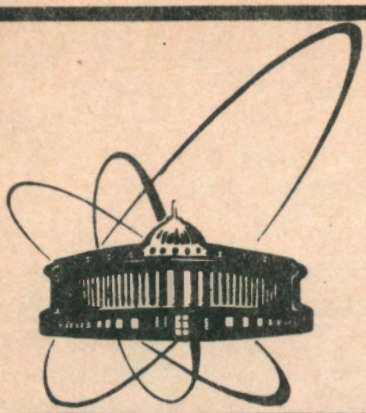

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STATISTICAL DESCRIPTION OF HOT AND DENSE CLUSTERING MATTER

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1. INTRODUCTION

The study of the hot and dense nuclear matter has many important applications to nucleus-nucleus and ion-ion collisions, the early Universe, neutron stars and other stellar models. A major obstacle for the description of nuclear matter in the whole range of temperatures and densities is that a large variety of very complicated bound structures are favoured. A different way to express this inherent problem is that many-particle correlations are very important, and so standard methods that systematically improve the mean-field description are impractical. Indeed, it is necessary to invoke non-perturbative approaches.

For example, at low temperatures and densities the nucleons congregate into clusters embedded in a lower density nucleon fluid, which has been shown using phenomenological models [1,2] and the canonical Metropolis simulations of nuclear matter at finite temperature [3]. In this case the clusters are formed of nucleons.

Another limiting situation happens at high temperatures or densities when the nuclear matter passes into the quark-gluon state. Then, the hadrons themselves are to be considered as clusters of the so-called partons, i.e., quarks and gluons. The temperatures and densities that can be reached in relativistic collisions of nucleons or nuclei amount to $T \approx 140$ MeV and $\rho \approx 5\rho_0$, respectively [4-7], where ρ_0 is the normal nuclear density. The disintegration of hadrons into unbound quarks and gluons is called the deconfinement; and the process of clustering of hadrons, the confinement transition. As is clear, these processes also require non-perturbative approaches for their description, which up to now has not been achieved with the standard chromodynamic equations. This is why a number of statistical models [7-12] for describing the deconfinement-confinement transition has been invoked. Another way is to resort to the Monte Carlo simulation techniques using a discrete space-time lattice [12-18]. However, because of computational difficulties, the latter approach has been applied solely to the matter with zero baryon density [19], while the statistical modelling, being much simpler, has no such limitation. Moreover, it is understood [20] that statistical models are, and will be, necessary for interpreting computer calculations giving just figures.

Contrary to the case of low temperatures and densities, where there exists even more than qualitative agreement between phenomenological models [1,2] and computer molecular dynamics simulation [3] of clustering nuclear matter,

the situation at high temperature or density is hitherto controversial, if not to say dramatic. This is because there is no statistical model which would fit, at least qualitatively, all predictions of the lattice simulations made for three known cases: the quarkless SU(2) and SU(3) systems, and the SU(3) system with physical quarks. Really, the majority of statistical models, following either Baym and Chin [9] or Källman [21], describe the deconfinement as a first order phase transition for all three types of the systems, quarkless SU(2), quarkless SU(3), and SU(3) with physical quarks. This has nothing to do with the results of the lattice simulation displaying the *second order* phase transition for the quarkless SU(2) system [12], the *first order* transition for the quarkless SU(3) system [13-15], and a *continuous crossover* for the SU(3) system with physical quarks [16-18]. Note that under physical quarks one means the quarks with physically realistic values of masses. Another group of statistical models [12, 22-24] incorporates the postulate about the second order deconfinement transition, which can agree solely with the lattice simulation for the quarkless SU(2) system, but contradicts the simulation results for the other two types of the systems mentioned.

This inability of any of the statistical models to describe all three kinds of the deconfinement transition is caused, to all appearance, by the following fact: Each of the statistical models, to simplify the analysis, has limited the number of possible states so that the restricted space of states has characterized a sole kind of the existing deconfinement transitions. The behaviour of high-temperature systems could be much richer if one would take into account, in a consistent way, the possibility of cluster formation and desintegration, for instance as it has been done for low-temperature nuclear matter with nucleon [1,2] and multiquark [25-27] clusters. The more so, as the recent lattice simulations [28] testify that hadron objects do exist simultaneously with unbound quarks and gluons.

One interesting attempt to construct a statistical model treating nucleons as three-quark clusters coexisting with unbound quarks has been accomplished by Clark, Cleymans, and Rafelski [29]. However, this model is too oversimplified: quarks are considered by means of perturbation theory, hadrons are presented solely by a nucleon gas in the excluded-volume approximation; the thermodynamic correctness of the model is not checked. It is, presumably, just this oversimplification which has resulted in the conclusion [30] that such a clustered state cannot exist at high temperatures.

