\ \theta = \widehat{\vec{p}_1 \vec{p}_1}' \ll 1,$$
$$-t = 2(p_1p_1')^2 \gg m^2 \gg m_e^2,\ s\sim -t \sim -u,\ y = \varepsilon_1'/\varepsilon_1 \sim 1.$$

Common but somewhat cumbersome calculation yields:

$$\left(-\frac{4\pi\alpha}{16\pi^3}\right)\int_{k<\Delta\varepsilon}\frac{d^3k}{\omega}\left\{\frac{m_e^2}{(p_1k)^2},\frac{m_e^2}{(p_1'k)^2},\frac{m^2}{(p_2k)^2},\frac{m^2}{(p_2'k)^2}\right\} =$$
$$= -\frac{\alpha}{\pi}\left\{\ln\frac{m_e\Delta\varepsilon}{\lambda\varepsilon_1},\ \ln\frac{m_e\Delta\varepsilon}{\lambda\varepsilon_1'},\ \ln\frac{2\Delta\varepsilon}{m}-1,\ \ln\frac{m\Delta\varepsilon}{\lambda\varepsilon_1}\right\},$$

$$\left(-\frac{4\pi\alpha}{16\pi^3}\right)\int_{k<\Delta\varepsilon}\frac{d^3k}{\omega}\frac{p_1p_1'}{(p_1k)(p_1'k)} = -\frac{\alpha}{\pi}\left\{\ln\frac{m_e\Delta\varepsilon}{\lambda\varepsilon_1}\ln\left(\frac{-t}{m_e^2}\right)+\right.$$
$$\left.+\frac{1}{4}\ln^2\left(\frac{-t}{m_e^2}\right) - \frac{1}{2}\ln y\ln\left(\frac{-t}{m_e^2}\right) - \frac{1}{4}\ln^2 y - \frac{\pi^2}{12}\right\},$$

$$\left(-\frac{4\pi\alpha}{16\pi^3}\right)\int_{k<\Delta\varepsilon}\frac{d^3k}{\omega}\frac{p_1'p_2'}{(p_1'k)(p_2'k)} = -\frac{\alpha}{\pi}\left\{\ln\frac{\sqrt{mm_e}\Delta\varepsilon}{\lambda\sqrt{\varepsilon_1'\varepsilon_2'}}\ln\frac{s}{mm_e}-\right.$$
$$\left.-\frac{1}{4}\ln^2\left(\frac{\varepsilon_2'}{\varepsilon_1'}\right) - \frac{1}{2}\ln\left(\frac{\varepsilon_2'm_e}{\varepsilon_1'm}\right)\ln\frac{m_e}{m} - \frac{\pi^2}{12} + \frac{1}{8}\ln^2\left(\frac{s}{m^2}\right) + \frac{1}{8}\ln^2\left(\frac{s}{m_e^2}\right)\right\},$$
$$s = (p_1+p_2)^2 = (p_1'+p_2')^2,$$

$$\left(-\frac{4\pi\alpha}{16\pi^3}\right)\int_{k<\Delta\varepsilon}\frac{d^3k}{\omega}\frac{p_1p_2'}{(p_1k)(p_2'k)} = -\frac{\alpha}{\pi}\left\{\ln\frac{\sqrt{mm_e}\Delta\varepsilon}{\lambda\sqrt{\varepsilon_1\varepsilon_2'}}\ln\frac{-u}{mm_e}-\right.$$
$$\left.-\frac{1}{4}\ln^2\left(\frac{\varepsilon_2'}{\varepsilon_1}\right) - \frac{1}{2}\ln\left(\frac{\varepsilon_2'm_e}{\varepsilon_1 m}\right)\ln\frac{m_e}{m} - \frac{\pi^2}{12} + \frac{1}{8}\ln^2\left(\frac{-u}{m^2}\right) + \frac{1}{8}\ln^2\left(\frac{-u}{m_e^2}\right)\right\},$$

