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WEAK FORM FACTORS OF BEAUTY BARYONS

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1. Introduction

Study of semileptonic (s.l.) decays of hadrons containing a single heavy quark has a great popularity [1]-[23]: it is a unique tool to determine of Cabibbo-Kabayashi-Maskawa matrix elements and an original source to probe hadronic structure.

During last years considerable progress has been made in experimental and theoretical investigation of weak heavy-baryon decays. The unique experiments were fulfilled by $ARGUS$ and $CLEO$ (study the properties of the Λ_c^+-baryon [16, 17]) and at the $CERN$ $p\bar{p}$ collider by $UA1$ Collaboration where the first observation of the beauty Λ_b-baryon was made [18]. Recently Isgur and Wise have discovered an additional kind of QCD spin-flavour symmetry [3]-[5]. The Isgur-Wise symmetry manifests itself in QCD when the mass of heavy quark m_Q goes to infinity ($m_Q \to \infty$). By exploiting this symmetry, model-independent predictions for heavy-hadron weak form factors and relations between them have been obtained [3]-[5]. It was shown that form factors arising in heavy-meson decays are defined by a single universal function $\xi(\omega)$ of argument ω with $\xi(1) = 1$ [3]. Dimensionless quantity ω is the dot product of the four-velocities of the initial and final heavy-hadron states. The Isgur-Wise function $\xi(\omega)$ was calculated in the framework of the Valence Quark Model (QM) [5], Relativistic Oscilator Model (ROM) [10], QCD Sum Rules (QCD SR) [15] and Quark Confinement Model (QCM) [19].

The s.l. decays of $\frac{1}{2}^+$ heavy baryons to $\frac{1}{2}^+$ and $\frac{3}{2}^+$ heavy baryon final states have been considered in [4] in the limit of large heavy quark masses. It was shown that the heavy-baryon weak form factors may be expressed in this limit through the three universal unknown functions $\xi(\omega)$, $\eta(\omega)$ and $\tau(\omega)$. The exclusive s.l. decays of heavy baryons have been worked out by J.G. Körner and collaborators (see,[2], [12]-[14]) in the relativistic spectator quark model.

In the papers [19]-[29], we have developed the Quark Confinement Model (QCM) based on some assumptions about the hadronization and confinement of light quarks. By assuming hadrons to be colourless excitations of quark-gluon interactions, the transition to hadron variables in the QCD functional was made following [30]. The hadron interactions are described by quark diagrams averaged over vacuum gluon backgrounds. The confinement hypothesis means that this averaging leads to that quarks do not appear in the observable hadron spectrum. Strong, weak and electromagnetic interactions of hadrons (both mesons and baryons) can be described in the QCM from a unified point of view. The calculations [24]-[29] of low-energy meson and baryon processes have shown that the model reproduces the quark structure of light hadrons quite accurately.

An extension of the QCM to heavy quark physics has been done in [19]-[23]. It was based on that heavy quarks weakly interact with vacuum background fields, and therefore, they can be considered as the static Fermi particles with large constituent masses. At the same time, interactions of the light quarks are defined by the confinement forces. Form factors of the s.l. heavy-meson decays were calculated in ref.[19]-[21]. The heavy quark limit $m_Q \to \infty$ (Isgur-Wise symmetry) for these form factors was examined in papers [19, 21].

S.l. decays of heavy baryons with $J^P = \frac{1}{2}^+$ were studied in papers [22, 23]. In ref.[22] we calculated weak form factors of $\Lambda_b^0 \to \Lambda_c^+ e\bar{\nu}$ and $\Sigma_b^+ \to \Sigma_c^{++} e\bar{\nu}$ decays. Also we obtained as explicit their asymptotics in the heavy quark limit. It was shown that explicit form factors practically coinside with their asymptotics and $1/m_Q$-corrections can be neglected. This confirms that the Isgur-Wise symmetry works well for the $b \to c$ transition. In [23] we analyzed the s.l. decays of heavy baryons corresponding to $b \to u$, $c \to s$ and $c \to d$ flavour exchange.

In this paper, we give more full analysis of s.l. decays of heavy baryons with $J^P = \frac{1}{2}^+$ and $\frac{3}{2}^+$ containing a single b-quark into heavy baryons with a single c-quark. We obtain explicit expressions for these form factors and analyse them in the heavy quark limit. Also we calculate the $1/m_Q$-corrections and check the Ademollo-Gatto theorem. By making use form factors obtaned in the heavy-quark limit we compute the decay rates Γ. It is shown that our results are close to the Free Quark Model predictions.

2. Matrix Elements of Semileptonic Heavy-Baryon Decays

Here we give the basic formulas defining the s.l. decays of heavy baryons $B_b(\frac{1}{2}^+)$ and $B_b^*(\frac{3}{2}^+)$ containing a single b-quark into baryons $B_c(\frac{1}{2}^+)$ and $B_c^*(\frac{3}{2}^+)$ containing a single c-quark.

The corresponding matrix element is written as

$$M(b \to c) = \frac{G_F}{\sqrt{2}} V_{bc} \ell_\mu(q) J^\mu(p,p') \tag{1}$$

where p is the momentum of a initial baryon,

p' is the momentum of a final baryon,

q is the total momentum of a lepton pair,

G_F is the Fermi constant,

$\ell_\mu = \bar{u}_\nu O_\mu u_\ell$ is a lepton current,

V_{bc} is the Kobayashi-Mascawa matrix element.

Hereafter we use the unified definition of weak O_μ-matrix:

$$O_\mu = \gamma_\mu(1 - \gamma_5) \qquad \gamma_5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

The general form for the weak hadron current $J^\mu(p,p')$ is

⋆ $B_b \to B_c$ transition

$$\begin{aligned} J^\mu(p,p') &= \bar{B}_c(p')\Lambda^\mu(p,p')B_b(p), \\ \Lambda^\mu(p,p') &= F_1(\omega)\gamma^\mu + F_2(\omega)v^\mu + +F_3(\omega)v'^\mu \\ &+ G_1(\omega)\gamma^\mu\gamma^5 + G_2(\omega)v^\mu\gamma^5 + +G_3(\omega)v'^\mu\gamma^5 \end{aligned} \tag{2}$$

⋆ $B_b \to B_c^*$ transition

$$\begin{aligned} J^\mu(p,p') &= \bar{B}_c^{*\alpha}(p')\Lambda^{\mu\alpha}(p,p')B_b(p), \\ \Lambda^{\mu\alpha}(p,p') &= \sum_{i=1}^{4} T_i^{\mu\alpha} \cdot [K_i(\omega) + \gamma^5 N_i(\omega)] \end{aligned} \tag{3}$$

where

$$T_1^{\mu\alpha} = \gamma^\mu v^\alpha \qquad T_3^{\mu\alpha} = v'^\mu v^\alpha$$
$$T_2^{\mu\alpha} = v^\mu v^\alpha \qquad T_4^{\mu\alpha} = g^{\mu\alpha}$$

