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THE FORM FACTOR OF THE PROCESS $\gamma^*\gamma^*\to\pi^0$ FOR SMALL VIRTUALITY OF ONE OF THE PHOTONS AND QCD SUM RULES (I): THE STRUCTURE OF THE INFRARED SINGULARITIES

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1 Introductory remarks

In the past ten years the processes $\gamma^*\gamma^* \to$hadrons ($\gamma^*\gamma^*$ subprocess of $ll \to ll$hadrons) have been of increasing interest among experimentalists [1]. For any luminosity of the e^+e^- beams, the best situation from the point of view of producing hadrons occurs when one of the photons (or both) is near its mass shell ($q_1^2 \to 0$)[2]. Let us mention the role of the future experiments with real photon obtained by converting a $e^+(e^-)$ beam into γ-beam without any lose of the energy or luminosity [3].

As the "quark content" of the photon is well known, the theoretical analysis of two-photon scattering is clearer than hadronic scattering. We calculate the form factor (FF) $F_{\gamma^*\gamma^*\to\pi^0}(q_1^2, q_2^2)$ of the transition $\gamma^*\gamma^* \to \pi^0$ in particular, in the kinematic region when $q_1^2 \to 0$. Theoretically the process $\gamma^*\gamma^* \to \pi^0$ is a playground for comparing different approaches.

In the framework of perturbative QCD the FF $F_{\gamma^*\gamma^*\to\pi^0}$ was first calculated for an asymptotically high virtuality of one of the photons in [4]. The case of asymptotically high and equal virtualities of both the photons ($Q^2 = -q_1^2 = -q_2^2$), was investigated by Voloshin [5] where the next-to-leading power correction was calculated as well. As the virtualities of the photons decrease, the nonperturbative QCD-effects play an increasingly important role. These effects may be taken into account by the QCD sum rules (SR) [6]. In Ref.[7] the FF $F_{\gamma^*\gamma^*\to\pi^0}(Q^2)$, obtained by the QCD SR method and by the perturbative approach were compared and the two approaches were shown to be mutually consistent in the asymptotic region: $Q^2 \geq 2$ GeV^2.

In the asymmetric case ($q_1^2 \neq q_2^2$) the total $O(\alpha_s)$ corrections in the asymptotic limit for $F_{\gamma^*\gamma^*\to\pi^0}(q_1^2, q_2^2)$ were obtained in Ref.[8][1]. In Ref.[10] the SR for FF were obtained for the range of moderate virtualities of the photons ($-q_1^2, -q_2^2 \geq 1$ GeV^2) taking into account the nonleading quark vacuum average (VA) of dimension 8. Only the leading in $1/Q^2$ terms, where $Q^2 = -(q_1^2 + q_2^2)/2$, were considered there. In particular, it was established that such SR work in the restricted kinematic region $\omega = (q_1^2 - q_2^2)/(q_1^2 + q_2^2) \leq 0.5$.

The authors of Ref.[11], extending their pioneering papers [12], have built a nonlocal version of the SR for FF $F_{\gamma^*\gamma^*\to\pi^0}$, which is a generalization of the standard SR for a meson FF [13, 14] in the case of essentially nonsymmetric kinematics[2] $Q_1^2 \gg Q_2^2 \geq 1$ GeV^2. Introducing nonlocal condensates corresponds to an effective summation of the whole series of power corrections (not limiting ourself to the VA of lowest dimension $\langle 0|\bar\psi\psi|0\rangle$ and $\langle 0|G_{\mu\nu}G_{\mu\nu}|0\rangle$). Thus it turns out to expand the SR to nearly all kinematical region of ω ($0 \leq \omega \leq 0,95$). For $\omega \geq 0.8$, the FF is most sensitive to the value of the effective virtuality of the "vacuum quarks" $\sigma^2 \sim \lambda_q^2 = (0,3 \div 0,6)$ GeV^2 and an experiment being performed in this kinematical region, could serve as a measurement of the nonlocality parameter λ_q^2.

In the present paper we consider the process $\gamma^*\gamma^* \to \pi^0$ in another essentially nonsymmetric kinematics: $Q_1^2 \ll Q_2^2 \geq 1$ GeV^2. In this region one needs to modify the SR, performing an additional factorization procedure to separate the contributions of large and small distances [17, 18, 19]. We shall follow the approach of analyzing the electromagnetic

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[1] see also Ref.[9],in which however, the evolution of the pion wave function was not taken into account
[2] see also the recent work [16]

FF of $\pi-$ and $K-$mesons in the low-Q^2 region developed in [19, 20].

In Sec.2 the main features of the method are briefly described. In Sec.3 the OPE for the three-point correlation function is calculated for the case of moderate virtualities. The corresponding SR is presented and its applicability is considered. For the case of essentially nonsymmetric kinematics, the OPE needs to be modified. In Sec.4 the most important steps in the calculations are demonstrated for the simple scalar example. For the realistic case of the process $\gamma^*\gamma^* \to \pi^0$ the structure of the infrared singularities is presented. The fully modified OPE and the corresponding SR will be presented elsewhere.

2 The Method

To write down the SR for the FF, we start, as usual, from the three-point correlation function (see [7]):

$$\mathcal{F}_{\alpha\mu\nu}(q_1,q_2) = i\int d^4x\, d^4y\; e^{-iq_1x}\, e^{-iq_2y} \langle 0|T\left\{J_\mu(x)\, J_\nu(y)\, j^5_\alpha(0)\right\}|0\rangle, \tag{2.1}$$

being considered at euclidean virtualities $q_1^2, q_2^2, p^2=(q_1+q_2)^2$. Here J_μ is the electromagnetic current of the two light quarks and j^5_α is the axial current, its projections onto the pion state $|\pi,\vec{p}\rangle$ being proportional to the pion decay constant f_π:

$$J_\mu = e\left(\frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d\right), j^5_\alpha = \frac{1}{\sqrt{2}}(\bar{u}\gamma_5\gamma_\alpha u - \bar{d}\gamma_5\gamma_\alpha d), \langle 0|j^5_\alpha(0)|\pi,\vec{p}\rangle = -ip_\alpha f_\pi \tag{2.2}$$

According to the general approach of factorization the small and large distances, based on the Feinman diagram asymptotic analysis [21, 22], the leading contribution in the correlator (see fig.1), which behaves like a power of $1/q_2^2$, $1/p^2$ ($p^2 \sim q_2^2 \sim -1$ GeV^2) comes from two different regions of integration: either from the SD(I)-region where all three currents are at small distances (i.e. the intervals $x^2, y^2, (x-y)^2$ are small) or from the SD(II)-region so that the electromagnetic current $J_\mu(x)$ is placed at large distances (i.e.y^2 is small, but $x^2, (x-y)^2$ are large)

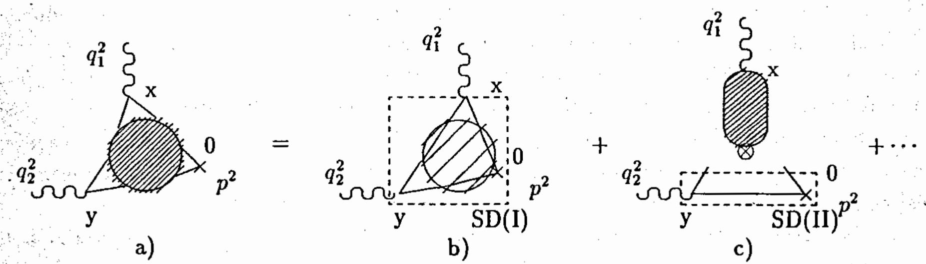


Fig. 1

Here fig.1a) presents the full correlator (2.1), whereas fig.1b,c) corresponds to the leading power contributions. In the fig.1c) the large distances are presented by a generalized

multiplier — the two-point correlator of the electromagnetic current $J_\mu(x)$ and some composite operator of quark and gluon fields with "n" derivatives, denoted as $\otimes$, the sum over "n" is undertaken. Let us emphasize that the twist, not the dimension of these operators, counts the value of the contribution for such two-point correlator. This proposes to take into account the operators of lowest twists for every "n". The corresponding, so called bilocal object is not calculable in the perturbation theory. Nevertheless, one can write down a dispersion relation for it (with a subtraction, strictly speaking) so that the parameters of the corresponding spectral density can be determined from a suitable "internal" borel SR (see sec. 4.1 for details).

Such a factorization of the small distances one needs to make for the triangle graph (see fig.2a,b,c) and also for all other diagrams (see fig.2d,e,f) which correspond to the condensate power corrections.

