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NEW FORMULATION OF THE UNITARY  
AND ANALYTIC VMD  
MODEL OF NUCLEON  
ELECTROMAGNETIC STRUCTURE

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Предложена новая унитарная и аналитическая ВМД модель электромагнитной структуры нуклонов, которая зависит только от восьми (в отличие от предыдущих 14) свободных параметров. Она опять конструируется включением правильных аналитических свойств нуклонных форм-факторов в приближении двух разрезов вместе с ненулевыми значениями ширин векторных мезонов в каноническую ВМД модель с теми самими векторными мезонами. До этого эта модель была автоматически нормализованной и обладала асимптотическим поведением, предсказанным (до логарифмических поправок) КХД для нуклонов.

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New Formulation of the Unitary and Analytic VMD  
Model of Nucleon Electromagnetic Structure

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A new unitary and analytic VMD model of the nucleon electromagnetic structure dependent only on eight (to be compared with previous fourteen) free parameters with clear physical meaning is presented. It is again constructed by incorporating of a two-cut approximation of the correct nucleon form factor analytic properties together with non-zero values of the vector meson widths into canonical VMD model with the same vector mesons which, however, before is automatically normalized and governs the asymptotic behaviour as predicted (up to logarithmic corrections) by QCD for nucleons.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

# 1 Introduction

A problem of a dynamical description of the hadron electromagnetic (e.m.) structure is solved [1] only partially. The perturbative QCD gives just the asymptotic behaviour of e.m. form factors (ff's). The QCD sum rules predict no more than the behaviour of e.m. ff's in a restricted region of space-like momenta. Therefore up to now in a global description of the existing experimental information on hadron e.m. ff's one deals with phenomenological models. Almost all of them are based on the well known experimental fact of creation of vector meson resonances in  $e^+e^-$  annihilation process, the first theoretical approximation of which is contained in the canonical zero-width vector meson-dominance (VMD) model

$$F_n(t) = \sum_v \frac{m_v^2}{m_v^2 - t} (f_{vh}/f_v), \quad (1)$$

where  $m_v$  is the vector meson mass,  $t = -Q^2$  is the photon four-momentum transfer squared,  $f_{vh}$  is a coupling constant of the vector-meson to hadron and  $f_v$  is the so-called universal vector meson coupling constant describing a transition between the virtual photon and the vector meson.

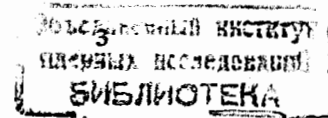
It is easy to see from (1) that the canonical VMD model governs the  $\sim 1/t$  asymptotic behaviour if  $|t| \rightarrow \infty$ . The latter coincides with perturbative QCD predictions (up to logarithmic corrections) for mesons. For baryons and more compound objects, like nuclei, the situation is more complicated. Generally, the QCD predicts the  $\sim t^{(1-n_q)}$  asymptotic behaviour for them if  $|t| \rightarrow \infty$ , where  $n_q$  is a number of constituent quarks of an object under consideration. As a result the asymptotic behaviour of (1) is in contradiction with perturbation QCD predictions for baryons and nuclei.

The latter is indicated also by existing experimental data on the nucleon and light nuclei e.m. ff's. Therefore many attempts were undertaken to solve theoretically the latter inconsistency in construction of phenomenological models for a global description of the e.m. structure of baryons and nuclei. However, extraordinary results were achieved [2-7] only recently by incorporating of a two-cut approximation of the correct ff analytic properties into (1), which leads to a factorization of resultant expressions into the pure asymptotic part and the finite energy part. Then the corresponding QCD asymptotic behaviour of e.m. ff's is made consistent just by a simple change of the power of the asymptotic part of a resultant expression.

In this way, the incorporation of analytic properties into (1) plays a two fold role. First, by means of the latter a unitarization of the canonical VMD model, leading to a perfect description of data, is realized, and second, it provides a powerful tool for a consistent incorporation of the correct ff asymptotic behaviour as predicted (up to logarithmic corrections) by the perturbative QCD.

However, the latter procedure, though nonviolating any of nice properties of the constructed global model, uses ill-founded mathematical methods. The correct asymptotic behaviour is achieved by changes carried by hand in the asymptotic part, leaving unchanged the finite energy part of resultant expressions. Moreover, it produces models with too many free parameters.

Here we formulate a new unitary and analytic VMD model of the nucleon e.m. structure which is again constructed by incorporating of the two-cut approximation of the correct nucleon ff analytic properties together with non-zero vector meson widths but now into a modified version of (1) that before is automatically normalized and governs the



asymptotic behaviour as predicted by QCD for nucleons. In this way an incorporation of analytic properties into the VMD model does not provide a consistent attainment of the correct asymptotic behaviour of the e.m. ff's, because now the latter is obtained automatically. However, it plays still a role of a unitarization of the canonical VMD model which leads to a new global model of the nucleon e.m. structure perfectly describing all the experimental information with only eight (to be compared with previous fourteen) free parameters, and moreover, it shows the change in the finite energy part of the resultant factorized expression explicitly.

In the next section we demonstrate how one can obtain the canonical VMD model (1) in the zero-width approximation to be automatically normalized and to govern the asymptotic behaviour as predicted by QCD for nucleons. Section 3 is devoted to a unitarization of the latter. In section 4 we analyze all the existing experimental information on the nucleon e.m. structure, including also the new LEAR data. Conclusions and summary are given in section 5.

## 2 Canonical VMD model of the nucleon electromagnetic structure with QCD asymptotics

The nucleon e.m. structure is completely described by four scalar functions, two electric  $G_E^p(t)$ ,  $G_E^n(t)$  and two magnetic  $G_M^p(t)$ ,  $G_M^n(t)$  ff's, which can be decomposed into isoscalar and isovector parts of the Dirac

and Pauli ff's as follows

$$\begin{aligned} G_E^p(t) &= [F_1^s(t) + F_1^v(t)] + \frac{t}{4m_p^2}[F_2^s(t) + F_2^v(t)] \\ G_M^p(t) &= [F_1^s(t) + F_1^v(t)] + [F_2^s(t) + F_2^v(t)] \\ G_E^n(t) &= [F_1^s(t) - F_1^v(t)] + \frac{t}{4m_n^2}[F_2^s(t) - F_2^v(t)] \\ G_M^n(t) &= [F_1^s(t) - F_1^v(t)] + [F_2^s(t) - F_2^v(t)], \end{aligned} \quad (2)$$

where  $m_p$  and  $m_n$  are masses of the proton and neutron, respectively. The Dirac and Pauli isoscalar and isovector ff's can be in the framework of the canonical VMD model (1) parametrized in the form

$$\begin{aligned} F_1^s(t) &= \sum_{s=\omega, \omega', \omega''} \frac{m_s^2}{m_s^2 - t} \cdot (f_{sNN}^{(1)}/f_s); \\ F_2^s(t) &= \sum_{s=\omega, \omega', \omega''} \frac{m_s^2}{m_s^2 - t} \cdot (f_{sNN}^{(2)}/f_s); \\ F_1^v(t) &= \sum_{v=\rho, \rho', \rho'', \rho'''} \frac{m_v^2}{m_v^2 - t} \cdot (f_{vNN}^{(1)}/f_v); \\ F_2^v(t) &= \sum_{v=\rho, \rho', \rho'', \rho'''} \frac{m_v^2}{m_v^2 - t} \cdot (f_{vNN}^{(2)}/f_v), \end{aligned} \quad (3)$$

where in the isoscalar ff's we require the couplings of  $\phi$  and  $\phi'$  mesons to nucleons to be zero, taking the OZI rule [8-10] into account strictly. The ratios of coupling constants in (3) are constrained by the relations

$$\begin{aligned} \sum_{s=\omega, \omega', \omega''} (f_{sNN}^{(1)}/f_s) &= \frac{1}{2}; & \sum_{s=\omega, \omega', \omega''} (f_{sNN}^{(2)}/f_s) &= \frac{1}{2}(\mu_p + \mu_n) \\ \sum_{v=\rho, \rho', \rho'', \rho'''} (f_{vNN}^{(1)}/f_v) &= \frac{1}{2}; & \sum_{v=\rho, \rho', \rho'', \rho'''} (f_{vNN}^{(2)}/f_v) &= \frac{1}{2}(\mu_p - \mu_n) \end{aligned} \quad (4)$$

following from the normalization conditions

$$\begin{aligned} G_E^p(0) &= 1; & G_M^p(0) &= 1 + \mu_p \\ G_E^n(0) &= 0; & G_M^n(0) &= \mu_n, \end{aligned} \quad (5)$$

where  $\mu_p$  and  $\mu_n$  are the anomalous magnetic moments of the proton and neutron, respectively.