⋆ $B_b^* \to B_c^*$ transition

$$\begin{aligned} J^\mu(p,p') &= \bar{B}_c^{*\alpha}(p')\Lambda^\mu_{\alpha\beta}(p,p')B_b^{*\beta}(p), \\ \Lambda^\mu_{\alpha\beta}(p,p') &= \sum_{i=1}^{8} R_i^{\mu\alpha\beta} \cdot [P_i(\omega) + \gamma^5 Q_i(\omega)] \end{aligned} \tag{4}$$

where

$$
\begin{aligned}
R_1^{\mu\alpha\beta} &= \gamma^\mu g^{\alpha\beta} & R_5^{\mu\alpha\beta} &= v^\mu v^\alpha v'^\beta \\
R_2^{\mu\alpha\beta} &= v^\mu g^{\alpha\beta} & R_6^{\mu\alpha\beta} &= v'^\mu v^\alpha v'^\beta \\
R_3^{\mu\alpha\beta} &= v'^\mu g^{\alpha\beta} & R_7^{\mu\alpha\beta} &= g^{\mu\beta} v^\alpha \\
R_4^{\mu\alpha\beta} &= \gamma^\mu v^\alpha v'^\beta & R_8^{\mu\alpha\beta} &= g^{\mu\alpha} v'^\beta
\end{aligned}
$$

One has to emphasize that all weak heavy-baryon form factors F_i, G_i, N_i, K_i, P_i and Q_i are defined as functions of dimensionless variable $\omega = v \cdot v'$ which is the dot product of the four velocities of initial (v) and final (v') baryons. This variable is related to the square of the total momentum of a lepton pair $q^2 = (p - p')^2$ as

$$\omega = \frac{pp'}{m_{B_b} m_{B_c}} = \frac{m_{B_b}^2 + m_{B_c}^2 - q^2}{2 m_{B_b} m_{B_c}} \tag{5}$$

where m_{B_b} and m_{B_c} are the masses of initial and final baryons, respectively.

3. Heavy Baryons in the QCM

Dynamical description of hadron processes in the QCM is based on interaction Lagrangians of hadrons with quarks. Particular interaction Lagrangians of heavy baryons with quarks have the following form

$$L_{B_Q} = g_{B_Q} \bar{B}_Q J_{B_Q} + h.c. \tag{6}$$

where B_Q is the baryon field.

The coupling constants g_{B_Q} are determined from the compositeness condition [31]

$$Z_{B_Q} = 1 + g_{B_Q}^2 \Sigma'_{B_Q}(m_{B_Q}) = 0, \tag{7}$$

where Σ'_{B_Q} is the derivative of the heavy baryon mass operator.

The J_{B_Q} are the three-quark currents with the quantum numbers of baryons (Λ_Q, Σ_Q or Σ_Q^*). It is known [22, 26] that there are two independent sets of three-quark currents for baryons with quantum numbers $J^P = \frac{1}{2}^+$. Here we use tensor current (see, ref.[26]) because it allows us obtain more successful description of low-energy hadron physics against vector current (see, ref. [26]). Tensor J_{Λ_Q} and J_{Σ_Q} have the forms

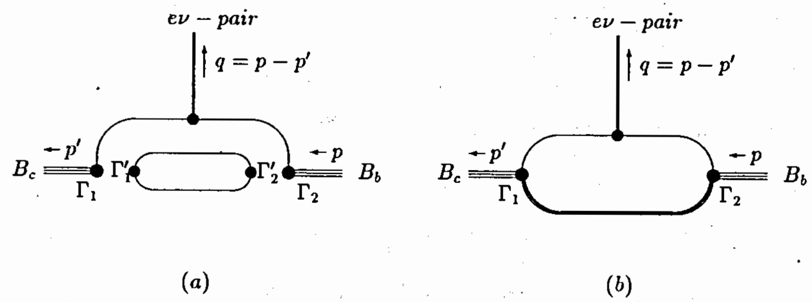

Fig.1 Heavy-Baryon Weak Form Factors in the QCM. a. Two-loop quark diagram, b. One-loop quark-diquark diagram.

$$
\begin{aligned}
J_{\Lambda_Q} &= \varepsilon^{abc} \cdot [Q^a(u^b C \gamma^5 d^c) + \gamma^5 Q^a (u^b C d^c)], \\
J_{\Sigma_Q} &= \varepsilon^{abc} \cdot \sigma^{\mu\nu} \gamma^5 Q^a (u^b C \sigma^{\mu\nu} u^c).
\end{aligned} \tag{8}
$$

where a, b, c are colour indices, $C = \gamma^0\gamma^2$ is the matrix of charge conjugation; $Q = c$ or b quark.

There is the only three-quark current for baryons with quantum numbers $J^P = \frac{3}{2}^+$. The current $J^\alpha_{\Sigma_Q^*}$ for baryon Σ_Q^* is written as

$$J^\alpha_{\Sigma_Q^*} = -\varepsilon^{abc} \cdot [Q^a(u^b C \gamma^\alpha d^c) - \frac{i}{2} \gamma^\beta Q^a (u^b C \sigma^{\alpha\beta} d^c)]. \tag{9}$$

S.l. decays of heavy baryons are described in the QCM by quark diagrams (Fig.1a) averaged over vacuum gluon backgrounds [24]. This averaging leads to that light quarks do not appear in the observable hadron spectrum. The light quark propagator in the external background field is

$$S_v(x) = \int \frac{d^4 k}{(2\pi)^4 i} \frac{e^{-ikx}}{v\Lambda_q - \not{k}} \tag{10}$$

where v is variable of integrating over gluon vacuum, Λ_q is dimensional parameter characterizing the confinement region of light quarks.

The averaging over the vacuum background is defined by formula

$$\int \frac{d\sigma_v}{v-z} = G(z) = a(-z^2) + zb(-z^2), \tag{11}$$

where the confinement functions are chosen to be [20]-[25]

$$\begin{aligned} a(u) &= \int d\sigma_v \frac{v}{v^2+u} = 2\exp\{-u^2-u\}, \\ b(u) &= \int d\sigma_v \frac{1}{v^2+u} = 2\exp\{-u^2+0.4u\}. \end{aligned}$$

The heavy quarks are suggested to describe as usual Fermi particles [19]-[22] because they weakly interact with external background fields. The behaviour of heavy quarks is described by standard free propagator of fermion particle with mass m_Q

$$S_Q(x) = \int \frac{d^4k}{(2\pi)^4 i} \frac{e^{-ikx}}{m_Q - \not k} \tag{12}$$

It was proposed in ref.[22, 26] to use the *Quark-Diquark Approximation of the Three-Quark Structure of Baryons* under calculation of the two-loop quark diagram in Fig.1a:

$$\int d\sigma_v \mathrm{Tr}[\Gamma_1' S_v(x_1-x_2)\Gamma_2' S_v(x_2-x_1)] \to D^{\Gamma_1\Gamma_2}(x_1-x_2) \tag{13}$$

where $D^{\Gamma_1\Gamma_2}(x_1-x_2)$ is considered to be the diquark propogator

$$\begin{aligned} D^{\Gamma_1\Gamma_2}(x) &= \int \frac{d^4k}{(2\pi)^4 i} e^{-ikx} \int d\sigma_v \frac{d^{\Gamma_1\Gamma_2}}{v^2\Lambda_D^2 - k^2} \\ &= \frac{d^{\Gamma_1\Gamma_2}}{\Lambda_D^2} \int \frac{d^4k}{(2\pi)^4 i} e^{-ikx} b(-\frac{k^2}{\Lambda_D^2}) \end{aligned} \tag{14}$$