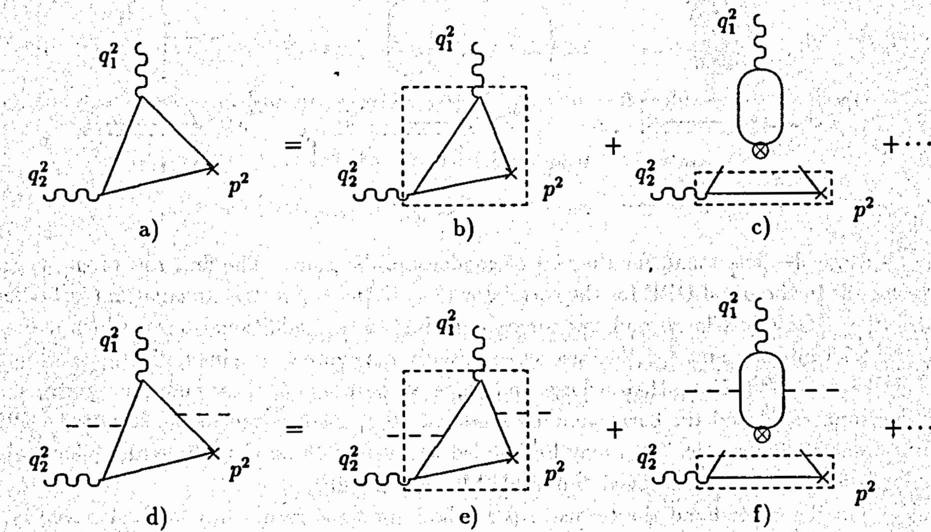


Fig. 2

Due to asymptotic freedom one may write down for the SD(I) contribution the following symbolical identity (fig.3):

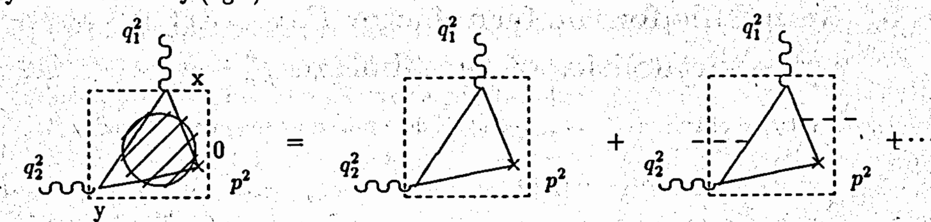


Fig. 3

Finally we obtain (fig.4):

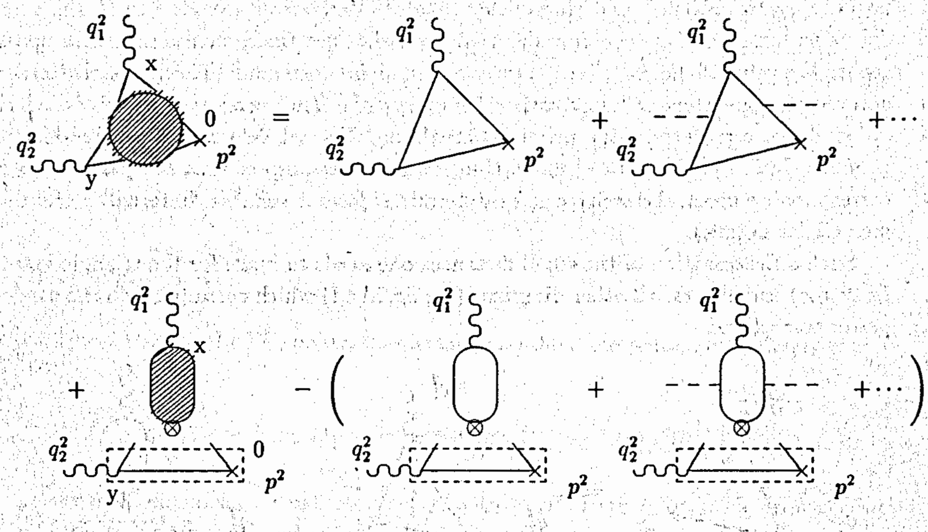


Fig. 4

Here the dots stand for the rest of condensate diagrams. The first row of (fig.4) corresponds to the usual OPE for the correlator (2.1) in the symmetric kinematics ($|q_1^2| \sim |q_2^2| \sim |p^2| \sim 1$ GeV^2). The second row corresponds just to that additional terms which one needs to take into account for the case of essentially nonsymmetric kinematics ($|q_1^2| \ll |q_2^2| \sim |p^2| \sim 1$ GeV^2). Note, that at large and moderate values of q_1^2 these additional terms should be suppressed and the corresponding modified OPE is converted to the standard OPE for symmetric kinematic. This may be fulfilled because at these virtualities the bilocal object satisfies the above mentioned “internal” SR (see [19, 20]).

On the other hand the terms in parenthesis into the second row of fig.4 have just the same behavior at $q_1^2 \to 0$ as the corresponding terms of the first row of fig.4 and therefore the full expression is regular at this kinematical limit. As it is shown in Sec.4, the singular in q_1^2 terms (so called massive or infrared singularities) appear from the lowest twists operators.

3 Sum Rule for the form factor $F_{\gamma^*\gamma^*\to\pi^0}(q_1^2, q_2^2)$ at moderate virtualities of the photons $q_1^2 \sim q_2^2 \sim -1$ GeV^2

The form factor of interest $F_{\gamma^*\gamma^*\to\pi^0}(q_1^2,q_2^2)$ is determined by the matrix element:

$$\int d^4x\, e^{-iq_1x}\langle \pi, \vec{p}\,|T\{J_\mu(x)J_\nu(0)\}|0\rangle = \epsilon_{\mu\nu q_1 q_2}\, F_{\gamma^*\gamma^*\to\pi^0}(q_1^2,q_2^2) \qquad (3.1)$$

Let us consider the Lorentz structure for the three-point correlator (2.1) to extract the tensor structure related to $F_{\gamma^*\gamma^*\to\pi^0}$. Using Lorentz invariance and Bose symmetry of the photons

the pseudotensor amplitude $\mathcal{F}_{\alpha\mu\nu}$ takes the form:

$$\begin{aligned}\mathcal{F}_{\alpha\mu\nu}(q_1,q_2) &= p_\alpha\epsilon_{\mu\nu q_1q_2}\mathcal{F}_1(p^2,q_1^2,q_2^2) + q_\alpha\epsilon_{\mu\nu q_1q_2}\mathcal{A}_1(p^2,q_1^2,q_2^2)\\ &+ [\epsilon_{\alpha\mu q_1q_2}q_{1\nu} - \epsilon_{\alpha\nu q_1q_2}q_{2\mu}]\mathcal{F}_2(p^2,q_1^2,q_2^2) + [\epsilon_{\alpha\mu q_1q_2}q_{2\nu} - \epsilon_{\alpha\nu q_1q_2}q_{1\mu}]\mathcal{F}_3(p^2,q_1^2,q_2^2)\\ &+ [\epsilon_{\alpha\mu q_1q_2}q_{1\nu} + \epsilon_{\alpha\nu q_1q_2}q_{2\mu}]\mathcal{A}_2(p^2,q_1^2,q_2^2) + [\epsilon_{\alpha\mu q_1q_2}q_{2\nu} + \epsilon_{\alpha\nu q_1q_2}q_{1\mu}]\mathcal{A}_3(p^2,q_1^2,q_2^2)\\ &+ \epsilon_{\alpha\mu\nu q}\left[\frac{p^2}{2}\mathcal{F}_4 + \frac{q^2}{2}\mathcal{F}_5 + (p.q)\mathcal{A}_6\right] + \epsilon_{\alpha\mu\nu p}\left[\frac{p^2}{2}\mathcal{A}_4 + \frac{q^2}{2}\mathcal{A}_5 + (p.q)\mathcal{F}_6\right]\end{aligned}$$

where

$$p = q_1 + q_2, \quad q = q_1 - q_2, \quad \mathcal{F}_i(p^2,q_1^2,q_2^2) = \mathcal{F}_i(p^2,q_2^2,q_1^2)$$
$$\epsilon_{\mu\nu q_1q_2} \equiv \epsilon_{\mu\nu\rho\sigma}{q_1}^\rho{q_2}^\sigma \quad \text{etc.}, \quad \mathcal{A}_i(p^2,q_1^2,q_2^2) = -\mathcal{A}_i(p^2,q_2^2,q_1^2)$$

Taking into account the nonexistence of rank 5 tensor in 4-dimension space-time:

$$\epsilon_{\alpha\mu\nu\gamma}\, g_{\delta\epsilon} + \epsilon_{\mu\nu\gamma\delta}\, g_{\alpha\epsilon} + \epsilon_{\nu\gamma\delta\alpha}\, g_{\mu\epsilon} + \epsilon_{\gamma\delta\alpha\mu}\, g_{\nu\epsilon} + \epsilon_{\delta\alpha\mu\nu}\, g_{\gamma\epsilon} = 0 \qquad (3.2)$$

and the gauge invariance conditions $q_1^\mu\mathcal{F}_{\alpha\mu\nu} = q_2^\nu\mathcal{F}_{\alpha\mu\nu} = 0$ we get finally:

$$\begin{aligned}\mathcal{F}_{\alpha\mu\nu}(q_1,q_2) &= \epsilon_{\mu\nu q_1q_2}[p_\alpha F_1 + q_\alpha A_1] + [q_{2\nu}\epsilon_{\alpha\mu q_1q_2} - q_{1\mu}\epsilon_{\alpha\nu q_1q_2}]F_2\\ &+ [q_{2\nu}\epsilon_{\alpha\mu q_1q_2} + q_{1\mu}\epsilon_{\alpha\nu q_1q_2}]A_2 + \epsilon_{\alpha\mu\nu q}\left[\frac{p^2+q^2}{4}F_2 - \frac{(p.q)}{2}A_2\right]\\ &+ \epsilon_{\alpha\mu\nu p}\left[-\frac{(p.q)}{2}F_2 + \frac{p^2+q^2}{4}A_2\right]\end{aligned} \qquad (3.3)$$

It is clear that on the photon mass shell the contributions come only from the structure $p_\alpha\epsilon_{\mu\nu q_1q_2}F_1(0,0,p^2)$. Therefore, and bearing in mind the expression (3.1), we shell study throughout this paper the invariant amplitude $F_1(q_1^2,q_2^2,p^2)$ (see [7]).

