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DECAY CONSTANTS OF HEAVY MESONS IN THE RELATIVISTIC POTENTIAL MODEL WITH VELOCITY DEPENDENT CORRECTIONS

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Константы распада тяжелых мезонов в релятивистской потенциальной модели с поправками, зависящими от скорости

В релятивистской модели с потенциалом, зависящим от скорости, вычислены массы и лептонные константы распада тяжелых псевдоскалярных и векторных мезонов. Обсуждается возможность использования этого потенциала.

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Decay Constants of Heavy Mesons in the Relativistic Potential Model with Velocity Dependent Corrections

In the relativistic model with the velocity dependent potential the masses and leptonic decay constants of heavy pseudoscalar and vector mesons are computed. The possibility of using this potential is discussed.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Over the last years considerable attention has been payed to the theoretical analysis of heavy quark $(Q\bar{Q})$ or heavy-light quark $(Q\bar{q})$ systems in relativistic models[1]. The consideration of relativistic models and consequently relativistic potentials must be important especially for $(Q\bar{q})$ systems (like B, D mesons) where one of the quarks is light. Increasing experimental and theoretical interest in heavy-light quark systems was a result of observation of $B^0-\overline{B}^0$ mixing. The mass difference $\Delta m = m_{B_L} - m_{B_S}$, which can be evaluated by the box diagram, depends on the square of the decay constant f_B of the B meson. There are several methods for evaluating pseudoscalar f_p and electromagnetic f_V decay constants. In this paper, the relativistic potential model is used to calculate the leptonic decay constants of heavy mesons.

The decay constants f_V and f_p are defined in the standard way

$$
\begin{aligned}
&< 0 \mid A^{\mu}_{ij} \mid P(p) >= ip^{\mu} Q_{ij}\sqrt{2} f_p \\
&< 0 \mid V^{\mu}_{ij} \mid V(\epsilon, p) >= \epsilon^{\mu} Q_{ij} f_V
\end{aligned}
\tag{1}
$$

where Q_{ij} is the meson flavor matrix, and the currents V^{μ}_{ij} and A_{ij} are given by

$$
\begin{aligned}
A^{\mu}_{ij} &= \bar{q}_i(x)\gamma^{\mu}\gamma^5 q_j(x) \\
V^{\mu}_{ij} &= \bar{q}_i(x)\gamma^{\mu} q_j(x)
\end{aligned}
\tag{2}
$$

Taking into consideration the relativistic effects;for pseudoscalar and electromagnetic decay constants the following expressions are valid [2]:

$$f_p = \sqrt{3/2}\frac{1}{2\pi M}\int_0^{\infty} dk \tilde{u}_0(k) k N^{1/2}\left[1 - \frac{k^2}{(E_1+m_1)(E_2+m_2)}\right]$$

$$f_V = \sqrt{3}\frac{1}{2\pi}\int_0^{\infty} dk \tilde{u}_0(k) k N^{1/2}\left[1 + \frac{k^2}{3(E_1+m_1)(E_2+m_2)}\right] \tag{3}$$

with $N = \frac{(E_1+m_1)(E_2+m_2)}{E_1E_2}$ and $E_i = (k^2+m_i^2)^{1/2}$

The wave function $\tilde{u}_0(k) = \int_0^{\infty} dr \sin(kr) u_0(r)$ is normalized as follows:

$$\int_0^{\infty} dk \mid \tilde{u}_0(k) \mid^2 = 2M \tag{4}$$

where M is the meson mass.

