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E2-92-388

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DEEP INELASTIC LEPTON SCATTERING ON NUCLEI: OPERATOR PRODUCT EXPANSION AND MESON-NUCLEON THEORY

Submitted to "Nuclear Physics A"

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*Partially supported by a grant from Ministry of Science, High School and Technical Policy of Russia

1992

1. The preliminaries

Since the EMC effect was discovered ten years ago, an extensive study of the deep inelastic scattering of leptons on nuclei was carried out. However, a short review of the experimental programs of the operating and forthcoming accelerators shows that the interest in lepton-nucleus deep inelastic reactions does not abate.

First of all, we mean experiments at CERN where the renowned EMC and BCDMS collaborations have already obtained a number of interesting results by means of nuclear targets. Among the recent achievements, it is worthwhile to mention the NMC precise data on the F_2^n/F_2^p ratio at small x extracted from the combined proton-deuteron measurements.

In view of the so-called "spin crisis", the SMC and NMC results on the μD, μ^3He reactions with polarized particles are anticipated impatiently. Next, the research program of the new electron accelerator CEBAF includes the study of deep inelastic scattering of electrons on nuclei near the boundary of the one-nucleon kinematics ($x \sim 1$). In the present context, the possibility of the detection of neutral and charged currents on nuclear targets (such as the deuteron or even heavier nuclei) at the HERA set up at DESY appears to be very important. Finally, new prospects in the investigation of the processes in question may emerge with the creation of hypothetical machines like UNK or the 10-20 Gev European project.

Since the information about the neutron structure is predominantly obtained by means of the nuclear processes, the study of the high energy lepton-nucleus reactions is important not only for the investigation of the "nuclear QCD effects" (short NN-distance phenomena, shadowing etc.) but for the particle physics as well. Both these aspects produce high requirements on the quantitative description of nuclear structure effects in deep inelastic scattering. Consequently, there is a need in an accurate method of taking into account such effects.

The discovery of the EMC effect initiated a large variety of theoretical works clarifying the gist of the detected phenomenon and the role of nuclear effects in deep inelastic scattering in general. Today, after a decade or so this amount of models could be conventionally divided into two large classes being the two faces of the basic idea of the change of nucleon properties in the nuclear medium. These are the well-known x-rescaling [1, 2] and Q^2-rescaling models [3].

The confiding parameters determining the EMC-like behavior of the A-dependence of the nuclear structure function were first identified by Vagradov's group [1] and, a while later, by Birbrair *et al* [2]. These are nothing else but slight shift ($\sim 5\%$) of the nucleon mass and the Fermi motion, that together lead to the x-rescaling. Here we mention that many authors utilized this fruitful idea [4, 5] and sometimes it happened that the x-rescaling was "rediscovered". We want to stress that despite the success of this type models they are phenomenological and the problem of their consistent theoretical investigation is still open.

In the present paper, we propose a rigorous theoretical treatment of deep inelastic scattering on nuclei. We consider the simplest nuclear system, the deuteron, within the meson-nucleon model. We analyze the deuteron ground state and the interaction operator in a consistent way [6, 7]. Applying the operator product expansion method we find the explicit form of the deuteron structure function moments. The inverse Mellin transformation turns the deuteron structure functions in the convolution form into terms of constituents relevant to determine the NN-potential [8]-[10] and the nuclear structure, viz. nucleons and mesons. The nucleon part (with corrections caused by the interaction) could be exactly reduced to the results of the x-rescaling model and the remaining part is corrections of the meson exchange currents.

The calculations are made in the following approximations:

1. Up to the second order in the meson-nucleon coupling constant g, which corresponds to the usual approximations in nuclear physics in deriving the potential and Schrödinger equation, g^2-approximation;
2. in the leading twist approximation, i.e. when corrections $\sim m^2/Q^2$ are negligible, *"twist two"*-approximation.

The results are extended to the scattering on heavy nuclei and formidable computations of the nuclear spectral function are avoided. The diagrams of the scattering on bound nucleons have also been computed numerically and the results conform to the earlier x-rescaling calculations.

2. The x-rescaling model

The main idea of the x-rescaling model is based on the well-known fact that the properties of quasiparticles, nucleons, differ from those of free nucleons. In particular, the bound nucleons have an effective mass depending on the shell energy. This leads to the renormalization of the Bjorken scaling variable $x \to m/m^*x$. The original formula

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of the model is [1]

$$F_2^{N/A}(x) = \int_x^{M_A/m} f^{N/A}(y) \cdot F_2^N(x/y)dy, \tag{1}$$

$$f^{N/A}(y) = \int \frac{d\mathbf{k}}{(2\pi)^3}\, d\varepsilon\, S(\mathbf{k},\varepsilon) \cdot \left(1+\frac{k_3}{m}\right)\delta\left(y-\left[1+\frac{\varepsilon}{m}+\frac{k_3}{m}\right]\right), \qquad A>2; \tag{2}$$

$$f^{N/D}(y) = \int \frac{d\mathbf{k}}{(2\pi)^3} \mid \Psi_D(\mathbf{k})|^2 \cdot \left(1+\frac{k_3}{m}\right)\delta\left(y-\left[1+\frac{\varepsilon_D}{m}-\frac{\mathbf{k}^2}{2m^2}+\frac{k_3}{m}\right]\right), \quad A=2; \tag{3}$$

where M_A and m are the nucleus and nucleon mass; $f^{N/A}(y)$ is the "nucleon distribution function" versus the "longitudinal fraction of the nuclear momentum"; $S(\mathbf{k},\varepsilon)$ and $\Psi_D(\mathbf{k})$ are the nuclear spectral function and the deuteron wave function, respectively; $\mathbf{k}$ and ε are the momentum and energy of the nucleon inside the nucleus. All the nuclear structure effects in (1,2,3) are encoded in the definition of y via δ-function. The essential distinction of the model is that the nucleons carry out only a part of the total momentum of the nuclear target. In fact, to estimate the nucleus structure behavior in the middle x region ($x \sim 0.3-0.7$), one could expand the integrand in (1) near $\langle y\rangle$:

$$F_2^{N/A}(x) \approx F_2^N(x/\langle y\rangle)dy, \tag{4}$$

where $\langle y\rangle = (1+\langle\varepsilon\rangle/m + \langle k_3^2\rangle/m^2) < 1$ (*vide supra* m^*/m !). The formula (4) makes clear the basic x-rescaling idea, which is a shift of the x argument of the bound nucleon structure function in comparison with the free one.

In conclusion of the section note once again that the x-rescaling approach given by the relations like (1), (2) is consistent with the experimental data (see e.g. [4, 5]), but to the point is phenomenlogical. However, it faithfully catches the essence of the effect.

3. The method

We start with the hadronic tensor $W_{\mu\nu}$ that is the imaginary part of the forward Compton amplitude $W_{\mu\nu}^D \propto \mathrm{Im}T_{\mu\nu}^D$. The amplitude $T_{\mu\nu}$ is given by the time-ordered product of two hadron currents:

$$T_{\mu\nu}^D(p_A,q) = i\int d^4x\, e^{iqx}\langle p_A \mid T(J_\mu(x)J_\nu(0)) \mid p_A\rangle, \tag{5}$$

[1]Here we omit detailed discussions, for details see e.g. [4, 5]

where q is the virtual photon momentum and p_A, the momentum of the nuclear target. To calculate the r.h.s (5), one needs a self-consistent model describing (i) the current operator and (ii) the nuclear ground state; both parts are to be treated within the same approach.

The most rigorous analysis of the product of two currents at high momentum transfers is accomplished by the Wilson's operator product expansion (OPE) method. For the deep inelastic scattering the leading operators in the OPE are twist two and the amplitude $T_{\mu\nu}$ can be represented in the form[11]:

$$T_{\mu\nu}^A(p_A,q) = \sum_{a;n=2,4,\dots}^{\infty} C_{a,n}^{(1)}\left(-g_{\mu\nu}+\frac{q_\mu q_\nu}{q^2}\right)\frac{2m_a 2^n q_{\mu_1}\dots q_{\mu_n}}{(-q^2)^n}\langle p_A \mid O_a^{\mu_1\dots\mu_n}(0)\mid p_A\rangle + \tag{6}$$

$$+\sum_{a;n=2,4,\dots}^{\infty} C_{a,n}^{(2)}\left(g_{\mu\mu_1}-\frac{q_\mu q_{\mu_1}}{q^2}\right)\left(g_{\nu\mu_2}-\frac{q_\nu q_{\mu_2}}{q^2}\right)\frac{2m_a 2^n q_{\mu_3}\dots q_{\mu_n}}{(-q^2)^{n-1}}\langle p_A \mid O_a^{\mu_1\dots\mu_n}(0)\mid p_A\rangle,$$

where a runs over the relevant interacting fields which are to determine both the twist two operators $O_a^{\mu_1\dots\mu_n}(0)$ and the nucleus ground state, m_a stands for corresponding masses, $C_{a,n}^{(i)}$ are the Wilson coefficients. We treat the nucleus in the framework of the effective meson-nucleon theory, i.e. the nucleus is represented as a system of interacting mesonic and nucleonic fields.

