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GAUGE DEPENDENT PART OF THE TWO-POINT GREEN'S FUNCTION OF THE GLUON FIELD

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1. When extracting quantitative theoretical predictions from quantum chromodynamics (QCD) using a perturbation expansion in the coupling constanst one often faces not only a problem of a choice of the renormalization scheme for parameters of the coupling constant (and, consequently, for the coefficients in the expansion), but also closely related topic - the problem of gauge dependence of the renormalized quantities in momentum-subtraction (MOM) schemes [1-8].

The new type of the Ward-Slavnov-Taylor identities (WI), extracted from a supergauge transformations, has been adopted as a starting point for the resolution of the problem of renormalization of the gauge-invariant operators in the non-Abelian Yang-Mills's theories [9]. In Ref.9 on the basis of these identities, the variations of the ordinary one-particle irreducible (1PI) Green's functions with respect to the gauge parameter have been related to the new 1PI- Green's functions for which the amputed legs represent the composite of operators, which correspond to the invariants of the gauge transformations. These new 1PI- Green's functions manifest explicitly the gauge dependence and with the help of them one can study the gauge-dependent part of the ordinary 1PI- Green's functions.

In the present paper we are going to present the results of calculation of the radiative corrections to the dimentionally regularized 1PI- Green's function (with the inserted operators) in the two-loop approximation of the PT, both in the arbitrary gauge and dimension space-time. That Green's function is related to the variation of the ordinary two-point Green's function of the gluon field (gluon propagator) over the gauge parameter in the case of the massless gauge Yang-Mills's theory. On the basis of the above-mentioned relation we derive the relation between the derivative with respect to the gauge parameter of the anomalous dimension of the gluon propagator , and anomalous dimension of the calculated Green's function. The result of calculation of the latter also in two-loop approximation but in renormalization MOM-scheme is presented.

2. Let us consider the Lagrangian of the pure Yang-Mills theory ,

$$\mathcal{L} = -\frac{1}{4} F^i_{\mu\nu} F^i_{\mu\nu} - \frac{1}{2a_o} \left(\partial_\mu A^i_\mu\right)^2 + \bar{C}^i \partial_\mu D^{ij}_\mu C^j \ , \tag{1}$$

$$F^i_{\mu\nu} = \partial_\mu A^i_\nu - \partial_\nu A^i_\mu + g_o f^{ijk} A^j_\mu A^k_\nu ,$$
$$D^{ij}_\mu C^j = \partial_\mu C^i + g_o f^{ijk} A^j_\mu C^k .$$

Here A^i_μ, C^i are the gauge and ghost fields,respectively; a_o and g_o are the bare gauge parameter and coupling constant, respectively; f^{ijk} are the fully antisymmetric structure constants of the underlying gauge group $\mathbf{G}$.

Let us consider briefly the generalized Slavnov-Taylor identities, modified by taking into account the variation of the gauge parameter. We refer the reader to Ref.9 for details.

In order to obtain the generating functional for the ordinary Green's functions, besides the source terms for the fields A^i_μ, $\bar{C}^i$, C^i and the Lagrangian eq.(1), we also take into account the source terms for the invariants of the BRS- transformations of the fields A^i_μ, C^i, that is, respectively, the terms

$$J^i_\mu(x) D^\mu_{ij} C^j(x) , \qquad \frac{1}{2} g_o K^i(x) \left(\vec{C} \times \vec{C}\right)^i (x) .$$

In order to take into account the effect of the variation of the gauge parameter in the generating functional for the Green's functions, we introduce one more term

$$L \left(\bar{C}^i(x) \partial_\mu A^i_\mu(x) + J^i_\mu(x) A^i_\mu(x)\right)$$

with the source L. Note, that the sources $\vec{J}_\mu$, $\vec{K}$ are nothing but the anticommuting classical and x-dependent quantities, and L is the x-independent and anticommuting one.The thus obtained generating functional Γ (by taking into account first, the Legendre transformation, and second, the transversality) for the 1PI- Green's functions leads us to the desired identities. Then one can write the modified generalized Slavnov-Taylor identities as