In the present paper, we formulate a more elaborate statistical model that can describe a clustering matter of arbitrary nature. When constructing a cluster Hamiltonian or, generally, any effective Hamiltonian [31], it is extremely important to choose its form so that it would satisfy all thermodynamic ralations. For this purpose, in Sec.2 we phrase the principle of thermodynamic equivalence yielding the conditions of thermodynamic correctness controlling the correct form of effective Hamiltonians. Note that the widely used

excluded-volume approximation does not satisfy these conditions of thermodynamic correctness although to some extent, but not always, the thermodynamic incorrectness of the excluded-volume approximation can be compensated by special restriction rules [31-33] used when differentiating thermodynamic quantities with respect to volume or density. To check the validity of our model, we have, first of all, to compare its predictions with the results of lattice simulations well known for pure gauge theories and for systems with quarks in the case of zero baryon density. To this end, in Sec.3 we consider the clustering gluon-glueball matter for the SU(2) and SU(3) systems, and in Sec.4 the clustering quark-hadron matter for the case of zero baryon density. Comparing our calculations with the lattice-simulation data, we have found a very good agreement between them. The main is that our statistical model is a unique one being able to describe all three kinds of the deconfinement-confinement transition. Sec.5 contains the conclusion. Everewhere the system of units is used in which $\hbar \equiv 1 \equiv c$. Some preliminary results of our approach have been reported at conferences [34-36].

2. THERMODYNAMIC EQUIVALENCE AND CORRECTNESS

The necessity of introducing cluster Hamiltonians is explained by the following fact. It is always difficult to depict bound states in the many-body problem, and to do this in quantum chromodynamics seems practically impossible because of a great variety of possible clusters. When constructing some cluster, or in general, any effective Hamiltonians [31], one must be extremely cautious seeing to it that the use of these Hamiltonians would not disturb the known thermodynamic relations. An incorrect choice of an effective Hamiltonian having, at first sight, quite a reasonable form but breaking some of thermodynamic relations, can lead to wrong conclusions, especially for describing phase transitions. To avoid a distortion of thermodynamic rules helps the principle of thermodynamic equivalence which we formulate below.

Consider a system of particles that can form clusters. Let an exact Hamiltonian of this system be $H \equiv H(\psi)$, where ψ is a set of field operators of these elementary particles. Being unable to solve the problem dealing with the Hamiltonian $H(\psi)$, we would like to separate out cluster degrees of freedom for constructing a more treatable effective cluster Hamiltonian $H_c = H_c(\psi_c)$ composed of a set

$$\psi_c \equiv \left\{ \psi_n : n = 1, 2, \ldots \right\} \tag{1}$$

of the cluster field operators ψ_n. A general connection of the latter with the field operators of generic particles can be written as

$$\psi_n(1) = \int A(1\,2\,3\,...n+1)\,\psi(2)\,\psi(3)\,...\,\psi(n+1)\,d(2\,3\,...n+1) +$$

$$+ \int B(1\,2\,3\,...\,n+)\,\overset{+}{\psi}(n+1)\,....\,\overset{+}{\psi}(3)\,\overset{+}{\psi}(2) d(2\,3\,...\,n+1) + C_n(1),$$

where the numbers in the brackets signify the corresponding dependence on variables and $A(...)$, $B(...)$, and $C_n(...)$ are nonoperator functions. Quite often, the constructed effective Hamiltonian acquires the dependence on thermodynamics parameters, say, on temperature T and a set $\rho_c \equiv \{\rho_n\}$ of the cluster densities ρ_n. Remember, e.g., the density dependent interaction potentials widely used in nuclear physics or the excluded-volume and some droplet models. Thus, the cluster Hamiltonian, being a kind of effective Hamiltonians, includes, in general, the dependence on thermodynamic parameters:

$$H_c = H_c(\psi_c; T, \rho_c);\ \rho_c \equiv \{\rho_n : n = 1, 2, ...\}. \qquad (2)$$

The introduced cluster Hamiltonian H_c would correctly describe the thermodynamics of the system under consideration provided it is, in some sense, thermodynamically equivalent to the initial Hamiltonian H.