The starting point of the consideration is the effective Lagrangian of interacting meson and nucleon fields. As an example, we present below the calculations with the scalar mesons described by the theory with the Lagrangian:

$$\mathcal{L} = \bar{N}\,(i\hat{\partial}-m)\,N + \frac{1}{2}\left((\partial\Phi)^2-\mu_\sigma^2\Phi^2\right) - g_\sigma \bar{N}N\Phi, \tag{7}$$

where $N(x)$ and $\Phi(x)$ are the nucleon and scalar meson (σ) fields, respectively; m and μ_σ are the corresponding masses. The generalization to arbitrary kinds of the mesons (isoscalar, vector, etc) contributing to the nucleon-nucleon interaction potential is straightforward.

For the unpolarized scattering within the theory with nucleon and scalar meson fields the operators of $\tau=2$ are:

$$O_N^{\mu_1\dots\mu_n} = \left(\frac{i}{2}\right)^{n-1}\mathcal{S}\left\{\bar{N}(0)\gamma^{\mu_1}\overleftrightarrow{\partial}^{\mu_2}\dots\overleftrightarrow{\partial}^{\mu_n}N(0)\right\}, \tag{8}$$

$$O_\Phi^{\mu_1\dots\mu_n} = \left(\frac{i}{2}\right)^{n}\mathcal{S}\left\{\Phi(0)\overleftrightarrow{\partial}^{\mu_1}\dots\overleftrightarrow{\partial}^{\mu_n}\Phi(0)\right\}, \tag{9}$$

where $\mathcal{S}$ symmetrizes the subsequent operator and removes all traces in $\mu_1 \dots \mu_n$.

For a large momentum transfer q OPE factorizes the amplitude (6) into pieces depending on short and long distance physics. The concrete scales are controlled by the properties of the chosen model. For instance, in asymptotically free theories these are perturbative and nonberturbative regions or the quark and hadron scales in QCD. In the effective meson-nucleon model the short and long distances correspond to the hadron and nuclear scales. In (6) these two pieces are $C^{(1,2)}_{a,n}$ and $\langle p_A \mid O^{\mu_1 \dots \mu_n}_a(0) \mid p_A \rangle$, respectively.

Further consideration of the amplitude (5) is based on the non-relativistic theoretical field approach suggested in refs. [12, 6, 7]. The approach gives the procedure of the non-relativistic reduction of the effective meson-nucleon theory with the elimination of irrelevant antinucleon degrees of freedom and allows us to describe the nuclear ground state and exchange effects with advantage. Besides, it gives a fit set of operators required in the OPE, hence the consistency of computations is maintained. Some technical details of computations are presented in Appendices A and B.

In order to obtain explicit expressions of the operators (9), and calculate the matrix elements (6) it is necessary, first of all, to define the Hamiltonian of the system which has to provide simultaneously the equation of motion for interacting fields and the target ground states:

$$\frac{\partial}{\partial t} N(\Phi) = i\,[H, N(\Phi)] \tag{10}$$

$$H \,|\, A \rangle = M_A \,|\, A \rangle. \tag{11}$$

The Hamiltonian can be obtained from the Lagrangian in a conventional manner and via the non-relativistic reduction we get:

$$\begin{aligned} H^\sigma_0 &= \frac{1}{2} \int d^3x \{ (\nabla \Phi(x))^2 + \dot\Phi(x)\dot\Phi(x) + {\mu_\sigma}^2 \Phi(x)\Phi(x) \}, \\ H^N_0 + H_{int} &= \int d^3x \{ \frac{1}{2m} \nabla\psi^*(x)\nabla\psi(x) + m\psi^*(x)\psi(x) \} + \\ &+ g_\sigma \int d^3x \{ \psi^*(x)\Phi(x)\psi(x) \}, \end{aligned} \tag{12}$$

where m and μ_σ are the bare nucleon and meson masses to be redefined by counterterms, ψ is the nonrelativistic nucleon field obeying the equation of motion (10) (see Appendix A). The other notation in (12) is obvious.

Since the operator of the number of mesons commutes with the Hamiltonian, the physical state can be represented as a superposition of states with bare nucleons and

a different number of free mesons, the Tamm-Dankoff method. For the deuteron one gets:

$$\mid D \rangle = \sqrt{1 - Z_D} \varphi^D_0 \mid \bar N \bar N \rangle + \varphi^D_1 \mid \bar N \bar N \sigma \rangle + \varphi^D_2 \mid \bar N \bar N \sigma\sigma \rangle + \cdots \tag{13}$$

where Z_D is the constant of renormalization of the wave function determined by the condition $\langle D | D \rangle = 1$. An analogous expression could be written for the state with any number of nucleons. The wave functions φ_i are defined through eq. (11) and details could be found in Appendices A-B.

We are now in a position to calculate the matrix elements given by (6). Note that in contrast to the QCD the matrix elements of the operators (9) sandwiched between the nuclear states (A.6) could be computed explicitly. By direct computation of the matrix elements over the bare states $\mid \bar N \rangle$ or $\mid \sigma \rangle$ one can easily show that the coefficients $C^{(1,2)}_{a,n}$ are identical with the moments of bare nucleons ($a = \bar N$) or mesons ($a = \Phi$). Since the coefficients $C^{(1,2)}_{a,n}$ are target-independent, the same quantities define the moments of the physical nucleon and nuclear structure functions. Below the nuclear moments are expressed in terms of the moments of the *physical* nucleons and the nuclear structure characteristics (such as the potential and kinetic energies, wave function, etc.), so that the dependence upon the unphysical bare moments falls out throughout. With this aim in mind we will separately compute the matrix elements for a physical nucleon and a physical nuclear target (the solutions to the system (A.7) for a physical nucleon and a physical deuteron are given in Appendix B). As an example, we take the deuteron as an exactly solvable nuclear model that allows us to derive analytic expressions in a closed form and employ the well-known results of studies of its basic properties[8]-[10] (wave functions, the potential of nucleon-nucleon interaction, etc.)

To present the results in a compact form, we define the reduced matrix elements $\bar a^D_{a,n}$:

$$\langle p_D \mid O^{\mu_1 \dots \mu_n}_a \mid p_D \rangle = p^{\mu_1}_D \dots p^{\mu_n}_D \cdot \bar a^D_{a,n}, \tag{14}$$

which are related with the structure function (e.g. F^D_2) moments and coefficients $C^{(i)}_{a,n}$, $i = 1, 2$ by:

$$\mathrm{M}_{n-1}(F^D_2) = \sum_a C^{(2)}_{a,n} \cdot \bar a^D_{a,n}, \quad \text{with} \quad \mathrm{M}_n(F) = \int_0^1 F(x) x^{n-1} dx. \tag{15}$$

The main problem here is to compute $\bar a^D_{a,n}$ explicitly. We do this by nonrelativistic reduction of the operators (9) and averaging over the nuclear physical states (A.6). In deep inelastic kinematics ($q = (\nu, 0, 0, -\nu - M_A x_A)$) it is convenient to operate with the convolutions of operators (8), (9) with respective kinematic factors:

$$
\begin{aligned}
O_N &= \frac{2^n q_{\mu_1}\cdots q_{\mu_n}}{(-q^2)^n} O_N^{\mu_1\dots\mu_n} = \left(\frac{i}{2}\right)^{n-1}\left(\frac{2\nu}{-q^2}\right)^n \bar{N}(0)(\gamma_0+\gamma_3)\left(\overleftrightarrow{\partial}_0+\overleftrightarrow{\partial}_3\right)^{n-1} N(0),\\
O_\Phi &= \frac{2^n q_{\mu_1}\cdots q_{\mu_{a,n}}}{(-q^2)^n} O_\Phi^{\mu_1\dots\mu_n} = \left(\frac{i}{2}\right)^{n}\left(\frac{2\nu}{-q^2}\right)^n \Phi(0)(\overleftrightarrow{\partial}_0+\overleftrightarrow{\partial}_3)^{n}\Phi(0).
\end{aligned} \tag{16}
$$