$$\begin{aligned} -2a_o \frac{\partial \Gamma}{\partial a_o}(x) &= \int d^4x \left\{ \vec{A}_\rho(x) \frac{\delta \Gamma}{\delta \vec{A}_\rho(x)} - \bar{C}^l(x) \frac{\delta \Gamma}{\delta \bar{C}^l(x)} - \vec{J}_\rho(x) \frac{\delta \Gamma}{\delta \vec{J}_\rho(x)} + \right. \\ &+ \left. \frac{\delta}{\delta L} \left[\frac{\delta \Gamma}{\delta \vec{A}_\rho(x)} \cdot \frac{\delta \Gamma}{\delta \vec{J}_\rho(x)} - \frac{\delta \Gamma}{\delta \vec{K}(x)} \cdot \frac{\delta \Gamma}{\delta \vec{C}(x)} \right] \right\} . \end{aligned} \quad (2)$$

If we had not introduced the term proportional to L, the resultant expressions would have been the ordinary generalized Slavnov-Taylor identities [9,10].

By differentiating eq.(2) with respect to both A^i_μ, and A^j_ν, and putting all the sources to be equal to zero; and, on the other hand, parametrizing the Green's functions which enter the resultant relations, we obtain two-point 1PI- Green's function which satisfy the equation

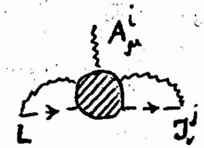
$$-2a_o \frac{\partial}{\partial a_o} I(p^2) = 2I \left[1 - X_1(p^2)\right], \quad (3)$$

or

$$-a_o \frac{\partial}{\partial a_o} \ln I(p^2) = \left[1 - X_1(p^2)\right], \quad (4)$$

where I and X_1 are the invariants,i.e. the form-factors which stand before the tensor structures of the following 1PI- Green's functions, respectively (the graphical representation corresponding to it):

$$\rightarrow \quad \frac{\delta^2 \Gamma}{\delta A^i_\mu \delta A^j_\nu} \equiv \Gamma^{ij}_{\mu\nu}(p) = \delta^{ij} \left[p_\mu p_\nu - g_{\mu\nu} p^2\right] I(p^2), \quad (5)$$



$$\rightarrow \quad \frac{\delta^3 \Gamma}{\delta A^i_\mu \delta L \delta J^j_\nu} = -\delta^{ij} \left[g_{\mu\nu} X_1(p^2) + p_\mu p_\nu X_2(p^2)\right] . \quad (6)$$

Here eq.(5) represents the transversal part of the gluon propagator. Thus, the eq.(3) expresses the variation of the two-point 1PI- Green's function, gluon propagator, with respect to the change of the gauge parameter. According to the ordinary generalized Slavnov-Taylor identities (therefore, to the transversality of the gluon propagator) in this equation appear only the form-factors I and X_1,which are superficially divergent and $(1 - X_1) = O(g^2)$. The form-factor $X_1(p^2) = 1 + O(g^2)$ (see the proof in the Appendix).

We note that in same way we can derive from the eq.(2) the three-point 1PI- Green's function of the gluon field (the vertex).For the other Green's functions with the other external fields (the ghost and the matter fields) one must also introduce some modifications in the generating functional, as we shall demonstrate in our future paper.

3. We now present the results of the calculations of the form-factors X_1 and X_2 for the 1PI- Green's function – eq.(5) both in the one- and two-loop approximation of expansion theory for the massless non-Abelian gauge theory in the general covariant gauge. All the results of calculations of Feynman diagrams have been obtained with the help of the computer package [11], elaborated by us on the basis of the algebraic computer system – FORM [12] by using the algorithm [13] for the analytical calculation of Feynman integrals. The calculations have been performed in the gauge-invariant dimentional regularization [1] in $n = 4 - 2\epsilon$ space-time dimension, which makes it possible to express the results in arbitrary dimension , n.

