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# QUASIPOTENTIAL IN THE FOURTH ORDER OF PERTURBATION THEORY.

## Unequal Mass Case

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At present the problem of an existence of the narrow resonances in $(e^+e^-)$- and (pp)-systems [1, 2] requires the adequate theoretical description [3]-[6]. The method proposed in the above papers is based on the quasipotential equation [7, 8] with the relativistic Coulomb potential modified with taking into account the binding effects in two-particle systems.

In this article the method of construction of the quasipotential by means of the two-time Green function $\widehat{G}(x_a, x_b, t, y_a, y_b, t')$ is applied to the investigation of processes of the two-photon exchange.

The quasipotential

$$\hat{V} = F^{-1} - (\widehat{G}^{+})^{-1} \tag{1}$$

$$F = \widehat{G}_0^{+} = (2\pi)^3\delta(\vec{p}-\vec{q})(E - \sqrt{\vec{p}^2 + m_1^2} - \sqrt{\vec{p}^2 + m_2^2})^{-1} \tag{2}$$

was used earlier for the calculation of corrections to the Fermi energy of the hyperfine splitting and for the analysis of the fine structure energy in hydrogen-like atoms [9]-[11].

Recently the papers [12, 13] [1] devoted to the calculation of the $O(\alpha^6 log\alpha)$ corrections in the two-body quantum electrodynamic systems have been appeared. The planned experiments will produce the question of eliminating the discrepancy in the results of calculation of the $O(\alpha^6 log\alpha)$ corrections between [12] and [13].

The expressions for the quasipotential in the fourth order of perturbation theory (unequal mass case), which are presented in this paper, could be used for verifying the results [12, 13] and solving the problem. The indices C and T describe the successive exchange of the Coulomb photons and the transverse photons, respectively [2] The index "it" marks the expressions corresponding to the iteration diagrams. The index "×" marks the expressions corresponding to the cross diagrams (**Fig.1b,1e,1f,1h** ).

$$\begin{aligned} \hat{V}_{CC}(\vec{p},\vec{q};E) &= -\frac{2\alpha^2}{\pi}u_1^*(\vec{p})u_2^*(-\vec{p})\int\frac{d\vec{k}}{k_p^2k_q^2}\left\{\frac{\Lambda_1^+(\vec{k})\Lambda_2^+(-\vec{k})}{\epsilon_{1k}+\epsilon_{2k}-E}+\right. \\ &+ \left.\frac{\Lambda_1^-(\vec{k})\Lambda_2^-(-\vec{k})}{\epsilon_{1k}+\epsilon_{2k}+E}\right\}u_1(\vec{q})u_2(-\vec{q}) \end{aligned} \tag{5}$$

$$\begin{aligned} \hat{V}^{\times}_{CC}(\vec{p},\vec{q};E) &= \frac{2\alpha^2}{\pi}u_1^*(\vec{p})u_2^*(-\vec{p})\int\frac{d\vec{k}}{k_p^2k_q^2}\left\{\frac{\Lambda_1^+(\vec{k})\Lambda_2^-(\vec{k}-\vec{p}-\vec{q})}{\epsilon_{2p}+\epsilon_{1k}+\epsilon_{2kpq}+\epsilon_{2q}-E}+\right. \\ &+ \left.\frac{\Lambda_1^-(\vec{k})\Lambda_2^+(\vec{k}-\vec{p}-\vec{q})}{\epsilon_{1p}+\epsilon_{1k}+\epsilon_{2kpq}+\epsilon_{1q}-E}\right\}u_1(\vec{q})u_2(-\vec{q}) \end{aligned} \tag{6}$$

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[1] The results of these papers are different for the positronium from each other:

$$\Delta E_{[12]} = \frac{1}{96}m\alpha^6 log\alpha^{-1}(3+5\vec{\sigma}_1\vec{\sigma}_2)\frac{\delta_{l0}}{n^3} \tag{3}$$

$$\Delta E_{[13]} = \frac{5}{96}m\alpha^6 log\alpha^{-1}(3+\vec{\sigma}_1\vec{\sigma}_2)\frac{\delta_{l0}}{n^3} \tag{4}$$