A subsequent transformation of the operators (16) is the transition to nonrelativistic fields ψ and computation of all the derivatives $(\overleftrightarrow{\partial}_0+\overleftrightarrow{\partial}_3)^n$ in accordance with the equations of motion of fields (10). Upon quite cumbersome transformations, the result of nonrelativistic reduction can be written as a sum of operators

$$O_N = O_N^n + O_N^{n\sigma} + O_N^{nn}, \tag{17}$$

$$O_\Phi = O_\Phi^\sigma + O_\Phi^{n\sigma} + O_\Phi^{nn}, \tag{18}$$

where the operators are written in the order of growth of the coupling constant g and their explicit form is given in Appendix C. By sandwiching the operators (18) between the state (A.6) for a deuteron target, $\bar{a}^D_{a,n}$ are expressed through the following matrix elements

$$
\begin{aligned}
\bar{a}^D_{N,n} &= (1-Z_D)\Psi_D^+\Psi_D\langle \bar{N}\bar{N} \mid O_N^n \mid \bar{N}\bar{N}\rangle + \varphi_1^{D+}\varphi_1^D\langle \bar{N}\bar{N}\sigma \mid O_N^n \mid \bar{N}\bar{N}\sigma\rangle +\\
&+\varphi_1^{D+}\Psi_D\langle \bar{N}\bar{N}\sigma \mid O_N^{n\sigma} \mid \bar{N}\bar{N}\rangle + \Psi_D^+\Psi_D\langle \bar{N}\bar{N} \mid O_N^{nn} \mid \bar{N}\bar{N}\rangle + c.c.\\
\bar{a}^D_{\sigma,n} &= \varphi_1^{D+}\varphi_1^D\langle \bar{N}\bar{N}\sigma \mid O_\Phi^\sigma \mid \bar{N}\bar{N}\sigma\rangle + \varphi_1^{D+}\Psi_D\langle \bar{N}\bar{N}\sigma \mid O_\Phi^{n\sigma} \mid \bar{N}\bar{N}\rangle +\\
&+\varphi_2^{D+}\Psi_D\langle \bar{N}\bar{N}\sigma\sigma \mid O_\Phi^\sigma \mid \bar{N}\bar{N}\rangle + \Psi_D^+\Psi_D\langle \bar{N}\bar{N} \mid O_\Phi^{nn} \mid \bar{N}\bar{N}\rangle + c.c.
\end{aligned} \tag{19}
$$

These matrix elements can be grouped as to separate different contributions to the deuteron moments as is depicted in fig. 1. The diagrams a) denote the moments of the physical nucleons moving inside the deuteron. As it is seen, they consist of the moments of bare nucleons plus the self-energy corrections. The diagrams b) are the so-called renormalization and recoil terms that cancel each other in the g^2-approximation. Next diagrams c) are the interaction corrections to the nucleons contribution. The remaining diagrams d) are the pure meson exchange current corrections. A special note is to be paid to diagrams in the impulse approximation. Traditionally, when inclusive processes of scattering of leptons on nuclei are examined within the impulse approximation, there arises the problem with the amplitude of interaction of leptons with a

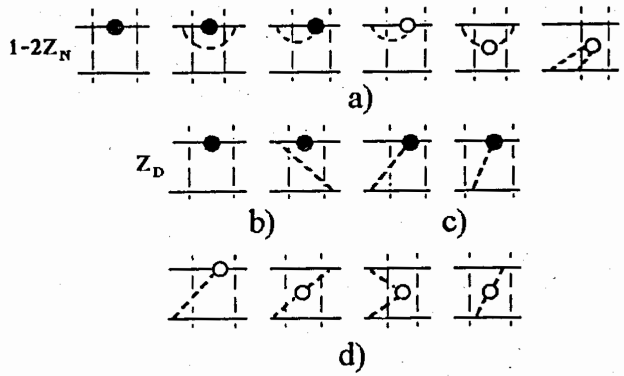

Figure 1: *The deuteron moments: a) - impulse approximation, b) - renormalization and recoil diagrams c) and d) - meson exchange currents. Closed and open circles denote the moments of a bare nucleon and a meson, respectively; the vertical dash-lines separate the operator and wave function in the corresponding matrix element.*

nuclear nucleon[13]. Usually it is suggested that the difference between the amplitudes of interaction with free nucleons and bound nucleons is small, and the nuclear effects are considered only in kinematics. This assumption is verified "a posteriori" by comparing numerical computations with experiment. As is seen from fig.1, diagrams of the impulse approximation contain moments of a physical nucleon measured experimentally. The nuclear structure is taken into account in terms of exchange diagrams , nucleon-nucleon interaction potential and binding energy. The corresponding explicit form of the moments of the deuteron structure function is:

$$\frac{1}{2}\left(\frac{M_D}{m}\right)^n \cdot \mathrm{M}_n(F_2^D) = \tag{20}$$

$$= \mathrm{M}_n(F_2^N)\int\frac{d^3\mathbf{p}}{(2\pi)^3}\mid\Psi_D(\mathbf{p})\mid^2\left(1+\frac{p_z}{m}\right)\cdot\left(1+\frac{p_z}{m}+\frac{\mathbf{p}^2}{2m^2}\right)^n + \tag{21}$$

$$+ \mathrm{M}_n(F_2^N)\int\frac{d^3\mathbf{p}d^3\mathbf{k}}{(2\pi)^6}\Psi_D^+(\mathbf{p})V(\mathbf{k})\Psi_D(\mathbf{p}+\mathbf{k})\frac{1}{k_z}\left[\left(1+\frac{k_z}{2m}\right)^n-\left(1-\frac{k_z}{2m}\right)^n\right]+ \tag{22}$$

$$- \mathrm{M}_n(F_2^M)\int\frac{d^3\mathbf{p}d^3\mathbf{k}}{(2\pi)^6}\Psi_D^+(\mathbf{p})V(\mathbf{k})\Psi_D(\mathbf{p}+\mathbf{k})\frac{m}{\omega^2(\mathbf{k})}\cdot\frac{(1+(-1)^{n+1})}{2}\cdot\left(\frac{k_z}{m}\right)^{n+1}, \tag{23}$$

where $\Psi_D \equiv \varphi_0^D$ is the conventional deuteron wave function; $\omega^2(\mathbf{k}) = (\mathbf{k}^2+\mu^2)$; $V(\mathbf{k})$ is the one-boson-exchange potential generated by the interaction term in the Hamiltonian of the theory.

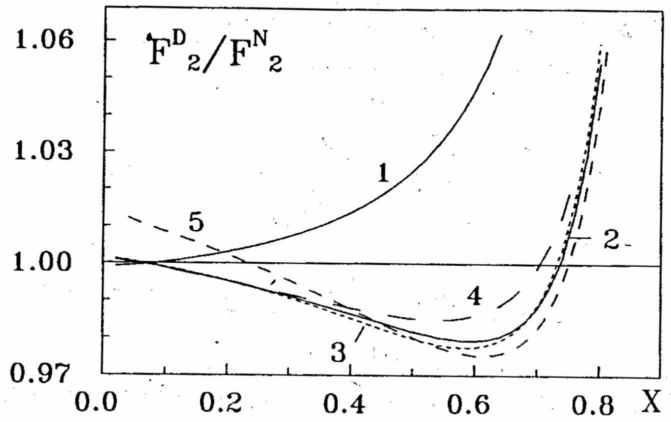

Figure 2: The ratio of the deuteron and isoscalar nucleon structure functions. Curves: 1 – the Fermi motion of the "on mass shell" nucleons; 2 – the Fermi motion with taking into account the boundness effects (full line); 3 – calculations on the basis of approximate formula (26) (short dash-line); 4 –impulse approximation ([6, 7] (long dash-line); 5 – the summary contribution of the impulse approximation and meson exchange currents obtained within present approach.

4. The nucleon contribution to the nuclear structure functions

The sum of terms (21) and (22) (diagrams a) and c) in fig. 1) is the contribution of the Fermi motion of interacting nucleons. Applying the inverse Mellin transformation to (21),(22) we reconstruct the nucleon contribution to the deuteron structure functions in the convolution form (1) with $f^{N/D} = f^{N/D}_{IA} + f^{N/D}_{int}$ given by:

$$f^{N/D}_{IA}(y) = \int \frac{d^3\mathbf{p}}{(2\pi)^3} | \Psi_D(\mathbf{p}) |^2 \left(1 + \frac{p_z}{m}\right) \delta\left(y - \left[1 + \frac{\mathbf{p}^2}{2m^2} + \frac{p_z}{m}\right]\right) \quad (24)$$

$$f^{N/D}_{int}(y) = \int \frac{d^3\mathbf{p}d^3\mathbf{k}}{(2\pi)^6} \Psi^+_D(\mathbf{p}) V(\mathbf{k}) \Psi_D(\mathbf{p}+\mathbf{k}) \times$$

$$\times \frac{1}{k_z} \left\{ \delta\left(y - \left|1 + \frac{k_z}{2m}\right|\right) - \delta\left(y - \left|1 - \frac{k_z}{2m}\right|\right) \right\}. \quad (25)$$

The distribution function $f^{N/D}_{IA}$ describes the Fermi motion of the on-mass-shell nucleons and it is quite similar to the conventional formula of nuclear physics usually referred to as the "impulse approximation". Taking into account only this type of "Fermi smearing" results in wrong nuclear structure functions. In particular, it breaks the sum rule for the four-momentum ($\langle y \rangle > 1$) and does not give the EMC-like A-dependence (see fig. 2, curve 1).