$$\left(-\frac{4\pi\alpha}{16\pi^3}\right)\int_{k<\Delta\varepsilon}\frac{d^3k}{\omega}\frac{p_1p_2}{(p_1k)(p_2k)} = -\frac{\alpha}{2\pi}\left\{\ln\frac{m_e\Delta\varepsilon}{\lambda\varepsilon_1}\ln\left(\frac{4\varepsilon_1^2}{m_e^2}\right)+\right.$$
$$\left.+\frac{1}{4}\ln^2\left(\frac{4\varepsilon_1^2}{m_e^2}\right) - \frac{\pi^2}{6}\right\} \equiv J(\varepsilon, m_e),$$

$$\left(-\frac{4\pi\alpha}{16\pi^3}\right)\int_{k<\Delta\varepsilon}\frac{d^3k}{\omega}\frac{p_1'p_2}{(p_1'k)(p_2k)} = J(\varepsilon_1', m_e),$$

$$\left(-\frac{4\pi\alpha}{16\pi^3}\right)\int_{k<\Delta\varepsilon}\frac{d^3k}{\omega}\frac{p_1p_2'}{(p_1k)(p_2'k)} = J(\varepsilon_2', m),$$

Appendix 3

The quantities Φ_i entering (18) may be expressed in terms of imaginary parts of coefficient functions of vector integrals listed in Appendix 2: $\Phi_i = \frac{s}{2}\mathrm{Im}\,\frac{\varphi}{\pi\tau}$. The first term $\Phi_1 = \frac{-s}{\pi}\mathrm{Im}\,\gamma_{2gg} = 1$ and the last one

$$\Phi_W = \frac{s^2t}{2\pi}(\frac{Q_q}{t} - \frac{a_e(a_q+v_q)}{\tau})(1-\xi)\mathrm{Im}\, I_{WW},\quad \tau = t - M_Z^2,$$

corresponds to $\gamma\gamma$ and W^+W^- exchange by leptons; ξ is the degree of longitudinal polarization of the initial electron. To simplify the notations we put $M_Z = 1$ and introduce the correspondence table

σ_1	σ_2	σ_3	σ_4	σ_5	σ_6	σ_7	σ_8
α_{ZZ}	β_{ZZ}	β_{Zg}	γ_{ZZ}	γ_{Zg}	γ_{gg}	γ_{2gg}	γ_{2gZ}

In terms of σ_i the remaining quantities may be written in the form: $\varphi_i = \sum_i C_{ij}\sigma_j$, $i = 2, ..., 10$. The nonzero elements C_{ij} are:

$C_{22} = -4t^2 + 4 + 8s$ $\qquad C_{23} = 8s(t-u) + 2t - 2$
$C_{26} = 4s(u-s)$ $\qquad C_{27} = 4t - 2$
$C_{28} = 2t - 2$ $\qquad C_{32} = 4(t-1)\xi$

$C_{35} = -2(t-1)\xi$
$C_{38} = -2(t-1)\xi$
$C_{43} = 4t^2$
$C_{45} = -8ts - 2t$
$C_{48} = -2t - 2(t-1)\xi$
$C_{57} = 4t\xi$
$C_{58} = 2(t-1) + 2(3t-1)\xi$
$C_{63} = 4t^2$
$C_{65} = -2t^2$
$C_{68} = -2t^2 - 2t(t-1)\xi$
$C_{78} = -2t\xi - 4t$
$C_{88} = 4t + 4t\xi$
$C_{98} = 2t$
$C_{101} = 4t$
$C_{108} = -2t$
$C_{54} = 2(t-1)(4s+1)(1+\xi)$
$C_{84} = 2t(1+4s) + 4t(1+s)\xi$