For F_1 we have a dispersion relation:

$$F_1(p^2,q_1^2,q_2^2) = \frac{1}{\pi}\int_0^\infty \frac{\rho^{ph}(\sigma,q_1^2,q_2^2)}{\sigma - p^2}\,d\sigma + \text{subtractions} \qquad (3.4)$$

where for the physical spectral density one may write the standard phenomenological anzatz, including π^0-meson and higher resonance state contributions (the continuum):

$$\rho^{ph}(\sigma,q_1^2,q_2^2) = \pi f_\pi\delta(\sigma)F_{\gamma^*\gamma^*\to\pi^0}(q_1^2,q_2^2) + \theta(\sigma-\sigma_o)\rho^{pt}(\sigma,q_1^2,q_2^2) \qquad (3.5)$$

In the chiral limit we neglect the pion mass and the masses of light quarks as well. The parameter σ_o stands for the threshold of the continuum with A_1-meson being included there.

Applying the SVZ-Borel transformation results in the SR:

$$\hat{B}(-p^2\to M^2)F_1(q_1^2,q_2^2,p^2) \equiv \Phi_1(q_1^2,q_2^2,M^2) = \frac{1}{\pi M^2}\int_0^\infty d\sigma e^{-\sigma/M^2}\rho^{ph}(\sigma,q_1^2,q_2^2) \qquad (3.6)$$

In the Euclidean region $q_1^2,q_2^2,p^2 < 0$, where the virtualities are large enough, due to asymptotic freedom, the main contribution comes from the perturbative triangle graph

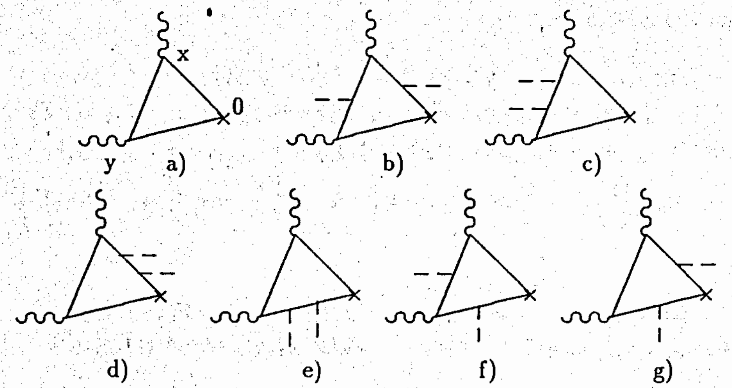


Fig. 5

(fig.5a). The perturbative loop make a contribution to most of the tensor structures of eq. (3.3). Extracting F_1 one gets:

$$F_1^{pt}(q_1^2,q_2^2,p^2) = \frac{\alpha_{e.m.}2\sqrt{2}}{\pi}\int_0^1 dx_1dx_2dx_3\ \delta(1-\sum_{i=1}^{3})\ \frac{x_1x_2}{[-q_1^2x_1x_3 - q_2^2x_2x_3 - p^2x_1x_2]} \quad (3.7)$$

For the borelized amplitude after simple calculation we obtain:

$$\Phi_1^{pt}(q_1^2,q_2^2,M^2) = \frac{1}{\pi M^2}\int_0^\infty d\sigma e^{-\sigma/M^2}\rho^{pt}(\sigma,q_1^2,q_2^2) \quad (3.8)$$

where the perturbative spectral density reads:

$$\rho^{pt}(\sigma,q_1^2,q_2^2) = 2\sqrt{2}\alpha_{e.m.}\int_0^1 dx\frac{x\bar{x}Q^4(1+\omega(x-\bar{x}))^2}{[\sigma x\bar{x}+Q^2(1+\omega(x-\bar{x}))]^3} \quad (3.9)$$

With decreasing the virtualities two type of corrections come into play. As was argued previously [6], for a system of light quarks the power corrections to Φ_1^{pt} ($\sim \langle 0|G^a_{\mu\nu}G^a_{\mu\nu}|0\rangle, \langle 0|\bar{\psi}\psi|0\rangle^2$, etc.), are of the most importance, whereas the perturbative ones may by neglected. The procedure to calculate the power corrections is well known [13, 14], but nevertheless it is reasonable to make it for the sake of subsequent analysis.

The contributions proportional to $\langle 0|G^a_{\mu\nu}G^a_{\mu\nu}|0\rangle$ condensate are depicted in fig.5b)–g) incorporating the fact that a quark propagates not in empty space, but interacting with the background field of nonperturbative gluon fluctuations. The most straightforward way is to take quark propagator in coordinate representation [23] (the Fock-Schwinger gauge for the background field $x_\mu A_\mu(x)=0$ is implied):

$$\hat{S}(x,y) = \frac{\hat{r}}{2\pi^2r^4} - \frac{1}{8\pi^2}\frac{r_\alpha}{r^2}\widetilde{G}_{\alpha\varphi}(0)\gamma_\varphi\gamma_5 + \left\{\frac{i}{4\pi^2}\frac{\hat{r}}{r^4}y_\rho x_\mu G_{\rho\mu}(0) - \frac{1}{192\pi^2}\frac{\hat{r}}{r^4}(x^2y^2-(xy)^2)G_{\varphi\chi}(0)G_{\varphi\chi}(0)\right\}. \quad (3.10)$$

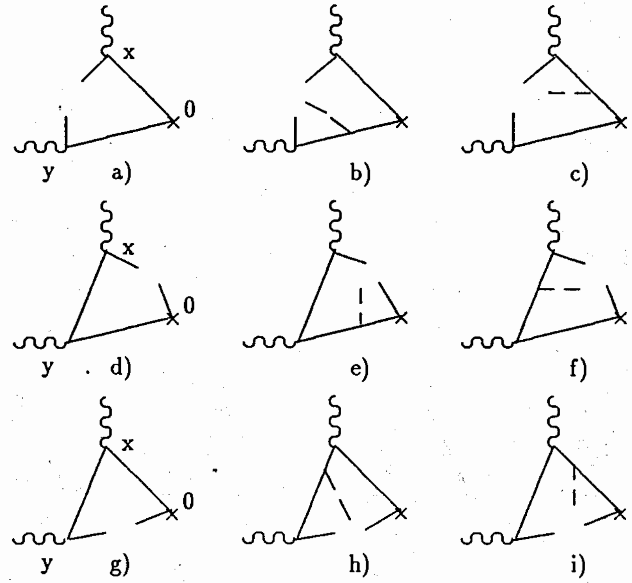


Fig. 6

where $r = x - y$ and $\widetilde{G}_{\alpha\varphi} = \frac{1}{2}\epsilon_{\alpha\varphi\sigma\rho}G_{\sigma\rho}$. The diagrams of fig.5b–g) obviously correspond to the different parts of the expression (3.10). From this representation it is straightforward to observe that the contributions d) and e) vanish due to the choice of the coordinate reference point. It should be noted that the diagrams b),c),f),g) are most easily to calculate by a direct transformation to α- representation with subsequent Fourier transform. Extracting the contribution to the relevant tensor structure and performing the Borel transform of the corresponding form factor $F_1^{\langle GG\rangle}(q_1^2,q_2^2,p^2)$, we get for the sum of the above mentioned diagrams:

$$\Phi_1^{\langle GG\rangle}(q_1^2,q_2^2,M^2) = \frac{\sqrt{2}\alpha_{e.m.}\pi}{9}\frac{\alpha_s}{\pi}\langle 0|GG|0\rangle\left(\frac{1}{Q^2M^4} - \frac{1}{Q^4M^2}\right)\frac{1}{1-\omega^2}. \quad (3.11)$$

Considering the contribution to quark condensate term, it should be mentioned that in the chiral limit they would start from the terms proportional to $\langle 0|\bar{\psi}\Gamma\psi\bar{\psi}\Gamma\psi|0\rangle$. For these VA it is supposed the dominance role of the vacuum intermediate state from the full set of hadronic states (see [6]). Two types of diagrams contribute to the term proportional to $\langle 0|\bar{\psi}\psi|0\rangle^2$. The first type diagrams are depicted in fig.6a–i) — the so called diagrams with a soft gluon. The calculation proceeds further in a standard way[13, 14].

