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E2-92-555

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GAUGE THEORY AND BRST QUANTIZATION OF THE MODELS FOR REGGE TRAJECTORIES

Submitted to «Physics Letters B»

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1992

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Калибровочная теория и БРСТ квантование в моделях траекторий Редже

Развита калибровочная теория для Пуанкаре- и репараметризационного инвариантных лагранжианов, в которых использованы твисторы для обеспечения массово-спиновых соотношений, соответствующих траекториям Редже. Показано, что лежащие в основе калибровочная и репараметризационная симметрии лагранжианов первого порядка являются эквивалентными. Эти классические модели последовательно квантуются с помощью БРСТ формализма. Сделаны полезные комментарии для обобщений одной из моделей на соответствующую струнную теорию, в которой аналог космологической константы зависит от произвольной функции. Также обсуждена калибровочная теория для двух неразложимых объектов, связанных с помощью гармонической силы.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна, 1992

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Gauge Theory and BRST Quantization of the Models for Regge Trajectories

A gauge theory is developed for the Poincaré and reparametrization invariant Lagrangians where twistors have been used to provide the mass-spin relationship corresponding to the Regge trajectories. It is demonstrated that the underlying gauge and reparametrization symmetries of the first order Lagrangians are equivalent. These classical models are subsequently quantized by invoking the methods of the BRST formalism. Useful comments are made for the generalization of one of the models to the corresponding string theory in which the analogue of the cosmological constant term depends on an arbitrary function. The gauge theory for two indecomposable objects, bound by the harmonic force, is also discussed.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna, 1992

The theories of the extended objects have attracted a great deal of interest during the past few years. The leading candidates for such an interest are primarily (super)string theories [1] and (super) p-brane ($p \geq 2$) theories [2,3]. Despite remarkable successes, there are longstanding questions which still defy their clear-cut resolutions in the framework of these theories. Among others, the unphysical critical dimensions of the quantum string theories and the loss of the Weyl invariance, even at the classical level in the case of the p-brane theories, are prime examples. These are precisely some of the reasons that few attempts [4,5] have been made to provide a viable alternatives to these theories. The model of ref.[5], where the ideas of the free relativistic top [6] and particle are combined together, has also been considered in the framework of the BRST formalism [7]. However, due to the presence of the entangled web of interaction terms, it is difficult to compute the critical dimension of the space-time in which the quantum "stringtop" theory, developed from this model, would be consistent.

Recently there has been an upsurge of interest in the twistor (two-component commuting spinors) formulation of the superparticle and superstrings[8,9]. The purpose of the present paper is to develop a gauge theory of the classical models for Regge-trajectories [10] where the role of the twistors has been highlighted in providing the mass-spin relationship. The first order Lagrangians of these models are endowed with gauge and reparametrization symmetries which are shown to be equivalent. These symmetries are, in turn, exploited to quantize these models in the framework of the BRST formalism [11,12]. One of the classical models is generalized to the corresponding 4D string theory in which the Weyl invariance is respected at the classical level under certain restrictions.

The reparametrization and Poincaré invariant "spinor" Lagrangian($\mathcal{L}_o^{(S)}$), describing the motion of the indecomposable object in the configuration space which is product of the Minkowski space and the first order internal space (FOS), is as follows [10]:

$$\mathcal{L}_o^{(S)} = -\frac{i}{2}(\phi^T \sigma_2 \dot{\phi} + \phi^\dagger \sigma_2 \dot{\phi}^*) - (-\dot{x}^2)^{\frac{1}{2}} f(\zeta) \tag{1}$$

where ϕ and ϕ^* are the commuting two-component spinors that transform as defining (1/2,0) and (0,1/2) representations of SL(2,C) and the usual dotted and undotted indices of the twistor components are suppressed here for the later convenience. The internal spaces that possess SL(2,C) invariant symplectic structure are called FOS and those which do not, are called second order spaces. All the dynamical variables are functions of the evolution parameter τ and $\dot{x}^\mu = \frac{dx^\mu}{d\tau}, \dot{\phi} = \frac{d\phi}{d\tau}$. The argument of the arbitrary function $f(\zeta)$ is the most general reparametrization invariant Lorentz scalar that can be constructed, besides $(\dot{x}^2)^{\frac{1}{2}}$, in the configuration space. This is given by:

$$\zeta = \frac{-\dot{x}_\mu V^\mu}{(-\dot{x}^2)^{\frac{1}{2}}} \equiv -\frac{\dot{x}\cdot V}{(-\dot{x}^2)^{\frac{1}{2}}}, \tag{2}$$

where positive light-like vector V^μ is expressed in terms of the twistor components ϕ, ϕ^* and Pauli matrices $\vec{\sigma}$ as:

$$V^\mu = \frac{1}{2}\phi^\dagger(1,\vec{\sigma})\phi \equiv \frac{1}{2}\phi^\dagger\sigma^\mu\phi. \tag{3}$$

