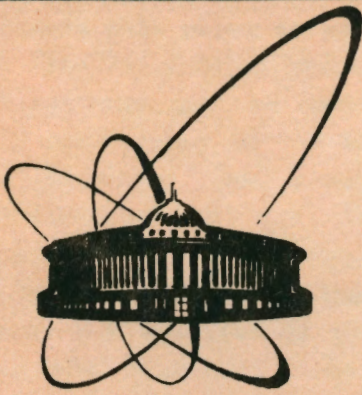


92-394



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E2-92-394

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**THE GRAVITATIONAL RADIUS
IN THE LOBACHEVSKY SPACE**

Submitted to "Quantum Nonlinear Phenomena"

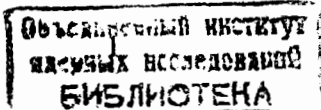
1992

The first of December of this year is the 200-th anniversary of the birthday of N.I.Lobachevsky. The author presents this paper to celebrate this great day.

The creator of the non-Euclidean geometry Lobachevsky did not consider the geometry alone. Having no doubt of the self-consistency of the new geometry and being convinced in its validity, he posed the problem of astronomical verification of the geometry of our visible world. Supplementing the available at that time data on parallaxes of stars by his own observations, he found out that the constant k specific of the non-Euclidean geometry is larger than the distances from the Earth to the nearest stars [1, pp. 207-210]. This unsatisfactory result did not however prevent him from posing the problem of what kind of changes will occur after introducing a new geometry in mechanics [1, p. 261]. The second problem inevitably follows the first one as soon as one starts considering celestial bodies under the conditions of the non-Euclidean geometry. But having introduced a new geometry into celestial mechanics, Lobachevsky went further and posed the problem of what changes this introduces into Newton's law of gravity. He himself answered this question giving a fundamental solution to the Poisson equation in the non-Euclidean space [2, pp. 158-160].

Those were completely new problems and Lobachevsky's idea " *one should not doubt that the forces produce all by themselves : the motion, velocity, time, mass, even distances and angles ... when it is true that forces depend on the distance, then lines can also depend on angles* " [2, p.159] leads us far beyond the scope of the Newton mechanics and theory of gravity. With this idea in mind one can go even far beyond Einstein's theories.

In the present paper, a new approach to the theory of gravity, based on Lobachevsky's geometry is expounded. We start with a fragmentary statement of Lobachevsky's differential geometry and end up with the exact solution of the Schwarzschild problem in the Lobachevsky space.



1. INTRODUCTION OF LOBACHEVSKY GEOMETRY

Lobachevsky's geometry is completely defined by the metric

$$d l^2 = L_{\alpha\beta} d x^\alpha d x^\beta, \quad (1)$$

in spherical coordinates ρ, θ, φ taking the next form:

$$d l^2 = d \rho^2 + \rho^2 (d \theta^2 + \sin^2 \theta d \varphi^2), \quad (2)$$

where

$$r = k \operatorname{sh} \frac{\rho}{k}. \quad (3)$$

Straight lines in the Lobachevsky space are geodesics with respect to affine connection with the components

$$L_{\mu\nu}^\alpha = \frac{1}{2} L^{\alpha\sigma} (\partial_\mu L_{\sigma\nu} + \partial_\nu L_{\sigma\mu} - \partial_\sigma L_{\mu\nu}), \quad (4)$$

where $L^{\alpha\sigma}$ is the cometric tensor determined by the metric tensor $L_{\sigma\beta}$ and unit affinor δ_β^α from the condition

$$L^{\alpha\sigma} L_{\sigma\beta} = \delta_\beta^\alpha, \quad (5)$$

∂_μ is the partial derivative with respect to the coordinate x^μ . Geodesics are determined upon deriving the system of equations

$$\frac{d}{d l} \frac{d x^\alpha}{d l} + L_{\mu\nu}^\alpha \frac{d x^\mu}{d l} \frac{d x^\nu}{d l} = 0, \quad \alpha \in \{1, 2, 3\}. \quad (6)$$

It is interesting that the components (4) remember all about the metric (1) they have been generated by. Indeed, they compose a tensor

$$L_{\mu\nu\beta}^\alpha = \partial_\mu L_{\nu\beta}^\alpha - \partial_\nu L_{\mu\beta}^\alpha + L_{\mu\sigma}^\alpha L_{\nu\beta}^\sigma - L_{\nu\sigma}^\alpha L_{\mu\beta}^\sigma \quad (7)$$

which equals

$$L_{\mu\nu\beta}^\alpha = (L_{\mu\beta}^\alpha \delta_\nu^\alpha - L_{\nu\beta}^\alpha \delta_\mu^\alpha) k^{-2}. \quad (8)$$

Consequently,

$$L_{\mu\beta}^\alpha = \frac{1}{2} k^2 L_{\mu\alpha\beta}^\alpha. \quad (9)$$

This is nontrivial as in the limit

$$k \rightarrow \infty \quad (10)$$

the components (4) forget much about the limit metric

$$E_{\alpha\beta} d x^\alpha d x^\beta = \lim_{k \rightarrow \infty} L_{\alpha\beta} d x^\alpha d x^\beta. \quad (11)$$

In this case, the limit tensor

$$E_{\mu\nu\beta}^\alpha = \partial_\mu E_{\nu\beta}^\alpha - \partial_\nu E_{\mu\beta}^\alpha + E_{\mu\sigma}^\alpha E_{\nu\beta}^\sigma - E_{\nu\sigma}^\alpha E_{\mu\beta}^\sigma, \quad (12)$$

composed, like (7), of the components of the limit connection

$$E_{\mu\nu}^\alpha = \frac{1}{2} E^{\alpha\sigma} (\partial_\mu E_{\sigma\nu} + \partial_\nu E_{\sigma\mu} - \partial_\sigma E_{\mu\nu}), \quad (13)$$

equals zero. Consequently, there can be found such coordinates y through which the connection (13) will be represented as

$$E_{\mu\nu}^\alpha = \frac{\partial x^\alpha}{\partial y^\sigma} \frac{\partial^2 y^\sigma}{\partial x^\mu \partial x^\nu}. \quad (14)$$

In the y coordinate map the components of the metric tensor $E_{\alpha\beta}$ are independent of the coordinates. This is probably all we can say about the metric (11) if only the connection components with the zero curvature tensor (12) are known.