So, our results in terms of the bare quantities for X_1 and X_2 are, respectively, (without counterterms)

$$
\begin{aligned}
X_1(p^2) = {} & 1 + \frac{g^2}{(4\pi)^2} B \left\{ C_A \left(\frac{1}{4}\right) \left[a_o^2 (n-4) + a_o (3n-10)\right] \cdot G_o(1,1) \right\} - \\
& - \frac{g^4}{(4\pi)^4} B^2 \left\{ C_A^2 \left[a_o^4 \left(\frac{1}{2} + \frac{n^2}{32} - \frac{n}{4}\right) \cdot \mathbf{I}_1 + a_o^3 \left(\left(\frac{27}{8} + \frac{9n^2}{32} - \frac{63n}{32}\right) \cdot \mathbf{I}_1 + \right.\right.\right. \\
& + \left. \left(-3 - \frac{3n^2}{16} + \frac{21n}{16} - \frac{3}{2}\frac{1}{(n-6)}\right) \cdot \mathbf{I}_2 \right) + a_o^2 \left(\left(\frac{25}{8} + \frac{11n^2}{32} - \frac{31n}{16} - \right.\right. \\
& - \left.\left. \frac{3}{8}\frac{1}{(n-1)}\right) \cdot \mathbf{I}_1 + \left(-21 - \frac{15n^2}{8} + \frac{89n}{8} + \frac{3}{5}\frac{1}{(n-1)} - \frac{53}{5}\frac{1}{(n-6)}\right) \cdot \mathbf{I}_2 \right) + \\
& + a_o \left(\left(-\frac{13}{2} - \frac{21n^2}{32} + \frac{133n}{32} + \frac{17}{24}\frac{1}{(n-1)} + \frac{2}{3}\frac{1}{(n-4)}\right) \cdot \mathbf{I}_1 \right. \\
& + \left(\frac{101}{6} + \frac{33n^2}{16} - \frac{199n}{16} - \frac{49}{15}\frac{1}{(n-1)} - \frac{4}{3}\frac{1}{(n-4)} - \frac{27}{10}\frac{1}{(n-6)} + \right. \\
& + \left.\left.\left. \frac{1}{15}\frac{1}{(3n-8)}\right) \cdot \mathbf{I}_2 \right)\right] + (C_A T N_f) \left[a_o^2 \left(-\frac{5}{2} + \frac{n}{2} + \frac{3}{2}\frac{1}{(n-1)}\right) \cdot \mathbf{I}_1 + \right. \\
& + a_o \left(\left(-\frac{11}{2} + \frac{3n}{2} + \frac{5}{2}\frac{1}{(n-1)}\right) \cdot \mathbf{I}_1 + \right. \\
& + \left.\left.\left. \left(24 - 6n - \frac{64}{5}\frac{1}{(n-1)} - \frac{16}{5}\frac{1}{(n-6)}\right) \cdot \mathbf{I}_2 \right)\right]\right\} ,
\end{aligned}
\tag{7}
$$

and

$$
\begin{aligned}
X_2(p^2) = {} & \frac{g^2}{(4\pi)^2} B \left\{ C_A \left(-\frac{1}{4}\right) (a_o^2 + a_o)(n-4) \cdot G_o(1,1) \right\} - \\
& - \frac{g^4}{(4\pi)^4} B^2 \left\{ C_A^2 \left[a_o^4 \left(-\frac{1}{2} - \frac{n^2}{32} + \frac{n}{4}\right) \cdot \mathbf{I}_1 + a_o^3 \left(\left(-\frac{27}{8} - \frac{9n^2}{32} + \frac{63n}{32}\right) \cdot \mathbf{I}_1 \right.\right.\right. \\
& + \left. \left(3 + \frac{3n^3}{16} - \frac{21n}{16} + \frac{3}{2}\frac{1}{(n-6)}\right) \cdot \mathbf{I}_2 \right) + a_o^2 \left(\left(-\frac{13}{4} - \frac{11n^2}{32} + 2n \right.\right. \\
& + \left.\left. \frac{3}{8}\frac{1}{(n-1)}\right) \cdot \mathbf{I}_1 + \left(\frac{47}{2} + 2n^2 - 12n - \frac{3}{5}\frac{1}{(n-1)} + \frac{121}{10}\frac{1}{(n-6)}\right) \cdot \mathbf{I}_2 \right)
\end{aligned}
\tag{8}
$$