The motion of the indecomposable object described by Lagrangian (1) is constrained because, eventhough, we have assumed the four components of the co-ordinate x^μ to be independent, the corresponding conjugate momenta p_μ are restricted to satisfy the following relation:

$$p^2 = -(f^2 - \zeta^2 f'^2). \tag{4}$$

The Pauli-Lubanski vector W_μ, which is one of the Casimir invariants of the Poincaré group, can be expressed in terms of the linear momenta p_μ and the light-like vector V^μ as follows:

$$W^2 = (-p_\mu V^\mu)^2 \equiv \xi^2 \tag{5}$$

where the spin operator ξ depends on the arbitrary function $f(\zeta)$ and the reparametrization invariant Lorentz scalar ζ as:

$$\xi = (f - \zeta f')\zeta. \tag{6}$$

For the generic function $f(\zeta)$, one can eliminate ζ from (4) and (6). As a consequence, the constraint equation can be re-expressed as:

$$p^2 + \alpha(\xi) = 0 \tag{7}$$

where $\alpha(\xi)$ satisfies following useful relations:

$$\alpha = f^2 - \zeta^2 f'^2; \qquad \alpha - \alpha'\xi = (f - \zeta f')^2. \tag{8}$$

One can get rid of the square root present in (1) by incorporating a metric e (or einbein) in the Lagrangian without spoiling the Poincaré and reparametrization invariance. The second order Lagrangian, thus obtained, is as follows:

$$\mathcal{L}_S^{(S)} = \frac{\dot{x}^2}{2e} - \frac{e}{2}\alpha\Big(\frac{-\dot{x}\cdot V}{e}\Big) - \frac{i}{2}(\phi^T\sigma_2\dot{\phi} + \phi^\dagger\sigma_2\dot{\phi}^*). \tag{9}$$

It can be readily seen that the use of equation of motion with respect to e and equation (8),lead us back to the original Lagrangian (1), we started with. The whole dynamics of the system under consideration is dictated by the constrained equation (7) because the canonical Hamiltonian derived from the Lagrangian (1) is zero. The first order Lagrangian incorporating this constraint and, equivalent to (1) and (9), is as given below:

$$\mathcal{L}_F^{(S)} = p_\mu\dot{x}^\mu - \frac{i}{2}(\phi^T\sigma_2\dot{\phi} + \phi^\dagger\sigma_2\dot{\phi}^*) - \frac{e}{2}(p^2 + \alpha(\xi)). \tag{10}$$

The equation of motion with respect to e yields the constraint equation which has been demonstrated in ref.[10] as the Regge-trajectory relation. (We shall discuss about the various subtleties involved in it a bit later).

Equivalent to the Lagrangian (1), there is a "tensor" FOS Lagrangian given by [10]:

$$\mathcal{L}_o^{(T)} = \frac{\vec{V}\cdot\vec{S'}}{V_o} - (-\dot{x}^2)^{\frac{1}{2}} f(\zeta) \tag{11}$$

where the component $\vec{S'}$ of the internal angular momentum $S_{\mu\nu}$ can be expressed in terms of the basic spinor fields as follows:

$$\vec{S'} = \frac{i}{4}(\phi^T\sigma_2\vec{\sigma}\phi - \phi^\dagger\vec{\sigma}\sigma_2\phi^*). \tag{12}$$

The second and first order Lagrangians corresponding to (11) can be written without square root. These are as follows:

$$\mathcal{L}_S^{(T)} = \frac{\dot{x}^2}{2e} - \frac{e}{2}\alpha\Big(\frac{-\dot{x}\cdot V}{e}\Big) + \frac{\vec{V}\cdot\vec{S'}}{V_o} \tag{13}$$

and,

$$\mathcal{L}_{\rm F}^{(T)} = p_\mu \dot{x}^\mu - \frac{e}{2}(p^2 + \alpha(\xi)) + \frac{\vec{V}\cdot\dot{\vec{S}'}}{V_o}. \quad (14)$$

The canonical Hamiltonian derived from the Lagrangians (9),(10),(13) and (14) is the same. This can be explicitly expressed as follows:

$$\mathcal{H} = \frac{e}{2}(p^2 + \alpha(\xi)). \quad (15)$$

The conjugate momenta (Π_e) corresponding to the variable e is the primary constraint $\Pi_e \approx 0$ (weakly zero) which, in turn, leads to the existence of the secondary constraint $\Pi_e^{(1)} = -\frac{1}{2}(p^2 + \alpha(\xi)) \approx 0$. The latter constraint is obtained by requiring the consistency of the former under the time evolution, generated by the Hamiltonian (15). Both the constraints are first class in the language of Dirac [13].