However, we can add that the metric (11) defines the Euclidean geometry as in spherical coordinates it takes the form

$$E_{\alpha\beta} d x^\alpha d x^\beta = d \rho^2 + \rho^2 (d \theta^2 + \sin^2 \theta d \varphi^2). \quad (15)$$

Meanwhile, the connection (14) is invariant with respect to

affine substitutions

$$y^\sigma = A_{\hat{y}}^\sigma \hat{y} + B^\sigma \quad (16)$$

and defines only the affine geometry rather than a much richer Euclidean geometry.

2. LOBACHEVSKY'S GEOMETRY AND LORENTZ GROUP

Four functions

$$\begin{aligned} x &= k \operatorname{sh} \frac{\rho}{k} \sin \theta \cos \varphi, & y &= k \operatorname{sh} \frac{\rho}{k} \sin \theta \sin \varphi, \\ z &= k \operatorname{ch} \frac{\rho}{k} \cos \theta, & u &= \operatorname{ch} \frac{\rho}{k} \end{aligned} \quad (17)$$

of the spherical coordinates ρ , θ and φ define the three-dimensional surface in the four-dimensional centro-affine space with the Cartesian coordinates x , y , z and u . The latter is the hyperboloid cavity

$$k^2 u^2 - x^2 - y^2 - z^2 = k^2, \quad (18)$$

in which $u > 0$. In the Cartesian coordinates it is defined by the equation

$$u = \sqrt{1 + (x^2 + y^2 + z^2) / k^2}. \quad (19)$$

We have obtained the one-to-one (or, as now it is called, bijective) mapping of the Lobachevsky space onto the surface (19). Differentiating the functions (3) and (17) we get

$$\begin{aligned} d x^2 + d y^2 + d z^2 &= d r^2 + r^2 (d \theta^2 + \sin^2 \theta d \varphi^2), \\ k^2 d u &= r d \rho, \quad d r = u d \rho. \end{aligned} \quad (20)$$

Hence, we find that in the coordinates x , y , and z Lobachevsky's metric (2) equals

$$d l^2 = d x^2 + d y^2 + d z^2 - k^2 d u^2, \quad (21)$$

where du is the differential function (19) so that

$$(k^2 + x^2 + y^2 + z^2) k^2 d u^2 = (x d x + y d y + z d z)^2. \quad (22)$$

Consequently, the internal geometry of the surface (19) in the four-dimensional pseudo-Euclidean space with the metric (21) coincides with the Lobachevsky geometry. Therefore, isometric transformations of the Lobachevsky space are given as linear transformations of the coordinates x , y , z and u conserving the quadratic form in the left-hand side of equality (18) and not changing the sign of the coordinate u . By the Poincare definition transformations of that type form a Lorentz group. Thus, the Lorentz group is isomorphic with respect to a group of isometries of the Lobachevsky space.

According to (21) and (22) the metric tensor components of the Lobachevsky space in the coordinates x , y , and z equal

$$L_{\alpha\beta} = c_{\alpha\beta} - k^{-2} u^{-2} x_\alpha x_\beta, \quad (23)$$

where $c_{\alpha\beta}$ are the constants (in the present case equal to 1 at $\alpha = \beta$ and 0 at $\alpha \neq \beta$),

$$x_\alpha = c_{\alpha\sigma} x^\sigma. \quad (24)$$

Consequently,

$$\frac{1}{2} (\partial_\mu L_{\sigma\nu} + \partial_\nu L_{\sigma\mu} - \partial_\sigma L_{\mu\nu}) = -k^{-2} u^{-2} x_\sigma L_{\mu\nu}. \quad (25)$$

The components of the cometric tensor in these coordinates are equal to

$$L^{\alpha\beta} = c^{\alpha\beta} + k^{-2} x^\alpha x^\beta, \quad (26)$$

where $c^{\alpha\sigma} c_{\sigma\beta} = \delta_\beta^\alpha$. Therefore,

$$L^{\alpha\sigma} x_\sigma = u^2 x^\alpha. \quad (27)$$

Consequently, .

$$L_{\mu\nu}^{\alpha} = -k^{-2} x^{\alpha} L_{\mu\nu} . \quad (28)$$

A simple form of the components (23) and (28) allows one to easily prove equality (8).

According to (6) and (28), in the x, y and z coordinates straight lines in the Lobachevsky space are determined after the solution of the system of equations

$$\frac{d}{d l} \frac{d x^{\alpha}}{d l} - k^{-2} x^{\alpha} L_{\mu\nu} \frac{d x^{\mu}}{d l} \frac{d x^{\nu}}{d l} = 0 , \quad (29)$$

$$\alpha \in \{1, 2, 3\} .$$

Calculating the second derivative of the function (19) we get

$$\frac{d}{d l} \frac{d u}{d l} - k^{-2} u L_{\mu\nu} \frac{d x^{\mu}}{d l} \frac{d x^{\nu}}{d l} = 0 . \quad (30)$$

Now denote by

$$r = \{x, y, z, u\} \quad (31)$$

the vector of the four-dimensional centro-affine space in which the surface (19) is the Lobachevsky space. The quadratic form (21) is denoted by $(d r, d r)$. In this notation the surface (19) is given by the conditions

$$(r, r) = -k^2 , \quad u > 0 , \quad (32)$$

and eqs. (29) and (30) are written in the form

$$\frac{d}{d l} \frac{d r}{d l} - k^{-2} r \left(\frac{d r}{d l}, \frac{d r}{d l} \right) = 0 . \quad (33)$$

Hence, it follows that a straight line of the Lobachevsky space lies at the intersection of the surface (19) with the two-dimensional plane of the centro-affine space, passing

through the center $r = 0$.

3. VELOCITY SPACE IN THE SPECIAL THEORY OF RELATIVITY

In the four-dimensional space-time of the special relativity theory the velocity of a material point can be represented by a bundle of timelike parallel straight lines. The combination of all these bundles is the three-dimensional space of velocities. In that space, the absolute geometry based on all the Euclidean postulates except for the fifth one is realised.

In the Lorentz case, we arrive at the Lobachevsky space of velocities assuming that the constant k equals the light velocity c , and the distance ρ equals the rapidity of a particle s . In this case, the quantities (17) are equal to the components of the four-velocity of a particle. The components of the usual velocity of a particle in the Lorentz case equal

$$v_1 = c \operatorname{th} \frac{s}{k} \sin \theta \cos \varphi , \quad v_2 = c \operatorname{th} \frac{s}{k} \sin \theta \sin \varphi ,$$

$$v_3 = c \operatorname{th} \frac{s}{k} \cos \theta . \quad (34)$$

In the Galilean case $c = \infty$ and the space of velocities is Euclidean. Instead of the surface (19) in this case there appears the hyperplane $u = 1$. As for the Galilean group, it is isomorphic to the group of isometries of the Euclidean space.

