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SPIN PHENOMENA IN ELASTIC PROCESSES

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In more detail the title is: spin formalism and some consequences of using dynamic amplitudes at high and low energies for any elastic processes. We will consider reactions with particles of any spin s_k, masses m_k, and helicities λ_k:

$$a(m_1, s_1, \lambda_1) + b(m_2, s_2, \lambda_2) \longrightarrow c(m_3, s_3, \lambda_3) + d(m_4, s_4, \lambda_4) \tag{1}$$

The conference participants are both theorists and experimentalists, and both particle and nuclear physicists, so I will try be simple as possible. On the other hand, because of the restrictions on the volume of the article I will consider in detal only points 1 and 2, much more shorter point 3 and will give just a survey (more exactly - an enumeration, and references) of some results that can be obtained on the basis of the suggested formalism.

1. FORMALISM

1.1. Helicity amplitudes

We are interested in kinematic aspects in binary processes related to spin. For particles with a zero spin the process is described by one amplitude, $A(s,t)$. This amplitude is decomposied in the polynomials in both invariant variables.For particles with a nonzero spin, the process (1) can be described by helicity amplitudes of Jacob and Wick [1].

The physical meaning of the helicity amplitudes is clear, the connection with physical quantities is simple. But conservation rules do impose definite restrictions on the helicity amplitudes. Besides they have the so colled kinematic singularities. This is manifested in particular by the fact that the generalisation of partial-wave decomposition to spin-particle scattering is the decomposition in the Wigner rotation functions. But thise functions are not polynomials, they contain some singularities, just the kinematic singularities of helicity amplitudes.

1.2. Conservation laws and dispersion amplitudes for any binary reaction

The total number of helicity (or other) amplitudes for scattering of massive particles is

$$N = (2s_1 + 1)(2s_2 + 1)(2s_3 + 1)(2s_4 + 1). \tag{2}$$

However, the conservation of the projection of angular momentum decreases the number of amplitudes for certain directions when the process has higher symmetry.

Consider the reaction in the s-channel described by the helicity amplitudes $f^s_{\lambda_3\lambda_4,\lambda_1,\lambda_2}(s,t)$. Introduce the quantities $\lambda = \lambda_1 - \lambda_2$ and $\mu = \lambda_3 - \lambda_4$. Their meaning will be clear if we recall that two particles in the centre-of-mass system are moving in opposite directions and thus λ and μ are projections of the total spin on the directions of motion prior to and after collision. Owing to the conservation of the projection of total angular momentum the amplitudes in the forward direction, $\theta_s \to 0$, should vanish in all cases except for $\lambda = \mu$. Analogously, for backward scattering; $\theta_s \to \pi$, the amplitudes should vanish for the same reasons in all cases except for $\lambda = -\mu$.

Thus, the binary processes involving particles with a nonzero spin may be described by N helicity amplitudes $f^s_{\lambda_3\lambda_4,\lambda_1,\lambda_2}(s,t)$, which are constrained by the conservation laws.

For forward scattering we have

$$f^{forward}_{\lambda_3\lambda_4,\lambda_1,\lambda_2} = \begin{cases} f_{\lambda_3\lambda_4,\lambda_1,\lambda_2}, & when \quad \lambda = \mu, \\ 0, & when \quad \lambda \neq \mu. \end{cases} \tag{3}$$

whereas for backward scattering

$$f^{backward}_{\lambda_3\lambda_4,\lambda_1,\lambda_2} = \begin{cases} f_{\lambda_3\lambda_4,\lambda_1,\lambda_2}, & when \quad \lambda = -\mu, \\ 0, & when \quad \lambda \neq -\mu. \end{cases} \tag{4}$$

Two questions arise: Can the helicity amplitudes be parametrized so as to satisfy the conditions (3) and (4) automatically? Can kinematic singularities of helicity amplitudes be found and separated in a simple way? The answer is "Yes".