The existence of the first class constraints on a system implies an underlying gauge symmetry for the corresponding Lagrangians[13,14]. The gauge symmetry generator for the first order Lagrangians (10) and (14) can be expressed in terms of the first class constraints as follows [14] :

$$G = \dot{\omega}\Pi_e - \omega\Pi_e^{(1)} \quad (16)$$

where the real function $\omega(\tau)$ is the gauge transformation parameter. The above generator results in following infinitesimal gauge transformation for the Lagrangian (10):

$$\begin{aligned} \delta x^\mu &= \omega(p^\mu - \frac{\alpha'}{2}V^\mu); \quad \delta p^\mu = 0; \quad \delta\alpha = 0; \quad \delta e = \dot{\omega} && (17) \\ \delta\phi_\sigma &= -\frac{\omega\alpha'}{4}\varepsilon_{\sigma\rho}(\sigma\cdot p)^T_{\rho\gamma}\phi^*_\gamma \quad \text{and} \quad \delta\phi^*_\sigma = -\frac{\omega\alpha'}{4}\varepsilon_{\sigma\rho}(\sigma\cdot p)_{\rho\gamma}\phi_\gamma, && (18) \end{aligned}$$

where ε is the 2x2 antisymmetric matrix with ε_{12}=+1. For the "tensor" FOS Lagrangian (14), the gauge symmetry transformations (17) remain the same but equations (18) are replaced by :

$$\delta V_\mu = -\frac{\omega\alpha'}{2}S_{\mu\nu}p^\nu \quad \text{and} \quad \delta S_{\mu\nu} = -\frac{\omega\alpha'}{2}(V_\mu p_\nu - V_\nu p_\mu) \quad (19)$$

where $S_{oj} = S'_j$ and $S_i = \frac{1}{2}\epsilon_{ijk}S_{jk}$ with $\vec{S} = -\frac{1}{4}(\phi^T\sigma_2\vec{\sigma}\phi + \phi^\dagger\vec{\sigma}\sigma_2\phi^*)$. The transformations (17), (18) and (19) are quasi-symmetry transformations [15] because the first order Lagrangians (10) and (14) undergo following change:

$$\delta\mathcal{L}_{\rm F}^{(S,T)} = \frac{d}{d\tau}\Big[\frac{\omega}{2}(p^2 - 2\alpha' p\cdot V - \alpha)\Big]. \quad (20)$$

At this juncture it is worthwhile to mention that, in addition to the local gauge symmetries, the system under consideration also respects reparametrization invariance under one-dimensional diffeomorphism ($\tau \to \tau - \epsilon(\tau)$). As pointed out in ref.[10], Lagrangians (1) and (11) are reparametrization invariant for arbitrary function $f(\zeta)$. This statement can be verified in a very transparent manner if we consider this symmetry for the first order Lagrangians (10) and (14). For instance, for the following infinitesimal diffeomorphism transformations:

$$\delta_r\Phi = \epsilon\dot{\Phi} \quad \text{and} \quad \delta_r e = \frac{d}{d\tau}(e\epsilon) \quad (21)$$

where ($\Phi = x_\mu, p_\mu, \alpha, \phi, \phi^*, \vec{V}$, and $\vec{S}'$), the first order Lagrangians transform as:

$$\delta_r\mathcal{L}_{\rm F}^{(S,T)} = \frac{d}{d\tau}\Big(\epsilon\mathcal{L}_F^{(S,T)}\Big). \quad (22)$$

As a consequence, the action changes only by a surface term which can be made equal to zero by putting appropriate boundary conditions on the diffeomorphism parameter ϵ.

The gauge symmetries as well as reparametrization symmetry are equivalent if we choose the gauge parameter to be $\omega = \epsilon e$ and use the following mass-shell conditions derived from the Lagrangians (10) and (14):

$$\begin{aligned} \dot{p}_\mu &= 0; \quad p_\mu = \frac{\dot{x}_\mu}{e} + \frac{\alpha'}{2}V^\mu; \quad p^2 + \alpha(\xi) = 0; \\ \dot{V}_\mu &= -\frac{e\alpha'}{2}(S_{\mu\nu}p^\nu); \quad \dot{S}_{\mu\nu} = -\frac{e\alpha'}{2}(V_\mu p_\nu - V_\nu p_\mu); && (23) \\ \dot{\phi}_\sigma &= -\frac{e\alpha'}{4}\varepsilon_{\sigma\rho}(\sigma\cdot p)^T_{\rho\gamma}\phi^*_\gamma; \quad \dot{\phi}^*_\sigma = -\frac{e\alpha'}{4}\varepsilon_{\sigma\rho}(\sigma\cdot p)_{\rho\gamma}\phi_\gamma. \end{aligned}$$