[2] The Coulomb gauge is used.

$$
\begin{aligned}
\hat V_{CT}(\vec p,\vec q;E) &= \frac{\alpha^2}{\pi}u_1^*(\vec p)u_2^*(-\vec p)\int\frac{d\vec k}{k_p^2k_q}\Bigg\{\frac{\Lambda_1^+(\vec k)\Lambda_2^+(-\vec k)}{\epsilon_{1k}+\epsilon_{2k}-E}\Big(\frac{1}{k_q+\epsilon_{2k}+\epsilon_{1q}-E}+\\
&+ \frac{1}{k_q+\epsilon_{1k}+\epsilon_{2q}-E}\Big)+\frac{\Lambda_1^-(\vec k)\Lambda_2^-(-\vec k)}{\epsilon_{1k}+\epsilon_{2k}+E}\Big(\frac{1}{k_q+\epsilon_{1k}+\epsilon_{1q}}+\frac{1}{k_q+\epsilon_{2k}+\epsilon_{2q}}\Big)-\\
&- \frac{\Lambda_1^-(\vec k)\Lambda_2^+(-\vec k)}{(k_q+\epsilon_{2k}+\epsilon_{1q}-E)(k_q+\epsilon_{1k}+\epsilon_{1q})}-\frac{\Lambda_1^+(\vec k)\Lambda_2^-(-\vec k)}{(k_q+\epsilon_{1k}+\epsilon_{2q}-E)(k_q+\epsilon_{2k}+\epsilon_{2q})}\Bigg\}\times\\
&\times \Gamma_{12}(\vec k-\vec q)u_1(\vec q)u_2(-\vec q)
\end{aligned}
\tag{7}
$$

$$
\begin{aligned}
\hat V^{\times}_{CT}(\vec p,\vec q;E) &= \frac{\alpha^2}{\pi}u_1^*(\vec p)u_2^*(-\vec p)\int\frac{d\vec k}{k_p^2k_q}\Bigg\{\frac{\Lambda_1^+(\vec k)\Gamma_{12}(\vec k-\vec q)\Lambda_2^+(\vec k-\vec p-\vec q)}{(k_q+\epsilon_{1p}+\epsilon_{2kpq}-E)(k_q+\epsilon_{1k}+\epsilon_{2q}-E)}+\\
&+ \frac{\Lambda_1^-(\vec k)\Gamma_{12}(\vec k-\vec q)\Lambda_2^-(\vec k-\vec p-\vec q)}{(k_q+\epsilon_{2p}+\epsilon_{2kpq})(k_q+\epsilon_{1k}+\epsilon_{1q})}-\frac{\Lambda_1^+(\vec k)\Gamma_{12}(\vec k-\vec q)\Lambda_2^-(\vec k-\vec p-\vec q)}{\epsilon_{2p}+\epsilon_{1k}+\epsilon_{2kpq}+\epsilon_{2q}-E}\times\\
&\times \Big(\frac{1}{k_q+\epsilon_{2p}+\epsilon_{2kpq}}+\frac{1}{k_q+\epsilon_{1k}+\epsilon_{2q}-E}\Big)-\frac{\Lambda_1^-(\vec k)\Gamma_{12}(\vec k-\vec q)\Lambda_2^+(\vec k-\vec p-\vec q)}{\epsilon_{1p}+\epsilon_{1k}+\epsilon_{2kpq}+\epsilon_{1q}-E}\times\\
&\times \Big(\frac{1}{k_q+\epsilon_{1p}+\epsilon_{2kpq}-E}+\frac{1}{k_q+\epsilon_{1k}+\epsilon_{1q}}\Big)\Bigg\}u_1(\vec q)u_2(-\vec q)
\end{aligned}
\tag{8}
$$