Instead of the modification of the impulse approximation by reasonable redefinition of the variable y [1, 2] we get the pure interaction term (25) of the exchange origin, $f^{N/D}_{int}$. The sum of $f^{N/D}_{IA}$ and $f^{N/D}_{int}$ gives a final result for the nucleon contribution to the deuteron compared with the result of the earlier x-rescaling calculation [6] in fig. 2 (curves 2 and 4, resp.). Expanding expressions (24) and (25) around the "on-mass-shell" $y = 1 + \mathbf{p}^2/2m^2 + p_z/m$ and formally keeping only the g^2-terms we derive the approximate expressions:

$$\frac{1}{2} F^{N/D}_2 = F^{N/D}_2(IA) - \frac{\langle V \rangle}{m} x \cdot \frac{dF^N_2}{dx} = F^{N/D}_2(IA) - \frac{\varepsilon_D - \langle T \rangle}{m} x \cdot \frac{dF^N_2}{dx}, \quad (26)$$

where $F^{N/D}_2(IA)$ is the impulse approximation contribution computed by (24). The structure function $F^{N/D}_2$ obtained by means of (26) is given in fig. 2 (curve 3). As is seen, the present result excellently fits the calculation by exact formulae (24) - (25).

Heavy nuclei. Notice that according to (26) the deuteron structure function is determined with high accuracy by the momentum distribution of nucleons and mean value of the potential $\langle V \rangle$. The Schrödinger equation allows one to express $\langle V \rangle$ through the binding energy and well defined mean value of the kinetic energy of nucleons. Due to this fact eq. (26) is straightforwardly extended to the scattering on heavy nuclei and formidable computations of the nuclear spectral function are avoided. With the two-body origin of $NN-$interaction in mind we have:

$$\langle V \rangle_A = 2(\varepsilon_A - \langle T \rangle_A), \quad (27)$$

where $\varepsilon_A \approx -8\, MeV$ is the binding energy of a nucleus per nucleon, $\langle T \rangle_A$ is the mean kinetic energy of a nucleon inside the nucleus. The structure function of ^{12}C calculated on the basis of (26), (27) is shown in fig. 3 (curve 3). The function $F^{N/A}_2(IA)$ and $\langle T \rangle_A$ has been calculated with the realistic momentum distribution obtained in the coherent-fluctuation density model ($CDFM$) [5]. This figure displays also the comparison of present calculations with the result of the $x-$rescaling model where the spectral function has been taken from $CDFM$ as well (curve 2) [5]. Both curves in

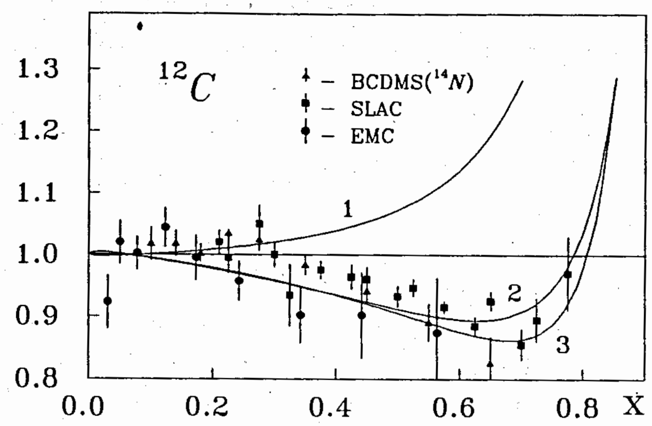

Figure 3: *The ratio of the carbon and isoscalar nucleon structure functions. Curves: 1 – the Fermi motion of the "on mass shell" nucleons; 2–impulse approximation within CDFM; 3 – the Fermi motion with taking into account the boundness effects within the present approach. Experimental data are taken from* [14].

fig. 3 are in a reasonable agreement with the data. The slight discrepancy between two curves is of the same origin as in the deuteron case.

5. The meson exchange corrections to the nuclear structure functions

The term (23) (diagrams d) in fig. 1) is the contribution of the meson exchange currents to the structure function moments of the deuteron. Applying the inverse Mellin transformation to (21), (22) we reconstruct the mesonic correction to the deuteron structure functions in the convolution form (1) with $f^{M/D}$ given by:

$$f^{M/D}(y) = -\int \frac{d^3\mathbf{p}d^3\mathbf{k}}{(2\pi)^6}\Psi_D^+(\mathbf{p})V(\mathbf{k})\Psi_D(\mathbf{p}+\mathbf{k})\frac{1}{\omega^2(\mathbf{k})} \times \times \left[k_z\cdot\delta\left(y-\frac{k_z}{m}\right)\cdot\theta(k_z) - k_z\cdot\delta\left(y+\frac{k_z}{m}\right)\cdot\theta(-k_z)\right]. \quad (28)$$

In the g^2-approximation this result is in agreement with that obtained earlier in ref. [6]. However, in contrast with [6] the accurate taking into account the relevant g^2-terms leads to full self-consistence of the approach. For instance, the energy sum rule

is exactly fulfilled in accordance with (B.17).

The numerical calculation by formulae (1) and (28) is presented in fig. 2 (curve 5). As is seen, the taking into account the mesonic exchange corrections increases the deuteron structure function at small $x(x \leq 0.3)$.

6. Examples of application: nuclear effects at small x and neutron structure function

The Gottfried Sum Rule.

Recently, the NMC data[15, 16] on the ratio F_2^n/F_2^p have been applied to derive the difference $F_2^p - F_2^n$ and to estimate the Gottfried Sum (GS) $S_G \equiv \int(F_2^p - F_2^n)dx/x$[17] experimentally. Its value has been found to be below the quark-parton model expectation of 1/3, namely:

$$S_G = 0.240 \pm 0.016 \quad (29)$$

Serious theoretical speculations have appeared as a consequence of this discrepancy, e.g. the strong isospin violation in the proton sea-quark distributions.

Note that the experimental value of the GS is sensitive to the procedure of extraction of the ratio F_2^n/F_2^p from the combined data on the deuteron and proton. Since the deuteron is a more complicated system than a simple sum of two free nucleons, a number of structure factors may change the ratio F_2^n/F_2^p. At least one should be careful while considering the influence of nuclear effects discussed above, such as Fermi motion, binding effects and mesonic exchanges in nuclei. Though in the integral characteristics of nuclear structure functions these corrections are small, it is not evident that they can be neglected in the procedure of determination of the neutron structure function $F_2^n(x)$ from the nuclear data. Moreover, the analysis of BCDMS data on the proton and the deuteron performed in ref.[18] has shown the noticeable influence of the deuteron structure factors on the extracted neutron structure function and the ratio F_2^n/F_2^p. It seems, the same corrections can also be expected for the NMC data.

In this section we estimate the typical value of the meson exchange corrections to the neutron structure function extracted from the combined proton-deuteron data. In particular, we demonstrate that in the presented theoretical approach it is possible to extract the neutron structure function so that the obtained value of the GS doesn't dramatically contradict the quark-parton predictions.

Since the structure functions have been measured not in the whole region of the scale variable x, it is useful to define the x-dependent Gottfried integral:

$$I_G(x_1 \div x_2) = \int_{x_1}^{x_2} (F_2^p - F_2^n)\, dx/x, \qquad (30)$$

and separately evaluate it in the measured and unmeasured regions of x. Thus, the GS may be written as a sum of three integrals (30) corresponding to three regions considered in ref.[15]:

$$\begin{array}{ccccccc} S_G & = & I_G^{NMC}(0 \div 0.004) & + & I_G^{NMC}(0.004 \div 0.8) & + & I_G^{NMC}(0.8 \div 1) \\ (0.240 \pm 0.016) & & (0.011 \pm 0.003) & & (0.227 \pm 0.014) & & (0.002 \pm 0.001) \end{array} \qquad (31)$$

The second term in (31) has been estimated experimentally by using the F_2^D from the fit of the published deuteron data and the ratio F_2^n/F_2^p has been taken from the *unsmeared* NMC experimental results[16]. The first and third terms correspond to the unmeasured regions and have been estimated by extrapolation. Thus, in all these three integrals the nuclear corrections have been missed. Let $F_2^{D(exp.)}$ be the experimental deuteron structure function (that obviously includes all the nuclear and other effects) and $F_2^{p(exp.)}$ the corresponding proton structure function. Then the unsmeared neutron structure function defined by:

$$\widetilde{F}_2^n = 2F_2^{D(exp.)} - F_2^{p(exp.)} \qquad (32)$$

is overestimated due to the mesonic contributions to the deuteron structure function. A more correct way to determine the neutron structure function is to solve the integral equation[2]:

$$\begin{aligned} F_2^n(x,Q^2) &= \left[2F_2^{D(exp.)}(x,Q^2) - \delta F_2^{mes.}(x,Q^2) - S_p^{-1}(x,Q^2)F_2^{p(exp.)}(x,Q^2)\right] S_n(x,Q^2), \\ S_{p(n)} &= \frac{F_2^{p(n)}(x,Q^2)}{\int F_2^{p(n)}(x/y,Q^2) f_{N/D}(y)dy}, \end{aligned} \qquad (33)$$

for $F_2^n(x)$. Here $\delta F_2^{mes.}(x,Q^2)$ is the inverse Mellin transformation of the moments (23), $f_{N/D}(y) \equiv f_{IA}^{N/D}(y) + f_{int}^{N/D}(y)$.