To construct the VMD model to be automatically normalized and to have the correct asymptotic behaviour, we first transform relations (3) to have common denominators. As a result, we get

$$\begin{aligned}
F_1^s(t) &= \frac{a_0^{1s} + a_1^{1s} \cdot t + a_2^{1s} \cdot t^2}{(m_\omega^2 - t)(m_{\omega'}^2 - t)(m_{\omega''}^2 - t)}; \\
F_2^s(t) &= \frac{a_0^{2s} + a_1^{2s} \cdot t + a_2^{2s} \cdot t^2}{(m_\omega^2 - t)(m_{\omega'}^2 - t)(m_{\omega''}^2 - t)}; \\
F_1^v(t) &= \frac{b_0^{1v} + b_1^{1v} \cdot t + b_2^{1v} \cdot t^2 + b_3^{1v} \cdot t^3}{(m_\rho^2 - t)(m_{\rho'}^2 - t)(m_{\rho''}^2 - t)(m_{\rho'''}^2 - t)}; \\
F_2^v(t) &= \frac{b_0^{2v} + b_1^{2v} \cdot t + b_2^{2v} \cdot t^2 + b_3^{2v} \cdot t^3}{(m_\rho^2 - t)(m_{\rho'}^2 - t)(m_{\rho''}^2 - t)(m_{\rho'''}^2 - t)},
\end{aligned} \tag{6}$$

where coefficients  $a$  and  $b$  are functions of the resonance masses and ratios of coupling constants.

Now, one can immediately see that the asymptotic behaviour

$$F_1^s(t) \sim F_1^v(t) \sim t^{-2} \Big|_{|t| \rightarrow \infty}; \quad F_2^s(t) \sim F_2^v(t) \sim t^{-3} \Big|_{|t| \rightarrow \infty} \tag{7}$$

predicted (up to logarithmic corrections) by QCD for nucleons leads to the requirements

$$\begin{aligned}
a_2^{1s} = 0; & \quad a_1^{2s} = 0; \quad a_2^{2s} = 0 \\
b_3^{1v} = 0; & \quad b_2^{2v} = 0; \quad b_3^{2v} = 0,
\end{aligned} \tag{8}$$

which together with (4) give the following four systems of algebraic equations for coupling ratios

$$\begin{aligned}
(f_{\omega NN}/f_\omega) + (f_{\omega' NN}/f_{\omega'}) + (f_{\omega'' NN}/f_{\omega''}) &= \frac{1}{2} \tag{9a} \\
(f_{\omega NN}/f_\omega)m_\omega^2 + (f_{\omega' NN}/f_{\omega'})m_{\omega'}^2 + (f_{\omega'' NN}/f_{\omega''})m_{\omega''}^2 &= 0
\end{aligned}$$

$$\begin{aligned}
(f_{\omega NN}/f_\omega) + (f_{\omega' NN}/f_{\omega'}) + (f_{\omega'' NN}/f_{\omega''}) &= \frac{1}{2}(\mu_p + \mu_n) \\
(f_{\omega NN}/f_\omega)m_\omega^2 + (f_{\omega' NN}/f_{\omega'})m_{\omega'}^2 + (f_{\omega'' NN}/f_{\omega''})m_{\omega''}^2 &= 0 \tag{9b}
\end{aligned}$$

$$\begin{aligned}
(f_{\omega NN}/f_\omega) m_\omega^2(m_{\omega'}^2 + m_{\omega''}^2) + (f_{\omega' NN}/f_{\omega'}) m_{\omega'}^2(m_\omega^2 + m_{\omega''}^2) + \\
+ (f_{\omega'' NN}/f_{\omega''}) m_{\omega''}^2(m_\omega^2 + m_{\omega'}^2) &= 0
\end{aligned}$$

$$\begin{aligned}
(f_{\rho NN}/f_\rho) + (f_{\rho' NN}/f_{\rho'}) + (f_{\rho'' NN}/f_{\rho''}) + (f_{\rho''' NN}/f_{\rho'''}) &= \frac{1}{2} \tag{9c} \\
(f_{\rho NN}/f_\rho)m_\rho^2 + (f_{\rho' NN}/f_{\rho'})m_{\rho'}^2 + (f_{\rho'' NN}/f_{\rho''})m_{\rho''}^2 + \\
+ (f_{\rho''' NN}/f_{\rho'''})m_{\rho'''}^2 &= 0
\end{aligned}$$

$$\begin{aligned}
(f_{\rho NN}/f_\rho) + (f_{\rho' NN}/f_{\rho'}) + (f_{\rho'' NN}/f_{\rho''}) + \\
(f_{\rho''' NN}/f_{\rho'''}) &= \frac{1}{2}(\mu_p - \mu_n) \\
(f_{\rho NN}/f_\rho) m_\rho^2 + (f_{\rho' NN}/f_{\rho'}) m_{\rho'}^2 + (f_{\rho'' NN}/f_{\rho''}) m_{\rho''}^2 + \\
(f_{\rho''' NN}/f_{\rho'''}) m_{\rho'''}^2 &= 0 \tag{9d}
\end{aligned}$$

$$\begin{aligned}
(f_{\rho NN}/f_\rho) m_\rho^2(m_{\rho'}^2 + m_{\rho''}^2 + m_{\rho'''}^2) + \\
+ (f_{\rho' NN}/f_{\rho'}) m_{\rho'}^2(m_\rho^2 + m_{\rho''}^2 + m_{\rho'''}^2) + \\
+ (f_{\rho'' NN}/f_{\rho''}) m_{\rho''}^2(m_\rho^2 + m_{\rho'}^2 + m_{\rho'''}^2) + \\
(f_{\rho''' NN}/f_{\rho'''}) m_{\rho'''}^2(m_\rho^2 + m_{\rho'}^2 + m_{\rho''}^2) &= 0.
\end{aligned}$$

Their solutions take the following form (see also ref. [11])

$$\begin{aligned}
(f_{\omega' NN}/f_{\omega'}) &= \frac{m_{\omega''}^2}{2(m_{\omega''}^2 - m_{\omega'}^2)} - \frac{m_{\omega''}^2 - m_\omega^2}{m_{\omega''}^2 - m_{\omega'}^2} (f_{\omega NN}/f_\omega) \tag{10a} \\
(f_{\omega'' NN}/f_{\omega''}) &= -\frac{m_{\omega'}^2}{2(m_{\omega''}^2 - m_{\omega'}^2)} - \frac{m_{\omega'}^2 - m_\omega^2}{m_{\omega''}^2 - m_{\omega'}^2} (f_{\omega NN}/f_\omega)
\end{aligned}$$

$$\begin{aligned}
(f_{\omega NN}/f_{\omega}) &= \frac{1}{2}(\mu_p + \mu_n) \frac{m_{\omega'}^2 m_{\omega''}^2}{(m_{\omega'}^2 - m_{\omega}^2)(m_{\omega''}^2 - m_{\omega}^2)} \\
(f_{\omega' NN}/f_{\omega'}) &= \frac{1}{2}(\mu_p + \mu_n) \frac{m_{\omega}^2 m_{\omega''}^2}{(m_{\omega''}^2 - m_{\omega'}^2)(m_{\omega}^2 - m_{\omega'}^2)} \\
(f_{\omega'' NN}/f_{\omega''}) &= \frac{1}{2}(\mu_p + \mu_n) \frac{m_{\omega}^2 m_{\omega'}^2}{(m_{\omega}^2 - m_{\omega''}^2)(m_{\omega'}^2 - m_{\omega''}^2)}
\end{aligned} \tag{10b}$$

$$\begin{aligned}
(f_{\rho' NN}/f_{\rho'}) &= \frac{m_{\rho''}^2}{2(m_{\rho''}^2 - m_{\rho'}^2)} - \frac{m_{\rho''}^2 - m_{\rho}^2}{m_{\rho''}^2 - m_{\rho'}^2} (f_{\rho NN}/f_{\rho}) + \\
&\quad + \frac{m_{\rho''}^2 - m_{\rho'}^2}{m_{\rho''}^2 - m_{\rho'}^2} (f_{\rho'' NN}/f_{\rho''}) \\
(f_{\rho'' NN}/f_{\rho''}) &= -\frac{m_{\rho'}^2}{2(m_{\rho''}^2 - m_{\rho'}^2)} + \frac{m_{\rho'}^2 - m_{\rho}^2}{m_{\rho''}^2 - m_{\rho'}^2} (f_{\rho NN}/f_{\rho}) - \\
&\quad - \frac{m_{\rho''}^2 - m_{\rho'}^2}{m_{\rho''}^2 - m_{\rho'}^2} (f_{\rho'' NN}/f_{\rho''})
\end{aligned} \tag{10c}$$