So the light velocity plays the role of the Lobachevsky constant in the space of velocities. This is the essence of the special relativity theory. It is interesting that a the perimeter of a circle in the Lobachevsky space is a particle momentum; and the area of the circle, its energy.

As in cosmic rays one can observe and at accelerators achieve rapidities much exceeding the light velocity c , in the high energy physics one cannot do without the

Lobachevsky geometry.

4. NEWTON'S THEORY OF GRAVITATION
IN THE LOBACHEVSKY SPACE

The Lagrange function of a material point, if it is influenced only by the gravity force with the potential U, equals

$$l = \frac{1}{2} L_{\mu\nu} \frac{d x^\mu}{d t} \frac{d x^\nu}{d t}. \quad (35)$$

Consequently, the Lagrange equations of motion are

$$\frac{d}{d t} (L_{\alpha\nu} \frac{d x^\nu}{d t}) - \frac{1}{2} (\partial_\alpha L_{\mu\nu}) \frac{d x^\mu}{d t} \frac{d x^\nu}{d t} + \partial_\alpha U = 0, \quad (36)$$

i.e.

$$\frac{d}{d t} \frac{d x^\alpha}{d t} + L_{\mu\nu}^\alpha \frac{d x^\mu}{d t} \frac{d x^\nu}{d t} + L^{\alpha\nu} \partial_\nu U = 0. \quad (37)$$

Here we have introduced the absolutely stationary (at rest), according to Newton, Lobachevsky space and the absolute, according to Newton also, time t, which is equivalent to denial of the Euclidean postulate of parallel lines in the visible world. In this case, the special relativity principle becomes invalid though the principle of kinematic relativity is conserved.

In the space-time SxT with the coordinates maps x^1, x^2, x^3, x^4 , where $x^4 = t$, the absolute time gives the differential form $\Theta = d t$. Writing down this form as

$$\Theta = \theta_a d x^a, \quad (38)$$

we introduce in SxT the covector field with the components

$$\theta_1 = 0, \quad \theta_2 = 0, \quad \theta_3 = 0, \quad \theta_4 = 1 \quad (39)$$

and the factorising metric

$$\theta_{ab} d x^a d x^b = \Theta \Theta. \quad (40)$$

The factorising time tensor determined by it equals

$$\theta_{ab} = \theta_a \theta_b. \quad (41)$$

The introduction of the Lobachevsky geometry defines in SxT the cometric tensor h^{ab} with the components equal to

$$h^{\alpha\beta} = L^{\alpha\beta}, \quad h^{\alpha 4} = 0, \quad h^{4\beta} = 0, \quad h^{44} = 1. \quad (42)$$

As

$$h^{ab} \theta_b = 0, \quad (43)$$

the time tensor and cometric tensor are coupled by the condition

$$\theta_{as} h^{sb} = 0. \quad (44)$$

The equations of motion (37) define in SxT the affine connection. Indeed, they can be written as equations of geodesics

$$\frac{d}{d s} \frac{d x^a}{d s} + \Gamma_{mn}^a \frac{d x^m}{d s} \frac{d x^n}{d s} = 0, \quad (45)$$

by substituting $s = A t + B$, where A and B are constant. Hence, we find

$$\Gamma_{\mu\nu}^\alpha = L_{\mu\nu}^\alpha, \quad \Gamma_{44}^\alpha = L^{\alpha\nu} \partial_\nu U, \quad (46)$$

$$\Gamma_{\mu 4}^\alpha = 0, \quad \Gamma_{4\nu}^\alpha = 0, \quad \Gamma_{mn}^4 = 0.$$

Consequently, the world trajectory of a material point in the gravitational field U is a geodesic line in the world SxT with respect to the affine connection (46).

Essentially, the world trajectory of the material point, which is not influenced by any forces, is a geodesic line with respect to the affine connection with the components $\check{\Gamma}_{mn}^a$ equal to (46) at $U = \text{const}$. Thus, we have two connections at once.

When there are two connections (say Γ and $\check{\Gamma}$), then for each tensor field two covariant derivatives ∇ and $\check{\nabla}$ are to be composed. For the vector and covector fields one assumes that

$$\nabla_m T^a = \partial_m T^a + \Gamma_{mn}^a T^n, \quad \nabla_m T_n = \partial_m T_n - \Gamma_{mn}^a T_a, \quad (47)$$

$$\check{\nabla}_m T^a = \partial_m T^a + \check{\Gamma}_{mn}^a T^n, \quad \check{\nabla}_m T_n = \partial_m T_n - \check{\Gamma}_{mn}^a T_a.$$

The difference

$$P_{mn}^a = \check{\Gamma}_{mn}^a - \Gamma_{mn}^a \quad (48)$$

is called the affine deformation tensor.

In the present case, the affine deformation tensor equals

$$P_{mn}^a = -\theta_m \theta_n h^{as} \partial_s U. \quad (49)$$

For each affine connection Γ there are composed the torsion tensor

$$S_{mn}^a = \Gamma_{mn}^a - \Gamma_{nm}^a \quad (50)$$

and the curvature tensor

$$R_{mnb}^a = \partial_m \Gamma_{nb}^a - \partial_n \Gamma_{mb}^a + \Gamma_{ms}^a \Gamma_{nb}^s - \Gamma_{ns}^a \Gamma_{mb}^s. \quad (51)$$

It is obvious that

$$S_{mn}^a + S_{nm}^a = 0, \quad R_{mnb}^a + R_{nmb}^a = 0. \quad (52)$$

All the affine connections considered here (for instance, (46)) satisfy the condition

$$\check{\Gamma}_{mn}^a = \Gamma_{nm}^a, \quad (53)$$

so that their torsion tensor equals zero, and the curvature tensor obeys the algebraic identity

$$R_{mnb}^a + R_{bmn}^a + R_{nbn}^a = 0 \quad (54)$$

and the differential identity

$$\nabla_k R_{mnb}^a + \nabla_n R_{kmb}^a + \nabla_m R_{nkb}^a = 0. \quad (55)$$

Moreover, we will consider only the equiaffine connections. For them the contracted tensor of curvature

$$R_{mn} = R_{smn}^s \quad (56)$$

is symmetric, i.e.,

$$R_{mn} = R_{nm}. \quad (57)$$

Another contraction of the curvature tensor equals zero:

$$R_{mns}^s = 0. \quad (58)$$

In the case (46) the curvature tensor equals

$$\begin{aligned} R_{\mu\nu\beta}^\alpha &= L_{\mu\nu\beta}^\alpha, \quad R_{\mu\nu 4}^\alpha = 0, \quad R_{\mu 4\beta}^\alpha = 0, \\ R_{\mu 44}^\alpha &= \partial_\mu U^\alpha + L_{\mu\nu}^\alpha U^\nu, \quad R_{mnb}^4 = 0. \end{aligned} \quad (59)$$

where

$$U^\alpha = L^{\alpha\sigma} \partial_\sigma U, \quad (60)$$

and the contracted curvature tensor, according to (8) and (9), equals

$$R_{\mu\nu} = -2 k^{-2} L_{\mu\nu}, \quad R_{44} = \Delta U, \quad R_{\mu 4} = 0, \quad R_{4\nu} = 0, \quad (61)$$

where

$$\Delta = L^{\mu\nu} (\partial_\mu \partial_\nu - L_{\mu\nu}^\sigma \partial_\sigma). \quad (62)$$

Note that the gravitational potential U is a scalar function in the Lobachevsky space with the metric (1), U^α is the vector in this space, $R_{\mu 4 4}^\alpha$ is the covariant derivative of the vector U^α generated by the connection (4), $R_{\mu\nu\beta}^\alpha$ is the curvature tensor (7) for the connection (4), and Δ is the differential Laplace operator generated by the metric (1).

Now let us recall the connection resulting from (46) at $U = \text{const}$ and denoted by $\check{\Gamma}_{mn}^a$. We will call it the background one. The curvature tensor \check{R}_{mnb}^a of the background connection and the contracted tensor \check{R}_{mn}^a result from (59) and (61) at $U = \text{const}$.

Consequently,

$$R_{mn} - \check{R}_{mn} = \theta_m \theta_n \Delta U. \quad (63)$$

It is obvious that the equation

$$R_{mn} - \check{R}_{mn} = 4 \pi \gamma M_{mn}, \quad (64)$$

where M_{mn} is the mass tensor equal to

$$M_{mn} = \rho \theta_m \theta_n, \quad (65)$$

and ρ is the mass density, is equivalent to the Poisson equation

$$\Delta U = 4 \pi \gamma \rho \quad (66)$$

in the Lobachevsky space.

Note that γ is the gravitational Newton constant. Mass

embedded in the region of the Lobachevsky space, is equal to the integral

$$m = \int \int \int \rho \sqrt{L} d x^1 d x^2 d x^3 \quad (67)$$

over this region where L is the determinant of the matrix $(L_{\mu\nu})$.

It is interesting to note that the gravitational equation (64) includes the contracted tensor of curvature \check{R}_{mn}^a of the background connection $\check{\Gamma}_{mn}^a$. In the limit (10) eq.(64) turns into

$$R_{mn} = 4 \pi \gamma M_{mn}, \quad (68)$$

in which nothing reminds the background connection. The thing is that in the limit (10) the contracted tensor of curvature \check{R}_{mn}^a equals zero. To the point, the complete tensor of curvature \check{R}_{mnb}^a also equals zero in this limit.

5. FUNDAMENTAL SOLUTION TO THE POISSON EQUATION IN THE LOBACHEVSKY SPACE

In the sphere of radius ρ at small values of ρ / k one can approximately use the Euclidean geometry. The mass density M lying at the origin of coordinates x, y, z (see (17)) equals

$$\rho = M \delta(x) \delta(y) \delta(z), \quad (69)$$

where $\delta(x)$ is the Dirac function as in these coordinates

$$\sqrt{L} = \frac{1}{u}, \quad (70)$$

and at the origin of coordinates $u = 1$. The solution satisfying the Poisson equation

$$\Delta U = 4 \pi \gamma M \delta(x) \delta(y) \delta(z) \quad (71)$$

and the condition

$$\lim_{\rho \rightarrow \infty} U = 0 \quad (72)$$

is called the fundamental one.

Lobachevsky has shown [2, p.159] that the "attractive force" is directed towards the center and is reciprocal to the area of the sphere; moreover, if the radius of the sphere equals ρ , then its area equals $4\pi r^2$ where the quantity r equals (3). Consequently, in the spherical coordinates the fundamental solution has the following partial derivatives:

$$\frac{\partial U}{\partial \rho} = A r^{-2}, \quad \frac{\partial U}{\partial \theta} = 0, \quad \frac{\partial U}{\partial \varphi} = 0, \quad (73)$$

where A is some constant. Integrating (73) we get

$$U = - \frac{A}{k} \operatorname{cth} \frac{\rho}{k} + B, \quad (74)$$

where B is the integration constant. From condition (72) it follows that $B = A$. As in the small vicinity of the source the function turns into the Newton potential $U = - \gamma M \rho^{-1}$, then $A = \gamma M$. Consequently

$$U = \frac{\gamma M}{k} (1 - \operatorname{cth} \frac{\rho}{k}). \quad (75)$$

Note that outside the source the fundamental solution satisfies the Laplace equation

$$\Delta U = 0. \quad (76)$$

In accordance with (62)

$$\Delta U = \frac{1}{\sqrt{L}} \partial_{\mu} (\sqrt{L} L^{\mu\nu} \partial_{\nu} U). \quad (77)$$

In the spherical coordinates the operator (62) equals

$$\frac{1}{r^2} \frac{\partial}{\partial \rho} r^2 \frac{\partial}{\partial \rho} + \frac{1}{r^2} \left(\frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \sin \theta \frac{\partial}{\partial \theta} + \frac{1}{\sin^2 \theta} \frac{\partial^2}{\partial \varphi^2} \right). \quad (78)$$

6. THEORY OF GRAVITY WITH TWO CONNECTIONS

In the Einstein theory of gravity, the principal geometric object is the cometric

$$g^{ab} \partial_a \partial_b \quad (79)$$

defined in space-time X . It is of normal hyperbolic type. In the vicinity of every point $x \in X$ one can choose the coordinates

$$x^1 = x, \quad x^2 = y, \quad x^3 = z, \quad x^4 = t, \quad (80)$$

so that at the point x the quadratic form (79) becomes equal to

$$g^{ab} \partial_a \partial_b = \partial_1 \partial_1 + \partial_2 \partial_2 + \partial_3 \partial_3 - c^{-2} \partial_4 \partial_4, \quad (81)$$

where c is the light velocity. Like in the special theory of relativity, the light velocity c is the Lobachevsky constant in the space of velocities. But in the general case we should now speak about the space of velocities of a particle at the given point $x \in X$.