[2]The present version of approach gives the description of the nuclear corrections coming from the fermi-motion and meson exchange currents in the deuteron. A more complete analysis should include also the shadowing as $x \to 0$ and the contributions of other non-nucleon degrees of freedom (multiquarks, Δ-isobar ...) as $x \to 1$.

To extract the neutron structure function by solving the integral equation (33), we should parametrize the proton, deuteron and neutron structure functions in the full region of x and experimental values of Q^2. At this moment we are free in the choice of the parameters and we can from the very beginning make them to obey the Gottfried Sum Rule exactly. That kind of analysis has been done in[18] to extract the neutron structure function from the combined BCDMS data[19]. From that analysis we can compute the corresponding Gottfried integrals (31):

$$\begin{aligned} I_G^{BCDMS}(0 \div 0.004) &= 0.036 \\ I_G^{BCDMS}(0.004 \div 0.8) &= 0.297 \\ I_G^{BCDMS}(0.8 \div 1) &= 0.0004 \end{aligned} \qquad (34)$$

Note that in eq.(34) the Gottfried Sum Rule is exactly fulfilled. Comparison of (34) with (31) shows that here is a systematic difference in the NMC treatment of the experimental data with the results obtained from BCDMS experiments. To achieve the agreement between them, it is necessary to take into account the following:

i) In the region $0.004 \le x \le 0.8$ for the moment we will neglect the role of the Fermi motion and estimate the correction to the difference $F_2^p - F_2^n$ by adding the function $\delta F_2^{mes.}(x)$. As a result, the Gottfried integral in this region increases by adding:

$$\delta I_G^{(mes.)}(0.004 \div 0.8) = \int_{0.004}^{0.8} \delta F_2^{mes.}(x)\, dx/x = 0.03 \pm 0.002 \qquad (35)$$

To estimate the integral $I_G^{(mes.)}(0.004 \div 0.8)$ (35), we have used the numerical results for the mesonic corrections computed in the present work. In ref.[6] it was noted that numerically the mesonic contribution to the deuteron structure function $\delta F_2^{mes.}(x)$ was underestimated, owing to the approximate form of the current operator. This circumstance was reflected in our earlier analysis[18] as a large systematic error ± 0.01 in (35). However, the correct estimation of the absolute value of $I_G^{(mes.)}(0.004 \div 0.8)$ was presented.

ii) Besides, the meson corrections change the behavior of $F_2^p - F_2^n$ as $x \to 0$. Usually in the region $x \le 0.004$ one assumes the "non-singlet" power behavior of the difference $F_2^p - F_2^n$ as ax^α. The fit of the NMC data at small x ($x = 0.004 - 0.15$) gives $a = 0.21 \pm 0.03$, $\alpha = 0.62 \pm 0.05$ [15]. This yields $I_G(0 \div 0.004)$ as is shown in(31). Upon taking into account the mesonic corrections to the NMC data the parameters become $a = 0.143 \pm 0.013$, $\alpha = 0.423 \pm 0.048$. This situation is shown in fig.4 where the

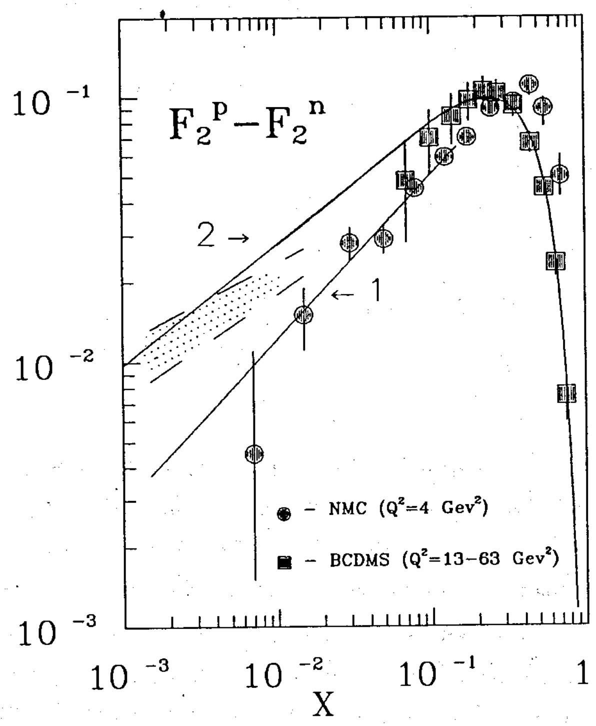

Figure 4: *The difference $F_2^p(x) - F_2^n(x)$. Solid lines: 1 - NMC data fit[15]; 2 - parametrization from ref.[18]. Dashed lines and shadow area - corrected NMC data fit with the taking into account of the mesonic corrections (see text). Data: circles - NMC[15], squares - BCDMS[19].*

dashed lines correspond to the new behavior of the data and the shadow area displays the ambiguities in computation of $\delta F_2^{mes.}(x)$ obtained in ref. [18].

Thus, the part of the Gottfried integral computed with the new parameters a and α becomes: $I_G(0 \div 0.004) = 0.0340 \pm 0.010$.

iii) At last in the region $0.8 \le x \le 1$ the mesonic contribution is negligible. Other nuclear effects, viz. Fermi motion and binding effects, in this region may be significant in the functional dependence of structure functions. However, since here the absolute values of structure functions are small, it is clear that their contributions to the integral characteristics are insignificant.

Gathering together the corrected integrals we obtain the corrected estimation of

the GS instead of (29):

$$S_G = (0.034\pm0.01)+(0.227\pm0.014)+(0.03\pm0.002)+(0.002\pm0.001) \approx 0.29\pm0.03, \quad (36)$$

that is close to the quark-parton predictions of 1/3.

Very important notice. Above we have neglected the Fermi motion effects in the deuteron structure function. However, since the nucleon structure function is connected with the deuteron one by the integral equation (33), the result for the extracted neutron structure function depends upon additional constraints imposed to this function. Examples of these constraints are the behavior as $x \to 0, x \to 1$ or integral relations, such as *Gottfried Sum Rule* ... Therefore, the ambiguities in the neutron structure function extracted from the combined proton-deuteron data are of the nature of both the nuclear and particle physics models.

Shadowing and mesonic contribution at very small x.

As we mentioned above, the problem of determination of neutron structure function is still more complicated because it is here necessary to solve the integral equation (33) in $F_{1,2}^n$, whose kernel depends on the model assumption and target structure and in this case it turns out that the sensitivity of the resultant structure functions $F_{1,2}^n$ to the model increases at boundaries of the kinematical region x: $x \to 1$ and $x \to 0$[18]. Moreover, the shadowing effect in DIS of leptons off nuclear targets [20, 21] has been firmly detected experimentally[22] for heavy nuclei. This important circumstance must be kept in mind when examining recent experiments performed by the NMC group [15] on measurements of the hydrogen and deuteron structure functions at very small x :$x \sim 10^{-3}$. As a result, the ratio $F_2^n(x)/F_2^p(x)$ in the interval $10^{-3} \div 10^{-2}$ was found to be about unity, i.e. at first sight there is no evidence of the virtual photon shadowing and of the contribution of nuclear effects in the deuteron for $x \sim 10^{-3}$. Here it is worth reminding that one should be careful as in these experiments directly measured in fact is the nuclear (deuteron), rather than neutron, structure function. So, it is relevant to consider the nuclear ratio $(F_2^D(x)/F_2^p(x) - 1)$ instead of $F_2^n(x)/F_2^p(x)$. Accurate consideration of the nuclear effects, even though they are opposite in sign and compensate each other, may lead to different conclusions. The mesonic contribution to the deuteron structure function is positive (see fig.2 curve 5) whereas the shadowing contribution is negative[21]:

$$F_2^D(x,Q^2) = F_2^{N/D}(x,Q^2) + \delta F_2^{mes.}(x,Q^2) - \delta F_2^{shad.}(x,Q^2) \quad (37)$$