The interplay between the gauge symmetry transformations as well as the reparametrization symmetry transformations can now be well understood because the generator (16) produces both the transformations with appropriate choice of the gauge parameter and use of the equations of motion (23). Moreover, to show that these symmetries are independent in their intrinsic nature, one has to make sure that they commute with each other;

namely, $[\delta, \delta_r] = 0$. This requirement can be fulfilled if the gauge parameter is chosen to be the Lorentz scalar under diffeomorphism transformations; namely: $\delta_r \omega = \epsilon \dot{\omega}$. The off-shell equivalence of the gauge symmetries and reparametrization symmetry can also be established as one can see that, for the choice $\omega = e\epsilon$, the transformations corresponding to $\delta_t = \delta_r - \delta$ are trivial symmetry transformations for any first order Lagrangian [16]. This can be seen explicitly for the action (S)corresponding to the Lagrangian (10) which remains invariant $(\delta_t S = 0)$ under following transformations:

$$\begin{aligned} \delta_t x^\mu &= \epsilon \frac{\delta S}{\delta p^\mu}; \quad \delta_t p^\mu = -\epsilon \frac{\delta S}{\delta x^\mu}; \quad \delta_t e = 0, \\ \delta_t \phi_\sigma &= \epsilon \varepsilon_{\sigma\rho} \frac{\delta S}{\delta \phi_\rho}; \quad \delta_t \phi^*_\sigma = \epsilon \varepsilon_{\sigma\rho} \frac{\delta S}{\delta \phi^*_\rho}. \end{aligned} \quad (24)$$

This type of symmetry is also respected by the "tensor" FOS Lagrangian (14). It will be noticed that the transformations generated by the orthogonal combination $\delta_r + \delta$ do not result in any such kind of general symmetry for our purpose and lead to the transformation of the Lagrangians as sum of (20) and (22).

The locally gauge invariant first order Lagrangians (10) and (14) are singular because the Hessian with respect to e is zero. As a result, we have primary constraint $\Pi_e \approx 0$ which subsequently leads to the existence of the secondary constraint $\Pi_e^{(1)} \approx 0$. To quantize such kind of constrained systems, one uses either Dirac bracket approach [13] or invokes the BRST formalism [11,12]. We follow here the latter approach which is covariant quantization procedure and most suitable for the gauge theories. The BRST prescription is to replace commuting gauge parameter ω (and reparametrization parameter ϵ) by an anticommuting number η and anticommuting ghost fields c (and λ). The total BRST invariant Lagrangian contains gauge fixing term and Faddeev-Popov ghost terms which can be obtained as follows [17]:

$$\eta(\mathcal{L}_{G.F.} + \mathcal{L}_{F.P.}) = \begin{pmatrix} \delta^B \\ \delta^B_r \end{pmatrix} \left[\begin{pmatrix} \bar{c} \\ \bar{\lambda} \end{pmatrix} \left\{ \dot{e} + \frac{1}{2} \begin{pmatrix} b \\ B \end{pmatrix} \right\} \right] \quad (25)$$

where $\bar{c}$ (and $\bar{\lambda}$) are anticommuting antighost fields and b (and B) are Nakanishi-Lautrup commuting auxiliary fields. The nilpotent BRST transformations corresponding to the

gauge symmetry for the total BRST invariant spinor Lagrangian $\mathcal{L}_g^{(S)}$ are as follows:

$$\begin{aligned} \delta^B x^\mu &= \eta c (p^\mu - \frac{\alpha'}{2} V^\mu); \quad \delta^B e = \eta \dot{c}; \quad \delta^B \alpha = 0; \\ \delta^B p^\mu &= 0; \delta^B c = 0; \quad \delta^B b = 0; \quad \delta^B \bar{c} = \eta b. \quad (26) \\ \delta^B \phi_\sigma &= -\frac{\eta c \alpha'}{4} \varepsilon_{\sigma\rho} (\sigma \cdot p)^T_{\rho\gamma} \phi^*_\gamma; \quad \delta^B \phi^*_\sigma = -\frac{\eta c \alpha'}{4} \varepsilon_{\sigma\rho} (\sigma \cdot p)_{\rho\gamma} \phi_\gamma \quad (27) \end{aligned}$$

and for the "tensor" BRST invariant Lagrangian, in addition to (26), following symmetry transformations are required:

$$\delta^B V_\mu = -\frac{\eta c \alpha'}{2} S_{\mu\nu} p^\nu \quad \text{and} \quad \delta^B S_{\mu\nu} = -\frac{\eta c \alpha'}{2} (V_\mu p_\nu - V_\nu p_\mu). \quad (28)$$