In the last few years several relativistic models and potentials were used for computing masses and decay constants of heavy mesons [1],[2]. One of the methods of deriving the quark-antiquark relativistic potential was developed in ref.[3], where from QCD, up to an order of $1/c^2$, the velocity dependent corrections of the $q\bar{q}$ potential were calculated. In the paper [3], using the static part of the potential

$$V_{st}(r) = -k/r + C + \sigma r \tag{5}$$

after developing a systematic method for deriving the $q\bar{q}$ potential in the form of an inverse quark mass expansion, the authors have obtained the velocity dependent part of this static potential:

$$
\begin{aligned}
V_{vd} = &\frac{1}{8}\left(\frac{1}{m_1^2}+\frac{1}{m_2^2}\right)\Delta(-k/r+\sigma r) - \frac{1}{9}\left(\frac{1}{m_1^2}+\frac{1}{m_2^2}-\frac{1}{m_1m_2}\right)\Delta\sigma r - \\
&-C\sum_{j=1}^{2}\frac{P_j^2}{4m_j^2} + \frac{k}{2m_1m_2}\left(P_1^h\frac{1}{r}P_2^h + P_2^h\frac{1}{r}P_1^h\right) + \\
&+\frac{1}{4m_1m_2r^2}\left(k/r+\frac{1}{3}\sigma r\right)\left(\vec{L}_1\vec{L}_2+\vec{L}_2\vec{L}_1\right) - \sum_{j=1}^{2}\frac{\sigma}{6m_j^2 r}L_j^2
\end{aligned}
\tag{6}
$$

Using this potential, below we will discuss how it can describe the spectra of $Q\overline{Q}$ and $Q\bar{q}$ mesons. In the reference system of the center of mass $\vec{P}_1 = -\vec{P}_2 \equiv \vec{P}$ the S wave part $(l=0)$ of the velocity dependent potential V_{vd} can be written in the form

$$V_{vd} = \frac{1}{2}\left(\frac{1}{m_1^2}+\frac{1}{m_2^2}+\frac{4}{m_1m_2}\right)\pi k\delta^3(\vec{r}) + \frac{\sigma}{36r}\left(\frac{1}{m_1^2}+\frac{1}{m_2^2}+\frac{8}{m_1m_2}\right) -$$

$$-C\frac{\vec{P}^2}{4}(\frac{1}{m_1^2}+\frac{1}{m_2^2})-\frac{k}{2m_1m_2}(\frac{1}{r}\vec{P}^2+\vec{P}^2\frac{1}{r}) \quad (7)$$

Below we consider only S wave states ($l=0$). According to [4], the spin dependent part of the potential for this case is given by

$$V_{sd}=\frac{8\pi k}{3m_1m_2}\vec{S}_1\vec{S}_2\delta^3(\vec{r}) \quad (8)$$

Following ref.[5] we assume an ad hoc spread in the color charge performing the replacement

$$\frac{1}{m^2}\delta^3(\vec{r})\rightarrow\frac{1}{m^2}(\frac{f^2m^2}{\pi})^{3/2}\exp(-f^2m^2r^2)$$
$$\frac{1}{m_1m_2}\delta^3(\vec{r})\rightarrow\frac{1}{m_1m_2}(\frac{f^2m_1m_2}{\pi})^{3/2}\exp(-f^2m_1m_2r^2) \quad (9)$$

with $f=\sqrt{2.3}$ [5].

We consider the wave equation

$$[(-\hbar\vec{\nabla}^2+m_1^2\ )^{1/2}\ +\ (-\hbar\vec{\nabla}^2+m_2^2\ )^{1/2}\ +V(\vec{r})]\,\Psi(\vec{r})=E\Psi(\vec{r}) \quad (10)$$

which arises from the Bethe-Salpeter equation in QCD by replacing the full interaction with the instantaneous local potential $V(r)$.

In the case of the potential (5),(7),(8) this equation gets the form ($r\equiv\mid\vec{r}\mid$)

$$\Big\{(-\hbar\vec{\nabla}^2+m_1^2)^{1/2}+(-\hbar\vec{\nabla}^2+m_2^2)^{1/2}-\left[\frac{C}{4}(\frac{1}{m_1^2}+\frac{1}{m_2^2})+\frac{k}{2m_1m_2}\frac{1}{r}\right](-\hbar\vec{\nabla}^2)-$$
$$-\frac{k}{2m_1m_2}(-\hbar\vec{\nabla}^2)\frac{1}{r}\Big\}\,\Psi(\vec{r})=[E-V_0(r)]\Psi(\vec{r}) \quad (11)$$

where $V_0(r)=V_{st}(r)+V_{sd}(r)+$

$$\frac{1}{2}(\frac{1}{m_1^2}+\frac{1}{m_2^2}+\frac{4}{m_1m_2})\pi k\delta^3(\vec{r})+\frac{\sigma}{36r}(\frac{1}{m_1^2}+\frac{1}{m_2^2}+\frac{8}{m_1m_2}) \quad (12)$$