The second type diagrams depicted in fig.7a–r) correspond to the situation with a hard gluon exchange. Only the diagrams a)–d) contribute to the form factor F_1. Summing the contributions to the form factor $F_1^{\langle\bar{q}q\rangle}$ one gets for the borelized amplitude:

$$\Phi_1^{\langle\bar{q}q\rangle} = \frac{\sqrt{2}\alpha_{e.m.}}{\pi M^2}\frac{64}{243}\pi^3\alpha_s\langle 0|\bar{\psi}\psi|0\rangle^2\left(\frac{11-3\omega^2}{Q^2M^4}+\frac{18}{Q^6}\right)\frac{1}{(1-\omega^2)^2} \quad (3.12)$$

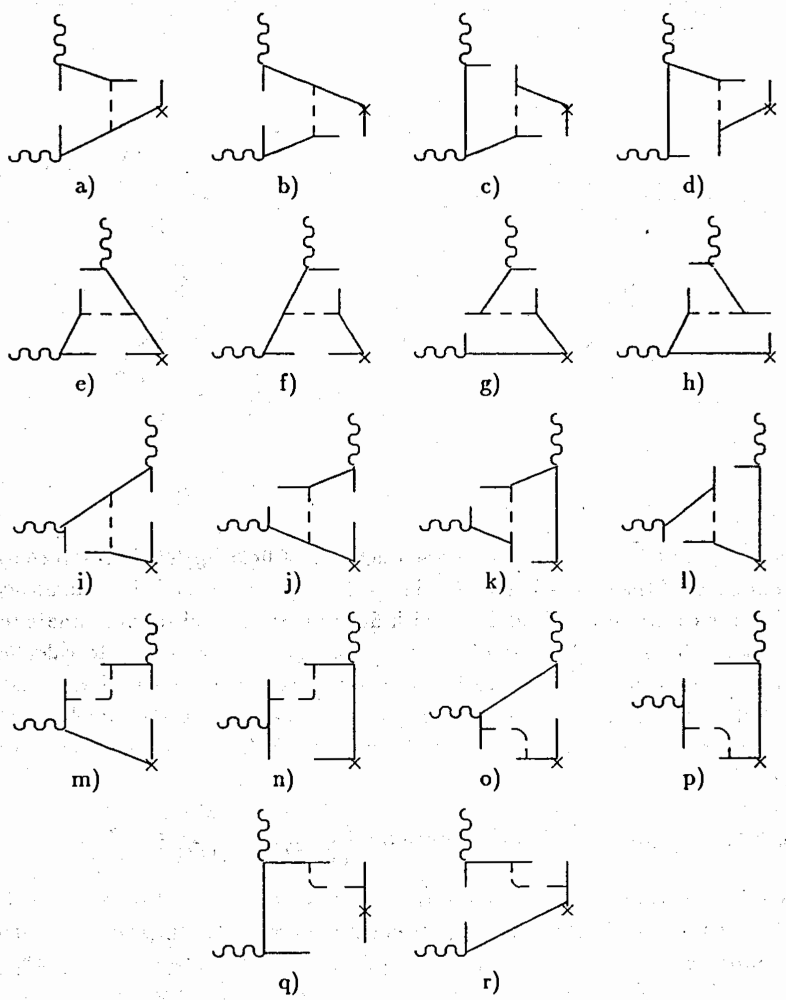


Fig. 7

Combining now Eqs.(3.6),(3.8),(3.11) and (3.12) we find the SR for the FF for moderate virtualities of the photons:

$$
\begin{aligned}
F_{\gamma^*\gamma^*\to\pi^0}(q_1^2,q_2^2) &= \frac{\sqrt{2}\alpha_{e.m.}}{\pi f_\pi}\left\{2\int_0^{\sigma_o} d\sigma e^{-\sigma/M^2}\int_0^1 dx \frac{x\bar{x}Q^4(1+\omega(x-\bar{x}))^2}{[\sigma x\bar{x}+Q^2(1+\omega(x-\bar{x}))]^3}\right.\\
&+ \frac{\pi^2}{9}\frac{\alpha_s}{\pi}\langle 0|GG|0\rangle\left(\frac{1}{Q^2M^2}-\frac{1}{Q^4}\right)\frac{1}{1-\omega^2}\\
&+ \left.\frac{64}{243}\pi^3\alpha_s\langle 0|\bar{\psi}\psi|0\rangle^2\left(\frac{11-3\omega^2}{Q^2M^4}+\frac{18}{Q^6}\right)\frac{1}{(1-\omega^2)^2}\right\} \qquad (3.13)
\end{aligned}
$$

In the symmetric kinematics $q_1^2 = q_2^2$ this SR coincides with previously calculated [7]. It should be mentioned that in Ref.[10] only the leading $1/Q^2$ terms have been calculated.

In fig.3 we give $F_{\gamma^*\gamma^*\to\pi^0}(Q^2,\omega)$ for $Q^2 = 2$ GeV^2 normalized by the value $F_{\gamma^*\gamma^*\to\pi^0}(0,0) = \sqrt{2}\alpha_{e.m.}/\pi f_\pi$ [24]. The sum rule method is applicable for all $0. \leq \omega \leq 0.7$. The scale σ_o, obtained by an explicit fitting procedure, varies from $\sigma_o = 0.75$ GeV^2 to $\sigma_o = 1.2$ GeV^2. For $\omega \geq 0.7$, the extrapolation is already invalid here. For $\omega \geq 0.8$, SR become nonstable. The increasing of the σ_o is not unexpected because with increasing of ω the relative value of the power corrections in (3.13) grows up and the scale σ_o play role of an effective duality interval. However, a universal, ω-independent value for σ_o is more natural [14, 15]. To make σ_o stable in the kinematics $Q_1^2 \gg Q_2^2 \geq 1$ GeV^2 one should take into account the whole series of higher power corrections [12, 11, 16] introducing nonlocal condensates. In the other essentially nonsymmetric kinematic one should use a modified form of the OPE [19] as well.

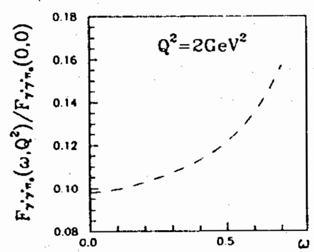


Fig. 8. *The normalized form factor*

4 The structure of the infrared singularities. A modified OPE in the case of essentially nonsymmetric kinematics: $|q_1^2| \ll |q_2^2| \sim 1$ GeV^2

It is more instructive to return to "old" variables q_1^2 and q_2^2 instead of ω and Q^2 in the SR (3.13), to investigate this kinematics. The condensate terms will contain singularities at $q_1^2 \to 0$ like $1/q_1^2$ or $1/q_1^4$. The perturbative expression contains nonanalytic in q_1^2 contribution as well. To make this clearly it is sufficient to substitute in (3.7):

$$x_1 = (1-x)\lambda \equiv \bar{x}\lambda, \; x_3 = x\lambda, \; x_2 = 1 - x_1 - x_3$$

and after borelization, to perform the integration over λ (see Sec. 4.1).

For $\Phi_{1,pt}(q^2, Q^2, M^2)$ we obtain an expression suitable to analyze the limit $q^2 \to 0$:

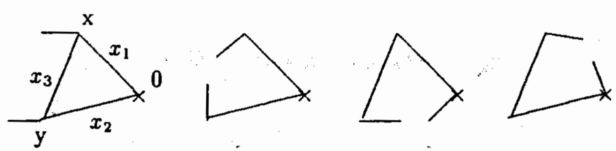
$$\Phi_{1,pt}(q^2,Q^2,M^2) = \frac{\alpha_{e.m.} 2\sqrt{2}}{\pi} \int_0^1 dx e^{-Q^2 x/M^2 \bar{x}} \frac{1}{2M^2} \times \left\{ (1 + \frac{q^2 x}{M^2} e^{q^2 x/M^2}) + e^{q^2 x/M^2} \ln \frac{q^2 x}{M^2} \left[2\frac{q^2 x}{M^2} + \frac{q^4 x^2}{M^4} \right] - \sum_{n=1}^{\infty} \left(\frac{q^2 x}{M^2} \right)^n \frac{\psi(n)(n+1)}{(n-1)!} \right\} \quad (4.1)$$

where $q^2 \equiv -q_1^2$, $Q^2 \equiv -q_2^2$, $\psi(z) \equiv \Gamma'(z)/\Gamma(z)$, $\Gamma(z) -$ the Euler Gamma function.