The time tensor in the Einstein theory equals

$$\theta_{ab} = - c^{-2} g_{ab}. \quad (82)$$

It is coupled with the cometric tensor by the condition

$$\theta_{as} g^{sb} = - c^{-2} \delta_a^b. \quad (83)$$

Another derivative geometric object of the cometric (79) is the volume tensor

$$dV_4 = \epsilon_{abmn} dx^a \wedge dx^b \wedge dx^m \wedge dx^n, \quad (84)$$

where

$$\epsilon_{1234} = c^{-1} \sqrt{-g}, \quad (85)$$

and g is the determinant of the matrix (g_{ab}) .

The third derivative geometric object of the cometric (79) is the Christoffel affine connection

$$\Gamma_{mn}^a = \frac{1}{2} g^{as} (\partial_m g_{sn} + \partial_n g_{sm} - \partial_s g_{mn}), \quad (86)$$

through which one can determine the Riemann-Christoffel tensor (51), Ricci tensor (56) and then the Hilbert scalar

$$R = g^{ab} R_{ab} \quad (87)$$

and the Einstein tensor

$$G_{mn} = R_{mn} - \frac{1}{2} R g_{mn}. \quad (88)$$

The latter together with the mass tensor M_{mn} enters into the gravitational equations

$$G_{mn} = 8\pi\gamma M_{mn}, \quad (89)$$

which are transformed into

$$R_{mn} = 8\pi\gamma (M_{mn} - \frac{1}{2} M \theta_{mn}), \quad (90)$$

where

$$M = -c^2 g^{ab} M_{ab}. \quad (91)$$

As is known, Hilbert derived gravitational equations (88) independently of Einstein, taking the variation

$$\delta H = \int G_{mn} \delta g^{mn} dV \quad (92)$$

of the integral

$$H = \int R dV. \quad (93)$$

Though independent of the choice of the coordinate map, the Hilbert integral does not contain any information on the gravitational field energy. Therefore, Einstein substituted it for the integral

$$E = \int \mathcal{L} dV, \quad (94)$$

where

$$\mathcal{L} = g^{mn} (\Gamma_{mb}^a \Gamma_{an}^b - \Gamma_{sa}^a \Gamma_{mn}^s). \quad (95)$$

which satisfies the necessary condition

$$\delta E = \delta H. \quad (96)$$

The Einstein integral (94) depends on the choice of the coordinate map, and thus, it disadvantageously differs from the Hilbert integral (91) but contains information on the gravitational field energy. Based on this integral Einstein determined the so-called pseudotensor of the gravitational field energy which, like the integral that generated it, depends on the choice of the coordinate map, which is in disagreement with his requirement to formulate the laws of Nature independently of the choice of coordinates. This point is in the focus of a seventy-year discussion.

The introduction of the energy pseudotensor cannot be justified also from the point of view of the tensor analysis unless one introduces one more object - the background affine connection $\tilde{\Gamma}_{mn}^a$ independent of the cometric tensor g^{ab} . Having introduced this connection, let us substitute the Einstein integral (94) for

$$E = \int \mathcal{L} dV, \quad (97)$$

where

$$\mathcal{L} = g^{mn} (P_{mb}^a P_{an}^b - P_{sa}^a P_{mn}^s), \quad (98)$$

and P_{mn}^a is the affine deformation tensor (48). Variation of the integral (97) at the fixed background connection equals

$$\delta \mathbb{I} = \int (S_{mn} - \frac{1}{2} S g_{mn}) \delta g^{mn} dV, \quad (99)$$

where

$$S_{mn} = R_{mn} - \frac{1}{2} (\check{R}_{mn} + \check{R}_{nm}), \quad (100)$$

$$S = g^{ab} S_{ab}. \quad (101)$$

(Here it is assumed that the background connection is symmetric but not necessarily equiaffine). Therefore, the gravitational equations (89), and correspondingly (90), are changed by

$$S_{mn} - \frac{1}{2} S g_{mn} = 8 \pi \gamma M_{mn}, \quad (102)$$

or, which is the same,

$$S_{mn} = 8 \pi \gamma (M_{mn} - \frac{1}{2} M \theta_{mn}). \quad (103)$$

As one can see, equations (89) are conserved provided that

$$\check{R}_{mn} + \check{R}_{nm} = 0. \quad (104)$$

But to return back to the Einstein case itself, the background connection should satisfy a stronger condition than (104). The fact is that the canonical tensor of the gravitational field energy, given by the Lagrangian (98), coincides with the Einstein pseudotensor provided that in a given coordinate map the following equality holds:

$$\check{\Gamma}_{mn}^a = 0, \quad (105)$$

but then

$$\check{R}_{mnb}^a = 0. \quad (106)$$

On the contrary, if the condition (106) is fulfilled, then

one can find such a coordinate map in which the coordinate equality (105) is valid.

So the theory of gravity with two affine connections developed here includes the Einstein theory as a special case with the condition (105).

Assuming that the action of sources of the gravitational field is independent of the choice of the background connection, we get the equality

$$\nabla_s g^{sm} M_{mn} = 0. \quad (107)$$

Therefore, from eq.(102) we get the corollary

$$\nabla_s g^{sm} (\check{R}_{mn} + \check{R}_{nm} - g^{ab} \check{R}_{ab} g_{mn}) = 0. \quad (108)$$

It is interesting that left-hand side of the last equality can be transformed into

$$(\check{\nabla}_s - P_s) [g^{sm} (\check{R}_{mn} + \check{R}_{nm})] - g^{ab} \check{R}_{ab}, \quad (109)$$

where

$$P_s = P_{sa}^a. \quad (110)$$

Therefore, if the background connection satisfies the condition

$$\check{\nabla}_s (\check{R}_{mn} + \check{R}_{nm}) = 0, \quad (111)$$

then according to (108) eq.(102) results in

$$(\check{R}_{mn} + \check{R}_{nm}) \phi^m = 0, \quad (112)$$

where

$$\phi^a = (\check{\nabla}_s - P_s) g^{sa} = g^{mn} P_{mn}^a. \quad (113)$$

The corollary (112) is of great interest in connection with the discussion of harmonic coordinates as the condition of harmonicity can be written as

$$\phi^a = 0. \quad (114)$$

Therefore, the vector Φ^a will be called the anharmonicity vector.