$C_{37} = 2(1-2t)\xi$
$C_{41} = 4(t-1)(t+2s+1)\xi$
$C_{44} = -2(t-1)\xi$
$C_{47} = -2t$
$C_{55} = 4t(1+2s)\xi$
$C_{53} = -4t(t+2s+1)\xi$
$C_{61} = 4t(t-1)\xi$
$C_{64} = -2t(t-1)\xi$
$C_{67} = -2t^2$
$C_{71} = 4t - 4tu(1+\xi)$
$C_{81} = 4t(s-t-1) - 4t(s+1)\xi$
$C_{91} = -4t(t+2s)$
$C_{94} = 2t(1-2s)$
$C_{104} = -2t$
$C_{51} = 4(t-1)(s-t-1)\xi + 4st + 4 - 8s$
$C_{74} = -2t(1+2s)\xi - 4t(1+s)$

The expression for ΔK_W have a form (only crossed W legs diagram works):

$$\Delta K_W = \left[\left(\frac{Q_q}{t} - \frac{a_e v_q}{\tau}\right) w_1 + \frac{a_e a_q}{\tau} w_2 \right] (1-\xi) \frac{\delta_{q,n}}{4\sin^4\theta_W}.$$

The quantities $w_i (i = 1,2)$ we put in the form

$$w_i = a_i + b_i j + c_i \tilde{I}_{WW} + d_i \tilde{I}_{dQW} + e_i I_{QWW},$$

where $\tilde{I}_i = I_i(s \to u)$, and the coefficient functions are equal ($M_W = 1$):

$$a_1 = s - \frac{s}{4}b - \frac{3s}{4}a;$$
$$a_2 = s + \frac{s}{2}\left(-\frac{1}{2} + \frac{t}{2u} - \frac{u}{t}\right)b - \frac{s}{2}\left(\frac{3}{2} + \frac{u}{t}\right)a;$$
$$b_1 = \frac{s}{2}(1+a); \qquad b_2 = \frac{s}{2}\left(1 + \frac{u}{t}b + \frac{s}{t}a\right);$$
$$c_1 = \frac{t-u}{2} - u^2 - \frac{t^2u}{4} + \frac{u^2s}{2} + \frac{u}{2}\left(\frac{1}{2} + \frac{t+tu}{4} + u + \frac{u^2}{2}\right)b$$
$$- \frac{s}{2}\left(\frac{1}{2} + \frac{3t}{2s} + \frac{5tu}{4s} + u - \frac{tu(t-s)}{4s} + \frac{u^2}{2}\right)a;$$
$$c_2 = \frac{t-u}{2} - u^2 - \frac{t^2u}{4} + \frac{u^2s}{2} + \frac{u}{2}\frac{6+t+6u-tu}{4}b$$

$$- \frac{s}{2}\left(\frac{u-1}{2} + \frac{2t+3tu-t^2u}{4s} + \frac{tu}{4} + u^2\right)a;$$
$$d_1 = -u - u^2 - \frac{tu}{2} + \frac{u}{2}\left(1 + u + \frac{t(u+1)}{2u}\right)b - \frac{s}{2}\left(1 + u + \frac{t(3+u)}{2s}\right)a;$$
$$d_2 = -u - u^2 - \frac{tu}{2} + \frac{u}{2}\left(2 + u + \frac{t(u+1)}{2u}\right)b - \frac{s}{2}\left(u + \frac{t(u+1)}{2s}\right)a;$$
$$e_1 = -t - us + \frac{t^2}{2} + \frac{u}{2}\left(-u - \frac{t+1}{2}\right)b - \frac{s}{2}\left(-s + \frac{1-t}{2} + \frac{t(t-3)}{2s}\right)a;$$
$$e_2 = -t - us + \frac{t^2}{2} + \frac{u}{2}\left(-\frac{u}{t} + \frac{t-5}{2}\right)b - \frac{s}{2}\left(-2u + \frac{3-t}{2} + \frac{t(t-1)}{2s} + \frac{u}{t}\right)a.$$

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