$$
\begin{aligned}
\hat V_{TT}(\vec p,\vec q;E) &= -\frac{\alpha^2}{2\pi}u_1^*(\vec p)u_2^*(-\vec p)\int\frac{d\vec k}{k_pk_q}\Gamma_{12}(\vec p-\vec k)\Bigg\{\frac{\Lambda_1^+(\vec k)\Lambda_2^+(-\vec k)}{\epsilon_{1k}+\epsilon_{2k}-E}\times\\
&\times \Big[\frac{1}{k_p+k_q+\epsilon_{1p}+\epsilon_{2q}-E}\Big(\frac{1}{k_p+\epsilon_{1p}+\epsilon_{2k}-E}+\frac{1}{k_q+\epsilon_{1k}+\epsilon_{2q}-E}\Big)+\\
&+ \frac{1}{k_p+k_q+\epsilon_{2p}+\epsilon_{1q}-E}\Big(\frac{1}{k_p+\epsilon_{2p}+\epsilon_{1k}-E}+\frac{1}{k_q+\epsilon_{2k}+\epsilon_{1q}-E}\Big)+\\
&+ \frac{1}{(k_p+\epsilon_{2p}+\epsilon_{1k}-E)(k_q+\epsilon_{1k}+\epsilon_{2q}-E)}+\frac{1}{(k_p+\epsilon_{1p}+\epsilon_{2k}-E)(k_q+\epsilon_{2k}+\epsilon_{1q}-E)}\Big]+\\
&+ \frac{\Lambda_1^-(\vec k)\Lambda_2^-(-\vec k)}{\epsilon_{1k}+\epsilon_{2k}+E}\Big[\frac{1}{k_p+k_q+\epsilon_{1p}+\epsilon_{2q}-E}\Big(\frac{1}{k_p+\epsilon_{1p}+\epsilon_{2k}-E_1+E_2}+\\
&+ \frac{1}{k_q+\epsilon_{1k}+\epsilon_{2q}+E_1-E_2}\Big)+\frac{1}{k_p+k_q+\epsilon_{2p}+\epsilon_{1q}-E}\times\\
&\times \Big(\frac{1}{k_p+\epsilon_{2p}+\epsilon_{1k}+E_1-E_2}+\frac{1}{k_q+\epsilon_{2k}+\epsilon_{1q}-E_1+E_2}\Big)+\\
&+ \frac{1}{(k_p+\epsilon_{2p}+\epsilon_{1k}+E_1-E_2)(k_q+\epsilon_{1k}+\epsilon_{2q}+E_1-E_2)}+\\
&+ \frac{1}{(k_p+\epsilon_{1p}+\epsilon_{2k}-E)(k_q+\epsilon_{2k}+\epsilon_{1q}-E)}\Big]+\frac{\Lambda_1^-(\vec k)\Lambda_2^+(-\vec k)}{-\epsilon_{1k}+\epsilon_{2k}-E}\times\\
&\times \Big[\frac{1}{k_p+k_q+\epsilon_{1p}+\epsilon_{2q}-E}\Big(\frac{1}{k_p+\epsilon_{1p}+\epsilon_{2k}-E}-\frac{1}{k_p+\epsilon_{1p}+\epsilon_{1k}}\Big)+\\
&+ \frac{1}{k_p+k_q+\epsilon_{2p}+\epsilon_{1q}-E}\Big(\frac{1}{k_q+\epsilon_{2k}+\epsilon_{1q}-E}-\frac{1}{k_q+\epsilon_{1k}+\epsilon_{1q}}\Big)+\\
&+ \frac{1}{(k_p+\epsilon_{1p}+\epsilon_{2k}-E)(k_q+\epsilon_{2k}+\epsilon_{1q}-E)}-\frac{1}{(k_p+\epsilon_{1p}+\epsilon_{1k})(k_q+\epsilon_{1k}+\epsilon_{1q})}\Big]+\\
&+ \frac{\Lambda_1^+(\vec k)\Lambda_2^-(-\vec k)}{\epsilon_{1k}-\epsilon_{2k}-E}\Big[\frac{1}{k_p+k_q+\epsilon_{1p}+\epsilon_{2q}-E}\Big(\frac{1}{k_q+\epsilon_{1k}+\epsilon_{2q}-E}-\frac{1}{k_q+\epsilon_{2k}+\epsilon_{2q}}\Big)+\\
&+ \frac{1}{k_p+k_q+\epsilon_{2p}+\epsilon_{1q}-E}\Big(\frac{1}{k_p+\epsilon_{2p}+\epsilon_{1k}-E}-\frac{1}{k_p+\epsilon_{2p}+\epsilon_{2k}}\Big)+\\
&+ \frac{1}{(k_p+\epsilon_{2p}+\epsilon_{1k}-E)(k_q+\epsilon_{1k}+\epsilon_{2q}-E)}-\frac{1}{(k_p+\epsilon_{2p}+\epsilon_{2k})(k_q+\epsilon_{2k}+\epsilon_{2q})}\Big]\Bigg\}\times\\
&\times \Gamma_{12}(\vec k-\vec q)u_1(\vec q)u_2(-\vec q)
\end{aligned}
\tag{9}
$$