Nonanalyticies of logarithmic type $\sim q^2 \ln q^2 \;\; \sim q^4 \ln q^2$ appear in the representation (4.1), which is connected with the possibility to create a massless $q\bar{q}$-pare by a photon with a virtuality q^2. Terms proportional to $\ln q^2$ signal the appearing of the so-called mass singularities [25] and therefore one needs to perform the factorization of the small and large distance contribution more precisely in accordance with the method described in Sec.2 (see fig.1,2).

4.1 The Scalar example $g\phi^3_{(4)}$

To make more clear the origin of the singularities and trace a program to remove it, consider a simple scalar example. In this example most of the elements of the real problem of the process $\gamma^*\gamma^* \to \pi^\circ$ will be considered as well.



a) b) c) d)

Fig. 9

Consider the $g\phi^3_{(4)}$-theory for definiteness. The scalar three-point correlator analogous to $\mathcal{F}_{\alpha\mu\nu}(q_1, q_2)$ reads:

$$\mathcal{F}(q_1,q_2) = \int d^4x\, d^4y\; e^{-iq_1 x} e^{-iq_2 y} \langle 0|T\{j(x)j(y)j(0)\}|0\rangle, \quad (4.2)$$

where $j(x) =: \phi(x)\phi(x) :$. The perturbative contribution and some of the power corrections are depicted in fig.9. As to the other power correction diagrams one not need to consider them for better understanding of the problem.

Calculating the perturbative contribution (fig.9a) in α-representation one gets[3]:

$$\Phi_{(a)} \equiv \hat{B}(-p^2 \to M^2)\mathcal{F}_{(a)} \propto \int_0^1 dx_1 dx_3 \theta(1 > x_1 + x_3) \frac{1}{x_2 x_3} e^{-(q^2 x_1 x_3 + Q^2 x_2 x_3)/x_1 x_2 M^2}$$
$$= \int_0^1 \frac{d\lambda dx}{\bar{\lambda} x M^2} e^{-q^2 \bar{x}\lambda/M^2 \bar{\lambda}} e^{-Q^2 \bar{x}/M^2 x} \quad (4.3)$$

If expanding formally the exponent in (4.3) in powers of q^2, the integral over λ for each term of the series diverges. Nevertheless, it is possible to perform the integration directly by using Mellin representation for the exponent:

$$e^{-A} = \frac{1}{2\pi i} \int_{C^+} A^J \Gamma(-J)\, dJ \quad (4.4)$$

where C^+ is the integration contour in the complex plane J placed on the left of all the poles of the integrand. Instead of (4.3) one gets the representation:

$$\Phi_{(a)} = \frac{1}{2\pi^2} \int_0^1 dx e^{-Q^2 \bar{x}/M^2 x} \frac{1}{x M^2} \left\{ -\ln \frac{q^2 \bar{x}}{M^2} e^{q^2 \bar{x}/M^2} + \sum_{n=0}^{\infty} \left(\frac{q^2 \bar{x}}{M^2} \right)^n \frac{\psi(n+1)}{n!} \right\}, \quad (4.5)$$

in which the singular in q^2 term is extracted evidently.

It is very easy to obtain the results for the diagrams of fig.9b,c,d), which simulate the power corrections:

$$\Phi_{(b)} = \hat{B}(-p^2 \to M^2) \left(-\frac{8\langle\phi^2\rangle}{q^2 Q^2} \right) = 0, \quad \Phi_{(c)} = -\frac{8\langle\phi^2\rangle}{Q^2 M^2}, \quad \Phi_{(d)} = -\frac{8\langle\phi^2\rangle}{q^2 M^2} \quad (4.6)$$

Obviously, one needs to perform an additional factorization of the small and large distance contributions for the diagrams of fig.9a,d), as just these diagrams contain singular in q^2 terms.

To get SD(II) contribution for the full amplitude (4.2), let us to extract the propagator $S(y) = i/4\pi^2(y^2 - i0)$ as a coefficient function (see fig.1c):

$$\mathcal{F}^{SD(II)} = \int d^4y\, e^{-iq_2 y} \frac{1}{\pi^2 y^2} \sum_{n=0}^{\infty} \frac{1}{n!} y^{\mu_1} \ldots y^{\mu_n} \times \int d^4x\, e^{-iq_1 x} \langle 0|T\left\{ j(x)\, \phi(0)(\overleftarrow{\partial}_{\mu_1} \ldots \overleftarrow{\partial}_{\mu_n})\phi(0) \right\}|0\rangle \quad (4.7)$$

[3] bearing in mind that all invariants q_1^2, q_2^2, p^2 are negative, we have made a Wick rotation in α-plane

Here the two-point correlator is responsible for the large distance contribution.

It should be noted, that for any "n" one may reexpand the current with derivatives over the set of traceless operators with definite twist [22]:

$$y^{\mu_1}\dots y^{\mu_n}\,(\phi\overleftarrow{\partial}_{\mu_1}\dots\overleftarrow{\partial}_{\mu_n}\phi) = $$
$$= \sum_{l=0}^{[n/2]}\frac{n!(n-2l+1)}{l!(n-l+1)!}\left(\frac{y^2}{4}\right)^l\{y^{\mu_1}\dots y^{\mu_{n-2l}}\}(\phi(\overleftarrow{\partial}^2)^l\{\overleftarrow{\partial}_{\mu_1}\dots\overleftarrow{\partial}_{\mu_{n-2l}}\}\phi) \qquad (4.8)$$

In (4.8) we have introduced the notation $\{\overleftarrow{\partial}_{\mu_1}\dots\overleftarrow{\partial}_{\mu_n}\}$ for a traceless group of indices $\mu_1\dots\mu_n$: $g^{\mu_i\mu_j}\hat{O}_{\{\dots\mu_i\dots\mu_j\dots\}}=0$. Then, obviously, the main contribution in (4.7) at $q^2\to 0$ comes from the lowest twist operators ($t=2$ in our case). The higher twist operators are accompanied by the multipliers $\sim y^2$, $(y^2)^2$, etc., which cancel singularity of the propagator $\sim 1/y^2$ and provide the regular in q^2 terms.

The factorization of the SD(II) regime for the perturbative triangle loop corresponds to the expression (4.7) in which the perturbative contribution for the two-point correlator is taken into account. The main observation here is that, *all, singular in* q^2, *terms* in (4.5) is due to the operators of twist 2. Indeed, considering the two-point correlator in the perturbation theory we find:

$$\Pi_{\{\mu_1\dots\mu_n\}}(q_1) \equiv \int d^4x\,e^{-iq_1x}\,\langle 0|T\left\{j(x)\,\phi(0)\{\overleftarrow{\partial}_{\mu_1}\dots\overleftarrow{\partial}_{\mu_n}\}\phi(0)\right\}|0\rangle$$
$$= (-i)^n\ \{q_{1\mu_1}\dots q_{1\mu_n}\}\ \Pi_n(q_1^2) \qquad (4.9)$$

where

$$\Pi^{pt}_n(q^2) = \frac{i}{8\pi^2}\int_0^1 dx\,x^n\,\ln\frac{q^2x\bar{x}}{\mu^2} \qquad (4.10)$$

Substituting (4.9), (4.10) in (4.7), it is straightforward to sum over "n", bearing in mind that $\{q_{1\mu_1}\dots q_{1\mu_n}\}$ differ from $q_{1\mu_1}\dots q_{1\mu_n}$ in the terms $\sim y^2$. Then we find:

$$\Phi^{SD(II)}_{pt} = \frac{1}{2\pi^2}\int_0^1\frac{dx}{xM^2}e^{-Q^2\bar{x}/M^2x}\left\{-\ln\frac{q^2x\bar{x}}{\mu^2}e^{q^2\bar{x}/M^2}\right\}. \qquad (4.11)$$

This result should be compared with the exact expression (4.5). Here μ^2 is the parameter of UV regularization for the composite operator $\phi\{\overleftarrow{\partial}_{\mu_1}\dots\overleftarrow{\partial}_{\mu_n}\}\phi$. Thus, the coefficient function of the SD(I) regime (see fig.2b), fig.3,4) does not contain nonanalytic in q^2 terms (see [25, 26]) due to the exact cancelation of the singular terms from (4.5) and (4.11). As to the factorization of SD(II) region for the diagram of fig.9d) proportional to $\langle\phi^2\rangle$, the corresponding contribution of the lowest twist operators reproduce the singular term (4.6) $\sim 1/q^2$ at all and thus, this singularity shall not appear in the modified OPE in the case of essentially nonsymmetric kinematics $q^2 \ll Q^2$ as well.

As have been mentioned above, the two-point correlator in (4.7) is responsible for the large distance contribution $\sim 1/|q_1|$ and is not calculable in the perturbation theory.