The nilpotent BRST transformations δ^B_r, corresponding to the reparametrization symmetry of the total BRST invariant spinor and tensor FOS Lagrangians $\mathcal{L}_r^{(S,T)}$, can be concisely expressed as:

$$\begin{aligned} \delta^B_r \Phi &= \eta \lambda \dot{\Phi}; \quad \delta^B_r e = \eta \frac{d}{d\tau}(e\lambda) \quad (29) \\ \delta^B_r \lambda &= \eta \lambda \dot{\lambda}; \quad \delta^B_r \bar{\lambda} = \eta B; \quad \delta^B_r B = 0 \end{aligned}$$

where $(\Phi = x_\mu, p_\mu, \alpha, \phi, \phi^*, V_\mu \text{and} S_{\mu\nu})$. Under above gauge and reparametrization BRST transformations, the total Lagrangians undergo following change:

$$\delta^B \mathcal{L}_g^{(S,T)} = \eta \frac{d}{d\tau} \left[\frac{c}{2}(p^2 - 2\alpha' p \cdot V - \alpha) + b\dot{c} \right] \quad (30)$$

and,

$$\delta^B_r \mathcal{L}_r^{(S,T)} = \eta \frac{d}{d\tau} \left[\lambda \mathcal{L}_F^{(S,T)} + B \frac{d}{d\tau}(e\lambda) \right]. \quad (31)$$

The application of the Noether theorem leads to the following conserved BRST charges$Q$ and Q_r corresponding to both the above cited symmetries:

$$Q = \frac{c}{2}(p^2 + \alpha(\xi)) + b\dot{c} \quad \text{and} \quad Q_r = \frac{\lambda e}{2}(p^2 + \alpha(\xi)) + B \frac{d}{d\tau}(\lambda e). \quad (32)$$

As is clear from the BRST prescription, there are two-fermionic ghost fields corresponding to one commuting gauge parameter. This symmetry in the field content results in another conserved charge which is christened as anti-BRST charge ($\bar{Q}$). The transformations generated by $\bar{Q}$ can be obtained by replacing ghost fields by anti-ghost fields and requiring anticommutativity of the BRST and anti-BRST charges. Recently, the usefulness of the latter has been pointed out in refs. [16,18]. It can be explicitly seen from equation (32) that the replacement of c by λe in the expression for Q yields the expression for Q_r. This is not surprising because the following Euler-Lagrange equations (for $e \neq 0$),in addition to (23), have been used to derive Q and Q_r :

$$\begin{aligned} \frac{d^2}{d\tau^2}c &= \frac{d^2}{d\tau^2}\bar{c} = \frac{d^2}{d\tau^2}\bar{\lambda} = \frac{d^2}{d\tau^2}(\lambda e) = 0, \\ (b, B) &= -\dot{e}, \qquad (\dot{b}, \dot{B}) = -\frac{1}{2}(p^2 + \alpha(\xi)). \end{aligned} \tag{33}$$

The canonical Hamiltonian (H_B) derived from the total BRST invariant gauge Lagrangian $\mathcal{L}_g^{(S,T)}$:

$$H_B = \frac{e}{2}(p^2 + \alpha(\xi)) + \dot{\bar{c}}\dot{c} - \frac{b^2}{2} \tag{34}$$

is decoupled in terms of the conserved ghost momenta. Furthermore, it will be noticed that the ghost fields have no interaction with the rest of the fields of the theory. Thus, the quantum states in the total Hilbert space are direct product of the matter states and the ghost states (i.e. $|phys>=|matt> \otimes |ghost>$) [19,20]. The ghost states are designated by the ghost number which corresponds to the eigen value of the conserved operator:

$$Q_g = c\dot{\bar{c}} + \bar{c}\dot{c}, \tag{35}$$

emerging due to the global scale invariance of the ghost term in the action. By exploiting following BRST quantization relations:

$$\begin{aligned} &[x_\mu, p_\nu] = i\eta_{\mu\nu}; \quad [e, b] = i; \quad \{c, \dot{\bar{c}}\} = -i; \quad \{\bar{c}, \dot{c}\} = i \\ &[\phi_\alpha, \phi_\beta] = [\phi^*_\alpha, \phi^*_\beta] = i\varepsilon_{\alpha\beta}; \qquad [\vec{S}', \vec{V}] = iV_o, \end{aligned} \tag{36}$$

it can be seen that the following BRST algebra is satisfied:

$$\begin{aligned} &\{Q, Q\} = 0; \qquad [iQ_g, Q] = Q; \qquad [Q_g, Q_g] = 0 \\ &[H_B, Q] = 0; \qquad [H_{F.P.}, Q_g] = 0, \end{aligned} \tag{37}$$

where $H_{F.P.} = \dot{\bar{c}}\dot{c}$ is the Hamiltonian corresponding to the ghost term. This algebra together with physical state condition ($Q|phys>=0$) provides the key ingredients for the

proof of unitarity of the S-matrix [20], framework for the confinement of the unphysical particles through quartet mechanism [21] etc.