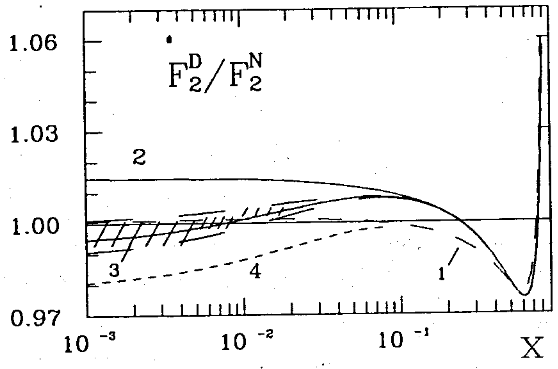

Figure 5: *The ratio of the deuteron and isoscalar nucleon structure functions. Lines: 1 - the contribution of bound nucleons to the deuteron structure function; 2 - the deuteron structure function within the present approach: impulse approximation+binding effects+mesonic corrections; 3 - the total deuteron structure function: shadow area corresponds to the uncertainties in the calculations; 4 - Fermi-motion of bound nucleons + shadowing effects*

The estimations of the shadowing contribution to the deuteron structure function and our calculations of the mesonic exchange contribution (diagrams c) and d) in fig.1 and curve 5 in fig.2) show that

$$\delta F_2^{mes.}(x,Q^2) \simeq \delta F_2^{shad.}(x,Q^2), \quad x \to 0.$$

For a more quantitative comparison of these two effects we consider the ratio of the deuteron and isoscalar nucleon structure functions, $F_2^D(x)/F_2^N(x)$ which is less dependent on ambiguities in the determination of the neutron structure function F_2^n and whose behavior as a function of x is quite similar to the behavior of the $(F_2^D(x)/F_2^p(x) - 1)$. The mesonic contribution $\delta F_2^{mes.}(x)$ has been computed within the approach described above, and for $\delta F_2^{shad.}(x)$ we use the parametrization:

$$\delta F_2^{shad.}(x) = 0.004\ \exp(-28x),$$

which gives a reasonable fit to the calculation performed in ref. [21]. The result is depicted in fig. 5 (a shadow area corresponds to the uncertainties in the calculations of the $\delta F_2^{shad.}(x)$). One can see from fig. 5 that the shadowing and mesonic contributions in the deuteron are opposite in sign and practically cancel each other, so that the

final deuteron structure function looks like there are neither shadowing nor exchange effects in the deuteron. This quite qualitative analysis allows one to conclude that in the experiments of the NMC group an evidence of the existence of both *mesonic and shadowing effects* has rather been detected than their absence in the deuteron.

7. Concluding remarks

1. We have proposed a quite rigorous theoretical method that describes the nuclear structure effects in deep inelastic lepton-nucleus scattering and substantiates the x-rescaling idea. The self-consistency peculiar to the method assures the energy-momentum conservation, the role of the meson exchange currents being found to be important. The explicit expression for the mesonic correction derived from (23) in the g^2-approximation coincides with the one presented in ref. [6]. An almost model-independent representation of the nuclear structure functions has been achieved.

2. The performed analysis of the structure function moments persuades us that there exist a tight relation between Q^2- and $x-$rescaling models. Equating the nuclear moments M_n found in two approaches makes it possible to obtain the QCD motivated parameters of the Q^2-rescaling in terms of the nuclear structure.

3. The study of the nuclear structure functions at $x \sim 1$ and beyond is an interesting theoretical problem. Obviously, the nuclear structure effects are here predominant and the application of our approach is rather appropriate. In this region other degrees of freedom become relevant ($\Delta-$isobars[23], multiquarks[24], ...) and the OPE and Tamm-Dankoff methods should merely be applied with taking into account such fields.

4. The enlargement of the basis (9) of twist two operators by attaching the axial operators (γ_5-terms) [25] allows us to extend our method to the consideration of polarization processes and the spin-dependent structure functions of nuclei.

We would like to thank K. Kazakov for fruitful collaboration. We also benefited from stimulating discussions with S. Dorkin, A. Efremov and N. Krasnikov.

Appendix A. The Non-Relativistic Field Theoretical Approach to the Exchange Effects in Nuclei.[12, 6, 7]

The classical equation of motion for the theory with Lagrangian (7) has the form:

$$(i\hat{\partial} - m)\, N(x) = g_\sigma \Phi(x) N(x), \qquad \text{(A.1)}$$

$$(\Box + \mu_\sigma^2)\Phi(x) = -g_\sigma \bar{N}(x) N(x), \qquad \text{(A.2)}$$

where the Dirac bispinor field $N(x)$ could be determined in terms of big and small components $f(x)$, $\chi(x)$.

Antinucleon degrees of freedom nonessential for the nuclear physics are eliminated by a nonrelativistic reduction of matrix eq. (A.1). For this aim we employ eq. (A.1) to express a small component $\chi(x)$ of the spinor field $N(x)$ in terms of a large component $f(x)$:

$$\chi(x) = -\frac{i}{2m}\,\sigma\partial\, f(x) \tag{A.3}$$

Then in the first order of the expansion in powers of $1/m$ we obtain the following nonrelativistic equation of motion for $f(x)$:

$$i\dot{f}(x) = mf(x) - \frac{\Delta}{2m} f(x) + g_\sigma \Phi(x) f(x). \tag{A.4}$$

Note that the expansion $1/m$ corresponds to the approximation in coupling constant, g^2 , accepted in nuclear physics in deriving the interaction potential and Schrödinger equation.

In principle, eq. (A.4) can be utilized for quantization and determination of physical properties of the system. However, it has been noticed [26, 12] that the fields $f(x)$ do not obey the conditions of normalization of the probability density and charge and consequently cannot serve true second-quantized fields. Usually, new fields, $\psi(x) = (\hat{I} + \hat{F})f(x)$, are introduced where $\hat{I}$ is a unit operator. The operator $\hat{F}$ is defined so that the fields $\psi(x)$ should obey all the conditions for canonical second-quantized fields. For the case under consideration of nucleon and σ-meson fields we have:

$$\psi(x) = \left(\hat{I} - \frac{\Delta}{8m^2}\right) f(x). \tag{A.5}$$

To compute any observable, one could first write the respective fully relativistic expression, employ eq. (A.3) for small components and rewrite $f(x)$ in terms of $\psi(x)$ through eq. (A.5). For instance, the Hamiltonian is found from the Lagrangian (7) and is of the form (12).

Since the operator of the number of mesons commutes with the Hamiltonian, the physical state $|A\rangle$ of A nucleons can be represented as a superposition of states with A bare nucleons and a different number of free mesons (the Tamm-Dankoff method):

$$|\,A\rangle_q = \sqrt{1-Z_A}\varphi_0^A\,|\,\bar{N}_1\cdots\bar{N}_A\rangle_q + \varphi_1^A\,|\,\bar{N}_1\cdots\bar{N}_A\sigma\rangle_q + \varphi_2^A\,|\,\bar{N}_1\cdots\bar{N}_A\sigma\sigma\rangle_q + \cdots, \tag{A.6}$$

where Z_A is the constant of renormalization of the wave function determined by the condition $\langle A|A\rangle = 1$. Projecting (11) onto various components of the Fock space we arrive at the system of equations for the wave functions φ_i^A:

$$\begin{aligned}
&\langle\bar{N}_1\cdots\bar{N}_A\,|\,(H_0-E_A)\,|\,\bar{N}_1\cdots\bar{N}_A\rangle\varphi_0^A + |\,\langle\bar{N}_1\cdots\bar{N}_A\sigma\,|\,H_{int}\,|\,\bar{N}_1\cdots\bar{N}_A\rangle\,|^2\,\varphi_0^A/\Delta_1 = 0,\\
&\varphi_1^A = \langle\bar{N}_1\cdots\bar{N}_A\sigma\,|\,H_{int}\,|\,\bar{N}_1\cdots\bar{N}_A\rangle\varphi_0^A/\Delta_1 + O(g^3),\\
&\varphi_2^A = \langle\bar{N}_1\cdots\bar{N}_A\sigma\sigma\,|\,H_{int}\,|\,\bar{N}_1\cdots\bar{N}_A\sigma\rangle\langle\bar{N}_1\cdots\bar{N}_A\sigma\,|\,H_{int}\,|\,\bar{N}_1\cdots\bar{N}_A\rangle\varphi_0^A/\Delta_1\Delta_2 + O(g^3),
\end{aligned} \tag{A.7}$$

where

$$\begin{aligned}
\Delta_1 &= \langle\bar{N}_1\cdots\bar{N}_A\sigma\,|\,H_0\,|\,\bar{N}_1\cdots\bar{N}_A\sigma\rangle - E_A,\\
\Delta_2 &= \langle\bar{N}_1\cdots\bar{N}_A\sigma\sigma\,|\,H_0\,|\,\bar{N}_1\cdots\bar{N}_A\sigma\sigma\rangle - E_A.
\end{aligned} \tag{A.8}$$

In view of the approximation adopted when deriving the Hamiltonian (12) we retain in (A.6)-(A.7) only terms of order g^2.

Appendix B. The Nucleon and Deuteron Ground State.

Here we write the solution of (A.7) for nucleon and deuteron, determine the physical nucleon mass and the trace of the energy-momentum tensor.

Nucleon.