7. CHOICE OF THE BACKGROUND CONNECTION

In the case when there is no gravity we assume that

$$\Gamma_{mn}^a = \check{\Gamma}_{mn}^a. \quad (115)$$

This is a trivial solution of the gravitational equations (103). Indeed, in this case everywhere on the manifold the mass tensor should be equal to zero, and consequently, these equations should take the form

$$S_{mn} = 0. \quad (116)$$

The condition (115) means that the background connection can be represented in the form of the Christoffel connection. This connection should necessarily be equiaffine. Consequently, eq.(116) should take the form

$$R_{mn} = \check{R}_{mn}. \quad (117)$$

The theory expounded in sect.6 does not impose any other conditions on the background connection and admits a large freedom in choosing them.

Let us use this freedom and assume that in the absence of gravity the metric takes the form

$$g_{ab} dx^a dx^b = c^2 dt^2 - L_{\alpha\beta} dx^\alpha dx^\beta, \quad (118)$$

where the components $L_{\alpha\beta}$ are independent of the coordinate $x^4 = t$ and form the metric (1). Consequently, the background connection is assumed to be equal to the Christoffel connection, given by the metric (118), and we find it to be equal to

$$\begin{aligned} \check{\Gamma}_{\mu\nu}^\alpha &= L_{\mu\nu}^\alpha, & \check{\Gamma}_{44}^\alpha &= 0, \\ \check{\Gamma}_{\mu 4}^\alpha &= 0, & \check{\Gamma}_{4\nu}^\alpha &= 0, & \check{\Gamma}_{mn}^4 &= 0. \end{aligned} \quad (119)$$

It is interesting that the light velocity c didn't enter into the background connection in spite of the fact that it explicitly enters into the metric (118). Therefore, the

background connection (119) coincides with the previously chosen one, i.e., with the connection (46) at $U = \text{const}$.

Correspondingly, the curvature tensor of the background connection equals the tensor (59) at $U = \text{const}$, i.e.,

$$\begin{aligned} \check{R}_{mnb}^4 &= 0, & \check{R}_{4nb}^a &= 0, & \check{R}_{m4b}^a &= 0, & \check{R}_{mn4}^a &= 0, \\ \check{R}_{\mu\nu\beta}^\alpha &= L_{\mu\nu\beta}^\alpha = (L_{\mu\beta}^\alpha \delta_\nu^\alpha - L_{\nu\beta}^\alpha \delta_\mu^\alpha) k^{-2}. \end{aligned} \quad (120)$$

The contracted curvature tensor of the background connection equals the tensor (61) at $U = \text{const}$, i.e.,

$$\check{R}_{\mu\nu} = -2 k^{-2} L_{\mu\nu}, \quad \check{R}_{44} = 0, \quad \check{R}_{\mu 4} = 0, \quad \check{R}_{4\nu} = 0. \quad (121)$$

It is remarkable that the covariant derivative of the tensor field (120) with respect to the background connection (119) equals zero:

$$\check{\nabla}_s \check{R}_{mnb} = 0. \quad (122)$$

Therefore, the condition (111) is fulfilled and we have the corollary (112) that in the present case means

$$L_{\mu\nu} \Phi^\nu = 0. \quad (123)$$

As the determinant of the matrix $(L_{\mu\nu})$ is not equal to zero, of four conditions of harmonicity (114) three of them

$$\Phi^\alpha = 0 \quad (124)$$

are corollary of the gravitational equations (103).

8. SOLUTION OF THE SCHWARZSCHILD PROBLEM

IN THE LOBACHEVSKY SPACE

Let us solve the equations

$$R_{4n} = 0, \quad R_{m4} = 0, \quad R_{\mu\nu} = -2 k^{-2} L_{\mu\nu}, \quad (125)$$

assuming that

$$g_{ab} dx^a dx^b = V^2 dt^2 - F^2 d\rho^2 - H^2 d\Omega^2, \quad (126)$$

where

$$d\Omega^2 = d\theta^2 + \sin^2 \theta d\varphi^2, \quad (127)$$

and the functions V, F, H depend only on the coordinate ρ .

In this case, nonzero components of the connection (86) equal

$$\Gamma_{41}^4 = V^{-1} \frac{dV}{d\rho} = \Gamma_{14}^4; \quad \Gamma_{44}^1 = F^{-2} V \frac{dV}{d\rho}, \quad (128)$$

$$\Gamma_{11}^1 = F^{-1} \frac{dF}{d\rho}, \quad \Gamma_{22}^1 = -F^{-2} H \frac{dH}{d\rho}, \quad \Gamma_{33}^1 = \Gamma_{22}^1 \sin^2 \theta;$$

$$\Gamma_{12}^2 = H^{-1} \frac{dH}{d\rho} = \Gamma_{21}^2, \quad \Gamma_{33}^2 = -\sin \theta \cos \theta;$$

$$\Gamma_{13}^3 = H^{-1} \frac{dH}{d\rho} = \Gamma_{31}^3, \quad \Gamma_{23}^3 = \text{ctg} \theta = \Gamma_{32}^3.$$

In the same coordinates nonzero components of the background connection equal

$$\check{\Gamma}_{22}^1 = -k \text{sh} \frac{\rho}{k} \text{ch} \frac{\rho}{k}, \quad \check{\Gamma}_{33}^1 = \check{\Gamma}_{22}^1 \sin^2 \theta; \quad (129)$$

$$\check{\Gamma}_{12}^2 = k^{-1} \text{cth} \frac{\rho}{k} = \check{\Gamma}_{21}^2, \quad \check{\Gamma}_{33}^2 = -\sin \theta \cos \theta;$$

$$\check{\Gamma}_{13}^3 = k^{-1} \text{cth} \frac{\rho}{k} = \check{\Gamma}_{31}^3, \quad \check{\Gamma}_{23}^3 = \text{ctg} \theta = \check{\Gamma}_{32}^3.$$

Hence, we find the anharmonicity vector

$$\phi^1 = \frac{1}{V F H^2} \left[k F V \text{sh} \frac{2\rho}{k} - \frac{d}{d\rho} \left(\frac{V H^2}{F} \right) \right], \quad (130)$$

$$\phi^2 = 0, \quad \phi^3 = 0, \quad \phi^4 = 0.$$

According to (124), eqs.(125) result in the equality $\phi^1 = 0$, which is equivalent to the equality

$$\frac{d}{d\rho} (F^{-1} V H^2) = F V k \text{sh} \frac{2\rho}{k}. \quad (131)$$