One of the unsatisfactory part of the BRST quantization procedure is the fact that there exist more solutions to the quantum states than are physically allowed. To extract out only the physically meaningful states, one invokes additional constraint on the total quantum states by requiring that the conserved and nilpotent BRST charge Q must annihilate these states ($i.e. Q|phys>=0$). The dynamical origin of this outside constraint has been sought out in the framework of the local-BRST invariance [22]. However, this approach suffers from the requirement of "ghost-for-ghost" ad-infinitum. Recently, various subtleties involved with the supplementary conditions to be imposed on $Q|phys>=0$ have been discussed in refs.[23,24]. However, there are still some intricate issues that need transparent clarification. Here we shall pursue the usual procedure of picking out the physical states [20] and remark on some salient features relevant to the auxiliary conditions, following the discussion of ref. [24]. Besides $\Pi_e|matt>=0$, the physical state condition leads to the following restriction on the quantum states:

$$(p^2 + \alpha(\xi))|matt>=0. \tag{38}$$

It has been demonstrated in ref.[10] that this condition coupled with certain restriction on $f(\zeta)\Big(\text{namely; } f, (f\zeta)', (\frac{\zeta}{f})' > 0\Big)$ can be translated into the Regge-trajectory relationship which amounts to the dynamical grouping together of the strongly interacting particles of varying mass and spin, but with the same hadronic quantum numbers, into single families [6]. To be more precise, the specific choice of $f(\zeta) = f_o\zeta^a$ ($f_o > 0$) in powers of its argument,leads to the mass M ($M = (-p^2)^{\frac{1}{2}}$) and the spin s ($Ms = (W^2)^{\frac{1}{2}}$) relationship in the functional form : $\mathcal{M}(s) = \mathcal{M}_o s^a$ where a is shown to vary as: $-1 < a < +1$. Here $M = \mathcal{M}(s)$ is the functional solution of the constraint equation (38) expressed as : $M^2 = \alpha(Ms)$ and $\mathcal{M}_o$ is positive multiple of f_o. Ultimately, it has been shown in ref.[10] that for each quantum state with spin s, there exists only one Regge trajectory.

A very subtle point is the discussion about the operator and state cohomology of the physical state condition (38) in view of the recent results of refs. [23,24] where general auxiliary conditions, including well-known supplementary restrictions such as anti-BRST [17] and co-BRST [25] etc., have been derived. As indicated in ref. [24], it can be readily seen in the models under consideration that the gauge-fixing condition $\bar{\chi} = \frac{x^o - \tau}{p^o}$ ($p^o \neq 0$) leads to the BRST quartet $\left(p^2 + \alpha(\xi); \bar{\chi}; c; \dot{\bar{c}}\right)$ which finally splits into two-doublets$\left(p^2 + \alpha(\xi); \dot{\bar{c}}\right)$ and $\left(\bar{\chi}; c\right)$. As a consequence of $Q|phys>= 0$, either of these doublets must annihilate the physical states on their own right. However, the manifestly Lorentz invariant constraint is still (38) which amounts to the annihilation of the quantum states by the Regge-trajectory constraint.

The generalization of the second order spinor Lagrangian (9) to the corresponding string theory action is straightforward because all the derivatives with respect to τ can be generalized to the two-dimensional sheet-derivative (∂_a) and the one dimensional metric e can be extended to the two-dimensional metric (g_{ab}) as follows [5]:

$$\begin{aligned} S &= \frac{1}{2}\int d^2\rho\sqrt{g}g^{ab}\partial_a X^\mu \partial_b X^\nu \eta_{\mu\nu} - \frac{1}{2}\int d^2\rho\sqrt{g}\alpha(-\sqrt{g}g^{ab}\partial_a X^\mu V_b^\nu \eta_{\mu\nu}) \\ &- \frac{i}{2}\int d^2\rho(\phi^T\sigma_2\gamma^a\partial_a\phi + \phi^\dagger\sigma_2\gamma^a\partial_a\phi^*) \end{aligned} \tag{39}$$