This assumption essentially simplifies calculations because it allows one to replace two-loop quark diagrams drawn in Fig.1a to one-loop quark-diquark diagrams in Fig.1b. From the physical point of view this prescription may be justified by that the baryons containing the only heavy quark may be considered as two-particle systems Qd where d is a light diquark. This approximation can be realized for baryons with quantum numbers $J^P = \frac{1}{2}^+$ and $J^P = \frac{3}{2}^+$ by unified manner. The parameters $d^{\Gamma_1\Gamma_2}$ are chosen so as to be convenient for calculations

$$\begin{aligned} &d^{PP} = c^{PP}, \qquad d^{SS} = c^{SS}, \qquad d^{TT}_{\mu\nu,\alpha\beta} = c^{TT}(g_{\mu\alpha}g_{\nu\beta} - g_{\nu\alpha}g_{\mu\beta}), \\ &d^{VV}_{\mu\nu} = c^{VV} g_{\mu\nu}, \qquad d^{VT}_{\mu,\alpha\beta} = d^{TV}_{\alpha\beta,\mu} = ic^{VT}(k_\alpha g_{\mu\beta} - k_\beta g_{\mu\alpha}) \end{aligned}$$

The choice of numerical coefficients $c^{\Gamma_1\Gamma_2}$ must satisfy two conditions: Ward identity and the identities for weak heavy-hadron form factors obtained by Isgur and Wise in ref. [4] which were followed from heavy-quark spin-flavour symmetry:

- ⋆ $\Lambda_b \to \Lambda_c$ decay

$$F_1(\omega) = -G_1(\omega), \quad F_2(\omega) = F_3(\omega), \quad G_2(\omega) = -G_3(\omega) \tag{15}$$

- ⋆ $\Sigma_b \to \Sigma_c(\Sigma_c^\star)$ decay

$$\begin{aligned} &F_1(\omega) = -G_1(\omega), \quad N_2(\omega) = K_2(\omega) = 0, \quad N_i(\omega) = K_i(\omega), \ i = 2 \text{ and } 3 \\ &F_2(\omega) = F_3(\omega) = \frac{2}{\sqrt{3}} N_1(\omega), \quad G_2(\omega) = -G_3(\omega) = \frac{2}{\sqrt{3}} K_1(\omega), \end{aligned} \tag{16}$$

With taking into account of (15) and (16) we find that the only set of the parameters $c^{\Gamma_1\Gamma_2}$ is

$$c^{PP} = c^{SS} = c^{TT} = 1, \qquad c^{VV} = c^{VT} = 0$$

The dimensional parameter Λ_D is a free parameter. In this paper we use for Λ_D value $\Lambda_D = 827.7$ Mev obtained in ref. [26] where light baryon physics was studied.

Using *the Quark-Diquark Approximation* gives the following expression for typical vertex function $\Lambda_\mu^{\Gamma_1\Gamma_2}(p,p')$:

$$\Lambda_\mu^{\Gamma_1\Gamma_2}(p,p') = g_{B_b}g_{B_c}d^{\Gamma_1\Gamma_2} \int \frac{d^4k}{(2\pi)^4 i} b(-k^2)\Gamma_1 \frac{1}{m_c - (\not k + \not p')} O_\mu \frac{1}{m_b - (\not k + \not p)} \Gamma_2 \tag{17}$$

Hereafter, all masses and momenta are given in the units of the dimensional parameter Λ_D.

The calculation technique of matrix element (15) was illustrated in refs. [19]-[23]. As in paper [22] we suppose that the mass of heavy baryon containing the only heavy quark is equal to the mass of heavy quark $m_{B_Q} = m_Q$. Additionaly we assume that masses of baryons containing an identical heavy quark are equal to each other:

$$m_{\Lambda_c} = m_{\Sigma_c} = m_{\Sigma_c^\star} = 2.28 \text{ GeV}, \qquad m_{\Lambda_b} = m_{\Sigma_b} = m_{\Sigma_b^\star} = 5.60 \text{ GeV}$$

4. Heavy-Baryon Weak Form Factors

Now we discuss the heavy-baryon weak form factors obtained in the QCM. We have for the transition $\frac{1}{2}^+_b \to \frac{1}{2}^+_c$ the following results

$$F\{G\}_i(\omega)=\frac{1}{\sqrt{R_0(m^2_{B_b})R_0(m^2_{B_c})}}\int\limits_0^\infty dub(u)\int\limits_0^1 d\alpha\tilde{F}\{\tilde{G}\}_i(u,D(\alpha))\frac{C_0(u,D(\alpha))}{\sqrt{u^2+4uD(\alpha)}} \tag{18}$$

where

$$D(\alpha)\equiv D(\alpha,m_{B_b},m_{B_c})=[m_{B_b}\alpha+m_{B_c}(1-\alpha)]^2+2m_{B_b}m_{B_c}\alpha(1-\alpha)(\omega-1)$$

$$C_0(u,x)=\frac{\sqrt{u^2+4ux}-u}{2x},$$

$$R_0(x)=\int\limits_0^\infty dub(u)\frac{C_0(u,x)[1-C_0(u,x)]}{\sqrt{u^2+4ux}}.$$

The functions $\tilde{F}\{\tilde{G}\}_i(u,D(\alpha))$ are written as

a. Decay $\Lambda^0_b\to\Lambda^+_c$

$$\begin{aligned}
\tilde{F}_1&=1-\frac{1}{2}\Big(1-\frac{\alpha}{r}-r(1-\alpha)\Big)C_0(u,D(\alpha)),\\
\tilde{F}_2&=-\frac{\alpha}{r}C_0(u,D(\alpha)),\\
\tilde{F}_3&=-r(1-\alpha)C_0(u,D(\alpha)),\\
\tilde{G}_1&=-1+\frac{1}{2}\Big(1+\frac{\alpha}{r}+r(1-\alpha)\Big)C_0(u,D(\alpha))\\
\tilde{G}_2&=-\tilde{F}_2,\qquad \tilde{G}_3=\tilde{F}_3
\end{aligned}$$

b. Decay $\Sigma^+_b\to\Sigma^{++}_c$

$$\begin{aligned}
\tilde{F}_1&=-\frac{1}{3}\Big[1-\frac{1}{2}\Big(1-\frac{\alpha}{r}-r(1-\alpha)\Big)C_0(u,D(\alpha))\Big],\\
\tilde{F}_2&=\frac{2}{3}\Big[1-\frac{\alpha}{2}\Big(1+2r\Big)C_0(u,D(\alpha))\Big],\\
\tilde{F}_3&=\frac{2}{3}\Big[1-\frac{1-\alpha}{2}\Big(1+\frac{2}{r}\Big)C_0(u,D(\alpha))\Big],\\
\tilde{G}_1&=\frac{1}{3}\Big[1-\frac{1}{2}\Big(1+\frac{\alpha}{r}+r(1-\alpha)\Big)C_0(u,D(\alpha))\Big],\\
\tilde{G}_2&=\frac{2}{3}\Big[1-\frac{\alpha}{2}\Big(1-2r\Big)C_0(u,D(\alpha))\Big],\\
\tilde{G}_3&=-\frac{2}{3}\Big[1+\frac{1-\alpha}{2}\Big(1-\frac{2}{r}\Big)C_0(u,D(\alpha))\Big],
\end{aligned}$$

Here and further r is the ratio of the masses of a final baryon and an initial one.