However, we may write down a dispersion relation for it, substituting a reasonable spectral density of the type: "the lowest resonance contribution"+ "continuum":

$$\Pi_n(q_1^2) = \frac{1}{\pi}\int_0^\infty ds\frac{\delta\Pi_n(s)}{s-q_1^2} + (\text{ subtractions }) \qquad (4.12)$$

where $\delta\Pi_n(s)\equiv(\Pi_n(s+i0)-\Pi_n(s-i0))/2i$.

A similar relation one may write down for the perturbative correlator (4.10) substituting the perturbative spectral density instead of exact one. It should be mentioned that the arbitrariness in the value of μ^2 (4.10),(4.11) corresponds to a finite arbitrariness in the UV subtraction procedure. However, the UV behavior of the exact and perturbative spectral density coincides and it is not necessary to specify the subtraction procedure because the corresponding correlators come into play in a difference in the OPE (see fig.4).

Using the completeness condition for the set of the "hadron" states in (4.12), it is easy to get the following representation:

$$i\Pi_n(q_1,y) \equiv i\Pi_{\{\mu_1\dots\mu_n\}}(q_1)\,y^{\mu_1}\dots y^{\mu_n} =$$
$$= \frac{f_\phi^2(-i)^n(yq_1)^n\langle x^n\rangle}{m_\phi^2-q_1^2}+\frac{1}{\pi}\int_{s_o}^\infty ds\frac{i\delta\Pi_n^{pt}(s)(-i)^n(yq_1)^n}{s-q_1^2} \qquad (4.13)$$
$$+\ (\text{ subtractions })$$

where we define $\langle 0|j(0)|\phi,\vec{p}\rangle = if_\phi$ by analogy with the known matrix element for the π-meson; $\langle x^n\rangle\equiv\int_0^1 dx x^n\phi(x)$; $\phi(x)$ — the twist 2 wave function (WF) of the scalar "meson" ϕ, the moments of which are defined by the matrix element:

$$\langle\phi,\vec{p}|\phi(0)(y\overrightarrow{\partial})^n\phi(0)|0\rangle = i^n(yp)^n(-if_\phi)\int_0^1 dx x^n\phi(x) \qquad (4.14)$$

The continuum contribution of the higher exited states, as usual, is approximated by the perturbative spectral density $\delta\Pi_n^{pt}(s) = -(i/8\pi^2)\int_0^1 dx x^n\pi$ starting from the continuum threshold s_o in this chanel.

As it have been already mentioned in Sec.2, the spectral density parameters of the exact correlator (4.13) (namely: the mass m_ϕ, residue f_ϕ, moments of the WF $\langle x^n\rangle$ and s_o) one may determine from the corresponding sum rule for the moments of the scalar meson WF, which provides the additional terms in the OPE to be suppressed with increasing q^2. So the modified SR will be in accordance with the standard one for the three-point correlator in the case of a symmetric kinematics (see fig.4). Considering these parameters as known one may substitute (4.13) in (4.7) and define the "bilocal" contribution in the r.h.s. of the SR for the three-point correlator. Finally, all additional terms in the OPE (see the second row of fig.4) we may write down in the form:

$$\Delta\Phi \equiv \Phi^{bilocal}-\left(\Phi^{SD(II)}_{(a)}+\Phi^{SD(II)}_{(d)}+\dots\right)$$
$$= \int_0^1\frac{1}{xM^2}e^{-Q^2\bar{x}/M^2x}e^{q^2\bar{x}/M^2}$$
$$\times\left[\frac{4f_\phi^2\phi(x)}{m_\phi^2+q^2}+\frac{1}{2\pi^2}\ln\left(\frac{q^2}{s_o+q^2}\right)+8\frac{\langle\phi^2\rangle}{q^2}(\delta(x)+\delta(\bar{x}))\right] \qquad (4.15)$$

Thus the full expression for the r.h.s. of the SR reads:

$$\begin{aligned}\Phi(q^2,Q^2,M^2) &= \Phi_{(a)}+\Phi_{(b)}+\Phi_{(c)}+\Phi_{(d)}+\Delta\Phi = \\ &= \int_0^1 \frac{1}{xM^2} e^{-Q^2\bar{x}/M^2x} \left\{ \frac{4f_\phi^2\phi(x)}{m_\phi^2+q^2} e^{q^2\bar{x}/M^2} - \frac{1}{2\pi^2}\ln\frac{(s_o+q^2)\bar{x}}{M^2} e^{q^2\bar{x}/M^2} + \right. \\ &\left. + \sum_{n=0}^{\infty}\left(\frac{q^2\bar{x}}{M^2}\right)^n \frac{\psi(n+1)}{n!} \right\} - \frac{8\langle\phi^2\rangle}{Q^2M^2} \end{aligned} \tag{4.16}$$

It is straightforward to observe that the last expression is well-defined for $q^2=0$. In our subsequent analysis of the SR for $F_{\gamma^*\gamma^*\to\pi^o}$ we shall follow a similar strategy. Instead of the scalar meson the ρ-meson will contribute.

4.2 Structure of the infrared singularities in the realistic case of the process $\gamma^*\gamma^* \to \pi^o$

Consider the SD(II) contribution for the correlator (2.1) (see fig.1b), where only the points y and 0 are at small distances. The small distance contribution is factorized in a coefficient function — a propagator or a multiplication of propagators. The large distance contribution is represented by a two-point correlator of the electromagnetic current $J_\mu(x)$ and some composite operator with derivatives (see [17, 19]).

Namely, if one extracts as a coefficient function the free quark propagator $S(y) = \hat{y}/2\pi^2(y^2-i0)^2$, for the contribution from fig.1b one get from (2.1):

$$\begin{aligned}\mathcal{F}^{SD(II)}_{\alpha\mu\nu} &= -\frac{e^2}{3\sqrt{2}}\int d^4y\, e^{-iq_2y}\frac{y^\beta}{2\pi^2y^4}\sum_{n=0}^{\infty}\frac{1}{n!}y^{\mu_1}\dots y^{\mu_n} \\ &\times\left\{ -S_{\nu\beta\alpha\sigma}\int d^4x\, e^{-iq_1x}\langle 0|T\left\{J_\mu(x)\bar{u}(0)(\overleftarrow{\partial}_{\mu_1}\dots\overleftarrow{\partial}_{\mu_n})\gamma_\sigma\gamma_5u(0)\right\}|0\rangle \right. \\ &\left. +i\epsilon_{\nu\beta\alpha\sigma}\int d^4x\, e^{-iq_1x}\langle 0|T\left\{J_\mu(x)\bar{u}(0)(\overleftarrow{\partial}_{\mu_1}\dots\overleftarrow{\partial}_{\mu_n})\gamma_\sigma u(0)\right\}|0\rangle \right\} \\ &+ \text{charge conjugate contribution}\end{aligned} \tag{4.17}$$

where $S_{\nu\beta\alpha\sigma} \equiv (g_{\nu\beta}\,g_{\alpha\sigma} - g_{\nu\alpha}\,g_{\beta\sigma} + g_{\nu\sigma}\,g_{\alpha\beta})$.

For any definite "n" we may expand the current with derivatives over the set of traceless operators. More precisely, one needs to deal with traceless combinations of the indices $\beta,\mu_1,\dots,\mu_n$. Therefore the main contribution to (4.17) will come from the operators with lowest two twists, which corresponds to the traceless $\{\beta,\mu_1,\dots,\mu_n\}$ combination and to combination with one contraction $\sim g_{\beta\mu_i}$ or $\sim g_{\mu_i\mu_j}$. The higher twist operators yields the multipliers like $(y^2)^2$, $(y^2)^3$, etc., which cancels the singularity of the quark propagator $1/y^4$ and leads to regular terms in (4.17).

Consider the factorization for the perturbative loop (see fig.2a,b,c). Diagram of fig.2c) corresponds to the expression (4.17) with a substitution of the perturbative two-point correlators.

The notations read:

$$\begin{aligned}\Pi L_n^5(q_1,y) &= \int d^4x\, e^{-iq_1x}\langle 0|T\left\{J_\mu(x)\bar{u}(0)(y\overleftarrow{\partial})^n\gamma_\sigma\gamma_5u(0)\right\}|0\rangle \\ \Pi L_n(q_1,y) &= \int d^4x\, e^{-iq_1x}\langle 0|T\left\{J_\mu(x)\bar{u}(0)(y\overleftarrow{\partial})^n\gamma_\sigma u(0)\right\}|0\rangle\,.\end{aligned} \tag{4.18}$$

Analogously, $\Pi R_n^5(q_1,y)$ and $\Pi R_n(q_1,y)$ denote correlators of the type (4.18), but with derivatives on the right. Calculating these correlators in the perturbative theory we obtain for arbitrary "n":

$$\left\{\begin{array}{c}\Pi L_n^5(q_1,y)\\ \Pi L_n(q_1,y)\end{array}\right\} = (-i)^n 3\left[\begin{array}{c}-4i\epsilon_{\mu\alpha\sigma\beta}\\ 4S_{\mu\alpha\sigma\beta}\end{array}\right]\int d^D\hat{k}\,\frac{(yk)^n[k_\alpha k_\beta - k_\alpha q_{1\beta}]}{k^2(k-q_1)^2} \tag{4.19}$$

where $d^D\hat{k} \equiv d^Dk/(2\pi)^D$, $D=4-2\varepsilon$; here and in the following we use the dimensional regularization and the $\overline{MS}$ subtraction scheme.