where $(\rho^o;\rho^1)=(\tau;\sigma)$ are the parameters characterizing the world- sheet traced out by the string; a, b are the world-sheet indices and γ^a are the matrices defined on this sheet which can be taken as $\gamma^o = \sigma_2$ and $\gamma_1 = i\sigma_1$ in the Majorana representation. These matrices, in turn, generalize the light-like vector V^μ to pick-up one sheet index (i.e.$V_a^\mu = \frac{1}{2}\phi^\dagger\sigma^\mu\gamma_a\phi$). The propagation of this string sweeps out a helical two-dimensional world- sheet in the 4D target space. This is due to the fact that equations (23) describe the helical motion [26] of the indecomposable object that corresponds to the infinite tension limit of the string action (39).

It would be interesting to note that the second term in (39), which plays the role of the cosmological constant term, depends on an arbitrary function $f(\zeta)$. The degenarate case [10] of the arbitrary function :$f(\zeta) = C_1\zeta + \frac{C_2}{\zeta}$ (where C_1 and C_2 are constants) leads to the constant value of $\alpha(\xi) = 4C_1C_2$. In this limit the second term of the equation (39) becomes identical to the cosmological constant term of the usual string theory [27]. Furthermore, the condition $C_2 = 0$ (or $C_1 = 0$) corresponds to the string action in which

the Weyl invariance is respected at the classical level. In view of this interesting feature, the action (39) has an edge over the usual p-brane theories in which Weyl invariance is not respected even at the classical level. The parameter dependence of the second term in the action (39) might shed light on the cosmological constant problem in the framework of the string theory. The interesting problem for future investigation for this action (39) is to determine the critical dimension of the space-time in which the quantum version of this string theory is consistent, following the procedures of refs.[28].

Another related topic of interest is to consider the gauge theory of N-relativistic indecomposable objects described by the Lagrangian (10) and coupled by linear harmonic forces [29]. One of the key ingredients in ref.[29] is to firstly obtain a "rudimentary" Lagrangian and look for its rigid (global) symmetries. The first order Lagrangian (10), without the gauge field e, can be recast in the following manifest global gauge invariant form:

$$\mathcal{L}_F^{(S)} = \frac{1}{2}\Psi^T C(\partial_\tau - H_R)\Psi - \frac{1}{2}(\phi^T C\dot{\phi} + \phi^\dagger C\dot{\phi}^*) \tag{40}$$

where $C = i\sigma_2$,$H_R = \frac{1}{2}(\sigma_1 - i\sigma_2)$ and the column vector Ψ with elements :$\Psi_{11} = p_A$ $\Psi_{21} = x_A$ bear indices A=0,1,2,3 as 4D Minkowski indices and an extra Euclidean dimension (i.e.A = 4) for the introduction of the cosmological constant term $\alpha(\xi)$ by dimensional reduction ($p_4 = \sqrt{\alpha(\xi)}$ and $\dot{x}_4 = 0$). We have dropped a total-derivative term $\frac{d}{d\tau}(px)$ because it does not influence the equation of motion. The above Lagrangian is invariant under following rigid gauge transformations:

$$\delta\Psi = F\Psi; \quad \delta\phi = l^*\phi^*; \quad \delta\phi^* = l\phi \tag{41}$$

with restrictions on the gauge parameters as listed below:

$$\begin{aligned} F^T C + CF &= 0; \qquad [H_R, F] = 0; \\ Cl^* + l^T C &= 0; \qquad Cl + l^\dagger C = 0. \end{aligned} \tag{42}$$

These restrictions are satisfied by following choice of matrices, in addition to C and H_R as quoted above:

$$F = \frac{1}{2}(\sigma_1 - i\sigma_2)\Omega; \quad l_{\rho\sigma} = \varepsilon_{\rho\gamma}(\sigma\cdot\kappa)_{\gamma\sigma}; \quad l^*_{\rho\sigma} = \varepsilon_{\rho\gamma}(\sigma\cdot\kappa)^T_{\gamma\sigma} \tag{43}$$

where Ω is the global (τ independent) gauge parameter and κ is a global four-vector. Now the Lagrangian (40) can be genaralized to the multi-relativistic indecomposable objects coupled by harmonic potential. To see this, let us consider the simplest case of two such objects bound by the harmonic potential (i.e. $\frac{1}{2}kx^2$) with force constant k. We shall use here the center of mass co-ordinates (P, X) and the relative co-ordinates (p, x). The spinors (ϕ, ϕ^*) and (φ, φ^*) are attached to these co-ordinates. The Lagrangian for such a coupled system is as follows:

$$\begin{aligned} \mathcal{L} &= P \cdot \dot{X} - \frac{e_o}{2}(P^2 + \alpha) - \frac{i}{2}(\phi^T \sigma_2 \dot{\phi} + \phi^\dagger \sigma_2 \dot{\phi}^*) \\ &+ p \cdot \dot{x} - \frac{e_1}{2}(p^2 + \alpha_s + kx^2) - \frac{i}{2}(\varphi^T \sigma_2 \dot{\varphi} + \varphi^\dagger \sigma_2 \dot{\varphi}^*) \end{aligned} \tag{44}$$

where the arguments of α and α_s are $(-P \cdot V)$ and $(-p \cdot v)$. The light-like vector V^μ is given by equation (3) and v^μ is obtained from it by replacement: $(\phi, \phi^*) \rightarrow (\varphi, \varphi^*)$. It will be noticed that the harmonic potential is coupled to the metric e_1 in such a way that the total Lagrangian is Lorentz and reparametrization invariant. The dynamics of this system is dictated by the four first-class constraints. The generator $\mathcal{G}$, given by:

$$\mathcal{G} = \dot{\omega}_o \Pi_{e_o} + \dot{\omega}_1 \Pi_{e_1} + \frac{\omega_o}{2}(P^2 + \alpha) + \frac{\omega_1}{2}(p^2 + \alpha_s + kx^2) \tag{45}$$

leads to the following gauge transformations:

$$\begin{aligned} \delta X^\mu &= \omega_o(P^\mu - \frac{\alpha'}{2}V^\mu); \delta P^\mu = 0; \delta\alpha = 0; \delta e_o = \dot{\omega}_o; \delta e_1 = \dot{\omega}_1 \\ \delta x^\mu &= \omega_1(p^\mu - \frac{\alpha'_s}{2}v^\mu); \quad \delta p^\mu = -\omega_1 k x^\mu; \quad \delta\alpha_s = \omega_1 k \alpha'_s v \cdot x; \\ \delta\phi_\sigma &= -\frac{\omega_o \alpha'}{4}\varepsilon_{\sigma\rho}(\sigma \cdot P)^T_{\rho\gamma}\phi^*_\gamma; \qquad \delta\phi^*_\sigma = -\frac{\omega_o \alpha'}{4}\varepsilon_{\sigma\rho}(\sigma \cdot P)_{\rho\gamma}\phi_\gamma \\ \delta\varphi_\sigma &= -\frac{\omega_1 \alpha'_s}{4}\varepsilon_{\sigma\rho}(\sigma \cdot p)^T_{\rho\gamma}\varphi^*_\gamma; \qquad \delta\varphi^*_\sigma = -\frac{\omega_1 \alpha'_s}{4}\varepsilon_{\sigma\rho}(\sigma \cdot p)_{\rho\gamma}\varphi_\gamma \end{aligned} \tag{46}$$

where ω_o and ω_1 are the gauge parameters. Under above transformations, the Lagrangian undergoes following change:

$$\delta\mathcal{L} = \frac{d}{d\tau}\Big[\frac{\omega_o}{2}(P^2 - 2\alpha' P \cdot V - \alpha) + \frac{\omega_1}{2}(p^2 - 2\alpha'_s p \cdot v - \alpha_s - kx^2)\Big]. \tag{47}$$

The BRST quantization of this system can be performed as outlined earlier for the single indecomposable object. The nilpotent BRST charge(Q_B) is as follows:

$$Q_B = \frac{c_o}{2}(P^2 + \alpha) + \frac{c_1}{2}(p^2 + \alpha_s + kx^2) + b_o \dot{c}_o + b_1 \dot{c}_1 \tag{48}$$

where b_i and c_i (i=0,1) are Nakanishi-Lautrup auxiliary fields and Faddeev-Popov ghost fields respectively. This approach can be generalized in a straightforward manner to the multi-relativistic indecomposable objects.

Another unsolved problem in the context of our discussion is to consider these Regge-trajectory models and their generalization to string models in the non-trivial background fields following the methodology of refs. [30]. The supersymmetrization of these models is yet another interesting project for future investigations. We hope to come to all the problems, indicated above, in future.

Part of this work was done at Center for Theoretical Studies, I.I.Sc.-Bangalore (India) and Mehta Research Institute, Allahabad (India). Fruitful discussions with N.Mukunda, H.S.Mani, R.K.Kaul, J.Maharana and D.Sen are gratefully acknowledged. It is pleasure to thank A.Pashnev, A.Filippov and A.Isaev for discussions on twistors and N-particle gauge theory. My thanks are also due to Boris, Ruchir Shukla and G.R.Nagabhushan for helps and encouragements.

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Received by Publishing Department
on December 25, 1992.