For the $\frac{1}{2}^+_b\to\frac{3}{2}^+_c$ transition one obtains

$$N\{K\}_i(\omega)=\frac{1}{\sqrt{3R_0(m^2_{B_b})R_0(m^2_{B^*_c})}}\int\limits_0^\infty dub(u)\int\limits_0^1 d\alpha\tilde{N}\{\tilde{K}\}_i(u,D(\alpha))\frac{C_0(u,D(\alpha))}{\sqrt{u^2+4uD(\alpha)}} \tag{19}$$

The functions $\tilde{N}\{\tilde{K}\}_i(u,D(\alpha))$ have the following form

$$\begin{aligned}
\tilde{N}_1=\tilde{K}_1&=C_0(u,D(\alpha))\Big[1-C_0(u,D(\alpha))\Big],\\
\tilde{N}_2=\tilde{K}_2&=2\frac{\alpha}{r}C_0(u,D(\alpha))\Big[C_0(u,D(\alpha))+\frac{u}{D(\alpha)}\Big(1-C_0(u,D(\alpha))\Big)\Big]\\
\tilde{N}_3=\tilde{K}_3&=-2C_0(u,D(\alpha))\Big[1-C_0(u,D(\alpha))\Big]\Big[1+\frac{\alpha(1-\alpha)u}{D(\alpha)}\Big],\\
\tilde{N}_4=\tilde{K}_4&=C_0(u,D(\alpha))\Big[2\omega\Big(1-C_0(u,D(\alpha))\Big)\\
&+C_0(u,D(\alpha))\Big(1-\frac{\alpha}{r}-r(1-\alpha)-\frac{u}{2m_{B_b}m_{B^*_c}}\Big)\Big].
\end{aligned}$$

For the $\frac{3}{2}^+_b\to\frac{3}{2}^+_c$ transition we obtain

$$P\{Q\}_i(\omega)=\frac{1}{\sqrt{R_0(m^2_{B^*_b})R_0(m^2_{B^*_c})}}\int\limits_0^\infty dub(u)\int\limits_0^1 d\alpha\tilde{P}\{\tilde{Q}\}_i(u,D(\alpha))\frac{C_0(u,D(\alpha))}{\sqrt{u^2+4uD(\alpha)}} \tag{20}$$

The functions $\tilde{P}\{\tilde{Q}\}_i(u,D(\alpha))$ have the following form

$$\tilde{P}_1=C_0(u,D(\alpha))\Big[\omega\Big(1-C_0(u,D(\alpha))\Big)-\frac{C_0(u,D(\alpha))}{2}\Big(1+\frac{\alpha}{r}+r(1-\alpha)\Big)\Big]$$

$$\tilde{P}_2 = \tilde{P}_3 = -\tilde{Q}_2 = \tilde{Q}_3 = \alpha C_0^2(u, D(\alpha))$$
$$\tilde{P}_4 = -\tilde{Q}_4 = -C_0(u, D(\alpha))\Big[1 - C_0(u, D(\alpha))\Big]$$
$$\tilde{P}_5 = \tilde{P}_6 = \tilde{Q}_5 = \tilde{Q}_6 = 0$$
$$\tilde{P}_7 = -C_0^2(u, D(\alpha))\Big[1 - \alpha + \frac{\alpha}{r}\Big]$$
$$\tilde{P}_8 = -C_0^2(u, D(\alpha))\Big[\alpha + r(1 - \alpha)\Big]$$
$$\tilde{Q}_7 = C_0^2(u, D(\alpha))\Big[1 - \alpha - \frac{\alpha}{r}\Big]$$
$$\tilde{Q}_8 = -C_0^2(u, D(\alpha))\Big[\alpha - r(1 - \alpha)\Big]$$

Let us discuss the heavy quark limit $m_Q = m_{B_Q} \to \infty$ in the heavy-baryon weak form factors obtained in the QCM. It is easily to get this limit in the typical integral

$$\int\limits_0^\infty dub(u) \int\limits_0^1 d\alpha \frac{C_0(u, D(\alpha))}{\sqrt{u^2 + 4uD(\alpha)}}$$
$$= \int\limits_0^\infty dub(u) \int\limits_0^1 d\alpha \frac{1}{2D(\alpha)} \Big[1 + \sum_{n=0}^\infty (-)^{n+1} \Big(\frac{u}{4D(\alpha)}\Big)^{n+\frac{1}{2}} \frac{\Gamma(n+\frac{1}{2})}{\Gamma(n)\Gamma(\frac{1}{2})}\Big]$$
$$= \int\limits_0^\infty dub(u) \int\limits_0^1 d\alpha \frac{1}{2D(\alpha)} \Big[1 + O\Big(\sqrt{\frac{u}{D(\alpha)}}\Big)\Big]$$
$$= \frac{1}{2m_b m_c}\{\Phi(\omega) + O(1/m)\}$$

where
$$\Phi(\omega) = \frac{1}{\sqrt{\omega^2 - 1}} \ln(\omega + \sqrt{\omega^2 - 1}).$$

Here ω is the dot product of the four-velocities $v = p/m_b$ and $v' = p'/m_c$.

The results for our form factors are listed in Tables 1-4. For a comparison, we give the Isgur-Wise [4], Georgi [6] and Körner [2] results. In the definiton of the Isgur-Wise form factors we omit the coefficient function $C^{ji}(\omega)$ (see, ref. [4]).

One has to remark that all form factors obtained in the QCM are expressed through the universal function $\Phi(\omega)$ which is independent on the model parameters. It is differed from the heavy meson form factors having the dependence on the integrals of the confinement functions $A_0 = \int\limits_0^\infty dua(u)$ and $B_{1/2} = \int\limits_0^\infty du\sqrt{u}b(u)$ (see, ref.[19]). Also the QCM form factors satisfy to identities between heavy-baryon form factors obtained in [4].

It is easily to express the Isgur-Wise, Georgi and Körner form factors through the function

Φ. We have

$$\xi(\omega) = \Phi(\omega) \qquad \zeta(\omega) = \Phi(\omega) \qquad F_\Lambda(\omega) = \Phi(\omega)$$
$$\eta(\omega) = \Phi(\omega) \qquad \zeta_1(\omega) = -\omega\Phi(\omega) \qquad F_L(\omega) = \Phi(\omega)$$
$$\tau(\omega) = -2\Phi(\omega) \qquad \zeta_2(\omega) = -\Phi(\omega) \qquad F_T(\omega) = \omega\Phi(\omega)$$

5. $1/m_Q$-Corrections in the Asymptotics of Heavy-Baryon Weak Form Factors and the Ademollo-Gatto Theorem

As it was shown in our recent paper [22], that Isgur-Wise symmetry manifests itself very clearly in $b \to c$ flavour transitions of $\frac{1}{2}^+$ heavy baryons. In particular, the explicit form factor $F_1(\omega)$ arising in the decay $\Lambda_b^0 \to \Lambda_c^+$ practically coincides with its asymptotics $\Phi(\omega)$. This means that the $1/m_Q$-corrections to the form factors in the leading order are small. However, from a theoretical point of view it is interesting to calculate these corrections. For this purpose we use the formula

$$\int\limits_0^\infty dub(u) \int\limits_0^1 d\alpha \frac{C_0(u, D(\alpha))}{\sqrt{u^2 + 4uD(\alpha)}} \tag{21}$$
$$= \frac{1}{2m_{B_b} m_{B_c}} \int\limits_0^\infty dub(u) \Big\{\Phi(\omega) - \frac{\sqrt{u}}{2(1+\omega)}\Big(\frac{1}{m_{B_b}} + \frac{1}{m_{B_c}}\Big) + O\Big(\frac{1}{m_{B_Q}^2}\Big)\Big\}$$

and

$$\int\limits_0^\infty dub(u) \int\limits_0^1 d\alpha\alpha^n \frac{C_0^2(u, D(\alpha))}{\sqrt{u^2 + 4uD(\alpha)}} = \frac{1}{m_{B_b} m_{B_c}} \int\limits_0^\infty dub(u) \Big\{\frac{x_n\sqrt{u}}{\omega + 1} + O\Big(\frac{1}{m_{B_Q}^2}\Big)\Big\} \tag{22}$$

where

$$n = 0, 1 \qquad x_0 = \frac{1}{m_{B_b}} + \frac{1}{m_{B_c}} \qquad x_1 = \frac{1}{m_{B_b}}.$$