$$
\begin{aligned}
\hat V^{\times}_{TT}(\vec p,\vec q;E) &= -\frac{\alpha^2}{2\pi}u_1^*(\vec p)u_2^*(-\vec p)\int\frac{d\vec k}{k_pk_q}\alpha_{1i}\Big(\delta_{ij}-\frac{(\vec p-\vec k)_i(\vec p-\vec k)_j}{(\vec p-\vec k)^2}\Big)\times\\
&\times \Bigg\{\Lambda_1^+(\vec k)\Gamma_{12}(\vec k-\vec q)\Lambda_2^+(\vec k-\vec p-\vec q)\Big[\frac{1}{k_p+k_q+\epsilon_{1k}+\epsilon_{2kpq}-E}\Big(\frac{1}{k_p+\epsilon_{2p}+\epsilon_{1k}-E}+\\
&+ \frac{1}{k_q+\epsilon_{1p}+\epsilon_{2kpq}-E}\Big)\Big(\frac{1}{k_p+\epsilon_{2kpq}+\epsilon_{1q}-E}+\frac{1}{k_q+\epsilon_{1k}+\epsilon_{2q}-E}\Big)+\\
&+ \frac{1}{(k_p+k_q+\epsilon_{1p}+\epsilon_{2q}-E)(k_q+\epsilon_{1p}+\epsilon_{2kpq}-E)(k_q+\epsilon_{1k}+\epsilon_{2q}-E)}+\\
&+ \frac{1}{(k_p+k_q+\epsilon_{2p}+\epsilon_{1q}-E)(k_p+\epsilon_{2p}+\epsilon_{1k}-E)(k_p+\epsilon_{2kpq}+\epsilon_{1q}-E)}\Big]+\\
&+ \Lambda_1^-(\vec k)\Gamma_{12}(\vec k-\vec q)\Lambda_2^-(\vec k-\vec p-\vec q)\Big[\frac{1}{k_p+k_q+\epsilon_{2kpq}+\epsilon_{1k}+E}\Big(\frac{1}{k_p+\epsilon_{1p}+\epsilon_{1k}}+\\
&+ \frac{1}{k_q+\epsilon_{2p}+\epsilon_{2kpq}}\Big)\Big(\frac{1}{k_q+\epsilon_{1k}+\epsilon_{1q}}+\frac{1}{k_p+\epsilon_{2kpq}+\epsilon_{2q}}\Big)+\\
&+ \frac{1}{(k_p+k_q+\epsilon_{1p}+\epsilon_{2q}-E)(k_p+\epsilon_{1p}+\epsilon_{1k})(k_p+\epsilon_{2kpq}+\epsilon_{2q})}+\\
&+ \frac{1}{(k_p+k_q+\epsilon_{2p}+\epsilon_{1q}-E)(k_q+\epsilon_{2p}+\epsilon_{2kpq})(k_q+\epsilon_{1k}+\epsilon_{1q})}\Big]-\\
&- \Lambda_1^+(\vec k)\Gamma_{12}(\vec k-\vec q)\Lambda_2^-(\vec k-\vec p-\vec q)\Big[\frac{1}{\epsilon_{2p}+\epsilon_{1k}+\epsilon_{2kpq}+\epsilon_{2q}-E}\Big(\frac{1}{k_p+\epsilon_{2kpq}+\epsilon_{2q}}+\\
&+ \frac{1}{k_p+\epsilon_{2p}+\epsilon_{1k}-E}\Big)\Big(\frac{1}{k_q+\epsilon_{2p}+\epsilon_{2kpq}}+\frac{1}{k_q+\epsilon_{1k}+\epsilon_{2q}-E}\Big)+\\
&+ \frac{1}{(k_p+k_q+\epsilon_{1p}+\epsilon_{2q}-E)(k_p+\epsilon_{2kpq}+\epsilon_{2q})(k_q+\epsilon_{1k}+\epsilon_{2q}-E)}+\\
&+ \frac{1}{(k_p+k_q+\epsilon_{2p}+\epsilon_{1q}-E)(k_p+\epsilon_{2p}+\epsilon_{1k}-E)(k_q+\epsilon_{2p}+\epsilon_{2kpq})}\Big]-\\
&- \Lambda_1^-(\vec k)\Gamma_{12}(\vec k-\vec q)\Lambda_2^+(\vec k-\vec p-\vec q)\Big[\frac{1}{\epsilon_{1p}+\epsilon_{1k}+\epsilon_{2kpq}+\epsilon_{1q}-E}\Big(\frac{1}{k_p+\epsilon_{1p}+\epsilon_{1k}}+\\
&+ \frac{1}{k_p+\epsilon_{2kpq}+\epsilon_{1q}-E}\Big)\Big(\frac{1}{k_p+\epsilon_{1k}+\epsilon_{1q}}+\frac{1}{k_q+\epsilon_{1p}+\epsilon_{2kpq}-E}\Big)+
\end{aligned}
$$