$$
\begin{aligned}
& + a_o \left(\left(\frac{55}{8} + \frac{21n^2}{32} - \frac{135n}{32} - \frac{17}{24}\frac{1}{(n-1)} - \frac{2}{3}\frac{1}{(n-4)}\right) \cdot \mathbf{I}_1 \right. \\
& + \left(-\frac{319}{18} - \frac{35n^2}{16} + \frac{647n}{48} + \frac{49}{15}\frac{1}{(n-1)} + \frac{4}{3}\frac{1}{(n-4)} + \frac{21}{5}\frac{1}{(n-6)} - \right. \\
& - \left.\left.\left. \frac{8}{45}\frac{1}{(3n-8)}\right) \cdot \mathbf{I}_2 \right)\right] + (C_A T N_f) \left[a_o^2 \left(\frac{5}{2} - \frac{n}{2} - \frac{3}{2}\frac{1}{(n-1)}\right) \cdot \mathbf{I}_1 + \right. \\
& + a_o \left(\left(\frac{11}{2} - \frac{3n}{2} - \frac{5}{2}\frac{1}{(n-1)}\right) \cdot \mathbf{I}_1 + \right. \\
& + \left.\left.\left. \left(-24 + 6n + \frac{64}{5}\frac{1}{(n-1)} - \frac{16}{5}\frac{1}{(n-6)}\right) \cdot \mathbf{I}_2 \right)\right]\right\} .
\end{aligned}
$$

The one-loop results correspond to a sum of contributions of the two diagramms; the two-loop ones – to a sum of 35 ones. Here $B \equiv \left(\frac{-p^2}{4\pi\mu^2}\right)^{\frac{n-4}{2}} \equiv l^{\frac{n-4}{2}}$. By $G_o(1,1)$, $\mathbf{I}_1 \equiv \mathbf{I}_1(1,1,1,1)$ and $\mathbf{I}_2 \equiv \mathbf{I}_2(1,1,1,1)$ we represent the one- and two- loop "standard" Feynman integrals, defined through the Euler $\Gamma-$ function (see Appendix).

In terms of $\epsilon = \frac{4-n}{2}$ we have (taking into account the counterterms), respectively

$$
\begin{aligned}
X_1(p^2) \; = \; & 1 + \frac{g^2}{(4\pi)^2} \left\{ C_A \left[\frac{a_o}{2}\left(\frac{1}{\epsilon} - \ln l\right) - \frac{1}{2}(a_o^2 + a_o) - (a_o^2 + a_o)\,\epsilon - \right.\right. \\
& - \left.\left. (2a_o^2 + 6a_o)\,\epsilon^2 \right] \right\} + \frac{g^4}{(4\pi)^4} \left\{ C_A^2 \left[\frac{1}{\epsilon^2}\left(\frac{a_o^2}{4} + \frac{3a_o}{8}\right) + \frac{1}{\epsilon}\left(-\frac{a_o^2}{4} - \frac{11a_o}{16}\right) + \right.\right. \\
& + \left(-\frac{3a_o^3}{4} + \frac{a_o^2}{3} + \frac{5a_o}{8}\right) \cdot \ln l + \left(\frac{17a^2}{64} - \frac{3a}{8}\right) \cdot \ln^2 l + \left(-\frac{a_o^4}{8} + \frac{9a_o^3}{16} + \frac{5a_o^2}{18} \right. \\
& + \left.\left. \left(-\frac{25}{96} + 2\zeta(3)\right) a_o \right)\right] + (C_A T N_f) \left.\left[\left(-\frac{2a_o^2}{3}\right) \cdot \ln l + \left(\frac{10a_o^2}{9}\right) \right] \right\} ,
\end{aligned}
\tag{9}
$$