It is convenient to expand helicity amplitudes into a set of the Wigner rotation functions. In this decomposition the scattering amplitude in the c.m.system of the s-channel, $f^s_{\lambda_3\lambda_4,\lambda_1,\lambda_2}(s,t)$, is splitted into two parts; one part is defined by the symmetry properties and enters into the Wigner functions $d^J_{\lambda\mu}(\cos\theta)$ that make the conservation laws of the angular momentum valid, and the other part has a dynamic nature and enters into the partial helicity amplitudes $f^J_{\lambda_3\lambda_4,\lambda_1,\lambda_2}(s)$:

$$f^s_{\lambda_3\lambda_4,\lambda_1,\lambda_2}(s,t) = \sum_J (2J+1) f^J_{\lambda_3\lambda_4,\lambda_1,\lambda_2}(s) d^J_{\lambda\mu}(\cos\theta). \tag{5}$$

This decomposition is physically distinguished because the basis functions of the decomposition are eigenfunctions of the total angular momentum (which is a conservative quantitiy). This is the kinematic part in decomposition, while dynamics is contained in coefficient functions.

In the group theory language this expansion in the physical region of the s-channel implies that the amplitude is expressed in terms of basis elements of an irreducible representation of the Poincare group. The rotation matrices $d^J_{\lambda\mu}(\cos\theta)$ are representations of a small group of the three-dimensional rotation group $O(3)$ or its universal covering $SU(2)$. According to the group theory [2,3], matrix elements of irreducible representations of the group $SU(2)$ form a complete basis through elements of which any function quadratically integrable on a group manifold can be expressed.

Expressing $\cos\theta$ in terms of s and t, one can represent the Wigner function as dimensionless factors which do not depend on the summation index multiplied by polinomials in the variable t [4]:

$$d^J_{\lambda\mu}(\cos\theta) \sim A^{|\lambda-\mu|} B^{|\lambda+\mu|} Polinom(t). \tag{6}$$

Where

$$A = \frac{\sqrt{L^2 - a^2}}{(m_1+m_2)(m_3+m_4)}, B = \frac{\sqrt{L^2 + a^2}}{(m_1+m_2)(m_3+m_4)},$$
$$L^2 = \{[s-(m_1-m_2)][s-(m_1+m_2)][s-(m_3-m_4)][s-(m_3+m_4)]\}^{1/2}$$
$$a^2 = 2st + s^2 - s\sum m_k^2 + (m_1^2 - m_2^2)(m_3^2 - m_4^2) \tag{7}$$

The mass factors in the denominators make A and B dimensionless without introducing additional singularities in the variable s .

Using this property we can separate the common factors which do not depend on J and define dispersion amplitudes for any binary processes:

$$f^s_{\lambda_3\lambda_4,\lambda_1,\lambda_2}(s,t) = A^{|\lambda-\mu|} B^{|\lambda+\mu|} \bar{f}^s_{\lambda_3\lambda_4,\lambda_1,\lambda_2}(s,t). \tag{8}$$

For elastic processes ($m_1 = m_3 = \mu; m_2 = m_4 = m$) we can introduce dispersion amplitudes, avoiding an additional singularity at $s = 0$ point by the equation:

$$f^s_{\lambda_3\lambda_4,\lambda_1,\lambda_2}(s,t) = \left(\frac{\sqrt{-t}}{m+\mu}\right)^{|\lambda-\mu|} \left(\frac{\sqrt{L^2+st}}{(m+\mu)^2}\right)^{|\lambda+\mu|} \bar{f}^s_{\lambda_3\lambda_4,\lambda_1,\lambda_2}(s,t). \tag{9}$$

Under these parametrizations the conditions (3) and (4) are fulfilled automatically. All kinematic singularities in variable t and no false singularities in s are introduced. The amplitudes $\bar{f}^s_{\lambda_3\lambda_4,\lambda_1,\lambda_2}(s,t)$ suit well for studying the analytic properties of amplitudes at fixed s becous they obey dispersion relations. Therefore, we call them the dispersion amplitudes [5]. They still may have the kinematic singularities in the variable s.

1.3. Dynamic amplitudes for gravitino-pion scattering

Let us consider as an example a kinematically simple and physically interesting supersymmetric process, scattering of a gravitino on a spinless massive target [6].