$$\begin{aligned}
(f_{\rho NN}/f_{\rho}) &= \frac{1}{2}(\mu_p - \mu_n) \frac{m_{\rho'}^2 m_{\rho''}^2}{(m_{\rho'}^2 - m_{\rho}^2)(m_{\rho''}^2 - m_{\rho}^2)} - \\
&\quad - \frac{(m_{\rho''}^2 - m_{\rho'}^2)(m_{\rho''}^2 - m_{\rho}^2)}{(m_{\rho''}^2 - m_{\rho}^2)(m_{\rho'}^2 - m_{\rho}^2)} (f_{\rho'' NN}/f_{\rho''}) \\
(f_{\rho' NN}/f_{\rho'}) &= -\frac{1}{2}(\mu_p - \mu_n) \frac{m_{\rho}^2 m_{\rho''}^2}{(m_{\rho''}^2 - m_{\rho'}^2)(m_{\rho}^2 - m_{\rho'}^2)} + \\
&\quad + \frac{(m_{\rho''}^2 - m_{\rho'}^2)(m_{\rho''}^2 - m_{\rho}^2)}{(m_{\rho''}^2 - m_{\rho'}^2)(m_{\rho}^2 - m_{\rho'}^2)} (f_{\rho'' NN}/f_{\rho''}) \\
(f_{\rho'' NN}/f_{\rho''}) &= \frac{1}{2}(\mu_p - \mu_n) \frac{m_{\rho'}^2 m_{\rho}^2}{(m_{\rho''}^2 - m_{\rho'}^2)(m_{\rho''}^2 - m_{\rho}^2)} - \\
&\quad - \frac{(m_{\rho''}^2 - m_{\rho'}^2)(m_{\rho''}^2 - m_{\rho}^2)}{(m_{\rho''}^2 - m_{\rho'}^2)(m_{\rho''}^2 - m_{\rho}^2)} (f_{\rho'' NN}/f_{\rho''}).
\end{aligned} \tag{10d}$$

Substituting (10a - 10d) into (3) one gets the canonical VMD model in the zero-width approximation

$$\begin{aligned}
F_1^s(s) &= \frac{m_{\omega'}^2 m_{\omega''}^2}{(m_{\omega'}^2 - t)(m_{\omega''}^2 - t)} \left[ \frac{1}{2} - (f_{\omega NN}/f_{\omega}) \right] + \\
&\quad + \left[ \frac{m_{\omega}^2 m_{\omega''}^2}{(m_{\omega}^2 - t)(m_{\omega''}^2 - t)} \frac{m_{\omega''}^2 - m_{\omega}^2}{m_{\omega''}^2 - m_{\omega'}^2} - \right. \\
&\quad \left. - \frac{m_{\omega}^2 m_{\omega'}^2}{(m_{\omega}^2 - t)(m_{\omega'}^2 - t)} \frac{m_{\omega'}^2 - m_{\omega}^2}{m_{\omega''}^2 - m_{\omega'}^2} \right] (f_{\omega NN}/f_{\omega})
\end{aligned} \tag{11a}$$

$$F_2^s(t) = \frac{1}{2}(\mu_p + \mu_n) \frac{m_{\omega}^2 m_{\omega'}^2 m_{\omega''}^2}{(m_{\omega}^2 - t)(m_{\omega'}^2 - t)(m_{\omega''}^2 - t)} \tag{11b}$$

$$\begin{aligned}
F_1^v(t) &= \frac{m_{\rho'}^2 m_{\rho''}^2}{(m_{\rho'}^2 - t)(m_{\rho''}^2 - t)} \left[ \frac{1}{2} - (f_{\rho NN}/f_{\rho}) - (f_{\rho'' NN}/f_{\rho''}) \right] + \\
&\quad + \left[ \frac{m_{\rho}^2 m_{\rho''}^2}{(m_{\rho}^2 - t)(m_{\rho''}^2 - t)} \frac{m_{\rho''}^2 - m_{\rho}^2}{m_{\rho''}^2 - m_{\rho'}^2} - \right. \\
&\quad \left. - \frac{m_{\rho}^2 m_{\rho'}^2}{(m_{\rho}^2 - t)(m_{\rho'}^2 - t)} \frac{m_{\rho'}^2 - m_{\rho}^2}{m_{\rho''}^2 - m_{\rho'}^2} \right] (f_{\rho NN}/f_{\rho}) + \\
&\quad + \left[ \frac{m_{\rho'}^2 m_{\rho''}^2}{(m_{\rho'}^2 - t)(m_{\rho''}^2 - t)} \frac{m_{\rho''}^2 - m_{\rho'}^2}{m_{\rho''}^2 - m_{\rho}^2} - \right. \\
&\quad \left. - \frac{m_{\rho'}^2 m_{\rho}^2}{(m_{\rho'}^2 - t)(m_{\rho}^2 - t)} \frac{m_{\rho}^2 - m_{\rho'}^2}{m_{\rho''}^2 - m_{\rho}^2} \right] (f_{\rho'' NN}/f_{\rho''})
\end{aligned} \tag{11c}$$

$$\begin{aligned}
F_2^v(t) &= \frac{1}{2}(\mu_p - \mu_n) \frac{m_{\rho}^2 m_{\rho'}^2 m_{\rho''}^2}{(m_{\rho}^2 - t)(m_{\rho'}^2 - t)(m_{\rho''}^2 - t)} + \\
&\quad + \left[ \frac{m_{\rho}^2 m_{\rho'}^2 m_{\rho''}^2}{(m_{\rho}^2 - t)(m_{\rho'}^2 - t)(m_{\rho''}^2 - t)} \frac{(m_{\rho''}^2 - m_{\rho}^2)(m_{\rho''}^2 - m_{\rho'}^2)}{(m_{\rho'}^2 - m_{\rho}^2)(m_{\rho''}^2 - m_{\rho'}^2)} - \right. \\
&\quad \left. - \frac{m_{\rho}^2 m_{\rho'}^2 m_{\rho''}^2}{(m_{\rho}^2 - t)(m_{\rho'}^2 - t)(m_{\rho''}^2 - t)} \frac{(m_{\rho''}^2 - m_{\rho}^2)(m_{\rho''}^2 - m_{\rho'}^2)}{(m_{\rho'}^2 - m_{\rho}^2)(m_{\rho''}^2 - m_{\rho'}^2)} - \right.
\end{aligned}$$

$$\begin{aligned}
& - \frac{m_\rho^2 m_{\rho'}^2 m_{\rho''}^2}{(m_\rho^2 - t)(m_{\rho'}^2 - t)(m_{\rho''}^2 - t)} + \\
& + \frac{m_{\rho'}^2 m_{\rho''}^2 m_{\rho'''}^2}{(m_{\rho'}^2 - t)(m_{\rho''}^2 - t)(m_{\rho'''}^2 - t)} \frac{(m_{\rho'''}^2 - m_{\rho'}^2)(m_{\rho'''}^2 - m_{\rho''}^2)}{(m_{\rho'} - m_\rho)(m_{\rho''} - m_\rho)} \Big] \\
& \cdot (f_{\rho'' NN}^{(2)} / f_{\rho''}),
\end{aligned} \tag{11d}$$

which is automatically normalized and governs the asymptotic behaviour (7). However, despite the latter properties, as it will be seen in Section 4, the model is unable to reproduce the existing experimental information. Only its unitarization, carried out in the next section, leads to a proper description of the data.

### 3 New unitary and analytic VMD model of nucleon electromagnetic structure

The unitarity requires the imaginary part of the nucleon e.m. ff's to be different from zero above the lowest branch point  $t_0$ , and moreover, it determines a smoothly varying behaviour of the imaginary part in the  $t_0 < t < \infty$  region. For  $t < t_0$  the nucleon e.m. ff imaginary part is equal to zero as a consequence of the hermiticity of the e.m. current.

All these properties are not fulfilled by the canonical VMD model (11).