and

$$
\begin{aligned}
X_2(p^2) \; = \; & \frac{g^2}{(4\pi)^2} \left\{ C_A \left[\frac{1}{2}(a_o^2 + 3a_o)\left(1 + (2 - \ln l)\,\epsilon + 4\epsilon^2\right) \right] \right\} + \\
& + \frac{g^4}{(4\pi)^4} \left\{ C_A^2 \left[\left(\frac{3a_o^3}{4} + \frac{7a_o^2}{6} + \frac{9a_o}{4}\right) \cdot \ln l + \left(\frac{a_o^4}{8} - \frac{9a_o^3}{16} - \frac{25a_o^2}{9} - \right.\right.\right. \\
& - \left.\left. \left(\frac{211}{48} + 2\zeta(3)\right) a_o \right)\right] + (C_A T N_f) \left.\left[\left(\frac{2a_o^2}{3}\right) \cdot \ln l + \left(-\frac{10a_o^2}{9}\right) \right] \right\} + \\
& + O(\epsilon, \ln^2 l) .
\end{aligned}
\tag{10}
$$

Here C_A is the $SU(N)$ Casimir operator in the adjoint representation (for $SU(3)$: $C_A = 3$ and $T \equiv T(R) = \frac{1}{2}$); N_f is the flavour number. We note that as consequence of the used invariant regularization of the Feynman integrals in our results the Euler constant γ_E and Riemann function $\zeta(2)$ do not occur.

Now integrating the eq.(4) over the gauge parameter $-a_o$, we get the following equation :

$$I\left(\epsilon,a_o,g^2\right) = I^*\left(\epsilon,a_o^*,g^2\right)\exp\left\{-\int_{a_o^*}^{a_o} d\tau\left[\frac{1}{\tau}\left(1-X_1(\epsilon,\tau,g^2)\right)\right]\right\}, \qquad (11)$$

where a_o^* is the fixed gauge parameter equal e.g. either 0 (Landau gauge) or 1 (Feynman gauge). I^* is the gluon propagator in the given gauge.

Using the expression (9) for X_1 ,performing the integration and further expanding the exponential, and I^* over the coupling constant , we readily arrive at the expression for the dimensionally regularized gluon propagator in the arbitrary gauge for the massless gauge theory. The gauge-dependent part is given by the X_1 and by the 1-loop expression for I^*. When we performe the expansion of the I^* in the one-loop approximation we must to take into account the expansion over ϵ up to $O(\epsilon^2)$.

4. We recall that according to the ordinary generalized Slavnov-Taylor identities [9,10] only the form-factor X_1 is superficially divergent and consequently multiplicatively renormalizable. We consider the renormalized quantities in momentum-subtraction scheme given in [2].That is, we define our renormalized quantities I^R and X_1^R at the point $p^2=-M^2$ (where the scale $M>0$ characterize the congifuration of external momenta) so that ,e.g.:

$$\lim_{\epsilon\to 0}\left[X_1\left(\frac{-p^2}{\mu^2},\ldots\right)-X_1\left(\frac{M^2}{\mu^2},\ldots\right)\right] = X_1^{mom}\left(\frac{-p^2}{M^2},\ldots\right).$$

Then referring to eq.(9) we get

$$X_1^{mom}\left(\frac{-p^2}{M^2},g^2,a\right) = 1+\frac{g^2_{mom}}{(4\pi)^2}\left\{C_A\left[\left(-\frac{a}{2}\right)\ln\frac{-p^2}{M^2}\right]\right\}+ \qquad (12)$$
$$\frac{g^4_{mom}}{(4\pi)^4}\left\{\left[C_A^2\left(-\frac{3a^3}{4}+\frac{a^2}{3}+\frac{5a}{8}\right)+C_ATN_f\left(-\frac{2a^2}{3}\right)\right]\ln\frac{-p^2}{M^2}\right\}.$$