Independent helicity amplitudes in the s-channel c.m.system are taken to be $f^s_{3/20,3/20}(s,t)$, the helicity-nonflip amplitude, and $f^s_{3/20,-3/20}(s,t)$, the helicity-flip amplitude.

Dispersion and helicity amplitudes in the s-channel are connected by the formulae

$$f^s_{3/20,3/20}(s,t) = \left(\frac{\sqrt{st+(s-m^2)^2}}{m^2}\right)^3 \bar{f}^s_{3/20,3/20}(s,t);$$
$$f^s_{3/20,-3/20}(s,t) = \left(\frac{\sqrt{-t}}{m}\right)^3 \bar{f}^s_{3/20,-3/20}(s,t). \tag{10}$$

The process in the t-channel is also decsribed by two amplitudes, and helicity and dispersion amplitudes are related as follows

$$f^t_{3/23/2,00}(s,t) = \bar{f}^t_{3/23/2,00}(s,t); f^t_{3/2-3/2,00}(s,t) = \left(\frac{\sqrt{st+(s-m^2)^2}}{m^2}\right)^3 \bar{f}^t_{3/2-3/2,00}(s,t). \tag{11}$$

Dispersion amplitudes of the annihilation channel can be expanded in polynomials in the variable s and are free of kinnematic singularities in this variable. In this channel, both for forward and backward scattering, the second helicity amplitude should turn into zero in accordance with conservation laws, which is provided by the corresponding factor in formula (11).

Because of the massless gravitino and spinness the pion the crossing relations for helicity amplitudes are extremely simple this is one of the reasons we consider this reaction)

$$f^s_{3/20,3/20}(s,t) = \alpha f^t_{3/2-3/2,00}(s,t), f^s_{3/20,-3/20}(s,t) = \beta f^t_{3/23/2,00}(s,t). \quad (12)$$

where α and β are constants of absolute value 1 (which are irrelevant for us). Owing to the crossing relations being simple we can easily separate the kinematic singularities of s-channel helicity amplitudes in the variable s. Using the above formulae we derive the following crossing relations for dispersion amplitudes

$$\bar{f}^s_{3/20,3/20}(s,t) = \alpha \bar{f}^t_{3/2-3/2,00}(s,t); \left(\frac{\sqrt{-t}}{m}\right)^3 \bar{f}^s_{3/20,-3/20}(s,t) = \beta \bar{f}^t_{3/23/2,00}(s,t). \quad (13)$$

The s-channel dispersion amplitudes are free of kinematic singularities in t; in principle, they may have kinematic singularities in s. The t-channel dispersion amplitudes are free of kinematic singularities in s. From formula (13) it is seen that both the dispersion amplitudes for the process under consideration are free of kinematic singularities in s. Thus, we have found the dynamic amplitudes that are free of kinnematic variables in both independent invariant variables. The dynamic amplitudes of the considered reaction coincide with the reduced amplitudes and are connected with helicity amplitudes in the following way

$$f^s_{3/20,3/20}(s,t) = \left(\frac{\sqrt{st + (s-m^2)^2}}{m^2}\right)^3 D^s_{3/20,3/20}(s,t), \quad (14)$$

$$f^s_{3/20,-3/20}(s,t) = \left(\frac{\sqrt{-t}}{m}\right)^3 D^s_{3/20,-3/20}(s,t). \quad (15)$$

1.4. Crossing and dynamic amplitudes

During crossover from one channel to another in the case of helicity amplitudes, analytic continuation from a physical region of variables s and t of one channel to a physical region of another channel should be accompanied by transition from the c.m.system of the s-channel to the c.m.system of the t-channel with the use of the Lorentz complex transformation. The latter results in requantization of spins; and the amplitude is expressed in terms of the Wigner functions.