Let us expand the integral (4.19) in powers of y^2 keeping only the terms up to y^2 (see Appendix), because it is sufficient for the sake of extracting the contribution of the lowest two twist operators. Indeed, the contributions of the lowest twist operators in (4.17) one can get if suppose formally $y^2=0$ in the numerator of the integrand. The terms proportional to y^2 will provide the contribution of the next-to-leading operators. Taking into account these expressions and (4.17) after simple but too large calculations it is possible to sum over "n" and to integrate over d^4y with the help of Fourier transforms. As a result, for $\Phi^{SD(II)}_{1,pt}(q^2,Q^2,M^2)$ we find:

$$\begin{aligned}\Phi^{SD(II)}_{1,pt}(q^2,Q^2,M^2) &= \frac{\alpha_{e.m.}2\sqrt{2}}{\pi}\int_0^1 dx e^{-Q^2x/M^2\bar{x}}\frac{1}{2M^2} \\ &\times\left\{e^{q^2x/M^2}\ln\frac{q^2x\bar{x}}{\mu^2}\left[2\frac{q^2x}{M^2}+\frac{q^4x^2}{M^4}\right]+2\frac{q^2x}{M^2}-\frac{3}{2}\frac{q^4x^2}{M^4}\right\}\end{aligned} \tag{4.20}$$

We note that the contributions of the lowest two twist operators proportional to $q^2\ln q^2$ and $q^4\ln q^2$ respectively, just coincide with the nonanalytic terms in (4.1).

Now we may proceed further in a similar way as to the diagrams proportional to the gluon condensates (see fig.5). A similar study of the singularities appeared, is more convenient to make diagram by diagram because for the different groups of diagrams, after a factorization of the SD(II) regime, the different coefficient functions (CF) come into play. For the diagrams of fig.5b,c) the exact expression, after a transformation just similar to that the perturbative one have been made, has the form:

$$\begin{aligned}\Phi_{1,b}(q^2,Q^2,M^2) &= -\frac{\alpha_{e.m.}\sqrt{2}\pi}{18}\langle\frac{\alpha_s}{\pi}GG\rangle\frac{1}{M^6}\int_0^1 dx\frac{\bar{x}}{x^2}e^{-Q^2\bar{x}/M^2x} \\ &\times\left\{\frac{M^2}{q^2\bar{x}}+e^{q^2\bar{x}/M^2}\ln\frac{q^2\bar{x}}{M^2}-\sum_{n=1}^{\infty}\left(\frac{q^2\bar{x}}{M^2}\right)^n\frac{\psi(n+1)}{n!}\right\}\end{aligned} \tag{4.21}$$

$$\Phi_{1,c}(q^2,Q^2,M^2) = \frac{\alpha_{e.m.}\sqrt{2}\pi}{18}\langle\frac{\alpha_s}{\pi}GG\rangle\frac{1}{M^4}\left\{\frac{1}{q^2}+\frac{1}{Q^2}\right\}$$
$$-\frac{\alpha_{e.m.}\sqrt{2}\pi}{18}\langle\frac{\alpha_s}{\pi}GG\rangle\frac{2}{M^6}\int_0^1 dx e^{-Q^2\bar{x}/M^2 x}\left\{\frac{M^2}{q^2 x}+\left(\frac{\bar{x}}{x}-\frac{1}{x^2}\right)e^{q^2\bar{x}/M^2}\ln\frac{q^2\bar{x}}{M^2}\right.$$
$$\left.-\left(\frac{\bar{x}}{x}-\frac{1}{x^2}\right)\sum_{n=1}^{\infty}\left(\frac{q^2\bar{x}}{M^2}\right)^n\frac{\psi(n+1)}{n!}\right\} \quad (4.22)$$

The factorization for these diagrams may be performed as in the perturbative case. The obvious difference from that case is that instead of the perturbative contribution for the "bilocal" objects (4.18) mentioned above, it is necessary to take into account the corresponding $\langle GG\rangle$ contributions. The final result for the diagram of fig.5b) take the form:

$$\Phi_{1,b}^{SD(II)}(q^2,Q^2,M^2) = -\frac{\alpha_{e.m.}\sqrt{2}\pi}{18}\langle\frac{\alpha_s}{\pi}GG\rangle\frac{1}{M^6}\int_0^1 dx\frac{\bar{x}}{x^2}e^{-Q^2\bar{x}/M^2 x}$$
$$\times\left\{\frac{M^2}{q^2\bar{x}}e^{q^2\bar{x}/M^2}+e^{q^2\bar{x}/M^2}\ln\frac{q^2 x\bar{x}}{\mu^2}+e^{q^2\bar{x}/M^2}(2\bar{x}-\frac{7}{6})\right\}, \quad (4.23)$$

and for the diagram of fig.5c):

$$\Phi_{1,c}^{SD(II)}(q^2,Q^2,M^2) = \frac{\alpha_{e.m.}\sqrt{2}\pi}{18}\langle\frac{\alpha_s}{\pi}GG\rangle\frac{1}{M^4}\frac{1}{q^2}$$
$$-\frac{\alpha_{e.m.}\sqrt{2}\pi}{18}\langle\frac{\alpha_s}{\pi}GG\rangle\frac{2}{M^6}\int_0^1 dx e^{-Q^2\bar{x}/M^2 x}$$
$$\times\left\{\frac{M^2}{q^2 x}e^{q^2\bar{x}/M^2}+\left(\frac{\bar{x}}{x}-\frac{1}{x^2}\right)e^{q^2\bar{x}/M^2}\ln\frac{q^2 x\bar{x}}{\mu^2}\right\} \quad (4.24)$$

Note, that the terms proportional to $1/q^2$ are due to the traceless combination of indices $\beta,\mu_1,\ldots,\mu_n$ in (4.17), whereas the terms proportional to $\ln q^2$ correspond to the next-to-leading twist operators. A similar consideration of the diagrams of fig.5f,g) can be made analogously. The exact expression for the sum of these two diagrams reads:

$$\Phi_{1,f+g}(q^2,Q^2,M^2) = -\frac{\alpha_{e.m.}\sqrt{2}\pi}{18}\langle\frac{\alpha_s}{\pi}GG\rangle\frac{1}{M^6}\int_0^1 dx e^{-Q^2\bar{x}/M^2 x}$$
$$\times\left\{\frac{(2x^2+\bar{x})}{x^2}\left(e^{q^2\bar{x}/M^2}\ln\frac{q^2\bar{x}}{M^2}-\sum_{n=1}^{\infty}\left(\frac{q^2\bar{x}}{M^2}\right)^n\frac{\psi(n+1)}{n!}\right)\right.$$
$$\left.-\frac{(2x-1)}{x^2}\frac{M^2}{q^2}\right\} \quad (4.25)$$

To separate the contribution of the small distances for these diagrams one gets a multiplication of two quark propagators as a CF:

$$\mathcal{F}_{\alpha\mu\nu}^{SD(II)} = \frac{e^2}{3\sqrt{2}}\int d^4y\, e^{-iq_2y}\, d^4z\, \frac{(y-z)^\delta}{2\pi^2(y-z)^4}\,\frac{z^\varepsilon}{2\pi^2 z^4}\sum_{n,m=0}^{\infty}\frac{1}{n!m!}\,y^{\mu_1}\ldots y^{\mu_n}\, z^{\nu_1}\ldots z^{\nu_m} \quad (4.26)$$
$$\times\int d^4x\, e^{-iq_1x}\,\langle 0|T\left\{J_\mu(x)\,\bar{u}(0)(\overleftarrow{\partial}_{\mu_1}\ldots\overleftarrow{\partial}_{\mu_n})\gamma_\nu\gamma_\delta\gamma_\gamma g(\overrightarrow{\partial}_{\nu_1}\ldots\overrightarrow{\partial}_{\nu_n}A_\gamma^b(0))t^b\gamma_\varepsilon\gamma_5\gamma_\alpha u(0)\right\}|0\rangle$$

The $\langle GG\rangle$ contribution in (4.26) leads to the expression:

$$\Phi_{1,f+g}^{SD(II)}(q^2,Q^2,M^2) = -\frac{\alpha_{e.m.}\sqrt{2}\pi}{18}\langle\frac{\alpha_s}{\pi}GG\rangle\frac{1}{M^6}\int_0^1 dx e^{-Q^2\bar{x}/M^2 x}$$
$$\times\left\{\frac{(2x^2+\bar{x})}{x^2}e^{q^2\bar{x}/M^2}\ln\frac{q^2 x\bar{x}}{\mu^2}-\frac{(2x-1)}{x^2}\frac{M^2}{q^2}e^{q^2\bar{x}/M^2}\right.$$
$$\left.+e^{q^2\bar{x}/M^2}(\ldots)\right\} \quad (4.27)$$

Finally, an analogous procedure of factorization should be performed for the diagrams with a quark condensate. The situation here is very similar to that of the scalar example. As to the diagrams with a soft gluon (fig.6) , the contribution of the diagrams of fig.6a,b,c) in F_1 is independent on p^2 and vanishes after borelization. Diagrams of fig.6g,h,i) give a regular in q^2 contribution and thus they need no of additional factorization. The relevant diagrams are of fig.6d,e,f), for which the factorized expression (extracting the corresponding CF (see (4.17),(4.26)) coincides with the exact one.