The unitarization of (11) can be achieved by an incorporation of the correct ff analytic properties and nonzero values of vector meson widths into the latter. Practically it is realized (up to the two-cut

approximation) by the following special nonlinear transformations

$$\begin{aligned}
t &= t_0^s - \frac{4(t_{in}^{1s} - t_0^s)}{[1/V - V]^2} & t &= t_0^s - \frac{4(t_{in}^{2s} - t_0^s)}{[1/U - U]^2} \\
t &= t_0^v - \frac{4(t_{in}^{1v} - t_0^v)}{[1/W - W]^2} & t &= t_0^v - \frac{4(t_{in}^{2v} - t_0^v)}{[1/X - X]^2}
\end{aligned} \tag{12}$$

applied to (11a - 11d), where  $t_0^s = 9m_\pi^2$ ,  $t_0^v = 4m_\pi^2$  and  $t_{in}^{1s}$ ,  $t_{in}^{2s}$ ,  $t_{in}^{1v}$ ,  $t_{in}^{2v}$  are square-root branch points. The latter is transparent from the inverse transformation to (12)

$$V(t) = i \frac{\sqrt{\left(\frac{t_{in}^{1s} - t_0^s}{t_0^s}\right)^{1/2} + \left(\frac{t - t_0^s}{t_0^s}\right)^{1/2}} - \sqrt{\left(\frac{t_{in}^{1s} - t_0^s}{t_0^s}\right)^{1/2} - \left(\frac{t - t_0^s}{t_0^s}\right)^{1/2}}}{\sqrt{\left(\frac{t_{in}^{1s} - t_0^s}{t_0^s}\right)^{1/2} + \left(\frac{t - t_0^s}{t_0^s}\right)^{1/2}} + \sqrt{\left(\frac{t_{in}^{1s} - t_0^s}{t_0^s}\right)^{1/2} - \left(\frac{t - t_0^s}{t_0^s}\right)^{1/2}}} \tag{13}$$

and similarly for  $U(t)$ ,  $W(t)$  and  $X(t)$ .

To be more specific, in the incorporation of the two-cut approximation of the nucleon ff analytic properties into (11), besides (12), we use also expressions for the vector meson masses squared

$$\begin{aligned}
m_s^2 &= t_0^s - \frac{4(t_{in}^{1s} - t_0^s)}{[1/V_{s_0} - V_{s_0}]^2}; & m_s^2 &= t_0^s - \frac{4(t_{in}^{2s} - t_0^s)}{[1/U_{s_0} - U_{s_0}]^2} \\
m_v^2 &= t_0^v - \frac{4(t_{in}^{1v} - t_0^v)}{[1/W_{s_0} - W_{s_0}]^2}; & m_v^2 &= t_0^v - \frac{4(t_{in}^{2v} - t_0^v)}{[1/X_{s_0} - X_{s_0}]^2}
\end{aligned} \tag{14}$$

and identities

$$\begin{aligned}
0 &= t_0^s - \frac{4(t_{in}^{1s} - t_0^s)}{[1/V_N - V_N]^2}; & 0 &= t_0^s - \frac{4(t_{in}^{2s} - t_0^s)}{[1/U_N - U_N]^2} \\
0 &= t_0^v - \frac{4(t_{in}^{1v} - t_0^v)}{[1/W_N - W_N]^2}; & 0 &= t_0^v - \frac{4(t_{in}^{2v} - t_0^v)}{[1/X_N - X_N]^2}
\end{aligned} \tag{15}$$

following from (12) where  $V_{s_0}, U_{s_0}, W_{s_0}, X_{s_0}$  are the zero-width (therefore a subindex 0) VMD poles and  $V_N, U_N, W_N, X_N$  are the normalization points (corresponding to  $t = 0$ ) in the  $V, U, W, X$  planes, respectively.

The relations (12), (14) and (15) first transform every  $t$ -dependent term and every constant term consisting of a ratio of mass differences

in (11) into the corresponding new variable. For instance the term  $\frac{m_\rho^2}{m_\rho^2 - t}$  in (11c) is transformed into the following form

$$\frac{m_\rho^2 - 0}{m_\rho^2 - t} = \left( \frac{1 - W^2}{1 - W_N^2} \right)^2 \frac{(W_N - W_{\rho_0})(W_N + W_{\rho_0})(W_N - 1/W_{\rho_0})(W_N + 1/W_{\rho_0})}{(W - W_{\rho_0})(W + W_{\rho_0})(W - 1/W_{\rho_0})(W + 1/W_{\rho_0})} \quad (16)$$

The constant mass terms, e.g.  $\frac{m_{\omega'}^2 - m_\omega^2}{m_{\omega''}^2 - m_\omega^2}$  from (11a) become as follows

$$\begin{aligned} \frac{m_{\omega'}^2 - m_\omega^2}{m_{\omega''}^2 - m_\omega^2} &= \frac{(m_{\omega'}^2 - 0) - (m_\omega^2 - 0)}{(m_{\omega''}^2 - 0) - (m_\omega^2 - 0)} = \\ &= \left[ \frac{(V_N - V_{\omega'_0})(V_N + V_{\omega'_0})(V_N - 1/V_{\omega'_0})(V_N + 1/V_{\omega'_0})}{(V_{\omega'_0} - 1/V_{\omega'_0})^2} \right. \\ &\quad \left. - \frac{(V_N - V_{\omega_0})(V_N + V_{\omega_0})(V_N - 1/V_{\omega_0})(V_N + 1/V_{\omega_0})}{(V_{\omega_0} - 1/V_{\omega_0})^2} \right] / \\ &= \left[ \frac{(V_N - V_{\omega'_0})(V_N + V_{\omega'_0})(V_N - 1/V_{\omega'_0})(V_N + 1/V_{\omega'_0})}{(V_{\omega'_0} - 1/V_{\omega'_0})^2} \right. \\ &\quad \left. - \frac{(V_N - V_{\omega_0})(V_N + V_{\omega_0})(V_N - 1/V_{\omega_0})(V_N + 1/V_{\omega_0})}{(V_{\omega_0} - 1/V_{\omega_0})^2} \right] \end{aligned} \quad (17)$$

Then, utilizing the relations between complex and complex conjugate values of the corresponding zero-width VMD pole positions in the  $V$ ,  $U$ ,  $W$ ,  $X$  planes

$$\begin{aligned} V_{\omega_0} &= -V_{\omega_0}^*; & V_{\omega'_0} &= -V_{\omega'_0}^*; & V_{\omega''_0} &= -V_{\omega''_0}^* \\ U_{\omega_0} &= -U_{\omega_0}^*; & U_{\omega'_0} &= 1/U_{\omega'_0}^*; & U_{\omega''_0} &= 1/U_{\omega''_0}^* \\ W_{\rho_0} &= -W_{\rho_0}^*; & W_{\rho'_0} &= -W_{\rho'_0}^*; & W_{\rho''_0} &= 1/W_{\rho''_0}^* \\ X_{\rho_0} &= -X_{\rho_0}^*; & X_{\rho'_0} &= -X_{\rho'_0}^*; & X_{\rho''_0} &= 1/X_{\rho''_0}^* \end{aligned} \quad (18)$$

following from the fact that in a fitting procedure we find

$$\begin{aligned} (m_\omega^2 - \Gamma_\omega^2/4) &< t_{in}^{1s}; & (m_{\omega'}^2 - \Gamma_{\omega'}^2/4) &< t_{in}^{1s}; & (m_{\omega''}^2 - \Gamma_{\omega''}^2/4) &< t_{in}^{1s} \\ (m_\omega^2 - \Gamma_\omega^2/4) &< t_{in}^{2s}; & (m_{\omega'}^2 - \Gamma_{\omega'}^2/4) &> t_{in}^{2s}; & (m_{\omega''}^2 - \Gamma_{\omega''}^2/4) &> t_{in}^{2s} \\ (m_\rho^2 - \Gamma_\rho^2/4) &< t_{in}^{1v}; & (m_{\rho'}^2 - \Gamma_{\rho'}^2/4) &< t_{in}^{1v}; & (m_{\rho''}^2 - \Gamma_{\rho''}^2/4) &< t_{in}^{1v} \\ & & & & (m_{\rho'''}^2 - \Gamma_{\rho'''}^2/4) &> t_{in}^{1v} \\ (m_\rho^2 - \Gamma_\rho^2/4) &< t_{in}^{2v}; & (m_{\rho'}^2 - \Gamma_{\rho'}^2/4) &< t_{in}^{2v}; & (m_{\rho''}^2 - \Gamma_{\rho''}^2/4) &< t_{in}^{2v} \\ & & & & (m_{\rho'''}^2 - \Gamma_{\rho'''}^2/4) &> t_{in}^{2v} \end{aligned} \quad (19)$$