By definition, two Hamiltonians, say H and H_c, are thermodynamically equivalent if, and only if, in the thermodynamic limit their thermodynamic characteristics coincide, which requires that for any of the thermodynamic potentials $F(H)$, defined as

$$F(H) = -T \ln Tr \exp(-H/T)$$

the following limiting equalities are true:

$$\lim_{V\to\infty} \frac{1}{V}\left[F(H) - F(H_c)\right] = 0,$$

$$\lim_{V\to\infty} \frac{1}{V}\left[dF(H) - dF(H_c)\right] = 0, \qquad (3)$$

where V is the volume of the system, and the limit $V \to \infty$ implies the thermodynamic limit

$$V \to \infty,\ N_n \to \infty,\ \rho_n \equiv N_n/V \to \text{const},$$

N_n being a number of n-particle clusters. Because of its importance, Eq. (3) can be called the *principle of thermodynamic equivalence* of two Hamiltonians. Let us emphasize that the principle (3) is to hold for any thermodynamic potential expressed through its natural thermodynamic variables. For instance, if for $F(H)$ one takes the free energy, then

$$F(H) = F(T, V, N_c);\ \ N_c \equiv \{N_n : n = 1, 2, ...\}.$$

Using the known thermodynamic relations [37] it is not too difficult to show that the principle of thermodynamic equivalence (3) yields the validity of the conditions

$$\langle \frac{\partial H_c}{\partial T} \rangle = 0,\ \langle \frac{\partial H_c}{\partial \rho_n} \rangle = 0, \qquad (4)$$

in which the average of an operator $\hat{A}$ reads

$$\langle \hat{A} \rangle \equiv \frac{Tr\,\hat{A}\,\exp(-H_c/T)}{Tr\,\exp(-H_c/T)}.$$

These conditions are very important for the practical construction of cluster Hamiltonians which would satisfy all thermodynamic relations. This is why these conditions (4) can be called the conditions of thermodynamic correctness of cluster Hamiltonians.

As has been emphasized above, the mean-field approximation is inapplicable to the initial Hamiltonian H since this approximation is unable to describe the appearance of clusters. However, when the cluster degrees of freedom are already separated out, the mean-field approximation can be applied to cluster Hamiltonian H_c which then has the form

$$H_c = \sum_n \sum_s \int \overset{+}{\psi}_n(\vec{r}, s)\,(K_n + U_n)\,\psi_n(\vec{r}, s)\,d\vec{r} - B \cdot V. \qquad (5)$$

Here n enumerates the clusters; and s, their internal degrees of freedom; unbound particles are treated as trivial clusters with $n = 1$; K_n is the kinetic energy being either $K_n = -\nabla^2/2M_n$ or $K_n = \sqrt{-\nabla^2 + M_n^2}$ in the nonrelativistic or relativistic cases, respectively, and M_n is a mass of the corresponding cluster; $U_n = U_n(T, \rho_c)$ is a mean field acting on an n-particle cluster. The nonoperator term BV enters into (5) because of the general rule [37] according to which the mean-field approximation applied to a product of operators $\hat{A}_1 \cdot \hat{A}_2$ is equivalent to the substitution

$$\hat{A}_1 \cdot \hat{A}_2 \to \hat{A}_1 \langle \hat{A}_2 \rangle + \langle \hat{A}_1 \rangle \hat{A}_2 - \langle \hat{A}_1 \rangle \langle \hat{A}_2 \rangle,$$

so that a nonoperator term must always be included. Besides, as we will show below, the nonoperator term is necessary for the cluster Hamiltonian (5) to satisfy the conditions of thermodynamic correctness (4). This means that the

cluster Hamiltonian (5) is correctly defined in the sense of (3) only if it contains a function $B = B(T, \rho_c)$ allowing the correctness of conditions (4).

The conditions of thermodynamic correctness (4) for the cluster Hamiltonian (5) write

$$\sum_n \rho_n \frac{\partial U_n}{\partial \rho_m} - \frac{\partial B}{\partial \rho_m} = 0,$$

$$\sum_n \rho_n \frac{\partial U_n}{\partial T} - \frac{\partial B}{\partial T} = 0. \qquad (6)$$

If we assume, which is natural, that the functions $U_n(T, \rho_c)$ and $B(T, \rho_c)$ are smooth functions of their arguments so that the derivatives with respect to temperature and densities commute, then differentiating the first of equating (6) with respect to temperature and the second with respect to ρ_m and comparing the results we obtain

$$\frac{\partial U_n}{\partial T} = 0; \quad (\forall n).$$

Therefore, the mean field $U_n = U_n(\rho_c)$ cannot depend on temperature directly but this dependence may enter only through densities.