The expansion (A.6) for a physical nucleon (A=1) with momentum q is of the form

$$|\,N\rangle_q = \sqrt{1-Z_N}\varphi_0^N\,|\,\bar{N}\rangle_q + \varphi_1^N\,|\,\bar{N}\sigma\rangle_q + \varphi_2^N\,|\,\bar{N}\sigma\sigma\rangle_q + \cdots, \tag{B.1}$$

where $\varphi_0^N = 1$, and the wave functions $\varphi_{1,2}^N$ in the system with $q=0$ have the form

$$\varphi_1^N(\mathbf{k}) = -\frac{g_\sigma}{\sqrt{2\omega(\mathbf{k})}}\cdot\frac{1}{\omega(\mathbf{k})} \tag{B.2}$$

$$\varphi_2^N(\mathbf{k}_1,\mathbf{k}_2) = \frac{g_\sigma^2}{\sqrt{2\omega(\mathbf{k}_1)2\omega(\mathbf{k}_2)}}\cdot\frac{1}{\omega(\mathbf{k}_1)\omega(\mathbf{k}_2)}, \tag{B.3}$$

where $\omega(\mathbf{k}) = \sqrt{\mathbf{k}^2+\mu_\sigma^2}$ and $\Delta_1 \simeq -\omega(\mathbf{k})$, $\Delta_2 \simeq -\omega(\mathbf{k}_1)-\omega(\mathbf{k}_2)$. The renormalization constant Z_N is given by

$$Z_N = \int\frac{d^3\mathbf{k}}{(2\pi)^3}\cdot\frac{g_\sigma^2}{2\omega^3(\mathbf{k})}. \tag{B.4}$$

We obtained a diverging integral. To remove divergences, integrals of that type should be regularized and the corresponding counterterms are to be determined in the Lagrangian; for instance, the mass term is written as

$$m_{phys.} = m + \delta m,$$

where

$$\delta m = -\int \frac{d^3\mathbf{k}}{(2\pi)^3} \cdot \frac{g_\sigma^2}{2\omega^2(\mathbf{k})}. \tag{B.5}$$

and regularization is assumed.

Equations (B.1)-(B.5) completely determine the states of a physical observable nucleon. Let us, for example, compute the average trace of the energy-momentum tensor over nucleon states. This operator plays an important role in the theory as it is proportional to the operator $\hat{O}^{\mu_1\mu_2}$ in the OPE at $n = 2$. As it is obvious, we should obtain the nucleon physical mass in its rest frame of reference.

The trace of the energy-momentum tensor of a system of interacting nucleons and mesons in a covariant form looks as follows

$$\theta^\mu_\mu(x) = :m\bar{N}(x)N(x): + :\mu_\sigma^2\Phi^2(x):. \tag{B.6}$$

Nonrelativistic reduction of (B.6) gives

$$\theta^\mu_\mu(x) = -\frac{1}{2m} :\nabla\psi^+(x)\nabla\psi(x): + m:\psi(x)^+\psi(x): + :\mu_\sigma^2\Phi^2(x):, \tag{B.7}$$

and then the matrix element of (B.7) over the nucleon state (B.1) equals

$$\langle N \mid \theta^\mu_\mu(0) \mid N\rangle = m + \delta m_\theta, \tag{B.8}$$

where

$$\delta m_\theta = \int \frac{d^3\mathbf{k}}{(2\pi)^3} \cdot \frac{g_\sigma^2}{\omega^2(\mathbf{k})} - g_\sigma^2 \int \frac{d^3\mathbf{k}}{(2\pi)^3} \cdot \frac{\mathbf{k}^2}{\omega^4(\mathbf{k})} \neq \delta m. \tag{B.9}$$

At first sight, we arrive at the contradiction with (B.5), however, it is not difficult to verify that the regularized expressions (B.5) and (B.9) coincide.

Deuteron.

Below we determine basic observable characteristics of the deuteron and express them in terms of the corresponding observables of a realistic nucleon.

Following standard procedure we represent the deuteron ground state in the form

$$\mid D\rangle = \sqrt{1 - Z_D}\varphi_0^D \mid \bar{N}\bar{N}\rangle + \varphi_1^D \mid \bar{N}\bar{N}\sigma\rangle + \varphi_2^D \mid \bar{N}\bar{N}\sigma\sigma\rangle + \cdots \tag{B.10}$$

In the system with $q = 0$ it is convenient to redefine the wave function φ_0^D as follows

$$\varphi_0^D(\mathbf{p}_1, \mathbf{p}_2) = (2\pi)^3\delta^{(3)}(\mathbf{p}_1 + \mathbf{p}_2)\varphi_0^D(\mathbf{p}_1), \tag{B.11}$$

and then it obeys the usual Schrödinger equation with the one-boson exchange potential

$$2 \cdot \frac{\mathbf{p}^2}{2m_{phys.}}\varphi_0^D(\mathbf{p}) - \int \frac{d^3\mathbf{k}}{(2\pi)^3}\frac{g_\sigma^2}{\omega^2(\mathbf{k})}\varphi_0^D(\mathbf{p} + \mathbf{k}) = \epsilon_D\varphi_0^D(\mathbf{p}), \tag{B.12}$$

where $\epsilon_D = M_D - 2m_{phys.}$, and M_D is the deuteron physical mass. In what follows we will use for the deuteron wave function φ_0^D another notation Ψ_D bearing in mind that when other mesons $(\pi, \omega, \rho \dots)$ are included, Ψ_D will represent the well known deuteron function computed by Paris or Bonn groups[8, 9].

The wave functions $\varphi_{1,2}^D$ are given by the formulae

$$\begin{aligned}
\varphi_1(\mathbf{p}_1, \mathbf{p}_2, \mathbf{k}) &= -\frac{g_\sigma}{\sqrt{2\omega(\mathbf{k})}\omega(\mathbf{k})}(2\pi)^3\delta^{(3)}(\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{k})\left(\Psi_D(\mathbf{p}_1) - \Psi_D(\mathbf{p}_2)\right), \\
\varphi_2(\mathbf{p}_1, \mathbf{p}_2, \mathbf{k}_1, \mathbf{k}_2) &= \frac{g_\sigma^2}{2\sqrt{2\omega(\mathbf{k}_1)2\omega(\mathbf{k}_2)}\omega(\mathbf{k}_1)\omega(\mathbf{k}_2)}(2\pi)^3\delta^{(3)}(\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{k}_1 + \mathbf{k}_2) \times \\
&\times \left(\Psi_D(\mathbf{p}_1 + \mathbf{k}_1) - \Psi_D(\mathbf{p}_2 + \mathbf{k}_1) + \Psi_D(\mathbf{p}_1) - \Psi_D(\mathbf{p}_2)\right).
\end{aligned} \tag{B.13}$$

By a simple computation it is not difficult to verify that the renormalization constant of the deuteron wave function is connected with the corresponding nucleon constant

$$\begin{aligned}
Z_D &= 2Z_N + \widetilde{Z}_D, \\
\widetilde{Z}_D &= \int \frac{d^3\mathbf{k}d^3\mathbf{p}}{(2\pi)^6}\frac{g_\sigma^2}{\omega^3(\mathbf{k})}\Psi_D^+(\mathbf{p})\Psi_D(\mathbf{p} + \mathbf{k}).
\end{aligned} \tag{B.14}$$

Note that the purely deuteron part of the renormalization constant $\widetilde{Z}_D$ is finite and, as is seen from (B.13), it defines the total number of extra mesons in the deuteron mediating the interaction between nucleons.

Let us compute the matrix element of the operator of the energy-momentum tensor over the deuteron state and verify that its total mass is expressed through the nucleon mass and binding energy ϵ_D

$$\begin{aligned}
\langle D \mid \theta^\mu_\mu(0) \mid D\rangle &= 2\int \frac{d^3\mathbf{p}}{(2\pi)^3}\left(m - \frac{\mathbf{p}^2}{2m}\right)\Psi_D^+(\mathbf{p})\Psi_D(\mathbf{p}) + \\
&+ 2\mu_\sigma^2 \int \frac{d^3\mathbf{p}_1 d^3\mathbf{p}_2}{(2\pi)^6} \cdot \frac{g_\sigma^2}{\omega^4(\mathbf{k})}\Psi_D^+(\mathbf{p}_1)\Psi_D(\mathbf{p}_2) + 2\delta m_\theta,
\end{aligned} \tag{B.15}$$

where $\mathbf{k} = \mathbf{p}_1 + \mathbf{p}_2$. Now we rewrite (B.15) in the coordinate representation

$$\langle D \mid \theta^\mu_\mu(0) \mid D\rangle = 2m + \int d^3\mathbf{r}\Psi_D^+(\mathbf{r})\left(-\frac{\Delta}{m} + r\frac{d}{dr}V(r) + V(r)\right)\Psi_D(\mathbf{r}) + 2\delta m_\theta, \tag{B.16}$$

where V is the Yukawa potential of nucleon-nucleon interaction. Using the virial theorem $\langle rdV/dr\rangle = 2\langle T\rangle$ ($\langle T\rangle \equiv \langle p^2/m_{phys.}\rangle$ is the mean kinetic energy of nucleons in the deuteron) we get

$$\begin{aligned} M_D &= 2m_{phys} + \epsilon_D, \\ \epsilon_D &= \langle T\rangle + \langle V\rangle. \end{aligned} \qquad (B.17)$$

Thus, we see that basic characteristics of the deuteron in this approach (the mass, interaction potential, wave function) are described in a self-consistent way.