The crossing relations between the s- and t-channel helicity amplitudes lock as follows [7-10]

$$f_{\lambda_3\lambda_4,\lambda_1,\lambda_2}(s,t) = \sum_{\mu_1\mu_2\mu_3\mu_4} \alpha d^{s_1}_{\lambda_1\mu_1}(\chi_1) d^{s_2}_{\lambda_2\mu_2}(\chi_2) d^{s_3}_{\lambda_3\mu_3}(\chi_3) d^{s_4}_{\lambda_4\mu_4}(\chi_4) f^t_{\mu_3\mu_4,\mu_1\mu_2}(s,t). \quad (16)$$

When in a reaction particles with zero masses are involved, or the spin of the particle is sero, crossing relations get simplified. In such cases the d-matrix reduces to the Kronecker symbols, and there is no sum in formula for corresponding indices [11].

For elastic processes the crossing relations between dispersion amplitudes are of the form ($\lambda' = \mu_1 - \mu_2, \mu' = \mu_3 - \mu_4$):

$$\bar{f}_{\lambda_3\lambda_4,\lambda_1,\lambda_2}(s,t) = \left(\frac{\sqrt{-t}}{m+\mu}\right)^{-|\lambda-\mu|} \left(\frac{\sqrt{L^2+st}}{(m+\mu)^2}\right)^{-|\lambda+\mu|} \sum_{\mu_1\mu_2\mu_3\mu_4} d^{s_1}_{\lambda_1\mu_1}(\chi_1) d^{s_2}_{\lambda_2\mu_2}(\chi_2)$$
$$d^{s_3}_{\lambda_3\mu_3}(\pi+\chi_1) d^{s_4}_{\lambda_4\mu_4}(\pi+\chi_2) (A^t)^{1/2|\lambda'+\mu'|} (B^t)^{1/2|\lambda'-\mu'|} \bar{f}^t_{\mu_3\mu_4,\mu_1\mu_2}(s,t). \quad (17)$$

$$A^t = \frac{\sqrt{(t-4m^2)(t-4\mu^2)} - 2s - t + 2(m^2+\mu^2)}{(m+\mu)^2}, \quad (18)$$

$$B^t = \frac{\sqrt{(t-4m^2)(t-4\mu^2)} + 2s + t - 2(m^2+\mu^2)}{(m+\mu)^2}. \quad (19)$$

Crossing angles are given by the formulae

$$\cos\chi_1 = -\frac{(s+\mu^2-m^2)t}{L\tau_\mu}, \sin\chi_1 = \frac{2\mu\sqrt{\Phi}}{L\tau_\mu}, \quad (20)$$

$$\cos\chi_2 = \frac{(s-\mu^2+m^2)t}{L\tau_m}, \sin\chi_2 = \frac{2m\sqrt{\Phi}}{L\tau_m}, \quad (21)$$

where

$$\Phi(s,t,u) + stu - t(m^2-\mu^2), \tau_\mu = \sqrt{t(t-4\mu^2)}, \tau_m = \sqrt{t(t-4m^2)}, \quad (22)$$

As has been shown in the previous section for a kinematically simple process the crossing relations for dispersion amplitudes allow us to separate the kinematic singularities in s. In this way we can determine the dynamic amplitudes for alastic processes [12]:

$$f_{\lambda_3\lambda_4,\lambda_1,\lambda_2}(s,t) = \left(\frac{\sqrt{-t}}{m+\mu}\right)^{-|\lambda-\mu|} \left(\frac{\sqrt{L^2+st}}{(m+\mu)^2}\right)^{-|\lambda+\mu|} \left(\frac{L}{(m+\mu)^2}\right)^{-2(s_1-s_2)} D_{\lambda_3\lambda_4,\lambda_1,\lambda_2}(s,t). \quad (23)$$

2. OBSERVABLES

From this point an account will be shorter and more schematic. The observable quantities are simply expressed via the helicity amplitudes.

As we have already mentioned, the helicity amplitudes have a clear physical meaning, and physical observables (polarization cross sections, asymmetries, etc.) are simply expressed via them. As the connection between the helicity and dynamic amplitudes is one-to-one, every helicity amplitude for elastic scattering is expressed in terms of one dynamic amplitude. Hence it follows that all attractive features of the helicity amplitudes, a clear physical meaning, simple relations with observables, and equal dimensions, are also inherent in the dynamic amplitudes. The formalism of dynamic amplitudes is simple for low spins and remains such also for higher spins: the formalism is simple for any spins.