Note that the widely used excluded-volume approximation yields effective Hamiltonians which do not satisfy the conditions of thermodynamic correctness (4). Consequently, this approximation is, strictly speaking, thermodynamically incorrect, and although it can be nevertheless used being supplemented by special restriction rules [32,33], it can lead to a wrong thermodynamic behaviour of a system, especially around phase transitions.

3. CLUSTERING GLUON-GLUEBALL MATTER

After formulating the general principles of describing clustering matter, it is reasonable to demonstrate their applicability to relatively simple models. To this end we consider first the quarkless SU(2) and SU(3) systems consisting of gluons whose bound states are glueballs [38]. The latter can be considered as gluon clusters. The total density of gluons in a quarkless system is the sum

$$\rho = \rho_g + \sum_{nj} n \rho_{nj} \qquad (7)$$

of the density ρ_g of unbound gluons and of the densities ρ_{nj} of n-gluon glueballs of a j type multiplied by n. The concentrations of the corresponding components can be defined as

$$w_g \equiv \frac{\rho_g}{\rho}, \quad w_G \equiv \frac{1}{\rho} \sum_{nj} n \rho_{nj}. \qquad (8)$$

The mean potential field acting on unbound particles having the property of the asymptotic freedom can be approximated [32-36] by the form

$$U_g = C/\rho^{\alpha}; \quad 0 < \alpha < 1, \qquad (9)$$

in which C is a constant in units of $\mathrm{MeV}^{3\alpha+1}$. The system of unbound gluons that can cluster into glueballs is presentable, according to (5) and (9), by the Hamiltonian

$$H_{gG} = \sum_{\sigma} \int \overset{+}{\psi}_g(\vec{r}, \sigma) \left(\sqrt{-\nabla^2} + C/\rho^{\alpha} \right) \psi_g(\vec{r}, \sigma)\, d\vec{r} +$$

$$+ \sum_{nj} \sum_{s} \int \overset{+}{\psi}_{nj}(\vec{r}, s) \left(\sqrt{-\nabla^2 + M_{nj}^2} + U_{nj} \right) \psi_{nj}(\vec{r}, s)\, d\vec{r} - BV, \qquad (10)$$

in which $\psi_g(\vec{r}, \sigma)$ is a Bose field operator representing an unbound gluon (trivial cluster) in a quantum state σ, while $\psi_{nj}(\vec{r}, s)$ is a Bose field operator corresponding to a glueball (n-gluon cluster) of a type j in a quantum state s.

The interaction potentials of clusters can be related with each other using the energy-momentum conservation laws in the reactions of fusion and decay of clusters. A straightforward consideration [36,39,40] gives the connection

$$\Phi_{nj,\,mi}(r) = \frac{nm}{4} \Phi_{22}(r) \qquad (11)$$

for the interaction potential of nj and mi clusters with the interaction potential $\Phi_{22}(r) \equiv \Phi_{20,\,20}(r)$ of the simplest nontrivial clusters which are the ground-state two-gluon glueballs. Using the Hartree approximation for a pure glueball system with the interaction potentials (11) we have

$$\lim_{\rho_g \to 0} U_{nj} = \frac{n}{4} \Phi_2 \rho,$$

$$\lim_{\rho_g \to 0} B = \frac{1}{8} \Phi_2 \rho^2, \qquad (12)$$

where

$$\Phi_2 \equiv \int \Phi_{22}(r)\, d\vec{r}. \qquad (13)$$

The use of the Hartree approximation is known [41] to be admissible for those hadrons whose masses are much higher than the deconfinement temperature.

The complete expressions for U_{nj} and B are to be found from the conditions of thermodynamic correctness (4) expressed in the form of equations (6). These equations, to give a unique solution, require boundary conditions. The latter are to be defined by the following natural agreement: when the concentration of the gluon component, given by (8), is negligible, i.e. $w_g \to 0$, we have a pure glueball system, and vice versa, if $w_G \to 0$, we get a pure gluon system. In this way, we obtain

$$U_{nj} = \frac{nC}{\rho^\alpha} - \frac{nC}{(\rho - \rho_g)^\alpha} + \frac{n}{4}(\rho - \rho_g)\,\Phi_2,$$

$$B = -\frac{\alpha}{1-\alpha} C \rho^{1-\alpha} + \frac{\alpha}{1-\alpha} C (\rho - \rho_g)^{1-\alpha} + \frac{1}{8}(\rho - \rho_g)^2 \Phi_2. \qquad (14)$$