and subsequently incorporating the nonzero values of vector-meson widths  $\Gamma \neq 0$  in a correct way, one gets for every isoscalar and isovector Dirac and Pauli ff, one analytic function in the whole complex  $t$ -plane besides two right-hand cuts of the following form

$$\begin{aligned} F_1^s[V(t)] &= \left( \frac{1 - V^2}{1 - V_N^2} \right)^4 \cdot \left\{ \frac{(V_N - V_{\omega'}) (V_N - V_{\omega'}^*) (V_N - 1/V_{\omega'}) (V_N - 1/V_{\omega'}^*)}{(V - V_{\omega'}) (V - V_{\omega'}^*) (V - 1/V_{\omega'}) (V - 1/V_{\omega'}^*)} \right. \\ &\quad \cdot \frac{(V_N - V_{\omega''}) (V_N - V_{\omega''}^*) (V_N - 1/V_{\omega''}) (V_N - 1/V_{\omega''}^*)}{(V - V_{\omega''}) (V - V_{\omega''}^*) (V - 1/V_{\omega''}) (V - 1/V_{\omega''}^*)} \left[ \frac{1}{2} - (f_{\omega'' NN}^{(1)}/f_\omega) \right] + \\ &\quad + \frac{(V_N - V_\omega) (V_N - V_\omega^*) (V_N - 1/V_\omega) (V_N - 1/V_\omega^*)}{(V - V_\omega) (V - V_\omega^*) (V - 1/V_\omega) (V - 1/V_\omega^*)} \\ &\quad \cdot \left[ \frac{(V_N - V_{\omega''}) (V_N - V_{\omega''}^*) (V_N - 1/V_{\omega''}) (V_N - 1/V_{\omega''}^*)}{(V - V_{\omega''}) (V - V_{\omega''}^*) (V - 1/V_{\omega''}) (V - 1/V_{\omega''}^*)} \cdot \frac{(C_{\omega''}^{1s} - C_\omega^{1s})}{(C_{\omega''}^{1s} - C_{\omega'}^{1s})} \right. \\ &\quad \left. - \frac{(V_N - V_{\omega'}) (V_N - V_{\omega'}^*) (V_N - 1/V_{\omega'}) (V_N - 1/V_{\omega'}^*)}{(V - V_{\omega'}) (V - V_{\omega'}^*) (V - 1/V_{\omega'}) (V - 1/V_{\omega'}^*)} \cdot \frac{(C_{\omega'}^{1s} - C_\omega^{1s})}{(C_{\omega''}^{1s} - C_{\omega'}^{1s})} \right] \\ &\quad \left. \cdot (f_{\omega NN}^{(1)}/f_\omega) \right\} \end{aligned} \quad (20a)$$

$$C_i^{1s} = \frac{(V_N - V_i) (V_N - V_i^*) (V_N - 1/V_i) (V_N - 1/V_i^*)}{-(V_i - 1/V_i) (V_i^* - 1/V_i^*)}; \quad i = \omega, \omega', \omega''$$

$$\begin{aligned} F_2^s[U(t)] &= \left( \frac{1 - U^2}{1 - U_N^2} \right)^6 \cdot \frac{1}{2} [\mu_p + \mu_n] \cdot \\ &\quad \cdot \left\{ \frac{(U_N - U_\omega) (U_N - U_\omega^*) (U_N - 1/U_\omega) (U_N - 1/U_\omega^*)}{(U - U_\omega) (U - U_\omega^*) (U - 1/U_\omega) (U - 1/U_\omega^*)} \right. \\ &\quad \cdot \frac{(U_N - U_{\omega'}) (U_N - U_{\omega'}^*) (U_N + U_{\omega'}) (U_N + U_{\omega'}^*)}{(U - U_{\omega'}) (U - U_{\omega'}^*) (U + U_{\omega'}) (U + U_{\omega'}^*)} \\ &\quad \left. \cdot \frac{(U_N - U_{\omega''}) (U_N - U_{\omega''}^*) (U_N + U_{\omega''}) (U_N + U_{\omega''}^*)}{(U - U_{\omega''}) (U - U_{\omega''}^*) (U + U_{\omega''}) (U + U_{\omega''}^*)} \right\} \end{aligned} \quad (20b)$$

$$\begin{aligned} F_1^v[W(t)] &= \left( \frac{1 - W^2}{1 - W_N^2} \right)^4 \cdot \\ &\quad \cdot \left\{ \frac{(W_N - W_{\rho'}) (W_N - W_{\rho'}^*) (W_N - 1/W_{\rho'}) (W_N - 1/W_{\rho'}^*)}{(W - W_{\rho'}) (W - W_{\rho'}^*) (W - 1/W_{\rho'}) (W - 1/W_{\rho'}^*)} \right. \end{aligned}$$



$$\begin{aligned}
& \frac{(W_N - W_{\rho''})(W_N - W_{\rho''}^*)(W_N - 1/W_{\rho''})(W_N - 1/W_{\rho''}^*)}{(W - W_{\rho''})(W - W_{\rho''}^*)(W - 1/W_{\rho''})(W - 1/W_{\rho''}^*)} \\
& \cdot \left[ \frac{1}{2} - (f_{\rho NN}^{(1)}/f_{\rho}) - (f_{\rho'' NN}^{(1)}/f_{\rho''}) \right] + \\
& + \frac{(W_N - W_{\rho})(W_N - W_{\rho}^*)(W_N - 1/W_{\rho})(W_N - 1/W_{\rho}^*)}{(W - W_{\rho})(W - W_{\rho}^*)(W - 1/W_{\rho})(W - 1/W_{\rho}^*)} \quad (20c) \\
& \cdot \left[ \frac{(W_N - W_{\rho''})(W_N - W_{\rho''}^*)(W_N - 1/W_{\rho''})(W_N - 1/W_{\rho''}^*)}{(W - W_{\rho''})(W - W_{\rho''}^*)(W - 1/W_{\rho''})(W - 1/W_{\rho''}^*)} \frac{(C_{\rho''}^{1v} - C_{\rho}^{1v})}{(C_{\rho''}^{1v} - C_{\rho'}^{1v})} \right. \\
& \left. - \frac{(W_N - W_{\rho})(W_N - W_{\rho}^*)(W_N - 1/W_{\rho})(W_N - 1/W_{\rho}^*)}{(W - W_{\rho})(W - W_{\rho}^*)(W - 1/W_{\rho})(W - 1/W_{\rho}^*)} \frac{(C_{\rho'}^{1v} - C_{\rho}^{1v})}{(C_{\rho''}^{1v} - C_{\rho'}^{1v})} \right] \\
& \cdot (f_{\rho NN}^{(1)}/f_{\rho}) + \\
& + \frac{(W_N - W_{\rho''})(W_N - W_{\rho''}^*)(W_N + W_{\rho''})(W_N + W_{\rho''}^*)}{(W - W_{\rho''})(W - W_{\rho''}^*)(W + W_{\rho''})(W + W_{\rho''}^*)} \\
& \cdot \left[ \frac{(W_N - W_{\rho})(W_N - W_{\rho}^*)(W_N - 1/W_{\rho})(W_N - 1/W_{\rho}^*)}{(W - W_{\rho})(W - W_{\rho}^*)(W - 1/W_{\rho})(W - 1/W_{\rho}^*)} \frac{(C_{\rho''}^{1v} - C_{\rho'}^{1v})}{(C_{\rho''}^{1v} - C_{\rho'}^{1v})} \right. \\
& \left. - \frac{(W_N - W_{\rho''})(W_N - W_{\rho''}^*)(W_N - 1/W_{\rho''})(W_N - 1/W_{\rho''}^*)}{(W - W_{\rho''})(W - W_{\rho''}^*)(W - 1/W_{\rho''})(W - 1/W_{\rho''}^*)} \frac{(C_{\rho''}^{1v} - C_{\rho'}^{1v})}{(C_{\rho''}^{1v} - C_{\rho'}^{1v})} \right] \\
& \cdot (f_{\rho'' NN}^{(1)}/f_{\rho''}) \}
\end{aligned}$$

$$\begin{aligned}
C_i^{1v} &= \frac{(W_N - W_i)(W_N - W_i^*)(W_N - 1/W_i)(W_N - 1/W_i^*)}{-(W_i - 1/W_i)(W_i^* - 1/W_i^*)}; \quad i = \rho, \rho', \rho'' \\
C_{\rho''}^{1v} &= \frac{(W_N - W_{\rho''})(W_N - W_{\rho''}^*)(W_N + W_{\rho''})(W_N + W_{\rho''}^*)}{-(W_{\rho''} - 1/W_{\rho''})(W_{\rho''}^* - 1/W_{\rho''}^*)}
\end{aligned}$$