In equations (10) and (11), the operator $(-\hbar\vec{\nabla}^2+m^2)^{1/2}$ is defined by the following spectral representation:

$$(-\hbar\vec{\nabla}^2+m_1^2)^{1/2}\Psi(\vec{r})=\int\frac{d^3k}{(2\pi\hbar)^3}\int d^3r'\exp(i\vec{k}(\vec{r}-\vec{r}')/\hbar)(\vec{k}^2+m^2)^{1/2}\Psi(\vec{r}')$$

For the central potential $V(\vec{r})=V(r)$ the angular dependence in $\Psi(\vec{r})$ can be factorized

$$\Psi(\vec{r})=Y_l^m(\hat{r})\Phi(r)\equiv Y_l^m(\hat{r})u_l(r)/r \quad (13)$$

and after a simple transformation we get the following equation:

$$\left\{f(r)\left[\hbar^2\frac{d^2}{dr^2}-\frac{l(l+1)}{r^2}\right]+\frac{\hbar^2}{r^2}\frac{k}{m_1m_2}\left[\frac{d}{dr}-\frac{1}{r}\right]\right\}u_l(r)=[V_0(r)-E]u_l(r)+$$
$$+\frac{2}{\pi\hbar}\int_0^\infty dr'\int_0^\infty dk\,[(\vec{k}^2+m_1^2)^{1/2}\ +\ (\vec{k}^2+m_2^2)^{1/2}]\,\chi_l(\frac{kr}{\hbar})\chi_l(\frac{kr'}{\hbar})_l u_l(r') \quad (14)$$

with $f(r)=-\frac{C}{4}(\frac{1}{m_1^2}+\frac{1}{m_2^2})-\frac{k}{m_1m_2}\frac{1}{r};\chi_l(r)=rj_l(r)$ – where $j_l(r)$ are spherical Bessel functions. The origin of the left hand side of this equation is the velocity dependent part of the potential (7).

As it was noticed in ref.[6], in contrast with the Schrödinger equation, the Coulombic divergence for $r\rightarrow 0$ in the relativistic case (10) results in a logarithmic divergence of the wave function $\Phi_0(r)(l=0)$ at the origin. This is a result of the approximation used in the Bethe-Salpeter equation. In our case, we do not have such a problem and, consequently, we do not need to smooth the potential for very small r to a constant as it was done for Richardson potential in ref.[2]. It is easy to show that the wave function $\Phi_l(r)$ for this potential behaves as r^{l+1} at the origin. Therefore,

$u_0(r)=\frac{\Phi_0(r)}{r}{}_{r\rightarrow 0}\rightarrow$ const

The reason for this, of course, is the dependence of the potential on $\vec{P}^2(V_{vd})$ and consequently the existence of the left hand side of equation (14). Using $j_0(r)=\sin(r)/r$ and the "parity" relation

$$u_l(-r)=(-1)^{l+1}u_l(r)$$

obtained from (11), if we set $V(-r)=V(r)$, after replacing $l(l+1)$ by $(l+1/2)^2$ in equation (14), as it is accepted in the case of the WKB approximation in Quantum Mechanics[7], we get the following S-wave equation satisfied by $u_0(r)$:

$$\left\{f(r)\left[\hbar^2\frac{d^2}{dr^2}-\frac{0.25}{r^2}\right]+\frac{\hbar^2}{r^2}\frac{k}{m_1m_2}\left[\frac{d}{dr}-\frac{1}{r}\right]\right\}u_0(r)=[V_0(r)-E]u_0(r)+$$
$$+\frac{1}{2\pi\hbar}\int_{-\infty}^\infty dr'\int_{-\infty}^\infty dk\,[(\vec{k}^2+m_1^2)^{1/2}+(\vec{k}^2+m_2^2)^{1/2}]\exp(ik(r-r')/\hbar)u_0(r') \quad (15)$$