Inserting (24) and (25) into (21), (22) and (23) gives the expressions for the weak baryon form factors in the heavy quark limit with taking into account of $1/m_Q$-corrections. The results for $1/m_Q$-corrections are listed in Tables 5-8. Here we use notation

$$r_b = \frac{3B_{1/2}}{4B_0}, \quad \text{where} \quad B_n = \int\limits_0^\infty duu^n b(u)$$

As illustration we draw in Fig.2 graphics of various approximations for form factor F_1 arising in the processes $\Lambda_b \to \Lambda_c$:

a. Explicit form factor $F_1(\omega)$,

b. Its asymptotics in the heavy quark limit $\Phi(\omega)$,

c. $1/m_Q$-correction to $\Phi(\omega)$.

Let us now to verify the Ademollo-Gatto theorem [32, 33] in spin-flavour symmetry of heavy quarks. As it is known the theorem states that if the four-velocities of initial hadron (v) and final (v') are equal to each other

$$v^\mu = v'^\mu = V^\mu \tag{23}$$

then matrix elements of a charge operator $J^\mu(p,p')$ can deviate from their symmetry values only in second order in symmetry breaking. In other words there are no the terms of $O(1/m_Q)$-order in the weak heavy-hadron matrix elements, when $v^\mu = v'^\mu$. Using (26) we obtain the following identities on the mass-self of baryons

$$\bar{B}_c\gamma^\mu B_b = \bar{B}_c V^\mu B_b, \qquad \bar{B}_c\gamma^5 B_b = 0 \tag{24}$$

Also the additional conditions can be obtained from the Rarita-Schwinger equation for fermion particle with spin $J = \frac{3}{2}$

$$\bar{B}_c^{*\alpha}(p')v'^\alpha = \bar{B}_c^{*\alpha}(p')\gamma^\alpha \equiv 0, \quad v^\beta B_b^{*\beta}(p) = \gamma^\beta B_b^{*\beta}(p) \equiv 0 \tag{25}$$

With taking into account of (26)-(28) the $1/m_Q$-corrections $\Delta J^\mu(p,p')$ to weak heavy-baryon currents are written as

$\star\ \frac{1}{2}^+ \to \frac{1}{2}^+ e\bar{\nu}$ decay

$$\begin{aligned}\Delta J^\mu(p,p') &= \bar{B}_c(p')\Big\{\gamma^\mu\Delta F_1(1) + v^\mu\Delta F_2(1) + v'^\mu\Delta F_3 \\ &+ \gamma^\mu\gamma^5\Delta G_1(1) + v^\mu\gamma^5\Delta G_2(1) + v'^\mu\gamma^5\Delta G_3\Big\}B_b(p) \\ &= \bar{B}_c(p')\Big\{V^\mu[\Delta F_1(1) + \Delta F_2(1) + \Delta F_3(1)] + \gamma^\mu\gamma^5\Delta G_1(1)\Big\}B_b(p) \equiv 0\end{aligned}$$

where

$$\Delta F_1(1) + \Delta F_2(1) + \Delta F_3(1) \equiv 0 \ \text{ and } \ \Delta G_1(1) \equiv 0$$

$\star\ \frac{1}{2}^+ \to \frac{3}{2}^+ e\bar{\nu}$ decay

$$\begin{aligned}\Delta J^\mu(p,p') &= \bar{B}_c^{*\alpha}(p')\Big\{\gamma^\mu v^\alpha[\Delta K_1(1) + \gamma^5\Delta N_1(1)] + v^\mu v^\alpha[\Delta K_2(1) + \gamma^5\Delta N_2(1)] \\ &+ v'^\mu v^\alpha[\Delta K_3(1) + \gamma^5\Delta N_3(1)] + g^{\mu\alpha}[\Delta K_4(1) + \gamma^5\Delta N_4(1)]\Big\}B_b(p) \\ &= \bar{B}_c^{*\alpha}(p')\Big\{V^\alpha V^\mu[\Delta K_1(1) + \Delta K_2(1) + \Delta K_3(1)] \\ &+ V^\alpha\gamma^\mu\gamma^5\Delta N_1(1) + g^{\mu\alpha}\Delta K_4(1)\Big\}B_b(p) \equiv 0\end{aligned}$$

where

$$K_4(1) \equiv 0 \tag{26}$$

$\star\ \frac{3}{2}^+ \to \frac{3}{2}^+ e\bar{\nu}$ decay

$$\begin{aligned}\Delta J^\mu(p,p') &= \bar{B}_c^{*\alpha}(p')\Big\{\gamma^\mu g^{\alpha\beta}(\Delta P_1(1) + \gamma^5\Delta Q_1(1)) \\ &+ v^\mu g^{\alpha\beta}(\Delta P_2(1) + \gamma^5\Delta Q_2(1)) + v'^\mu g^{\alpha\beta}(\Delta P_3(1) + \gamma^5\Delta Q_3(1)) \\ &+ \gamma^\mu v^\alpha v'^\beta(\Delta P_4(1) + \gamma^5\Delta Q_4(1)) + g^{\mu\beta}v^\alpha\Delta P_7(1) + g^{\mu\alpha}v'^\beta\Delta P_8(1)\Big\}B_b^{*\beta}(p) \\ &= \bar{B}_c^{*\alpha}(p')\Big\{g^{\alpha\beta}V^\mu[\Delta P_1(1) + \Delta P_2(1) + \Delta P_3(1)] \\ &+ V^\mu V^\alpha V^\beta\Delta P_4(1) + \gamma^\mu\gamma^5[g^{\alpha\beta}Q_1(1) + V^\alpha V^\beta\Delta Q_4(1)] \\ &+ g^{\mu\beta}V^\alpha\Delta P_7(1) + g^{\mu\alpha}V^\beta\Delta P_8(1)\Big\}B_b^{*\beta}(p) \equiv 0\end{aligned}$$

where

$$\Delta P_1(1) + \Delta P_2(1) + \Delta P_3(1) \equiv 0, \quad \Delta Q_1(1) \equiv 0$$

Thus, all $1/m_Q$-corrections to the weak heavy-baryon matrix elements are equal to zero at $v^\mu = v'^\mu$. This means that the Ademollo-Gato theorem for the semileptonic $b \to c$ decays of heavy baryons takes place.

6. Decay Rates and Quark-Hadron Duality

Now we calculate the semileptonic heavy-baryon decay rates using the weak form factors in the heavy quark limit (see, Tables 1-4).