$$\begin{aligned}
F_2^v[X(t)] &= \left( \frac{1 - X^2}{1 - X_N^2} \right)^6 \\
& \cdot \left\{ \frac{(X_N - X_{\rho})(X_N - X_{\rho}^*)(X_N - 1/X_{\rho})(X_N - 1/X_{\rho}^*)}{(X - X_{\rho})(X - X_{\rho}^*)(X - 1/X_{\rho})(X - 1/X_{\rho}^*)} \right. \\
& \frac{(X_N - X_{\rho'})(X_N - X_{\rho'}^*)(X_N - 1/X_{\rho'})(X_N - 1/X_{\rho'}^*)}{(X - X_{\rho'})(X - X_{\rho'}^*)(X - 1/X_{\rho'})(X - 1/X_{\rho'}^*)} \\
& \left. \frac{(X_N - X_{\rho''})(X_N - X_{\rho''}^*)(X_N - 1/X_{\rho''})(X_N - 1/X_{\rho''}^*)}{(X - X_{\rho''})(X - X_{\rho''}^*)(X - 1/X_{\rho''})(X - 1/X_{\rho''}^*)} \right\}
\end{aligned}$$

$$\begin{aligned}
& \cdot \left[ \frac{1}{2} (\mu_p - \mu_n) - (f_{\rho'' NN}^{(2)}/f_{\rho''}) \right] \\
& + \left[ \frac{(X_N - X_{\rho})(X_N - X_{\rho}^*)(X_N - 1/X_{\rho})(X_N - 1/X_{\rho}^*)}{(X - X_{\rho})(X - X_{\rho}^*)(X - 1/X_{\rho})(X - 1/X_{\rho}^*)} \right. \\
& \frac{(X_N - X_{\rho'})(X_N - X_{\rho'}^*)(X_N - 1/X_{\rho'})(X_N - 1/X_{\rho'}^*)}{(X - X_{\rho'})(X - X_{\rho'}^*)(X - 1/X_{\rho'})(X - 1/X_{\rho'}^*)} \\
& \frac{(X_N - X_{\rho''})(X_N - X_{\rho''}^*)(X_N + X_{\rho''})(X_N + X_{\rho''}^*)}{(X - X_{\rho''})(X - X_{\rho''}^*)(X + X_{\rho''})(X + X_{\rho''}^*)} \\
& \left. - \frac{(C_{\rho''}^{2v} - C_{\rho}^{2v})}{(C_{\rho''}^{2v} - C_{\rho'}^{2v})} \cdot \frac{(C_{\rho''}^{2v} - C_{\rho'}^{2v})}{(C_{\rho''}^{2v} - C_{\rho'}^{2v})} \right] \\
& \frac{(X_N - X_{\rho})(X_N - X_{\rho}^*)(X_N - 1/X_{\rho})(X_N - 1/X_{\rho}^*)}{(X - X_{\rho})(X - X_{\rho}^*)(X - 1/X_{\rho})(X - 1/X_{\rho}^*)} \\
& \frac{(X_N - X_{\rho''})(X_N - X_{\rho''}^*)(X_N - 1/X_{\rho''})(X_N - 1/X_{\rho''}^*)}{(X - X_{\rho''})(X - X_{\rho''}^*)(X - 1/X_{\rho''})(X - 1/X_{\rho''}^*)} \\
& \frac{(X_N - X_{\rho''})(X_N - X_{\rho''}^*)(X_N + X_{\rho''})(X_N + X_{\rho''}^*)}{(X - X_{\rho''})(X - X_{\rho''}^*)(X + X_{\rho''})(X + X_{\rho''}^*)} \\
& \left. - \frac{(C_{\rho''}^{2v} - C_{\rho}^{2v})}{(C_{\rho''}^{2v} - C_{\rho'}^{2v})} \cdot \frac{(C_{\rho''}^{2v} - C_{\rho'}^{2v})}{(C_{\rho''}^{2v} - C_{\rho'}^{2v})} \right] + \\
& \frac{(X_N - X_{\rho})(X_N - X_{\rho}^*)(X_N - 1/X_{\rho})(X_N - 1/X_{\rho}^*)}{(X - X_{\rho})(X - X_{\rho}^*)(X - 1/X_{\rho})(X - 1/X_{\rho}^*)} \\
& \frac{(X_N - X_{\rho''})(X_N - X_{\rho''}^*)(X_N - 1/X_{\rho''})(X_N - 1/X_{\rho''}^*)}{(X - X_{\rho''})(X - X_{\rho''}^*)(X - 1/X_{\rho''})(X - 1/X_{\rho''}^*)} \\
& \frac{(X_N - X_{\rho''})(X_N - X_{\rho''}^*)(X_N + X_{\rho''})(X_N + X_{\rho''}^*)}{(X - X_{\rho''})(X - X_{\rho''}^*)(X + X_{\rho''})(X + X_{\rho''}^*)} \\
& \left. - \frac{(C_{\rho''}^{2v} - C_{\rho}^{2v})}{(C_{\rho''}^{2v} - C_{\rho'}^{2v})} \cdot \frac{(C_{\rho''}^{2v} - C_{\rho'}^{2v})}{(C_{\rho''}^{2v} - C_{\rho'}^{2v})} \right] + \\
& \frac{(X_N - X_{\rho})(X_N - X_{\rho}^*)(X_N - 1/X_{\rho})(X_N - 1/X_{\rho}^*)}{(X - X_{\rho})(X - X_{\rho}^*)(X - 1/X_{\rho})(X - 1/X_{\rho}^*)} \\
& \frac{(X_N - X_{\rho''})(X_N - X_{\rho''}^*)(X_N - 1/X_{\rho''})(X_N - 1/X_{\rho''}^*)}{(X - X_{\rho''})(X - X_{\rho''}^*)(X - 1/X_{\rho''})(X - 1/X_{\rho''}^*)} \\
& \frac{(X_N - X_{\rho''})(X_N - X_{\rho''}^*)(X_N + X_{\rho''})(X_N + X_{\rho''}^*)}{(X - X_{\rho''})(X - X_{\rho''}^*)(X + X_{\rho''})(X + X_{\rho''}^*)} \\
& \left. \cdot \frac{(C_{\rho''}^{2v} - C_{\rho}^{2v})}{(C_{\rho''}^{2v} - C_{\rho'}^{2v})} \cdot \frac{(C_{\rho''}^{2v} - C_{\rho'}^{2v})}{(C_{\rho''}^{2v} - C_{\rho'}^{2v})} \right] (f_{\rho'' NN}^{(2)}/f_{\rho''}) \}
\end{aligned} \quad (20d)$$

$$\begin{aligned}
C_i^{2v} &= \frac{(X_N - X_i)(X_N - X_i^*)(X_N - 1/X_i)(X_N - 1/X_i^*)}{-(X_i - 1/X_i)(X_i^* - 1/X_i^*)}; \quad i = \rho, \rho', \rho'' \\
C_{\rho''}^{2v} &= \frac{(X_N - X_{\rho''})(X_N - X_{\rho''}^*)(X_N + X_{\rho''})(X_N + X_{\rho''}^*)}{-(X_{\rho''} - 1/X_{\rho''})(X_{\rho''}^* - 1/X_{\rho''}^*)}
\end{aligned}$$

Every of them is defined on a four-sheeted Riemann surface in  $t$ -variable with poles corresponding to vector meson resonances placed

on unphysical sheets. The relations (20a-20d) and (2) represent now the new unitary and analytic VMD model of the e.m. structure of nucleons dependent only on eight (to be compared with fourteen in the previous most accomplished formulation of the UA-VMD model [4]) free parameters

$$t_{in}^{1s}, t_{in}^{2s}, t_{in}^{1v}, t_{in}^{2v}, (f_{\omega NN}^{(1)}/f_{\omega}), (f_{\rho NN}^{(1)}/f_{\rho}), (f_{\rho'' NN}^{(1)}/f_{\rho''}), (f_{\rho'' NN}^{(2)}/f_{\rho''}) \quad (21)$$

if masses and widths of all vector mesons under consideration are fixed at the world averaged values. Its application to a description of all existing data on the nucleon e.m. structure is carried out in the next section.

## 4 Analysis of data on nucleon electromagnetic structure

The whole measurable region  $-\infty < t < +\infty$  from the point of view of a method of obtaining data can be roughly divided into three parts

- i) the space-like region  $t < 0$
- ii) the unphysical region  $0 < t < 4m_N^2$

and

- iii) the time-like region  $t \geq 4m_N^2$ .

