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Об электрических тороидальных соленоидах

Дана явная реализация электрического соленоида. Обсуждаются его свойства.

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Afanasiev G.N. E2-92-39
On the Electrical Toroidal Solenoids

We give explicit construction of electrical toroidal solenoid and discuss its properties.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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1. Introduction

Electrical toroidal solenoids (ETS) are much less known objects than the magnetic ones. There are known theoretical realization of the infinitely small ETS in terms of δ -functions /1/ and the nonphysioal realization in terms of magnetio monopoles current /2/. It is the aim of this oommunication to present more realistic construction of ETS and discuss its properties.

2. Some faots concerning magnetic toroidal solenoids

Consider the torus T

$$(\rho-d)^2+z^2=R^2. \qquad (2.1)$$

Introduoe ooordinates $\tilde{R},\psi$: $\rho=d+\tilde{R}\cos\psi$, $z=\tilde{R}\cdot\sin\psi$. The value of $\tilde{R}=R$ oorresponds to the torus T. Let the constant poloidal current (fig.1) flow over its surface. The density of this current is

$$\vec{j}=-\frac{gc}{4\pi}\,\frac{\delta(\tilde{R}-R)}{d+R\cos\psi}\cdot\vec{n}_\psi . \qquad (2.2)$$

Here $g=2NI/c$, N is the total number of turns in the poloidal coil, I is the current flowing in a particular turn, $\vec{n}_\psi$ is the unit veotor defining the current direction on the torus surface $\vec{n}_\psi=\vec{n}_z\cdot\cos\psi-(\vec{n}_x\cdot\cos\varphi+\vec{n}_y\cdot\sin\varphi)\cdot\sin\psi$. The constant g may be also expressed through the magnetio flux Φ inside T : $g=\Phi\cdot[2\pi(d-\sqrt{d^2-R^2})]^{-1}$. Magnetic field (MF) $\vec{H}=0$ outside T and $\vec{H}=\vec{n}_\varphi\cdot g/\rho$ inside it. Here ρ is the distanoe of the particular point inside T from the torus symmetry axes ($\rho=d+\tilde{R}\cos\psi$). The vector potential (VP) of the magnetic toroidal solenoid (MTS) was obtained in ref. /3/.

Its properties were discussed in /4/. In the integral form the non-vanishing cylindrical components of VP are

$$A_z = \frac{g\sqrt{R}}{2\pi}\int_0^{2\pi} d\varphi\, \frac{d-\rho\cos\varphi}{q^{3/2}} \cdot Q_{\frac{1}{2}}(\mathrm{ch}\,\mu),$$

$$A_\rho = \frac{g\sqrt{R}}{2\pi}\, z\int_0^{2\pi} d\varphi\, \frac{\cos\varphi}{q^{3/2}} \cdot Q_{\frac{1}{2}}(\mathrm{ch}\,\mu) \qquad (2.3)$$

$(\mathrm{ch}\,\mu = (r^2+d^2+R^2-2d\rho\cos\varphi)/2Rq,\ q^2=(\rho\cos\varphi-d)^2+z^2,\ r^2=\rho^2+z^2;$ $Q_\nu(x)$ is the Legendre function of the 2nd kind). For the infinitely thin TS ($R \ll d$) these integrals can be taken in a closed form

$$A_z = \frac{gR^2}{2(d\rho)^{3/2}}\,\frac{1}{\mathrm{sh}\,\mu_1}\left[\rho\cdot Q^1_{\frac{1}{2}}(\mathrm{ch}\,\mu_1) - d\cdot Q^1_{-\frac{1}{2}}(\mathrm{ch}\,\mu_1)\right], \qquad (2.4)$$

$$A_\rho = -\frac{gR^2}{2(d\rho)^{3/2}}\,\frac{z}{\mathrm{sh}\,\mu_1}\cdot Q^1_{\frac{1}{2}}(\mathrm{ch}\,\mu_1),\quad \mathrm{ch}\,\mu_1 = \frac{r^2+d^2}{2d\rho}.$$

At large distances VP falls as r^{-3}

$$A_z \sim \frac{1}{8}\pi g d R^2\,\frac{1+3\cos2\theta}{r^3},\quad A_\rho \sim \frac{3}{8}\pi g d R^2\,\frac{\sin 2\theta}{r^3}.$$