The s.l. decay widths are calculated according to the formula

$$\Gamma = \frac{G_F^2}{192\pi^3}|V_{bc}|^2 m_{B_b}^5 r^4 \int\limits_1^{\omega_{max}} d\omega R(\omega), \quad \text{where} \quad \omega_{max} = \frac{1+r^2}{2r} \tag{27}$$

The function $R(w)$ is equal to

$\star$ $\Lambda_b \to \Lambda_c e\bar{\nu}$ decay

$$R(w) = 32\sqrt{\omega^2-1}\,\Phi^2(\omega)(-1+3\omega\omega_{max}-2\omega^2) \tag{28}$$

$\star$ $\Sigma_b \to \Sigma_c e\bar{\nu}$ decay

$$R(w) = \frac{32}{9}\sqrt{\omega^2-1}\,\Phi^2(\omega)(-1-\omega\omega_{max}-2\omega^2+4\omega^3\omega_{max}) \tag{29}$$

$\star$ $\Sigma_b \to \Sigma_c^* e\bar{\nu}$ decay

$$R(w) = \frac{64}{9}\sqrt{\omega^2-1}\,\Phi^2(\omega)(-1+5\omega\omega_{max}-5\omega^2+7\omega^3\omega_{max}-6\omega^4) \tag{30}$$

$\star$ $\Sigma_b^* \to \Sigma_c^* e\bar{\nu}$ decay

$$R(w) = \frac{32}{9}\sqrt{\omega^2-1}\,\Phi^2(\omega)(-2+4\omega\omega_{max}-7\omega^2+11\omega^3\omega_{max}-6\omega^4) \tag{31}$$

The ω integration of (28)-(31) can be done using the formula

$$\int\limits_0^{\omega_{max}} d\omega \frac{\ln^2(\omega+\sqrt{\omega^2-1})}{\sqrt{\omega^2-1}} = \frac{1}{3}\ln^3(\omega_{max}+\sqrt{(\omega_{max}^2-1)}) \tag{32}$$

Finally, the expressions for decay rates are written as

$$\begin{aligned}\Gamma(\Sigma_b\to\Sigma_c) &= \frac{G_F^2|V_{bc}|^2}{192\pi^3}m_{\Sigma_b}^5\left[\frac{64}{27}r^4\ln^3 r + \frac{8}{27}(1+4r^2-4r^6-r^8)\ln^2 r\right.\\ &+ \frac{8}{81}(2+29r^2+72r^4+29r^6+2r^8)\ln r\\ &+ \left.\frac{4}{243}(4+193r^2-193r^6-4r^8)\right],\end{aligned}$$

$$\begin{aligned}\Gamma(\Sigma_b\to\Sigma_c^*) &= \frac{G_F^2|V_{bc}|^2}{192\pi^3}m_{\Sigma_b}^5\left[\frac{368}{27}r^4\ln^3 r + \frac{2}{27}(5+128r^2-128r^6-5r^8)\ln^2 r\right.\\ &+ \frac{1}{81}(29+2216r^2+5904r^4+2216r^6+29r^8)\ln r\\ &+ \left.\frac{1}{972}(143+30896r^2-30896r^4-143r^6)\right],\end{aligned}$$

$$\begin{aligned}\Gamma(\Sigma_b^*\to\Sigma_c^*) &= \frac{G_F^2|V_{bc}|^2}{192\pi^3}m_{\Sigma_b^*}^5\left[\frac{68}{9}r^4\ln^3 r + \frac{1}{27}(13+160r^2-160r^6-13r^8)\ln^2 r\right.\\ &+ \frac{1}{162}(61+2680r^2+7056r^4+2680r^6+61r^8)\ln r\\ &+ \left.\frac{1}{1944}(271+37072r^2-37072r^6-271r^8)\right],\end{aligned}$$

The results for decay rates are presented in Table 9. For comparison, the results of Spectator Quark Model (SQM) [12] are shown. One can see our results approximately satisfy to the relationships which come from the quark-hadron duality

$$\Gamma(b\to c) \approx \Gamma(\Lambda_b\to\Lambda_c) \approx \Gamma(\Sigma_b\to\Sigma_c)+\Gamma(\Sigma_b\to\Sigma_c^*) \approx \Gamma(\Sigma_b^*\to\Sigma_c)+\Gamma(\Sigma_b^*\to\Sigma_c^*)$$

Here

$$\Gamma(b\to c) \equiv \frac{G_F^2|V_{bc}|^2}{192\pi^3}m_b^5\left[1-8r^2+8r^6-r^8-24r^4\ln r\right] = 7.67\cdot 10^{10}\text{sec}^{-1}$$

$$\text{for } m_b = 4.7 \text{ GeV} \text{ and } r = \frac{m_c}{m_b} = 0.32$$

Also we can show that the results of Free Quark Model are reproduced in quark models (SQM [2] and QCM) if the form factors $F_\Lambda(\omega) = \Phi(\omega)$, $F_L(\omega) = \Phi(\omega)$ and $F_T = \omega\Phi(\omega)$ are supposed to be equal to unity. We have

Table 1.

Form Factors of $\Lambda_b \to \Lambda_c$ Decay in the Heavy Quark Limit

Form Factors	Approaches			
	QCM	Isgur and Wise [4]	Körner [2]	Georgi [6]
F_1	$\Phi(\omega)$	$\xi(\omega)$	$F_\Lambda(\omega)$	$\zeta(\omega)$
F_2	0	0	0	0
F_3	0	0	0	0
G_1	$-\Phi(\omega)$	$-\xi(\omega)$	$-F_\Lambda(\omega)$	$-\zeta(\omega)$
G_2	0	0	0	0
G_3	0	0	0	0