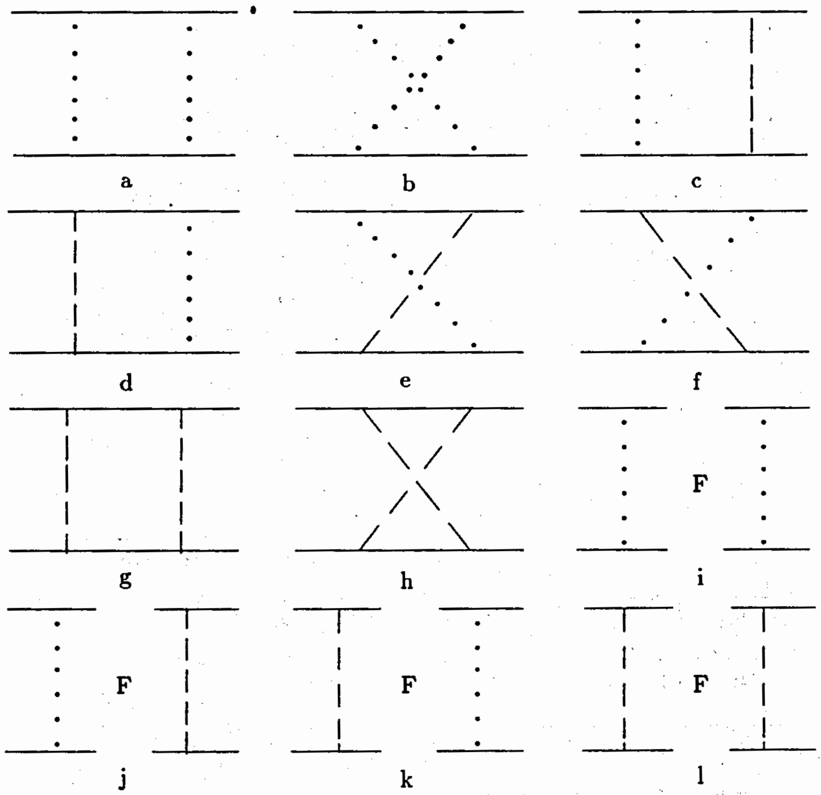


**Fig. 1.** The diagrams for calculation of the quasipotential in the fourth order of perturbation theory(unequal mass case). The dot line corresponds to the Coulomb photon; the dashed line, to the transverse photon.