To solve this equation we use the WKB approximation developed in ref.[8] and search for a solution in the form

$$u_0(r) = \exp[i\sigma_0(r)/\hbar + \sigma_1(r)] \tag{16}$$

The functions $\sigma_0(r)$ and $\sigma_1(r)$ are determined by the saddle point equations. At the zeroth order in $\hbar$ we obtain the following equation for $\sigma_0'(r) = d\sigma_0(r)/dr$:

$$E - V_0(r) - f(r)\frac{0.25}{r^2} - f(r)\sigma_0'^2(r) = (\sigma_0'^2 + m_1^2)^{1/2} + (\sigma_0'^2 + m_2^2)^{1/2} \tag{17}$$

The spectrum is determined in the standard way

$$\int_{r_1}^{r_2} \sigma_0'(r)dr = \pi(n+1/2) \qquad (\hbar = 1) \tag{18}$$

where r_1 and r_2 are "classical" turning points defined from (17) by the conditions

$$\sigma_0'(r_1) = \sigma_0'(r_2) = 0; \sigma_0'(r) > 0 \text{ when } r_1 < r < r_2$$

(In the cases discussed below for $\sigma_0'(r) = 0$ equation (17)

$$E - V_0(r) - f(r)\frac{0.25}{r^2} = 0$$

has two "classical" turning points r_1 and r_2).

Using the saddle point equations, after some calculations we obtain the following expression for $\exp(\sigma_1(r))$:

$$\exp(\sigma_1(r)) = \left|\sigma_0'(r)\left[2f(r) + \frac{1}{(\sigma_0'^2+m_1^2)^{1/2}} + \frac{1}{(\sigma_0'^2+m_2^2)^{1/2}}\right]\right|^{-1/2}$$

and, therefore, for the wave functions $u_0(r)$ in the WKB approximation we have ($\hbar = 1$):

$$u_I(r) = \frac{A}{2}\exp(\sigma_1(r))\exp\left[-\int_r^{r_1} \mid \sigma_0'(x) \mid dx\right] \qquad 0 < r < r_1$$

$$u_{II}(r) = A\exp(\sigma_1(r))\sin\left[\int_r^{r_2} \sigma_0'(x)dx\right] \qquad r_1 < r < r_2$$

$$u_{III}(r) = \frac{A}{2}\exp(\sigma_1(r))\exp\left[-\int_{r_2}^{r} \mid \sigma_0'(x) \mid dx\right] \qquad r > r_2 \tag{19}$$

where A is a normalization constant.

Using (17) and (18), from a fit of masses of $b\bar{b}$ mesons (S-states) we fix the values of the parameters k, σ of the potential and b quark mass. The value of the third parameter C in the potential (5) can be calculated according to the prescription given in ref.[9]:

$$C = -2\exp[-(\gamma - 0.5)]\sigma^{1/2} \tag{20}$$

where $\gamma = 0.5772\ldots$ is Euler's constant. The mass of c quark m_c was chosen by fitting the masses of $c\bar{c}$ mesons. We stress that, since we include in the potential the spin-dependent term (8), we are able to distinguish between pseudoscalar and vector states. We fix the values of the parameters as follows: $k = 0.410, \sigma = 0.173, m_b = 5.180, m_c = 1.777$. We have to note that these values differ a little from the corresponding values given in ref.[3].

Table I. - The mass spectrum of heavy ($Q\bar{Q}$) systems (GeV/c).$H(21)$ corresponds to the mass spectrum calculated by the WKB method with the Hamiltonian (21).