Though a lot of pretentious work has been carried out up to now, the experimental information is not at all complete.

The most rich data exist on the nucleon e.m. ff's in the space-like region ( $t < 0$ ). All data on the proton electric and magnetic ff's up to  $t = -33 \text{ GeV}^2$  were obtained from the elastic electron scattering on

the hydrogen target. Neutron electric and magnetic ff's were obtained mostly from cross-sections on the elastic and inelastic electron scattering on deuterons in a model-dependent way and therefore they are less reliable than the proton data.

Also the range of neutron data is poorer than in the proton case. There are only very dispersed data on the neutron electric ff for  $-4\text{GeV}^2 < t < 0$  and moreover with large errors. The neutron magnetic ff data exist for  $-10\text{GeV}^2 < t < 0$ . All references concerning the nucleon space-like data can be found in [2] besides the recent precise results on the proton magnetic ff measurements [12,13].

There are no data on nucleon electric and magnetic ff's in the unphysical region. However, they are expected to be measured from processes like

$$\begin{aligned} \pi^- p &\rightarrow n e^+ e^- \\ \bar{p} p &\rightarrow \pi^0 e^+ e^- \\ \bar{n} p &\rightarrow \pi^+ e^+ e^- \end{aligned} \quad (22)$$

in the near future.

In the time-like region the sources of information on the nucleon e.m. structure are  $e^+e^- \rightarrow N\bar{N}$  and  $\bar{p}p \rightarrow e^+e^-$  processes. There are 13 rather dispersed and not very precise data on the  $\sigma_{tot}(e^+e^- \rightarrow p\bar{p})$  [14-18] just above the  $p\bar{p}$  threshold. They seem not quite to be consistent with recent accurate 9 LEAR data [19] on  $|G_E^p| = |G_M^p| = |G^p|$  obtained from  $\bar{p}p \rightarrow e^+e^-$  process which show rather steep falling just above the  $p\bar{p}$  threshold. The latter effect caused a big discussion and its explanation led to assumptions [20-22] on the existence of a bound state (baryonium) in the  $p\bar{p}$  system.

There are also the first very valuable results [23] on the proton e.m. ff's at 3 large values of the c.m. energies squared obtained by FERMILAB experiment E760, measuring the cross-section of  $\bar{p}p \rightarrow e^+e^-$  process.

Finally, to be complete, we mention also the first result [24] on the neutron time-like e.m. ff's at  $t = 4.0\text{GeV}^2$  obtained in FENICE experiment performed at the ADONE  $e^+e^-$  storage ring in Frascati.

So, there are altogether about 400 experimental points on the nucleon e.m. structure, analysed by using the most accomplished up to now unitary and analytic VMD model of the nucleon e.m. ff's given by (2) and (20a-20d). We have eliminated from the analysis 13 rather dispersed data [14-18] on the  $\sigma_{tot}(e^+e^- \rightarrow p\bar{p})$  in order to avoid any troubles (they appear in the fitting procedure) with the reproduction of more precise LEAR data [19]. The results of the analysis are presented in Table 1. and are graphically shown in Figs. 1-4.

In Figs. 1-2, the best fit of the data by the VMD model (11a-11d) automatically normalized and with the asymptotic behaviour as predicted by QCD (up to logarithmic corrections), however, extended to complex masses  $m^2 \rightarrow (m - i\frac{\Gamma}{2})^2$ , is presented by dashed lines. Full lines show that only a unitarization of the latter by the incorporation of the two-cut approximation of the correct nucleon ff analytic properties leads to a perfect reproduction of the most existing data. For neutron electric and magnetic ff's in Fig. 2 we used the same scale for reference frame like for proton ff's. As a result, we give predictions in Fig. 2 for the behaviour of neutron electric and magnetic ff's up to  $t = -35\text{GeV}^2$ , following from our model.

Fig. 3a shows that the most accomplished unitary and analytic VMD model of the nucleon e.m. structure given by (2) and (11) is

flexible enough to reproduce steep falling of the new LEAR data [19] and thus, does not need any introduction of baryonium [20-22] in order to explain their behaviour. From the same Fig. 3a one can see clearly that our unitary and analytic VMD model is unable to reproduce the FERMILAB data at  $t = 8.9$ ;  $12.4$ ; and  $13.0\text{GeV}^2$ . The latter can be a sign of higher excited states of vector mesons under consideration, the inclusion of which into our model is not a technical problem.

From Fig. 3b it is seen that one point on the neutron e.m. ff's obtained in the FENICE experiment [24] is not reproduced by our model quite well. The situation may be changed if more experimental points will be measured and therefore, they will influence with a stronger statistical weight the numerical values of free parameters of the model under consideration determined in a fitting procedure.

The comparison of predicted behaviours of the  $\sigma_{tot}(e^+e^- \rightarrow p\bar{p})$  and  $\sigma_{tot}(e^+e^- \rightarrow n\bar{n})$  with existing 13 experimental points [14-18] on the  $\sigma_{tot}(e^+e^- \rightarrow p\bar{p})$  is shown in Fig. 4.

## 5 Summary and conclusions

We have constructed the most accomplished up to now model of the nucleon e.m. structure. First, we have reformulated the canonical VMD model in the zero-width approximation in order it to be automatically normalized and to govern the asymptotic behaviour as predicted (up to logarithmic corrections) by QCD for nucleons. Only then we have unitarized it by the incorporation of the two-cut approximation of the correct nucleon ff analytic properties and the nonzero width of vector meson resonances. As a result, we have obtained a new formulation of the unitary and analytic VMD model of the nucleon e.m. structure

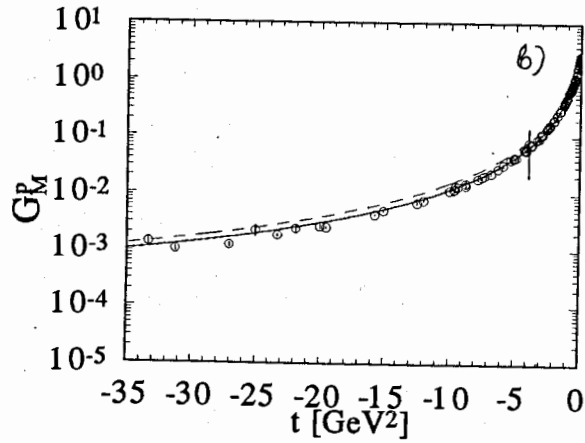
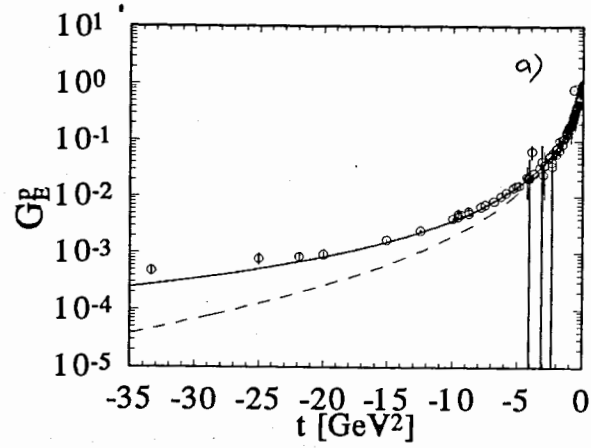


Fig 1.: A description of the proton electric and magnetic ff data by our new UA-VMD model. Dashed lines mean the best fit by means of the canonical VMD model (11) even with extension to complex masses of resonances.