Therefore,the corresponding renormalization constant (counterterm) is expressed as (by assuming that $X_1(-M^2)=1$):

$$Z_X\left(\epsilon^{-1},\frac{\mu^2}{M^2},g^2,a\right) = 1+\frac{g^2_{mom}}{(4\pi)^2}\left\{C_A\left[\frac{a}{2}\left(\frac{1}{\epsilon}+\ln 4\pi+\ln\frac{\mu^2}{M^2}\right)-\right.\right. \qquad (13)$$
$$\left.\left.-\frac{1}{2}\left(a^2+a\right)\right]\right\}+\frac{g^4_{mom}}{(4\pi)^4}\left\{C_A^2\left[\frac{1}{\epsilon^2}\left(\frac{a^2}{4}+\frac{3a}{8}\right)+\frac{1}{\epsilon}\left(-\frac{a^2}{4}-\frac{11a}{16}\right)-\right.\right.$$

$$-\left(-\frac{3a^3}{4}+\frac{a^2}{3}+\frac{5a}{8}\right)\cdot\left(\ln 4\pi+\ln\frac{\mu^2}{M^2}\right)+\left(-\frac{a^4}{8}+\frac{9a^3}{16}+\frac{15a}{18}-\right.$$
$$\left.\left.-\left(\frac{25}{96}-2\zeta(3)\right)a\right)\right]+C_ATN_f\left[\left(\frac{2a^2}{3}\right)\left(\ln 4\pi+\ln\frac{\mu^2}{M^2}\right)+\left(\frac{10a^2}{9}\right)\right]\Bigg\}$$
$$+O(\epsilon).$$

We analyse now our results in the framework of the renormalization group in the given scheme. Differentiating the eq.(4) with respect to p^2, we get the following renorm-group equations :

$$a\frac{\partial}{\partial a}\left[p^2\frac{d}{dp^2}\ln I^{ren}\left(\frac{-p^2}{M^2},g^2_{mom},a\right)\right] = p^2\frac{d}{dp^2}X_1^{ren}\left(\frac{-p^2}{M^2},g^2_{mom},a\right), \qquad (14)$$

or setting $-p^2=\mu^2$ $\left(\frac{-p^2}{M^2}=\frac{\mu^2}{M^2}\right)$ we obtain the differential relation for the anomalous dimension of the gluon field

$$\begin{aligned} a\frac{\partial}{\partial a}\gamma_I\left(g^2_{mom},a\right) &= p^2\frac{d}{dp^2}X_1^{ren}\left(\frac{-p^2}{M^2},g^2_{mom},a\right) \qquad (15)\\ &= \frac{g^2_{mom}}{(4\pi)^2}\left(-\frac{a}{2}\right)C_A+\frac{g^4_{mom}}{(4\pi)^4}\left[\left(-\frac{3a^3}{4}+\frac{a^2}{3}+\frac{5a}{8}\right)C_A^2+\right.\\ &+\left.\left(-\frac{2a^2}{3}\right)C_ATN_f\right].\end{aligned}$$

Integrating the Eq.(15) we give the anomalous dimension of the gluon propagator which can used for the analysis of the UV-behaviour.

5. Conclusions. First,as we have mentioned above the differential equation (3) derived on the basis of the modified Slavnov-Taylor identities give us explicitly the gauge dependence. Therefore, using it together with the analogous differential equations for the other two-point functions (of the ghost or of the fermion fields) and three-point functions (vertices) one can derive the differential equation with respect to the gauge parameter for the invariant charge (coupling constant) or for the renorm-group $\beta-function$. On the basis of the resulting relation one can study first ,the gauge dependence of these quantities and second,the condition(s) over which the gauge invariance of the $\beta-function$ is given in the different renormalization MOM-schemes. This program may be also be realized on the basis of the differential relations for the anomalous dimensions,like to Eq.(15).

Second, we want to attract attention to the obtained result for the quantity $X_2(p^2)$. That qauntity is related with the longitudinal part of the gluon propagator and therefore, in our view, we can to use it in the formalism proposed in [6] to "stop" the gauge.

We would like to thank Prof. D.V. Shirkov and Dr. D.I. Kazakov for valuable remarks.