In this case, all nondiagonal components of the tensor

R_{mn} are equal to zero, and nondiagonal ones satisfy the condition

$$R_{33} = R_{22} \sin^2 \theta. \quad (132)$$

Therefore, of eqs.(125) it remains to satisfy only the following three equations:

$$R_{44} = 0, \quad R_{11} = -2k^{-2}, \quad R_{22} = -2 \text{sh}^2 \frac{\rho}{k}. \quad (133)$$

In this case we have

$$R_{44} = F^{-1} H^{-2} V \frac{d}{d\rho} \left(\frac{H}{F} \frac{dV}{d\rho} \right),$$

$$R_{22} = 1 - \frac{1}{V F} \frac{d}{d\rho} \left(\frac{V H}{F} \frac{dH}{d\rho} \right), \quad (134)$$

$$\frac{1}{2} H (R_{11} + V^{-2} F^2 R_{44}) = \frac{dH}{d\rho} \frac{1}{V F} \frac{d(FV)}{d\rho} - \frac{d^2 H}{d\rho^2}.$$

According to (133) and (134) we get the following equations:

$$\frac{d^2 H}{d\rho^2} - \frac{H}{k^2} = \frac{dH}{d\rho} \frac{1}{V F} \frac{d(FV)}{d\rho}, \quad (135)$$

$$\frac{d}{d\rho} \left(\frac{H^2}{F} \frac{dV}{d\rho} \right) = 0, \quad (136)$$

$$\frac{d}{d\rho} \left(\frac{V H}{F} \frac{dH}{d\rho} \right) = F V \text{ch} \frac{2\rho}{k}. \quad (137)$$

Before solving these equations let us prove immediately that they result in the condition of harmonicity (131). Let us consider the covariant divergence

$$V_b G_a^b = \partial_b G_a^b - \Gamma_{ba}^s G_s^b + \Gamma_{bs}^b G_a^s \quad (138)$$

of the Einstein tensor

$$G_a^b = R_{as} g^{sb} - \frac{1}{2} R \delta_a^b, \quad (139)$$

which equals zero, as is known. In the considered case, the tensor (139) is diagonal and independent of angles and time. Therefore, the first component of the covector (138) equals

$$\frac{d}{d\rho} G_1^1 + \Gamma_{b1}^b G_1^1 - \Gamma_{11}^1 G_1^1 - \Gamma_{21}^2 G_2^2 - \Gamma_{31}^3 G_3^3 - \Gamma_{41}^4 G_4^4, \quad (140)$$

As in the considered case $\Gamma_{31}^3 = \Gamma_{21}^2$, $G_3^3 = G_2^2$, then

$$V_b G_1^b = \frac{d}{d\rho} G_1^1 + 2 \Gamma_{21}^2 (G_1^1 - G_2^2) + \Gamma_{41}^4 (G_1^1 - G_4^4). \quad (141)$$

Then, taking into account the next formulae for the differentiable tensor

$$G_1^1 = A - B, \quad G_2^2 = -A - C, \quad G_4^4 = -A - B, \quad (142)$$

where

$$A = H^{-1} F^{-2} [H'' - H' (FV)^{-1} (FV)'],$$

$$B = V^{-1} F^{-1} H^{-2} (V H F^{-1} H')' - H^{-2}, \quad (143)$$

$$C = V^{-1} F^{-1} H^{-2} (H^2 F^{-1} V')',$$

and the formulae

$$\Gamma_{21}^2 = H^{-1} H', \quad \Gamma_{41}^4 = V^{-1} V' \quad (144)$$

for connection, one can easily be convinced that the combination

$$V_b G_1^b = (A - B)' + 2 H^{-1} H' (2A - B + C) + 2 V^{-1} V' A \quad (145)$$

equals zero whatever the functions F , H , V be. If these functions satisfy the gravitational equations (135), (136) and (137), then

$$A = F^{-2} k^{-2}, \quad B = 2 H^{-2} sh^2 \frac{\rho}{k}, \quad C = 0. \quad (146)$$

Substituting these expressions into (145) we get

$$V_b G_1^b = 2 k^{-2} H^{-2} F^{-1} V^{-1} [(F^{-1} V H^2)' - k F V sh \frac{2\rho}{k}] \quad (147)$$

Since we have proved that (145) equals zero, we have also proved the corollary (131).

A partial solution of the system of equations (135), (136), (137) satisfies the condition

$$FV = C, \quad (148)$$

where $C = \text{const.}$

Indeed, it follows from eq.(136) that

$$H^2 \frac{dV}{d\rho} = BF, \quad (149)$$

where $B = \text{const.}$ Substituting into the latter the condition (148) we get

$$\frac{d}{d\rho} \left(\frac{1}{2} V^2 \right) = BC H^{-2}. \quad (150)$$

Substituting the condition (148) into (135) we get the equation

$$\frac{d^2 H}{d\rho^2} - \frac{H}{k^2} = 0, \quad (151)$$

whose general solution is

$$H = P k sh \frac{\rho + \hat{\rho}}{k}, \quad (152)$$

where P and $\hat{\rho}$ are the integration constants. Substituting this solution into (150), we get

$$\frac{1}{2} V^2 = N - BC P^{-2} k^{-1} cth \frac{\rho + \hat{\rho}}{k}, \quad (153)$$

where N is one more integration constant.

We have to consider only eq.(137) but instead we can consider the sum

$$\frac{d}{d\rho} [(FV)^{-1} \frac{d}{d\rho} \left(\frac{1}{2} V^2 H^2 \right)] = FV ch \frac{2\rho}{k} \quad (154)$$

of equations (136) and (137), which is more convenient. Substituting here the condition (148), we get the following equation:

$$\frac{d^2}{d\rho^2} \left(\frac{1}{2} V^2 H^2 \right) = C^2 ch \frac{2\rho}{k}. \quad (155)$$

This equation is to be put in conformity with the above

results (152) and (153) from which it follows that

$$\frac{1}{2} V^2 H^2 = \quad (156)$$

$$= [N P^2 k \operatorname{sh} \frac{\rho + \hat{\rho}}{k} - B C \operatorname{ch} \frac{\rho + \hat{\rho}}{k}] k \operatorname{sh} \frac{\rho + \hat{\rho}}{k} .$$

Differentiating this function we get its derivatives

$$\left(\frac{1}{2} V^2 H^2 \right)' = N P^2 k \operatorname{sh} \frac{2}{k} (\rho + \hat{\rho}) - B C \operatorname{ch} \frac{2}{k} (\rho + \hat{\rho}), \quad (157)$$

$$\left(\frac{1}{2} V^2 H^2 \right)'' = 2 N P^2 \operatorname{ch} \frac{2}{k} (\rho + \hat{\rho}) - 2 B C k^{-1} \operatorname{sh} \frac{2}{k} (\rho + \hat{\rho}).$$