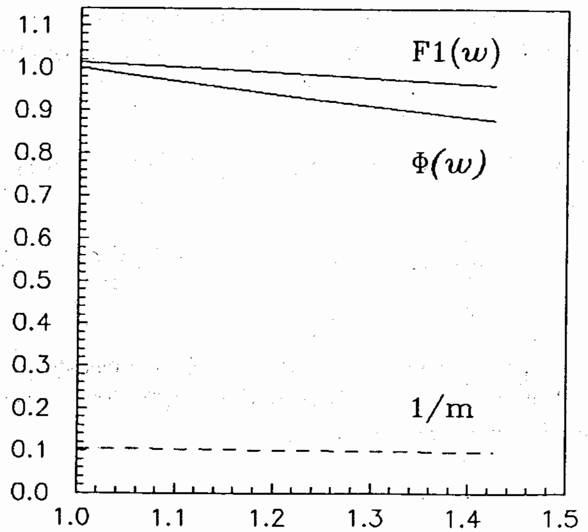

Fig.2. F_1 Form Factor of $\Lambda_b \to \Lambda_c$ decay

Table 2

Form Factors of $\Sigma_b \to \Sigma_c$ Decay In the Heavy Quark Limit

Form Factors	Approaches			
	QCM	Isgur and Wise [4]	Körner [2]	Georgi [6]
F_1	$-\frac{1}{3}\Phi(\omega)$	$-\frac{1}{6}(2\omega\eta(\omega)+(\omega-1)\tau(\omega))$	$-\frac{1}{3}F_L(\omega)$	$\frac{1}{3}(\omega\zeta_1(\omega)-(\omega^2-1)\zeta_2(\omega))$
F_2	$\frac{2}{3}\Phi(\omega)$	$\frac{2}{3}\eta(\omega)$	$\frac{2}{3(\omega+1)}(F_L(\omega)+F_T(\omega))$	$-\frac{2}{3}(\zeta_1(\omega)-(\omega-1)\zeta_2(\omega))$
F_3	$\frac{2}{3}\Phi(\omega)$	$\frac{2}{3}\eta(\omega)$	$\frac{2}{3(\omega+1)}(F_L(\omega)+F_T(\omega))$	$-\frac{2}{3}(\zeta_1(\omega)-(\omega-1)\zeta_2(\omega))$
G_1	$\frac{1}{3}\Phi(\omega)$	$\frac{1}{6}(2\omega\eta(\omega)+(\omega-1)\tau(\omega))$	$\frac{1}{3}F_L(\omega)$	$-\frac{1}{3}(\omega\zeta_1(\omega)-(\omega^2-1)\zeta_2(\omega))$
G_2	$\frac{2}{3}\Phi(\omega)$	$-\frac{2}{3}(\eta(\omega)+\tau(\omega))$	$-\frac{2}{3(\omega-1)}(F_L(\omega)-F_T(\omega))$	$\frac{2}{3}(\zeta_1(\omega)-(\omega+1)\zeta_2(\omega))$
G_3	$-\frac{2}{3}\Phi(\omega)$	$\frac{2}{3}(\eta(\omega)+\tau(\omega))$	$\frac{2}{3(\omega-1)}(F_L(\omega)-F_T(\omega))$	$-\frac{2}{3}(\zeta_1(\omega)-(\omega+1)\zeta_2(\omega))$

Table 3

Form Factors of $\Sigma_b \to \Sigma_c^*$ Decay In the Heavy Quark Limit

Form Factors	Approaches			
	QCM	Isgur and Wise [4]	Körner [2]	Georgi [6]
N_1	$\frac{1}{\sqrt{3}}\Phi(\omega)$	$\frac{1}{\sqrt{3}}\eta(\omega)$	$\frac{1}{(\omega+1)\sqrt{3}}(F_L(\omega)+F_T(\omega))$	$-\frac{1}{\sqrt{3}}(\zeta_1(\omega)-(\omega-1)\zeta_2(\omega))$
N_2	0	0	0	0
N_3	$-\frac{2}{\sqrt{3}}\Phi(\omega)$	$\frac{1}{\sqrt{3}}\tau(\omega)$	$-\frac{2}{(1-\omega^2)\sqrt{3}}(F_L(\omega)-\omega F_T(\omega))$	$\frac{2}{\sqrt{3}}\zeta_2(\omega)$
N_4	$\frac{2\omega}{\sqrt{3}}\Phi(\omega)$	$\frac{1}{\sqrt{3}}(2\eta(\omega)-(\omega-1)\tau(\omega))$	$\frac{2}{\sqrt{3}}F_T(\omega)$	$-\frac{2}{\sqrt{3}}\zeta_1(\omega)$
K_1	$\frac{1}{\sqrt{3}}\Phi(\omega)$	$-\frac{1}{\sqrt{3}}(\eta(\omega)+\tau(\omega))$	$-\frac{1}{(\omega-1)\sqrt{3}}(F_L(\omega)-F_T(\omega))$	$\frac{1}{\sqrt{3}}(\zeta_1(\omega)-(\omega+1)\zeta_2(\omega))$
K_2	0	0	0	0
K_3	$-\frac{2}{\sqrt{3}}\Phi(\omega)$	$\frac{1}{\sqrt{3}}\tau(\omega)$	$-\frac{2}{(1-\omega^2)\sqrt{3}}(F_L(\omega)-\omega F_T(\omega))$	$\frac{2}{\sqrt{3}}\zeta_2(\omega)$
K_4	$\frac{2\omega}{\sqrt{3}}\Phi(\omega)$	$\frac{1}{\sqrt{3}}(2\eta(\omega)-(\omega-1)\tau(\omega))$	$\frac{2}{\sqrt{3}}F_T(\omega)$	$-\frac{2}{\sqrt{3}}\zeta_1(\omega)$

Table 4

Form Factors of $\Sigma_b^* \to \Sigma_c^*$ Decay In the Heavy Quark Limit

Form Factors	Approaches		
	QCM	Körner [2]	Georgi [6]
P_1	$\omega\Phi(\omega)$	$F_T(\omega)$	$-\zeta_1(\omega)$
P_4	$-\Phi(\omega)$	$\frac{1}{(\omega-1)}(F_L(\omega)-F_T(\omega))$	$\zeta_2(\omega)$
$P_i\ (i=2,3,5...8)$	0	0	0
Q_1	$-\omega\Phi(\omega)$	$-F_T(\omega)$	$\zeta_1(\omega)$
Q_4	$\Phi(\omega)$	$-\frac{1}{(\omega-1)}(F_L(\omega)-F_T(\omega))$	$-\zeta_2(\omega)$
$Q_i\ (i=2,3,5...8)$	0	0	0

Table 5

$1/m_Q$-Correction to Form Factors of $\Lambda_b \to \Lambda_c$ Decay

Form Factors	QCM
ΔF_1	$r_b\cdot\left(\Phi(\omega)-\frac{2}{3(\omega+1)}\right)\cdot[1/m_{\Lambda_b}+1/m_{\Lambda_c}]$
ΔF_2	$-r_b\cdot\frac{4}{3(\omega+1)}\cdot 1/m_{\Lambda_c}$
ΔF_3	$-r_b\cdot\frac{4}{3(1+\omega)}1/m_{\Lambda_b}$
ΔG_1	$-r_b\cdot\left(\Phi(\omega)-\frac{2}{\omega+1}\right)\cdot[1/m_{\Lambda_b}+1/m_{\Lambda_c}]$
ΔG_2	$r_b\cdot\frac{4}{3(1+\omega)}\cdot 1/m_{\Lambda_c}$
ΔG_3	$-r_b\cdot\frac{4}{3(1+\omega)}\cdot 1/m_{\Lambda_b}$

Table 6

$1/m_Q$-Correction to Form Factors of $\Sigma_b \to \Sigma_c$ Decay

Form Factors	QCM
ΔF_1	$-r_b \cdot \frac{1}{3} \cdot [1/m_{\Sigma_b} + 1/m_{\Sigma_c}] \cdot \left(\Phi(\omega) - \frac{2}{3(\omega+1)}\right)$
ΔF_2	$r_b \cdot \frac{2}{3} \cdot \left(\Phi(\omega)[1/m_{\Sigma_b} + 1/m_{\Sigma_c}] - \frac{2}{3(1+\omega)}[3/m_{\Sigma_b} + 2/m_{\Sigma_c}]\right)$
ΔF_3	$r_b \cdot \frac{2}{3} \cdot \left(\Phi(\omega)[1/m_{\Sigma_b} + 1/m_{\Sigma_c}] - \frac{2}{3(1+\omega)}[2/m_{\Sigma_b} + 3/m_{\Sigma_c}]\right)$
ΔG_1	$r_b \cdot \frac{1}{3} \cdot [1/m_{\Sigma_b} + 1/m_{\Sigma_c}] \cdot \left(\Phi(\omega) - \frac{2}{\omega+1}\right)$
ΔG_2	$r_b \cdot \frac{2}{3} \cdot \left(\Phi(\omega)[1/m_{\Sigma_b} + 1/m_{\Sigma_c}] - \frac{2}{1+\omega} \cdot 1/m_{\Sigma_b}\right)$
ΔG_3	$-r_b \cdot \frac{2}{3} \cdot \left(\Phi(\omega)[1/m_{\Sigma_b} + 1/m_{\Sigma_c}] - \frac{2}{1+\omega} \cdot 1/m_{\Sigma_c}\right)$