•Appendix

1. *PROOF* of the equation $X_1 = 1+\tilde{X}_1+...$ (where $\tilde{X}_1 = O(g^2)$), i.e., the relation

$$\frac{\delta^3\Gamma}{\delta A^i_\mu \delta L \delta J^j_\nu} = -\delta^{ij} g_{\mu\nu} + O\left(g^2\right) \qquad (16)$$

Let us construct a generating functional Z for the Green's functions

$$Z\left(\eta^l_\rho, \xi^l, \bar{\xi}^l, J^l_\rho, K^l, L\right) = \int [d] exp\Big[i < \mathcal{L} + \eta^l_\rho A^l_\rho + \bar{\xi}^l C^l + \bar{C}^l \xi^l + J^l_\rho \nabla^{ls}_\rho C^s + \frac{1}{2} g_o K^l \left(\vec{C}\times\vec{C}\right)^l + L\left(\bar{C}^l A^l_\rho + J^l_\rho A^l_\rho\right) > (x')\Big],$$

and the generating functional W for the connected ones,

$$W(...) = i \ln Z(...).$$

Here $\mathcal{L} = eq.(1)$ without the matter fields; η^l_ρ, ξ^l,$\bar{\xi}^l$ are the anticommuting classical and x-dependent sources; J^l_ρ,K^l and L are given above. We use the following notation $[d] \equiv dAdCd\bar{C}$, $< ... > \equiv \int d^4x'(...)$ and $\nabla^\rho_{ls} \equiv D^\rho_{ls}$.

Varying $W(...)$ with respect to η^i_μ and expanding the exponential $exp[i < ... >]$ in a power series in the factor i up to $O(i)$ we obtain the following relation

$$-\frac{\delta W}{\delta \eta^i_\mu} \equiv \mathcal{A}^i_\mu = \frac{i}{Z}\int [d] A^i_\mu(x)\,[1+ < \eta A > + < LJA > + < L\bar{C}\partial A > + < J\nabla C > + ...]exp[i < ... >]. \qquad (17)$$

Here $\mathcal{A}^i_\mu \equiv \mathcal{A}^i_\mu[\eta,\xi,\bar{\xi},J,K,L]$ is the functional ; we omit the indices for the simplicity. Performing in eq.(20) the contraction (pairing) of the gluon fields and defining the functional $\eta^l_\rho(x') \equiv \eta^l_\rho[A,G,\bar{C},J,K,L]$ as

$$\eta^l_\rho(x') \simeq \int [D^{-1}_{\mu\rho}]^{li}(x-x')\, dx\, \mathcal{A}^i_\mu - (LJ)^i_\rho(x') - (LC\partial)^i_\rho(x') - (J\frac{\delta\nabla}{\delta A}C)^i_\rho(x'),$$

where $D^{li}_{\mu\rho}(x-x')$ is the two-point connected gluon Green's function.Now variating the η^l_ρ with respect to $J^j_\nu(y)$ and to L we have

$$\frac{\delta\eta^l_\rho(x')}{\delta L \delta J^j_\nu(y)} \simeq \delta^{lj} g_{\rho\nu}. \qquad (18)$$

Now we shall introduce a generating functional Γ for the 1PI Green's functions :

$$\Gamma(A,C,\bar{C},J,K,L) = -W(...) -,[< (\eta A + \bar{C}\xi + \bar{\xi}C) >]^i_\mu(x);$$

next we vary it with respect to $A^i_\mu(x)$, $J^j_\nu(y)$ and L. Then we get to the follwing relation :

$$\frac{\delta^3\Gamma}{\delta A^i_\mu(x)\delta L \delta J^j_\nu(y)} = -\frac{\delta^2\eta^i_\mu(x)}{\delta L \delta J^j_\nu(y)}. \qquad (19)$$

And lastly substituting here the r.h.s. by the eq.(21) we obtain the desired relation.

2. Here we give the formulas of the Feynman integrals.

2.1. <u>one-loop integration formula</u>

$$\int \frac{d^n q}{q^{2\alpha}(p-q)^{2\beta}} = \frac{i(-\pi)^{\frac{n}{2}}}{(p^2)^{\alpha+\beta-\frac{n}{2}}} \cdot G_o(\alpha,\beta),$$

$$G_o(\alpha,\beta) = \frac{\Gamma(\alpha+\beta-\frac{n}{2})\,\Gamma(\frac{n}{2}-\alpha)\,\Gamma(\frac{n}{2}-\beta)}{\Gamma(\alpha)\,\Gamma(\beta)\,\Gamma(n-\alpha-\beta)}.$$