Comparing this result with eq.(155) we find that the integration constants should satisfy the following conditions:

$$2 N P^2 \operatorname{ch} \frac{2 \hat{\rho}}{k} - 2 B C k^{-1} \operatorname{sh} \frac{2 \hat{\rho}}{k} = C^2, \quad (158)$$

$$2 N P^2 \operatorname{sh} \frac{2 \hat{\rho}}{k} - 2 B C k^{-1} \operatorname{ch} \frac{2 \hat{\rho}}{k} = 0 .$$

Hence, we find

$$2 B = C k \operatorname{sh} \frac{2 \hat{\rho}}{k}, \quad 2 N P^2 = C^2 \operatorname{ch} \frac{2 \hat{\rho}}{k}. \quad (159)$$

Substituting this into (156) we get

$$V^2 H^2 = C^2 k^2 \operatorname{sh} \frac{\rho + \hat{\rho}}{k} \operatorname{sh} \frac{\rho - \hat{\rho}}{k}. \quad (160)$$

Hence, on the basis of (152) we get

$$V^2 = C^2 P^{-2} \operatorname{sh} \frac{\rho - \hat{\rho}}{k} / \operatorname{sh} \frac{\rho + \hat{\rho}}{k}, \quad (161)$$

and then on the basis of (148) we get

$$F^2 = P^2 \operatorname{sh} \frac{\rho + \hat{\rho}}{k} / \operatorname{sh} \frac{\rho - \hat{\rho}}{k}. \quad (162)$$

It is interesting to verify that the condition of harmonicity (131) is fulfilled. Indeed, according to (148) it means that

$$\frac{d}{d\rho} (V^2 H^2) = C^2 k \operatorname{sh} \frac{2\rho}{k}, \quad (163)$$

and according to (160)

$$V^2 H^2 = \frac{1}{2} C^2 k^2 \left[\operatorname{ch} \frac{2\rho}{k} - \operatorname{ch} \frac{2\hat{\rho}}{k} \right]. \quad (164)$$

Consequently, the condition (163) is fulfilled.

Thus, the metric (126), satisfying eqs.(125), is found in the form

$$C^2 P^{-2} \Xi d t^2 - P^2 k^2 \{ \Xi^{-1} d \xi^2 + \operatorname{sh}^2(\xi + \alpha) d \Omega^2 \}, \quad (165)$$

where

$$\Xi = \frac{\operatorname{sh}(\xi - \alpha)}{\operatorname{sh}(\xi + \alpha)}, \quad \xi = \rho / k, \quad \alpha = \hat{\rho} / k. \quad (166)$$

At a large distance from the source, i.e. at large values of ξ , it should asymptotically approach the metric (118), more exactly the metric

$$c^2 d t^2 - k^2 \{ d \xi^2 + \operatorname{sh}^2 \xi d \Omega^2 \}. \quad (167)$$

Hence, it follows that

$$C = c, \quad P = \exp \{ - \alpha \}. \quad (168)$$

It remains to elucidate what is the parameter α equal to. We find it from the condition that at large values of ξ the connection (128) should tend to the connection (46). In other words, all the components of the connection (128) should tend to the background connection (129), except for the component Γ_{44}^1 , that should tend not to zero but to U' , i.e.,

$$\Gamma_{44}^1 \longrightarrow U' = \gamma M (k \operatorname{sh} \xi)^{-2}. \quad (169)$$

Note that the latter includes the very "attractive force" about which Lobachevsky wrote in 1835 [p.159.]

This condition can be fulfilled as

$$F^{-1} F' = -H^{-1} H' = \frac{1}{2k} [\operatorname{cth}(\xi + \alpha) - \operatorname{cth}(\xi - \alpha)],$$

$$H^{-1} H' = k^{-1} \operatorname{cth}(\xi + \alpha), \quad (170)$$

$$H H' = k P^2 \operatorname{sh}(\xi + \alpha) \operatorname{ch}(\xi - \alpha),$$

$$V V' = \frac{1}{2} k c^2 \operatorname{sh} 2\alpha / [k P \operatorname{sh}(\xi + \alpha)]^2.$$

Comparing (169) with (170) we get that

$$\frac{1}{2} \operatorname{sh} 2\alpha = \frac{\gamma M}{k c^2}. \quad (171)$$

As the gravitational radius $\gamma M c^{-2}$ is much smaller than the constant k we can approximately put

$$\alpha = \frac{\gamma M}{k c^2}, \quad P = 1. \quad (172)$$

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Received by Publishing Department
 on September 21, 1992.

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E2-92-394

Гравитационный радиус в пространстве Лобачевского

Сформулированы уравнения тяготения в пространстве Лобачевского. Решена задача о гравитационном поле точечной массы в пространстве Лобачевского. В ньютоновском (нерелятивистском) случае эта задача была поставлена и решена самим Лобачевским. В релятивистском случае задача состоит в том, чтобы сначала найти адекватные уравнения для метрики, описывающей гравитационное поле, а затем найти и решение этих уравнений. Такие уравнения найдены автором на основе разработанной им ранее теории с двумя связностями, одна из которых называется кристоффелевой, а вторая — фоновой. Последняя задается уравнениями движения свободной материальной точки в пространстве Лобачевского. Она не зависит от скорости света c . Найденная здесь статическая сферически симметричная метрика зависит от отношения гравитационного радиуса $\gamma M c^{-2}$ массы M к константе k Лобачевского для видимого мира. В пределе $k \rightarrow \infty$ она переходит в известную метрику Шварцшильда.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1992

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E2-92-394

The Gravitational Radius in the Lobachevsky Space

Equations of gravitation in the Lobachevsky space are formulated. The problem of the gravitational field of point mass in the Lobachevsky space is solved. In the Newton (nonrelativistic) case, this problem was posed and solved by Lobachevsky himself. In the relativistic case, the problem consists in that one should first find adequate equations for the metric describing the gravitational field and then find their solutions. These equations are found by the author on the basis of the theory, developed by him, with two affine connections; one called Christoffel and other, background. The latter is given by the equations of motion of a free material particle in the Lobachevsky space. It is independent of the light velocity c . The static, spherically symmetric metric found here depends on the ratio of the gravitational radius $\gamma M c^{-2}$ of mass M to the Lobachevsky constant k for the visible world. In the limit $k \rightarrow \infty$ it turns into the known Schwarzschild metric.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1992