3. An alternative viewpoint on the MTS

Instead of the poloidal current (2.1) one may equally use /5,6/ the magnetization $\vec{j} = c\cdot \mathrm{rot}\,\vec{M}$. It is confined completely inside TS and given by

$$\vec{M} = \mathcal{M}\cdot\vec{n}_\varphi,\quad \mathcal{M} = \frac{g}{4\pi}\,\frac{\Theta(R-\tilde{R})}{d+\tilde{R}\cos\psi} = \frac{g}{4\pi}\,\frac{\Theta(R-\sqrt{(\rho-d)^2+z^2})}{\rho}. \qquad (3.1)$$

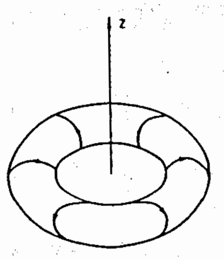

Fig.1. Poloidal current flowing on the surface of torus.

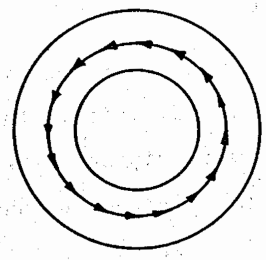

Fig.2. The explicit realization of magnetic (electric) toroidal solenoid by means of circular magnetic (electric) dipole chain.

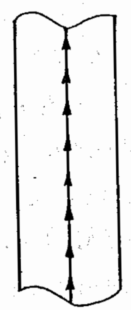

Fig.3. The explicit realization of magnetic (electric) cylindrical solenoid by means of linear magnetic (electric) dipole chain.

Here $\Theta(x)$ is the step function. For the infinitely thin TS ($R \ll d$) $\mathcal{M}$ reduces to

$$\mathcal{M} = \frac{1}{4\pi} \Phi \cdot \delta(\rho - d) \cdot \delta(z). \tag{3.2}$$

The VP is expressed through the magnetization as follows /6/:

$$\vec{A}(\vec{r}) = \int \mathrm{rot}\,\vec{M}(\vec{r}\,') \frac{1}{|\vec{r} - \vec{r}\,'|} dV' = \vec{\nabla} \times \int \frac{\vec{M}(\vec{r}\,')}{|\vec{r} - \vec{r}\,'|} dV' = \tag{3.3}$$

$$= \int \vec{M}(\vec{r}\,') \times \frac{\vec{r} - \vec{r}\,'}{|\vec{r} - \vec{r}\,'|^3} dV'.$$

What is the physical meaning of these relations? Eqs. (3.2) and (3.3) mean that infinitely thin MTS may be realized as the closed chain of magnetic dipoles (fig.2). In fact, the value of VP at the point $\vec{r}$ induced by the magnetic dipole situated at $\vec{r}_0$ is given by (see, e.g., /5/)

$$\vec{A}(\vec{r}) = m \frac{\vec{n} \times (\vec{r} - \vec{r}_0)}{|\vec{r} - \vec{r}_0|^3}. \tag{3.4}$$

Here $\vec{n}$ and m are the direction and power of dipole, resp. Integrating this Eq. over the circumference of the radius d lying in the $z=0$ plane ($\vec{n} = \vec{n}_\varphi$, $\vec{r}_0 = d \cdot \vec{n}_\rho$, $\vec{n}_\varphi = \vec{n}_y \cos\varphi - \vec{n}_x \sin\varphi$, $\vec{n}_\rho = \vec{n}_x \cos\varphi + \vec{n}_y \sin\varphi$) we arrive to (2.4) with $g = 4m/R^2$ or $\Phi = 4\pi m/d$. Eqs. (3.1) and (3.3) mean that finite MTS may be realized as a closed spin tube of the radius R. In fact integrating (3.4) over the volume of T where under m in (3.4) one should understand the spin density that coincides with the magnetization $\mathcal{M}$ given by (3.1) we get Eqs.(2.3). The closed spin tube (ferromagnetic ring with the magnetization independent of applied fields) was used /7/ for the experimental

investigation of the Aharonov – Bohm (AB) effect.

The simpler case presents the cylindrical solenoid. It may be realized (fig.3) as a straight line spin chain (or tube). In fact integrating Eq.(3.4) over z axis we arrive to the VP of the cylindrical solenoid $\vec{A} = \vec{n}_\varphi \cdot \Phi / 2\pi\rho$, $\Phi = 4\pi m$. Such spin chain (magnetized whisker) was used in earlier experiments testing AB effect (see, e.g., their review in /8/).