2.2. <u>two-loop integration formulas</u>

$$\int \frac{d^n q\, d^n k}{q^{2\alpha}k^{2\beta}(p-q)^{2\sigma}(p-k)^{2\tau}} = \frac{i^2(-\pi)^n}{(p^2)^{\alpha+\beta+\sigma+\tau-n}} \cdot \mathbf{I}_1(\alpha,\beta,\sigma,\tau),$$

$$\mathbf{I}_1(\alpha,\beta,\sigma,\tau) = \frac{\Gamma(\alpha+\sigma-\frac{n}{2})\Gamma(\frac{n}{2}-\alpha)\Gamma(\frac{n}{2}-\sigma)}{\Gamma(\alpha)\Gamma(\sigma)\Gamma(n-\alpha-\sigma)}\,\frac{\Gamma(\beta+\tau-\frac{n}{2})\Gamma(\frac{n}{2}-\beta)\Gamma(\frac{n}{2}-\tau)}{\Gamma(\beta)\Gamma(\tau)\Gamma(n-\beta-\tau)}.$$

$$\int \frac{d^n q\, d^n k}{q^{2\alpha}(p-q)^{2\sigma}(k-q)^{2\gamma}(p-k)^{2\tau}} = \frac{i^2(-\pi)^n}{(p^2)^{\alpha+\beta+\sigma+\tau-n}} \cdot \mathbf{I}_2(\alpha,\sigma,\gamma,\tau),$$

$$\mathbf{I}_2(\alpha,\sigma,\gamma,\tau) = \frac{\Gamma(\gamma+\tau-\frac{n}{2})\Gamma(\frac{n}{2}-\gamma)\Gamma(\frac{n}{2}-\tau)}{\Gamma(\gamma)\Gamma(\tau)\Gamma(n-\gamma-\tau)} \times \frac{\Gamma(\alpha+\sigma+\gamma+\tau-n)\Gamma(\frac{n}{2}-\alpha)\Gamma(n-\sigma-\gamma-\tau)}{\Gamma(\alpha)\Gamma(\sigma+\gamma+\tau-\frac{n}{2})\Gamma(\frac{3n}{2}-\alpha-\sigma-\gamma-\tau)}.$$

Here

$$\Gamma(z+1) = z\Gamma(z), \quad \Gamma(1) = \Gamma(2) = 1, \quad \Gamma(N+1) = N!,$$

$$\Gamma(1+z) = exp\left[-\gamma x + \sum_{n=2}^{\infty}(-)^n\frac{\zeta(n)}{n}x^n\right],$$

where γ is the Euler constant and the Riemann ζ-function.

References

[1] G. 'tHooft and M.Veltman,Nucl.Phys. B44 (1972) 189;
G.'tHooft,Nucl.Phys. B61 (1973) 455.

[2] W.Celmaster and R.J. Gonsalves, Phys. Rev. D20 (1979) 1420.

[3] E.Braaten and J.P. Leveille, Phys.Rev. D26 (1981) 1369.

[4] K. Hogiwara and T.Yoshino,Phys.Rev. D26 (1982) 2038.

[5] D.V. Shirkov and O.V.Tarasov, JINR Rapid Commm. N3[29] 18, Dubna(1988);
Proc. of the Tbilisi Conference "Quarks-88" May.1988
(World Scientific,Singapoure-1989).

[6] D.V. Shirkov,JINR preprint E2-89-288(1989),Dubna.

[7] .V.Dung,H.D.Phuoc and O.V.Tarasov, JINR preprint E2-89-415 (1989), Dubna.

[8] D.Espriu and R. Tarrach, Phys.Rev. D25 (1982) 1073.

[9] H.Kluberg-Stern and J.B.Zuber, Phys.Rev. D12 (1975) 467.

[10] See Refs.[11-14] in Ref.9 of our paper.

[11] That computer package in preparation to be published.

[12] J.Vermaseren,"FORM User's Guide",Amsterdam (1990).

[13] K.G. Chetyrkin,F.V.Tkachov, Nucl. Phys. B192 (1981) 159.

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