The same result we shall get for the diagrams with a hard gluon exchange (fig.7). It should be mentioned that the CF which corresponds to the diagram of fig.7a) (after extracting the SD(II) regime) is a multiplication of two quark propagators (see (4.26)), whereas the diagrams of fig.7b,c) in the same regime provide a new CF — a multiplication of two quark propagators and one gluon. The contribution of fig.7d) is a regular in q^2 function and it is no need of additional factorization.

As have already been mentioned above, the diagrams of fig.7e–r) do not contribute to the F_1 form factor. However, a more precise consideration of the bilocal objects which belong to the corresponding CF leads to possible nontrivial contributions — the so-called contact terms [17, 19, 20]. A subsequent analysis of such a terms leads to the correct normalization for the meson electromagnetic form factors at zero momentum transfer [19, 20]. A consideration of the contact type terms due to operators of the lowest twist (at an arbitrary "n" of the derivatives) will be performed in the next our paper.

As a conclusion it should be mentioned that as a result of the subtraction procedure, symbolically depicted in fig.4, all infrared singularities in the standard SR cancel with the corresponding singular contributions from the diagrams in which the SD(II) CF have been already extracted. Herewith, in the bilocal objects the contribution of the operators of the lowest two twist is taken into account. The regular in q^2 terms, which remain after this subtraction, will contribute to the desired SR.

A further step in the calculations is to determine the contributions of the bilocals themselves. It is of importance to emphasize that we take into account the contributions from composite operators in the correlators for an arbitrary "n", because the twist of these operators counts rather than their dimension. For the process in consideration these contributions will be obtain in the next our work and the derivation of the full SR for the process $\gamma^*\gamma^* \to \pi^0$ in the essentially nonsymmetric kinematics will be completed.

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A Appendix. Calculation of some useful momentum integrals expanding in powers of y^2

We shall calculate our integrals using dimensional regularization. The basic, well-known, integral reads:

$$I(L,r) = \int d^D\hat{p}\frac{(p^2)^r}{[p^2+S]^L} = \frac{i(-1)^{r-L}}{(4\pi)^{D/2}}\frac{\mu^{4-D}}{\Gamma(L)}\frac{\Gamma(r+D/2)}{\Gamma(D/2)}\frac{\Gamma(L-r-D/2)}{(-S)^{L-r-D/2}} \qquad (A.1)$$

where $D=4-2\varepsilon$ and $d^D\hat{p}\equiv d^Dp/(2\pi)^D$. It is more convenient to define the integral:

$$R(L,r)\equiv I(L,r)2^r\frac{\Gamma(D/2)}{\Gamma(r+D/2)} \qquad (A.2)$$

The integrals of interest are of the form:

$$\int d^D\hat{p}\frac{(py)^n\{1,p_\rho,p_\rho p_\sigma,\dots\}}{(p^2)^\alpha(p-q)^{2\beta}} = \int_0^1 dx x^{\alpha-1}\bar{x}^{\beta-1}\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\int d^D\hat{p}\frac{(py)^n\{1,p_\rho,p_\rho p_\sigma,\dots\}}{[(p-\bar{q})^2+S]^L} \qquad (A.3)$$

where $L=\alpha+\beta, \bar{q}=q\bar{x}, S=q^2x\bar{x}, \bar{x}\equiv 1-x$. Omitting for a moment the integration over x we are left with:

$$\{J(L,n),J_\rho(L,n),J_{\rho\sigma}(L,n),\dots\} = \int d^D\hat{p}\frac{(py)^n\{1,p_\rho,p_\rho p_\sigma,\dots\}}{[(p-\bar{q})^2+S]^L} \qquad (A.4)$$

After a shift of the integration variable we expand in a standard way:

$$(p+\bar{q}.y)^n = (\bar{q}.y)^n + C_n^1(\bar{q}.y)^{n-1}(py) + C_n^2(\bar{q}.y)^{n-2}(py)^2+\cdots \qquad (A.5)$$

where $C_n^m\equiv n!/m!(n-m)!$ are the binomial coefficients. Now the integration over $d^D\hat{p}$ is straightforward and we obtain up to terms $\sim y^2$:

$$J(L,n) = (\bar{q}.y)^nR(L,0) + C_n^2(\bar{q}.y)^{n-2}y^2R(L,1)+\bar{O}(y^4), \qquad (A.6)$$

$$\begin{aligned} J_\rho(L,n) &= \bar{q}_\rho\left\{(\bar{q}.y)^nR(L,0)+C_n^2(\bar{q}.y)^{n-2}y^2R(L,1)+\bar{O}(y^4)\right\}+\\ &+ y_\rho\left\{C_n^1(\bar{q}.y)^{n-1}R(L,1)+C_n^3(\bar{q}.y)^{n-3}3y^2R(L,2)+\bar{O}(y^4)\right\}, \end{aligned} \qquad (A.7)$$

$$\begin{aligned} J_{\rho\sigma}(L,n) &= \bar{q}_\rho\bar{q}_\sigma\left\{(\bar{q}.y)^nR(L,0)+C_n^2(\bar{q}.y)^{n-2}y^2R(L,1)+\bar{O}(y^4)\right\}+\\ &+ (\bar{q}_\rho y_\sigma+\bar{q}_\sigma y_\rho)\left\{C_n^1(\bar{q}.y)^{n-1}R(L,1)+C_n^3(\bar{q}.y)^{n-3}3y^2R(L,2)+\bar{O}(y^4)\right\}+\\ &+ g_{\rho\sigma}(\bar{q}.y)^nR(L,1)+C_n^2(\bar{q}.y)^{n-2}R(L,2)\left\{2y_\rho y_\sigma+g_{\rho\sigma}y^2\right\}+\\ &+ C_n^4(\bar{q}.y)^{n-4}R(L,3)\left\{12y_\rho y_\sigma y^2+\bar{O}(y^4)\right\} \end{aligned} \qquad (A.8)$$

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Формфактор процесса $\gamma^*\gamma^* \to \pi^0$ при малой виртуальности одного из фотонов и правила сумм КХД (I): структура инфракрасных сингулярностей

Методом правил сумм КХД мы исследуем формфактор $F_{\gamma^*\gamma^*\to\pi^0}(q_1^2, q_2^2)$ в области малых виртуальностей одного из фотонов: $|q_1^2| \ll |q_2^2| \geqslant 1$ ГэВ^2, где необходимо провести дополнительную факторизацию вкладов больших и малых расстояний. В качестве первого шага, формфактор исследуется в области умеренных виртуальностей фотонов: $|q_1^2| \sim |q_2^2| \geqslant 1$ ГэВ^2, где получены полные $\langle 0|G^a_{\mu\nu}G^a_{\mu\nu}|0\rangle$, $\langle 0|\bar\psi\psi|0\rangle^2$ поправки в правиле сумм. Показано, что инфракрасные (массовые) сингулярности могут быть вычтены при соответствующем операторном разложении для существенно несимметричной кинематической ситуации благодаря операторам двух нижайших твистов. Наиболее важные шаги дальнейших вычислений продемонстрированы на простом скалярном примере.

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Radyushkin A.V., Ruskov R. E2-92-425

The Form Factor of the Process $\gamma^*\gamma^* \to \pi^0$ for Small Virtuality of One of the Photons and QCD Sum Rules (I): The Structure of the Infrared Singularities

We extend the QCD sum rule analysis of the form factor $F_{\gamma^*\gamma^*\to\pi^0}(q_1^2, q_2^2)$ into the region of small virtuality of one of the photons: $|q_1^2| \ll |q_2^2| \geqslant$ GeV^2, where one should perform more precisely an OPE to factorize large and small distance contributions. As a first step the form factor is investigated in the region of moderate virtualities: $q_1^2 \sim q_2^2 \geqslant -1$ GeV^2 and the full $\langle 0|G^a_{\mu\nu}G^a_{\mu\nu}|0\rangle$, $\langle 0|\bar\psi\psi|0\rangle^2$ corrections in the sum rule are obtained. It is shown that the infrared mass singularities are subtracted in the corresponding OPE for essentially nonsymmetric kinematics due to the operators of lowest two twists. On a simple scalar example the most important steps of the further calculations are demonstrated.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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