4. Electrical toroidal solenoids

Now we substitute the magnetic dipoles by the electric ones. Then all Eqs. obtained in the previous section remain the same. Particularly, the electric VP is given by Eq.(3.3). The electric field (EF) equals zero outside the electric toroidal solenoid (ETS). Inside it $\vec{D} = \vec{n}_\varphi \Phi \cdot [2\pi\rho(d - \sqrt{d^2 - R^2})]^{-1}$ for the finite electric flux tube and $\vec{D} = \vec{n}_\varphi \cdot \Phi\, \delta(\rho - d)\, \delta(z)$ for the infinitely thin one. Here Φ is the EF flux through ETS cross-section: $\Phi = \int D_\varphi \cdot d\rho\, dz$. The EF is generated by the single electric dipole chain for the infinitely thin ETS (fig.2) and by their continuous superposition (with electrization $\vec{\mathcal{E}} = \mathcal{E} \cdot \vec{n}_\varphi$, $\mathcal{E} = g \cdot \Theta(R - \sqrt{(\rho - d)^2 + z^2})/4\pi\rho$) for the finite ETS.

How to verify the existence of EF inside ETS? There are the same means as for the MTS. We briefly enumerate then:

1) the electromagnetic field (EMF) strengths appear outside ETS when it moves uniformly in the medium with $\epsilon\mu \neq 1$ /9/;

2) EMF strengths appear outside the accelerated ETS (both in the vacuum or medium) /10/;

3) the interaction of the external EF with the electric dipoles confined inside ETS is given by

$$\mathcal{U} = -\int \vec{E}_{ext} \cdot \vec{\mathcal{E}}\, dV. \qquad (4.1)$$

At large distances from the source of the external EF (or for small dimensions of ETS) this Eq. reduces to

$$\mathcal{U} = -\frac{1}{2}\, \mathcal{E}_t \cdot \mathrm{rot}\, \vec{E}_{ext} = \frac{1}{2c}\, \frac{\partial \vec{H}_{ext}(\vec{r})}{\partial t} \cdot \vec{\mathcal{E}}_t . \qquad (4.2)$$

Here $\vec{\mathcal{E}}_t = \int \vec{r} \times \vec{\mathcal{E}}\, dV$ is the so-called toroidal electric moment /2/. For the electrization $\vec{\mathcal{E}}$ given above $\vec{\mathcal{E}}_t$ is directed along the ETS symmetry axis and is equal to $\mathcal{E}_t = \frac{1}{2}\pi g d R^2$. Eq. (4.2) means that at large distances ETS interacts with time varying MF /2/. Eq. (4.2) was used in ref. /11/ to explain the observed rotation of nonmagnetic molecules in the uniform MF slowly varying with time /12/. The situation looks much simpler for the cylindrical electric solenoid which is obtained by inserting the linear electric dipole chain into the cylinder (fig.3). This solenoid tends to be oriented along the external EF.

5. Discussion

In the examples considered at the end of the previous section we have either forced the EMF to come out of the ETS by putting it into the motion or permitted the external EF to penetrate inside ETS and interact with electric dipoles. Now we fix the position of ETS. There is nonvanishing electric VP outside it. This VP cannot be eliminated by the gauge transformation as $\oint A_e dl = \Phi$ for the closed contours passing through the torus hole. Can we prove the existence of electric VP outside ETS without penetrating inside it? (a suitable screen can be used to obtain impenetrability). We do not see here the obvious answer. In fact the analogue of AB effect for this case is the quantum scattering of

free magnetic charges on the electric VP outside ETS. However, these particles (monopoles) have not been found in Nature up to now. Something should be added about the technical realization of ETS. There exist dielectrics called electrets that carry nonzero static electric dipole moment /13/. Among the different types of electrets the most suitable seems to be the ferroelectrics which are the electric analogues of ferromagnetics. From these substances the electrified ring can be manufactured exactly in the same way as magnetized ring in Tonomura experiments /7/.

6. Conclusion

The main result obtained here is that there exists nonzero electric vector potential outside the electric toroidal solenoid. This VP, cannot be eliminated by the gauge transformation and thus it should have the physical meaning. The question is to find the physical effects in which this VP works. Probably this problem may be resolved by those scientists who seek the electric field produced by the closed constant currents /14,15/.

In conclusion, we rephrase the old question posed by Y.Aharonov and D.Bohm in their famous 1959 paper /16/: Does vector potential of the electrical toroidal solenoid have physical meaning?

It is rather curious that superposing the electric and magnetic dipoles distributions inside the torus T we get so-called electromagnetic toroidal solenoid. The electromagnetic field strengths $\vec{D}, \vec{B}$ differ from zero only inside T . Outside it there are nonvanishing electric and magnetic vector potentials.

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