		$c\bar{c}$			$b\bar{b}$		
			H(21)	exp		H(21)	exp
1	$^1S_0$	2.98	3.02	2.98	9.41		
2	$^1S_0$	3.58	3.72	3.59			
1	$^3S_1$	3.10	3.09	3.097	9.47	9.45	9.460
2	$^3S_1$	3.67	3.65	3.686	10.02	10.01	10.023
3	$^3S_1$	4.06	4.23	4.040	10.35	10.35	10.355
4	$^3S_1$	4.40	4.69	4.415	10.60	10.63	10.580
5	$^3S_1$				10.85	10.88	10.865
6	$^3S_1$				11.04	11.12	11.019

Table II. - The mass spectrum of heavy-light ($Q\bar{q}$) systems (GeV/c).

Mesons	Theory	Experiment
$B_c(0^-)$	5.54	
$B_c(1^-)$	6.34	
$B(0^-)$	5.32	5.279
$B^*(1-)$	5.57	
$D(0^-)$	1.77	1.864
$D^*(1^-)$	2.22	2.010
$D_s(0^-)$	1.87	1.969
$D_s^*(1^-)$	2.25	2.110

The results for the heavy meson (QQ̄) masses are given in Table I. There are given also the meson masses $H(21)$ calculated by the same method(WKB approximation) but using the Hamiltonian

$$H = 2m + p^2/m - p^2/4m^3 + V \tag{21}$$

which is an approximation of Hamiltonian presented in equation (10). As one can see, the agreement with the experimental data is quite good for $b\bar{b}$ and $c\bar{c}$ states. If we compare our results with the spectra evaluated from this potential in ref.[3] (with other values of the parameters), by using the Hamiltonian (21)instead of (10) some improvement in the agreement with the experimental data can be seen, expect of course for those states that have been used in ref.[3] as inputs. The reason for this is the existence in (21) of terms like p^2/m(in the case of the WKB approximation, $\sigma_0'^2/m$ term). Eq.(10) and, consequently, eq.(17) do not contain such terms. The only term like this presented in (17) is $\sigma_0'^2/m^2$ but not $\sigma_0'^2/m$. Since the WKB approximation works very good for $b\bar{b}$ and $c\bar{c}$ states ($\sigma_0'^2/m$ and of course $\sigma_0'^2/m^2$ are enough small) in both the cases ((10) and (21)), the agreement with the experimental data is satisfactory. But as one can see, the superiority of the Hamiltonian (10) is indeed, especially for $c\bar{c}$ states, since $m_c < m_b$. In the case of $Q\bar{q}$ states, eq. (10) gives much better results ($\sigma_0'^2/m$ is not enough small) than (21). Even for B_c meson the average value $< \sigma_0'^2/m > \simeq 1$.

On the other hand, it must be mentioned that even the Hamiltonian (10) is not able to describe the mass spectra of $Q\bar{q}$ systems with $q = u, d, s$ well. From the fit of B, D, D_s meson masses it follows that $m_u = m_d = 0.46$ GeV while $m_s = 0.36$ GeV, which, of course, does not correspond to the real situation ($m_s > m_d$!). The reason for this is the fact that the velocity dependent part of the potential has the form of an inverse quark mass expansion. Therefore, it seems impossible, using the velocity dependent relativistic potential (5)-(6), to describe well the B, D, D_s meson spectra and, consequently, to determine the light quark (u, d, s) masses from the fit of heavy meson spectra. In spite of this, the calculated values of these meson masses are given in Table II.

Using the wave function (19) and expressions (3) for decay constants we calculated the values f_p and f_v for different mesons. The results are given in Tables III and IV. For $Q\overline{Q}$ systems the decay constant f_v can be related to the $\Gamma(V \to e^+e^-)$

$$f_v = (3M^3\Gamma(V \to e^+e^-)/(4\pi e_q^2\alpha^2))^{1/2} \tag{22}$$

(The difference between (22) and the corresponding formula in [3] arises from a different definition of f_v). As one can see from Table III, the agreement

Table III. - Heavy (QQ̄) vector mesons decay constants f_v(GeV^2).

		$c\bar{c}$		$b\bar{b}$	
1	$^3S_1$	1.20	1.185 ± 0.05	6.72	6.765 ± 0.09
2	$^3S_1$	0.91	1.035 ± 0.05	4.92	4.89 ± 0.12
3	$^3S_1$	0.81	0.705 ± 0.08	4.62	4.44 ± 0.15
4	$^3S_1$	0.59	0.639 ± 0.07	3.36	3.39 ± 0.36
5	$^3S_1$			4.08	3.99 ± 0.45
6	$^3S_1$			3.18	2.64 ± 0.30

Table IV. - Decay constants of heavy-light (Qq̄) systems.