Appendix C. The Non-Relativistic Twist Two Operators of Deep Inelastic Scattering.

Upon nonrelativistic reduction, the operators (17),(18) assume the form:

The nucleon operator (17):

$$O_N^n = \int \frac{d^3\mathbf{p}_1 d^3\mathbf{p}_2}{(2\pi)^6} \mathcal{N}_n^{(1)}(\mathbf{p}_1,\mathbf{p}_2)\cdot a^+(\mathbf{p}_1)a(\mathbf{p}_2), \qquad (C.1)$$

$$O_N^{n\sigma} = g_\sigma \int \frac{d^3\mathbf{p}_1 d^3\mathbf{p}_2}{(2\pi)^9} \frac{d^3\mathbf{k}}{\sqrt{2\omega(\mathbf{k})}} \mathcal{N}_n^{(2)}(\mathbf{k})\cdot a^+(\mathbf{p}_1)a(\mathbf{p}_2)\left(b^+(\mathbf{k})+b(\mathbf{k})\right), \qquad (C.2)$$

$$O_N^{nn} = g_\sigma^2 \int \frac{d^3\mathbf{p}_1 d^3\mathbf{p}_2}{(2\pi)^6} \frac{d^3\mathbf{p}_3 d^3\mathbf{k}}{(2\pi)^6} \mathcal{N}_n^{(3)}(\mathbf{k})\cdot a^+(\mathbf{p}_1)a^+(\mathbf{p}_2)\left(a(\mathbf{p}_2-\mathbf{k})+a(\mathbf{p}_2+\mathbf{k})\right)a(\mathbf{p}_3), \qquad (C.3)$$

where $a^+(\mathbf{p}), a(\mathbf{p})$ and $b^+(\mathbf{k}), b(\mathbf{k})$ are nucleon and meson creation - annihilation operators, respectively, and $\mathcal{N}_n^{(i)}(\mathbf{k})$ equal:

$$\mathcal{N}_n^{(1)}(\mathbf{p}_1,\mathbf{p}_2) = 2m^n\left(1-\frac{\mathbf{p}_1^2+\mathbf{p}_2^2}{8m^2}+\frac{\mathbf{p}_1\mathbf{p}_2}{4m^2}+\frac{p_{1z}+p_{2z}}{2m}\right)\times \times\left[1+\frac{\mathbf{p}_1^2+\mathbf{p}_2^2}{4m^2}+\frac{p_{1z}+p_{2z}}{2m}\right]^{n-1},$$

$$\mathcal{N}_n^{(2)}(\mathbf{k}) = \frac{2m^n}{\omega_+(\mathbf{k})}\left(\left[1+\frac{\omega_+(\mathbf{k})}{2m}\right]^{n-1}-\left[1-\frac{\omega_+(\mathbf{k})}{2m}\right]^{n-1}\right),$$

$$\mathcal{N}_n^{(3)}(\mathbf{k}) = \frac{2m^n}{2\omega^2(\mathbf{k})\omega_+(\mathbf{k})}\times \times\left(\left[1+\frac{\omega_+(\mathbf{k})}{2m}\right]^{n-1}-\left[1-\frac{\omega_+(\mathbf{k})}{2m}\right]^{n-1}-\frac{\omega_+^2(\mathbf{k})}{k_z{}^2}\left(\left[1+\frac{k_z}{2m}\right]^{n-1}-\left[1-\frac{k_z}{2m}\right]^{n-1}\right)\right),$$

where $\omega_\pm(\mathbf{k}) = \omega(\mathbf{k}) \pm k_z$.

Analogously, the meson operator (18) can be represented in the form:

$$O_\Phi^\sigma = \int \frac{d^3\mathbf{k}_1 d^3\mathbf{k}_2}{(2\pi)^6} \frac{1}{\sqrt{2\omega(\mathbf{k}_1)2\omega(\mathbf{k}_2)}} \mathcal{M}_n^-(\mathbf{k}_1,\mathbf{k}_2)\times \times\left(b^+(\mathbf{k}_1)b^+(\mathbf{k}_2)+b(\mathbf{k}_1)b(\mathbf{k}_2)\right)+2\mathcal{M}_n^+(\mathbf{k}_1,\mathbf{k}_2)b^+(\mathbf{k}_1)b(\mathbf{k}_2), \qquad (C.4)$$

$$O_\Phi^{n\sigma} = g_\sigma \int \frac{d^3\mathbf{p} d^3\mathbf{k}_2}{(2\pi)^9} \frac{d^3\mathbf{k}_1}{\sqrt{2\omega(\mathbf{k}_1)}} \mathcal{M}_n^{(2)}(\mathbf{k}_1,\mathbf{k}_2)\times \times\left(a^+(\mathbf{p})a(\mathbf{p}-\mathbf{k}_2)b(\mathbf{k}_1)+a^+(\mathbf{p})a(\mathbf{p}+\mathbf{k}_2)b^+(\mathbf{k}_1)\right), \qquad (C.5)$$

$$O_\Phi^{nn} = g_\sigma^2 \int \frac{d^3\mathbf{k}_1 d^3\mathbf{p}_1}{(2\pi)^6} \frac{d^3\mathbf{k}_2 d^3\mathbf{p}_2}{(2\pi)^6} \mathcal{M}_n^{(3)}(\mathbf{k}_1,\mathbf{k}_2)\times \times a^+(\mathbf{p}_1)a^+(\mathbf{p}_2)a(\mathbf{p}_2+\mathbf{k}_2)a(\mathbf{p}_1+\mathbf{k}_1), \qquad (C.6)$$

where $\mathcal{M}_n^{(i)}(\mathbf{k}_1,\mathbf{k}_2)$ are as follows:

$$\mathcal{M}_n^\pm(\mathbf{k}_1,\mathbf{k}_2) = \frac{1}{2^n}(\omega_+(\mathbf{k}_1)\pm\omega_+(\mathbf{k}_2))^n,$$

$$\mathcal{M}_n^{(2)}(\mathbf{k}_1,\mathbf{k}_2) = \frac{1}{2^n\omega^2(\mathbf{k}_2)}\cdot\left((\omega_+(\mathbf{k}_1)+\omega_+(\mathbf{k}_2))^n+(\omega_+(\mathbf{k}_1)-\omega_-(\mathbf{k}_2))^n-2(\omega_+(\mathbf{k}_1)+k_{2z})^n\right),$$

$$\mathcal{M}_3(\mathbf{k}_1,\mathbf{k}_2) = \frac{1}{2^{n+2}\omega^2(\mathbf{k}_1)\omega(\mathbf{k}_2)}\cdot\left(2[\omega_-(\mathbf{k}_1)+\omega_+(\mathbf{k}_2)]^n+[\omega_-(\mathbf{k}_1)-\omega_-(\mathbf{k}_2)]^n +[\omega_+(\mathbf{k}_1)-\omega_+(\mathbf{k}_2)]^n-4[\omega_+(\mathbf{k}_1)-k_{2z}]^n+4[k_{1z}-k_{2z}]^n\right).$$

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Received by Publishing Department
on September 22, 1992.

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Глубоконеупругое рассеяние лептонов на ядрах:
операторное разложение и мезон-нуклонная теория

С новой точки зрения рассмотрено глубоконеупругое рассеяние лептонов на ядрах. В эффективной мезон-нуклонной теории методом операторного разложения исследовано глубоконеупругое рассеяние на дейтроне. Дан строгий вывод аналитического вида амплитуды рассматриваемого процесса и проведено детальное исследование ядерных структурных эффектов. Рассмотрены конкретные приложения метода для анализа экспериментальных данных.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1992

Kaptari L.P., Umnikov A.Yu. E2-92-388

Deep Inelastic Lepton Scattering on Nuclei:
Operator Product Expansion and Meson-Nucleon Theory

A novel point of view on the x-rescaling model in lepton-nucleus deep inetic scattering is suggested. Using the operator product expansion method within the effective meson-nucleon theory, we present a rigorous consideration of the scattering on the deuteron. We demonstrate that with the contributions interpreted as the Fermi motion corrections, the x-rescaling idea is exactly reproduced. The diagrams of scattering of bound nucleons have also been computed numerically. An example of application of the method, viz. the problem of extraction of the neutron structure function from the combined proton-deuteron data, is considered.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1992