The differential cross section for elastic scattering, when one measures the helicity of each particle expressed via helicity, invariant, and dynamic amplitudes has the following form:

$$\frac{d\sigma}{dt}(\lambda_3\lambda_4,\lambda_1\lambda_2) \sim \mid f_{\lambda_3\lambda_4,\lambda_1\lambda_2}(s,t)\mid^2=$$

$$=\mid \sum_{n=1}^{N} a^n_{\lambda_3\lambda_4,\lambda_1\lambda_2}(s,t)A_n(s,t)\mid^2=$$

$$=\mid \left(\frac{s-m^2}{m^2}\right)^{-2J}\left(\frac{\sqrt{-t}}{m}\right)^{|\lambda-\mu|}\left(\frac{\sqrt{st+(s-m^2)^2}}{m^2}\right)^{|\lambda+\mu|} D^s_{\lambda_3\lambda_4,\lambda_1,\lambda_2}(s,t)\mid^2. \quad (24)$$

Comments:

– First line in outward appearance is simplest, but helicity amplitudes contain kinematic singularities and the conservation laws do not fulfill automatically – so kinematics and dynamics are not separated. Here we have one term.

– In second line there is the sum of all invariant amplitudes. Here we have N^2 terms. For the spins equal to 3/2 there are 65536, and the spins equal to 11/2, there are more then 10^8 terms. In each term we have kinematic – dynamic separation, but there are so many such terms. To find such parametrisation is difficult for high spins. This variant is complicated – to definite one cross section with definite values of particle helicities one needs to know all invariant amplitudes. As in the previous variant, the consequences of conservation rules do not affect the parametrisation. As mentioned above, invariant amplitudes have different dimensions and have no physical meaning (the opposite will be much better). The nice point in using a set of invariant amplitudes is that they have no kinematic singularities.

– The parametrisation via dynamic amplitudes is best in our openion. In this case in the above formulae we have no summation! The differential cross section is expressed only via one dynamic amplitude with the kinematical factors which contain all kinematic singularities. Just thesa factors ensure that conservation laws are fulfilled automatically. In this case we have the simple separation of kinematics and dynamics in the description of the spin particle elastic scattering. The dynamic amplitudes have a clear physical meaning and the same dimensions. We have only one term.

For unpolarised reactions we have:

$$\frac{d\sigma}{dt} \sim \sum_{\lambda_i}^{N} \mid f_{\lambda_3\lambda_4,\lambda_1\lambda_2}(s,t)\mid^2=$$

$$=\sum_{\lambda_i}^{N}\mid \sum_{n=1}^{N} a^n_{\lambda_3\lambda_4,\lambda_1\lambda_2}(s,t)A_n(s,t)\mid^2=$$

$$=\sum_{\lambda_i}^{N}\mid \left(\frac{s-m^2}{m^2}\right)^{-2J}\left(\frac{\sqrt{-t}}{m}\right)^{|\lambda-\mu|}\left(\frac{\sqrt{st+(s-m^2)^2}}{m^2}\right)^{|\lambda+\mu|} D^s_{\lambda_3\lambda_4,\lambda_1,\lambda_2}(s,t)\mid^2. \quad (25)$$

We have depending of parametrisation N, N^2 and N terms. Because of "kinematic hierarchy" (see the next paragraph) at high energies we can reduce the number of terms, dominated in observables.

Other quantities, such as $P, A_{nn}, A_{ll}, A_{ss}$ in terms of the helicity amplitudes have the form [13,14]

$$\sim \frac{\sum c_{mn} f_m f_n^*}{\sum \mid f_m \mid^2} \quad (26)$$

Here m and n represent sets of helicity indices. $c_{mn} = \pm 1$. The sum is taken for all values of helicities. For briefty we did not write the above quantities in terms of invariant and dynamic amplitudes. Obviously the expressions, as for (24) and (25) will be most convenient in terms of dynamic amplitudes.