Pseudoscalar mesons f_p (GeV)		Vector mesons f_V (GeV)	
B_c	0.449	B_c^*	3.059
B	0.222	B^*	1.102
B_s	0.179	B_s^*	0.773
D	0.230	D	0.514
D_s	0.149	D_s^*	0.435
$\eta_c(c\bar{c})$	0.176		
$\eta_b(b\bar{b})$	0.300		

between theoretical results and experimental data is quite good for $c\bar{c}$ and $b\bar{b}$ states. As it has been mentioned above, it seems impossible to determine the light quark (u, d, s) masses using this potential (5),(6),(8). But it can be found that the heavy $Q\bar{q}$ meson decay constants are not very sensitive to the value of m_q.(The main contribution to the decay constants comes from the value of the wave function at the origin $\Phi_0(0), \Phi_0'(0)$). Therefore, in spite of the fact that the potential cannot describe the mass spectrum of heavy-light (Qq̄) mesons satisfactory, the values of the decay constants are determined quite well. That is why the approximation scheme proposed in the limit $m_Q \to \infty$ [10] gives the values of f_p and f_v in a quite good approximation. The values f_v, f_p given in Table IV do not change very much of u, d, s quark masses change a little.

Let us note that the value of the B meson decay constant $f_B = 222$ MeV is in agreement with the recent calculations in the lattice QCD [11].

In conclusion, we have discussed the model for heavy (QQ̄) and heavy-light (Qq̄) mesons based on the relativistic wave equation and the relativistic velocity dependent potential. We have calculated the mass spectra of heavy mesons and their leptonic decay constants. The analyses, with the use of the WKB approximation, have shown that this potential cannot describe the heavy-light

(Q$\bar{q}$) mesons spectra for $q = u, d, s$ quarks satisfactorily though can determine the values of the decay constants quite well.

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References

[1] S.N.Sinha, Phys.Lett.**B178**, 110, (1986)
M.Suzuki, Phys.Lett. **B162**, 392, (1985)
S.Gapstik, S.Godfrey, Phys.Rev. **D41**, 2856, (1990)
H.D.Trottier, R.R.Mendel, Phys.Lett. **B231**, 312, (1989); Phys.Rev. **D40**, 3709, (1989)

[2] P.Cea, P.Colangelo, L.Cosmai, G.Nardulli, Phys.Lett. **B206**, 692(1988)
P.Colangelo, G.Nardulli, preprint BARI TH/90-70, 1990

[3] A.Barchielli, E.Montaldi, G.M.Prosperi, Nucl.Phys. **B296**, 625, (1988)
A.Barchielli, N.Brambilla, G.M.Prosperi, Nuovo Cimento **103A**, (1190)

[4] E.Eichten, F.Feinberg, Phys.Rev. **D23**, 2724, (1981)

[5] R.McClary, N.Byers, Phys.Rev. **D28**, 1692, (1983)

[6] L.Durand, Phys.Rev. **D32**, 1257, (1985)

[7] L.D.Landau, E.M.Lifshitz, "Quantum Mechanics", (Pergamon, New York, 1958)

[8] P.Cea, P.Colangelo, G.Nardulli, G.Piano, G.Preparata, Phys.Rev. **D26**, 1157, (1982)

[9] D.Gromes, Z.Phys. **C11**, 147, (1981)

[10] M.B.Voloshin, M.A.Shifman, Sov. Journal of Nucl. Phys. **45**, (2), 292, (1987)
H.D.Politzer, M.B.Wise, Phys.Lett. **B206**, 681(1988)
B.Grinstein, Nucl.Phys. **B339**, 245, (1990)

[11] 11. C.Q.Geng, P.Turcotte, preprint UdeM-LPN-TH-66, Montreal, (1191)

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