$$+ \quad \frac{1}{(k_p+k_q+\epsilon_{1p}+\epsilon_{2q}-E)(k_p+\epsilon_{1p}+\epsilon_{1k})(k_q+\epsilon_{1p}+\epsilon_{2kpq}-E)}+$$
$$+ \quad \frac{1}{(k_p+k_q+\epsilon_{2p}+\epsilon_{1q}-E)(k_p+\epsilon_{2kpq}+\epsilon_{1q}-E)(k_q+\epsilon_{1k}+\epsilon_{1q})}\Big]\Big\} \times$$
$$\times \quad \alpha_{2j} u_1(\vec{q}) u_2(-\vec{q}) \qquad (10)$$

$$\hat{V}^{it}_{CC}(\vec{p},\vec{q};E) = -\frac{2\alpha^2}{\pi} u_1^*(\vec{p}) u_2^*(-\vec{p}) \int \frac{d\vec{k}}{k_p^2 k_q^2} \frac{\Lambda_1^+(\vec{k})\Lambda_2^+(-\vec{k})}{\epsilon_{1k}+\epsilon_{2k}-E} u_1(\vec{q}) u_2(-\vec{q}) \qquad (11)$$

$$\hat{V}^{it}_{CT}(\vec{p},\vec{q};E) = \frac{\alpha^2}{\pi} u_1^*(\vec{p}) u_2^*(-\vec{p}) \int \frac{d\vec{k}}{k_p^2 k_q} \frac{\Lambda_1^+(\vec{k})\Lambda_2^+(-\vec{k})}{\epsilon_{1k}+\epsilon_{2k}-E} \times$$
$$\times \quad \Big(\frac{1}{k_q+\epsilon_{1k}+\epsilon_{2q}-E} + \frac{1}{k_q+\epsilon_{2k}+\epsilon_{1q}-E}\Big)\Gamma_{12}(\vec{k}-\vec{q}) u_1(\vec{q}) u_2(-\vec{q}) \qquad (12)$$

$$\hat{V}^{it}_{TT}(\vec{p},\vec{q};E) = -\frac{\alpha^2}{2\pi} u_1^*(\vec{p}) u_2^*(-\vec{p}) \int \frac{d\vec{k}}{k_p k_q} \Gamma_{12}(\vec{p}-\vec{k}) \frac{\Lambda_1^+(\vec{k})\Lambda_2^+(-\vec{k})}{\epsilon_{1k}+\epsilon_{2k}-E} \times$$
$$\times \quad \Big(\frac{1}{k_p+\epsilon_{2p}+\epsilon_{1k}-E} + \frac{1}{k_p+\epsilon_{1p}+\epsilon_{2k}-E}\Big)\Big(\frac{1}{k_q+\epsilon_{1k}+\epsilon_{2q}-E} +$$
$$+ \quad \frac{1}{k_q+\epsilon_{2k}+\epsilon_{1q}-E}\Big)\Gamma_{12}(\vec{k}-\vec{q}) u_1(\vec{q}) u_2(-\vec{q}) \qquad (13)$$

Here $\Lambda^{\pm}$ are the projecting operators, $k_p = |\,\vec{p}-\vec{k}\,|$, $k_q = |\,\vec{k}-\vec{q}\,|$, $k_{pq} = |\,\vec{k}-\vec{p}-\vec{q}\,|$, $\Gamma_{12}(\vec{k}) = \vec{\gamma}_1\vec{\gamma}_2 - (\vec{\gamma}_1\vec{k})(\vec{\gamma}_2\vec{k})/\vec{k}^2$, $\vec{\alpha}_i = \gamma_i^0\vec{\gamma}_i$ and $\gamma_\mu = (\gamma^0, \vec{\gamma})$ – Dirac's matrices.

The expressions corresponding to **Fig.**1d,1e,1k are obtained from the expressions (7), (8), (12) by means of the substitutions $p \Leftrightarrow q$ and the displacement of the proecting operators and $\Gamma_{12}$. The quasipotentials for the diagrams with a vertex part and a self-energy part are easily deduced from the one-photon exchange quasipotential presented e.g in [6, 9, 11, 14, 15].

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