3. HIGH ENERGY BEHAVIOUR

At high energies it was often assumed the simple assumption that spin effects are died out, and consequently, the helicity amplitudes do not depend on spin. However, this cannot be assumed directly. Simplifications like that are not correct, as the obligatory kinematic conditions are not taken into account. One cannot neglect the kinematic factors or consider them to be all equal. In that case the helicity amplitudes could possess kinematic singularities, and the angular momentum would not be conserved, say, for forward scattering "forbidden" amplitudes would not vanish. Besides the experiment gives that at high energies the spin effects are considerable.

In study the binary processes at fixed scattering angles and high energies it is convenient to represent kinematic factors in the definition of dynamic amplitudes as functions of the scattering angle in the c.m. system θ and invariant variable s. Kinematic factors expressed in terms of θ and s are factorizable, and we can write

$$f_{\lambda_3\lambda_4,\lambda_1\lambda_2}(s,t) = P_{\lambda_3\lambda_4,\lambda_1\lambda_2}(s)F_{\lambda_3\lambda_4,\lambda_1\lambda_2}(\theta)D_{\lambda_3\lambda_4,\lambda_1\lambda_2}(s,t) \quad (27)$$

As $s \to \infty$ we get the small kinematic factor

$$P_{\lambda_3\lambda_4,\lambda_1\lambda_2}(s) \sim \left(\frac{m+\mu}{\sqrt{s}}\right)^{l(\lambda_3\lambda_4,\lambda_1\lambda_2)}. \quad (28)$$

For different values of helicities

$$l_{min} \le l(\lambda_3\lambda_4,\lambda_1\lambda_2) \le l_{max} \quad (29)$$

In observables, some of contributions of amplitudes are kinematically increased (such amplitudes will give leading contributions) whereas others are suppressed (and can be neglected in the first approximation). So we have the "kinematic hierarchy"

– the helicity amplitudes are divided into classes giving the leading contribution, the first corrections, second corrections , and so on.

For nucleon-nucleon scattering we have five independent amplitudes. In the high-energy large-fixed-angle region the helicity amplitudes are splitted into three classes in the order of smallness determined by the kinematic factors. So we obtain [15,16]

$$f_{1/2,1/2;1/2,-1/2} \gg f_{1/2,-1/2;1/2,-1/2} \sim f_{1/2,-1/2;-1/2,1/2} \gg f_{1/2,1/2;1/2,1/2} \sim f_{1/2,1/2;-1/2,1/2}. \quad (30)$$

or, in terms of dynamic amplitudes:

$$\frac{\sqrt{s}}{2m}\sin\theta D_{1/2,1/2;1/2,-1/2} \gg \cos^2\frac{\theta}{2}D_{1/2,-1/2;1/2,-1/2)} \sim \sin^2\frac{\theta}{2}D_{1/2,-1/2;-1/2,1/2} \gg$$
$$\gg \frac{m^2}{p^2}D_{1/2,1/2;1/2,1/2} \sim \frac{m2}{p^2}D_{1/2,1/2;-1/2,1/2}. \quad (31)$$

where $p = \sqrt{s-4m^2}/2$, "$a \gg b$" means that the contribution of b is suppressed relative to the contribution of a in the observables.

The predictions of perturbative QCD, taking into account the helicity conservation rule [17] may be written in the form

$$f_{1/2,1/2;1/2,1/2} \sim f_{1/2,-1/2;1/2,-1/2} \sim f_{1/2,-1/2;-1/2,1/2} \gg f_{1/2,1/2;1/2,-1/2} \gg f_{1/2,1/2;-1/2,1/2}. \quad (32)$$

For pp scattering at $\theta_{c.m} = 90°$ we have from $s-u$ crossing symmetry that

$$f_{1/2,1/2;1/2,-1/2}(90°) = 0, f_{1/2,-1/2;1/2,-1/2}(90°) = f_{1/2,-1/2;-1/2,1/2}(90°) \quad (33)$$