Table 7

$1/m_Q$-Correction to Form Factors of $\Sigma_b \to \Sigma_c^*$ Decay

Form Factors	QCM
ΔN_1	$r_b \cdot \frac{1}{\sqrt{3}} \cdot \left(\Phi(\omega) - \frac{2}{1+\omega}\right) \cdot [1/m_{\Sigma_b} + 1/m_{\Sigma_c^*}]$
ΔN_2	$r_b \cdot \frac{8}{\sqrt{3}(1+\omega)} \cdot 1/m_{\Sigma_c^*}$
ΔN_3	$-r_b \cdot \frac{2}{\sqrt{3}} \cdot \left(\Phi(\omega) - \frac{2}{1+\omega}\right) \cdot [1/m_{\Sigma_b} + 1/m_{\Sigma_c^*}]$
ΔN_4	$r_b \cdot \frac{2}{\sqrt{3}} \cdot \left(\Phi(\omega)\omega - \frac{6\omega+4}{3(\omega+1)}\right) \cdot [1/m_{\Sigma_b} + 1/m_{\Sigma_c^*}]$
ΔK_1	$r_b \cdot \frac{1}{\sqrt{3}} \cdot \left(\Phi(\omega) - \frac{2}{1+\omega}\right) \cdot [1/m_{\Sigma_b} + 1/m_{\Sigma_c^*}]$
ΔK_2	$r_b \cdot \frac{8}{\sqrt{3}(1+\omega)} \cdot 1/m_{\Sigma_c^*}$
ΔK_3	$-r_b \cdot \frac{2}{\sqrt{3}} \cdot \left(\Phi(\omega) - \frac{2}{1+\omega}\right) \cdot [1/m_{\Sigma_b} + 1/m_{\Sigma_c^*}]$
ΔK_4	$r_b \cdot \frac{2\omega}{\sqrt{3}} \cdot \left(\Phi(\omega) - \frac{2}{1+\omega}\right) \cdot [1/m_{\Sigma_b} + 1/m_{\Sigma_c^*}]$

Table 8

$1/m_Q$-Correction to Form Factors of $\Sigma_b^* \to \Sigma_c^*$ Decay

Form Factors	QCM
ΔP_1	$r_b \cdot \left(\Phi(\omega) - \frac{6\omega+4}{3(\omega+1)}\right) \cdot [1/m_{\Sigma_b^*} + 1/m_{\Sigma_c^*}]$
ΔP_2	$r_b \cdot \frac{4}{3(1+\omega)} \cdot 1/m_{\Sigma_c^*}$
ΔP_3	$r_b \cdot \frac{4}{3(1+\omega)} \cdot 1/m_{\Sigma_b^*}$
ΔP_4	$-r_b \cdot \left(\Phi(\omega) - \frac{2}{\omega+1}\right) \cdot [1/m_{\Sigma_b^*} + 1/m_{\Sigma_c^*}]$
$\Delta P_i (i = 5, 6)$	0
ΔP_7	$-r_b \cdot \frac{8}{3(1+\omega)} \cdot 1/m_{\Sigma_c^*}$
ΔP_8	$-r_b \cdot \frac{8}{3(1+\omega)} \cdot 1/m_{\Sigma_b^*}$
ΔQ_1	$-r_b \cdot \left(\Phi(\omega) - \frac{2}{\omega+1}\right) \cdot [1/m_{\Sigma_b^*} + 1/m_{\Sigma_c^*}]$
ΔQ_2	$-r_b \cdot \frac{4}{3(1+\omega)} \cdot 1/m_{\Sigma_c^*}$
ΔQ_3	$r_b \cdot \frac{4}{3(1+\omega)} \cdot 1/m_{\Sigma_b^*}$
ΔQ_4	$r_b \cdot \left(\Phi(\omega) - \frac{2}{\omega+1}\right) \cdot [1/m_{\Sigma_b^*} + 1/m_{\Sigma_c^*}]$
$\Delta Q_i (i = 5, ...8)$	0

for $m_b = 4.7$ GeV and $r = \frac{m_c}{m_b} = 0.32$.

Also we can show that the results of Free Quark Model are reproduced in quark models (SQM [2] and QCM) if the form factors $F_\Lambda(\omega) = \Phi(\omega)$, $F_L(\omega) = \Phi(\omega)$ and $F_T = \omega\Phi(\omega)$ are supposed to be equal to unity. We have

$$\Lambda^\mu(\Lambda_b \to \Lambda_c) = O^\mu,$$
$$\Lambda^\mu(\Sigma_b \to \Sigma_c) = -\frac{1}{3} \cdot \left[O^\mu + \frac{2}{\omega+1}(v^\mu + v'^\mu)\right],$$
$$\Lambda^{\mu\alpha}(\Sigma_b \to \Sigma_c^*) = \frac{2}{\sqrt{3}} \cdot \left[\frac{\gamma^\mu v^\alpha}{\omega+1} + g^{\mu\alpha}\right] \cdot (1+\gamma^5),$$
$$\Lambda^{\mu\alpha\beta}(\Sigma_b^* \to \Sigma_c^*) = \left[g^{\alpha\beta} - \frac{v^\alpha v'^\beta}{\omega+1}\right] \cdot O^\mu.$$

Then

$$\Gamma(b \to c) \equiv \Gamma(\Lambda_b \to \Lambda_c) \equiv \Gamma(\Sigma_b \to \Sigma_c) + \Gamma(\Sigma_b \to \Sigma_c^*) \equiv \Gamma(\Sigma_b^* \to \Sigma_c) + \Gamma(\Sigma_b^* \to \Sigma_c^*) .$$

This is exactly the result of the quark-hadron duality.

Table 9

Decay Rates of Semileptonic Heavy-Baryon Decays

Process	Decay Rates Γ, $10^{10} sec^{-1}$	
	QCM	SQM[12]
$\Lambda_b \to \Lambda_c$	10	5.24
$\Sigma_b \to \Sigma_c$	5.13	1.42
$\Sigma_b \to \Sigma_c^*$	8.66	4.40
$\Sigma_b^* \to \Sigma_c$	4.33	
$\Sigma_b^* \to \Sigma_c^*$	9.50	

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Иванов М.А. и др. E2-92-437
Форм-факторы полулептонных
распадов тяжелых барионов

Дается полный анализ полулептонных распадов тяжелых барионов с квантовыми числами $J^P=1/2^+$ и $J^P=3/2^+$ с изменением флэйвора $b \to c$ в рамках модели конфайнмированных кварков. Получены выражения для форм-факторов этих распадов. В пределе бесконечно больших масс $m_Q \to \infty$ вычислены асимптотики форм-факторов и $1/m_Q$-поправки к ним. Показано, что при равенстве скоростей начального и конечного барионов ($v^\mu = v'^\mu$) имеет место теорема Адемолло-Гатто.

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Weak Form Factors of Beauty Baryons

We report full analysis of semileptonic decays of beauty baryons with $J^P=1/2^+$ and $J^P=3/2^+$ into charmed ones within the Quark Confinement Model. Weak form factors and decay rates are calculated. Also the heavy quark limit $m_Q \to \infty$ (Isgur-Wise symmetry) is examined. We compute weak heavy-baryon form factors in the Isgur-Wise limit and $1/m_Q$-corrections to them. The Ademollo-Gatto theorem is spin-flavour symmetry of heavy quarks is checked.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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