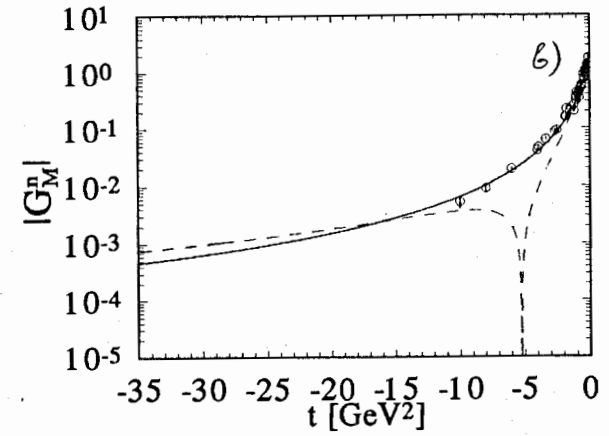
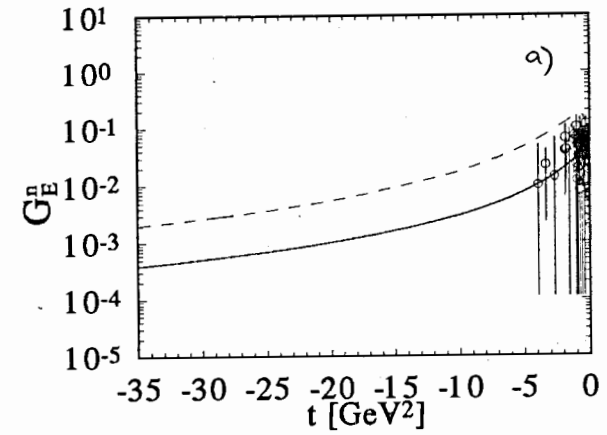


Fig 2.: A prediction of the neutron electric and magnetic ff behaviour up to  $t = -35\text{GeV}^2$  by the new UA-VMD model. Dashed lines mean the same like in Fig. 1.

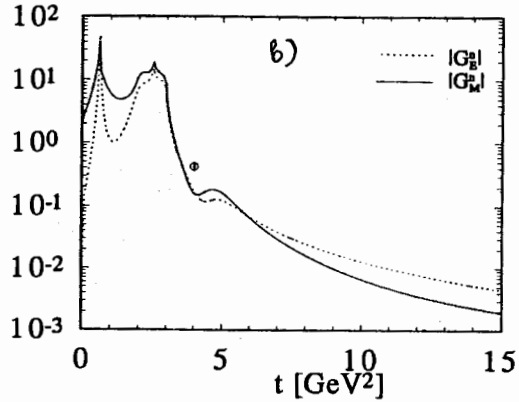
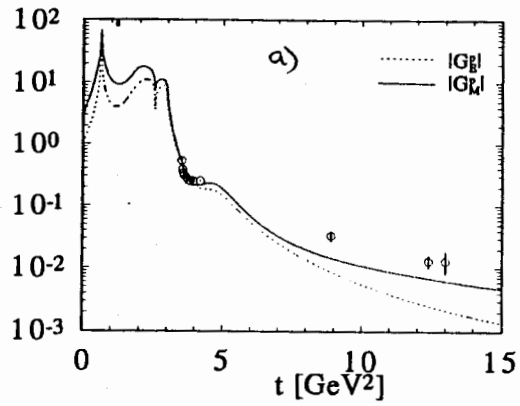


Fig 3.: a) A reproduction of new LEAR data [19] on the proton e.m. ff's by the new UA-VMD model. The last three experimental points from FERMILAB [23] are not described quite well. The latter indicates for an existence of further excited states of vector mesons under consideration.

b) A prediction of the neutron electric and magnetic ff behaviour by the new UA-VMD model. The experimental point at  $t = 4.0\text{GeV}^2$  is the first result from FENICE experiment in Frascati.

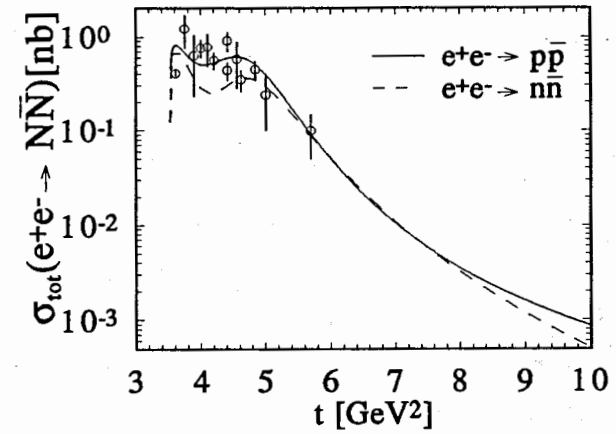


Fig 4.: A comparison of a predicted behaviour of  $\sigma_{tot}(e^+e^- \rightarrow p\bar{p})$  and  $\sigma_{tot}(e^+e^- \rightarrow n\bar{n})$  by our new UA-VMD model with older 13 data [14-18] on  $\sigma_{tot}(e^+e^- \rightarrow p\bar{p})$ .

Table 1: The results of the analysis of data on nucleon e.m. structure with the VMD model (11) and the new UA-VMD model (20).

Free parameters of the model	VMD model (11)	The new UA-VMD model (20)
$t_{in}^{1s}$	—	2.54106
$t_{in}^{2s}$	—	1.93608
$t_{in}^{1v}$	—	2.91078
$t_{in}^{2v}$	—	2.98067
$(f_{\omega NN}^{(1)}/f_{\omega})$	0.43219	0.84927
$(f_{\rho NN}^{(1)}/f_{\rho})$	0.74977	0.08120
$(f_{\rho'' NN}^{(1)}/f_{\rho''})$	0.07318	-0.23370
$(f_{\rho'' NN}^{(2)}/f_{\rho''})$	-0.06190	-0.00359

that reproduces (see Figs. 1-3.) all the existing experimental information quite well with only eight (to be compared with fourteen in the previous formulation [4]) free parameters with clear physical meaning.

We note that the VMD model (11), though automatically normalized and governing the correct asymptotic behaviour, even extended to complex masses, is unable (see dashed lines in Figs. 1-2) to reproduce the existing experimental information on the nucleon e.m. structure. The latter fact points to the relevance of the unitarization procedure of the canonical VMD model in describing the existing data.

New LEAR accurate proton e.m. ff data [19] are reproduced by our unitary and analytic VMD model too. As a result one needs no further assumption about the existence of baryonia for explaining their steep fall-off just above the proton-antiproton threshold and all speculations like in ref. [20-22] can be disregarded, because the behaviour of new data means just the creation of the third excited state of the  $\rho(770)$  meson at  $\sqrt{t} = 2.150\text{GeV}$  in the  $\bar{p}p \rightarrow e^+e^-$  process.

## References

- [1] A.V. Radyushkin, Nucl.Phys. **A532** 141c(1991).
- [2] S. Dubnička, Nuovo Cim. **A100** 1(1988) ibid **A103** 469(1990).
- [3] S. Dubnička, Nuovo Cim. **A103** 1417(1990) ibid **A104** 1075(1991).
- [4] S.I. Bilenkaya, S. Dubnička, A.Z. Dubničková, and P. Stríženec, Preprint JINR, E2-91-475, Dubna(1991), Nuovo Cim. **A105** (1992).

- [5] S. Dubnička and Ľ. Lúčan, Preprint ICTP, IC/88/422, Trieste(1988), J.Phys.G: Nuclear and Particle Physics **15** 1359(1989).
- [6] A.Z. Dubničková and S. Dubnička, Preprint ICTP, IC/91/149, Trieste(1991).
- [7] A.Z. Dubničková, S. Dubnička, M.E. Biagini, A. Castro, Czech.J.Phys. **B41** 1177(1991).
- [8] S.Okubo, Phys.Lett. **5** 163(1963).
- [9] G.Zweig, CERN Report No.8419/TH418 (1964).
- [10] J.Iizuka, Prog.Theor.Phys.Supl. **37-38** 21(1966).
- [11] A.Z. Dubničková, S. Dubnička and E. Kruáč, Phys.Lett. **B261** 127(1991).
- [12] R.C. Walker et al., Phys.Lett. **B224** 353(1989).
- [13] P.E. Bosted et al., Phys.Rev. **C42** 38(1990).
- [14] G. Bassompierre et al., Nuovo Cimento **A73** 347(1983).
- [15] B. Delcourt et al., Phys.Lett. **B86** 395(1979).
- [16] D. Bisello et al., Nucl.Phys **B224** 379(1983).
- [17] M. Castellano et al., Nuovo Cimento **A14** 1(1973).
- [18] D. Bisello et al., Z.Phys. **C48** 23(1990).
- [19] P. Dalpiaz, Yad.Fiz. **55** 1563(1992).
- [20] O.D. Dalkarov and K.V. Protasov, Nucl.Phys. **A504** 845(1989).
- [21] O.D. Dalkarov and K.V. Protasov, Phys.Lett. **B280** 107(1992).

- [22] V.A. Meshcheryakov and G.V. Meshcheryakov, Preprint JINR, P2-92-217, Dubna(1992).
- [23] T. Armstrong et al., Paper presented by D. Betoni at the XXVI. ICHEP, August 6-12, 1992 Dallas(USA).
- [24] A. Antonelli et al., Paper presented by M. Bertani at "International Workshop on Quark Cluster Dynamics", June 29-July 1, 1992, Bad Honnef (Germany).

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