Taking into account dominated amplitudes we get for asymmetries

$$A_{nn} = A_{ss} = \frac{2Re f_{1/2,-1/2;1/2,-1/2} f^*_{1/2,-1/2;-1/2,1/2}}{\mid f_{1/2,-1/2;1/2,-1/2}\mid^2 + \mid f_{1/2,-1/2;-1/2,1/2}\mid^2} \to 1. \quad (34)$$

QCD gives for thise quantities the value 1/3 [18,19]. The massive quark model [20] gives $A_{nn} = 0.97; A_{ss} = -0.01$

For experiment we have at 90°, for $A_{nn}(p_\perp^2(GeV)^2)$ the following values [21]

$$A_{nn}(3.81) = 0.26; A_{nn}(4.79) = 0.52; A_{nn}(5.56) = 0.59. \quad (35)$$

For the masseless gravitino scattering on the nucleon we have 6 independent amplitudes and the following hierarchy from them [22]:

$$f_{1/2,-3/2;1/2,-3/2} \gg f_{1/2,3/2;-1/2,3/2} \gg f_{1/2,3/2;1/2,-3/2} \gg$$
$$\gg f_{1/2,-3/2;-1/2,3/2} \sim f_{1/2,3/2;1/2,3/2} \gg f_{1/2,3/2;-1/2,-3/2}. \quad (36)$$

Two comments:

–We recomend for elastic scattering processes at high energies and large fixed angles first of all to measure dominating amplitudes.

–The kinematic hierarchy gives connections between various asymmetry parameters.

4. SOME APPLICATIONS

4.1. Low-energy theorems

The spin kinematics allows one to obtain the low-energy theorems for photon-hadron processes [23,24] and gravitino scattering on spin-0 target. For the latter process at low energies the helicity amplitudes up to $0(E^3)$ are determined by their t-cannel Born terms with the photon exchange [6,22].

4.2. Model-independent inequalities

The dynamic amplitudes, or more simply, the t-channel dispersion amplitudes can be used to prove model-independent dispersion inequalities for the Compton effect on the pion and nucleon target, including the case of the polarised photon scattering [25,26].

4.3. Other applications

Here we only mention other possible applications of dynamic amplitudes. These are the dispersion relations for individual helicity amplitudes for any elastic scattering and sum rules (espesially dual sume rules) also for any elastic scattering. The dynamic amplitudes for inelastic processes will be considered separately.

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Спиновые явления в упругих процессах

Обсуждаются спиновые явления в упругих процессах. Краткий обзор состоит из четырех частей.

1. Формализм. Сначала мы рассматриваем законы сохранения и проблему разделения кинематики и динамики. Основным результатом является введение динамических амплитуд для произвольных упругих процессов рассеяния.

2. Наблюдаемые. Выписываем выражения наблюдаемых величин посредством спиральных, инвариантных и динамических амплитуд и сравниваем их.

3. Высокоэнергетическое поведение. Получаем кинематическую иерархию вкладов амплитуд в наблюдаемые. В качестве примера рассматриваем протон-протонное рассеяние при высоких энергиях и больших фиксированных углах ($\sim 90^0$).

4. Некоторые приложения. Очень кратко обсуждаем низкоэнергетические теоремы, включая случай рассеяния суперсимметричной частицы (гравитино). Перечисляем другие возможности использования формализма (модельно независимые неравенства, дисперсионные соотношения, правила сумм и т.д.).

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1992

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Spin Phenomena in Elastic Processes

We will discuss spin phenomena in elastic processes. A short review consists of four paragraphs:

1. Formalism. First we discuss conservation laws and the problem of separation of the kinematics and dynamics. The main result is the introduction of the Dynamic Amplitudes for any elastic scattering.

2. Observables. We write out expressions of observables quantities in terms of helicity, invariant and dynamic amplitudes and compare them.

3. High energy behaviour. We obtain Kinematic Hierarchy of contributions of amplitudes in observables. As an example, we consider proton-proton scattering at high energies and large fixed angles ($\sim 90^0$).

4. Some applications. We shortly discuss the low-energy theorems, including the case of SUSY particle (gravitino) scattering. Some other possible applications of the formalism (model independent inequalities, dispersion relations, sum rules, and so on) are expounded.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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