It is worth noting that speaking of pure systems, either glueball or gluon, we imply their thermodynamic behaviour, that is the corresponding forms of their thermodynamic potentials. Analysing these two limiting situations directly in (14), we should use the following substitutions. The passage to a pure glueball system, when $w_g \to 0$, is equivalent to the limiting transition $\rho_g \to 0$; then (14) returns to (12). Another limiting case, when $w_G \to 0$, means that one comes to a pure gluon system in which there are no glueballs, so that $\rho \to \rho_g$; then from (14) we have

$$\lim_{w_G \to 0} B = -\frac{\alpha}{1-\alpha} C \rho_g^{1-\alpha}.$$

However, in the general case we have to invoke the total form of (14) letting the system itself to choose a more thermodynamically profitable state. Then, the gluon and glueball densities are defined by the equations

$$\rho_g = \frac{1}{V} \sum_\sigma \int \langle \overset{+}{\psi}_g(\vec{r}, \sigma)\, \psi_g(\vec{r}, \sigma) \rangle\, d\vec{r},$$

$$\rho_{nj} = \frac{1}{V} \sum_s \int \langle \overset{+}{\psi}_{nj}(\vec{r}, s)\, \psi_{nj}(\vec{r}, s) \rangle\, d\vec{r}. \qquad (15)$$

The thermodynamics of the clustering gluon-glueball system is completely defined by its free energy

$$F(H_{gG}) = -T \ln Tr \exp(-H_{gG}/T),$$

which can be easily calculated for the mean-field form of the cluster Hamiltinian (10).

It seems to be clear that the main contribution to thermodynamics should be due to the lightest glueballs, which is also supported by the consideration of a model with restriction rules [32,33]. This is why among all possible glueballs we choose those five that have the lightest masses [42,43] denoted by M_{nj} where n is the number of gluons bound into a glueball cluster, j enumerates the glueball excited states, corresponding to the ground state. The number of internal quantum states for each of the nj-glueballs will be written as ξ_{nj}. Thus, we take into consideration three 2-gluon glueballs with

$$M_{20} = 960 \text{ MeV}, \quad \xi_{20} = 6;$$
$$M_{21} = 1290 \text{ MeV}, \quad \xi_{21} = 6;$$
$$M_{22} = 1590 \text{ MeV}, \quad \xi_{22} = 6,$$

and two 3-gluon glueballs with

$$M_{30} = 1460 \text{ MeV}, \quad \xi_{30} = 11,$$
$$M_{31} = 1800 \text{ MeV}, \quad \xi_{31} = 39.$$

These values are in agreement with the lattice estimates [12,44] for both SU(3) and SU(3) cases. The number of quantum states of unbound gluons for the SU(2) system is $\xi_g = 6$ and for the SU(3) system is $\xi_g = 16$.

After specifying these data, we have in the model only three free parameters: C, α and Φ_2. Fixing the values of the latter, we can calculate all thermodynamic characteristics. In lattice simulations one usually considers the relative internal energy $\varepsilon/\varepsilon_{SB}$ and pressure P/P_{SB}, where ε_{SB} and P_{SB} are the Stefan — Boltzmann energy density and pressure, respectively. Our results for

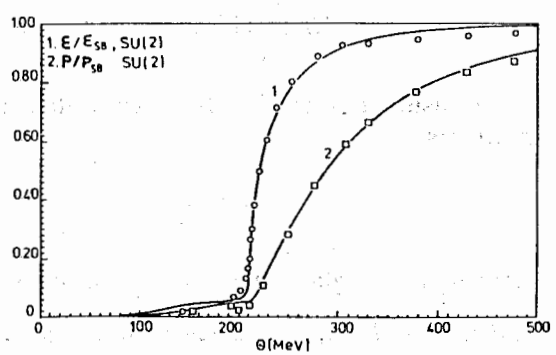

Fig.1. Relative internal energy $\varepsilon/\varepsilon_{SB}$ and pressure P/P_{SB} versus temperature for the quarkless SU(2) system: the solid curves present our calculaltions as compared with the lattice data (○ and □) taken from Ref.12

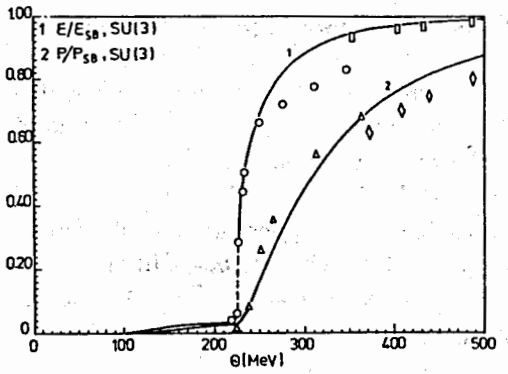

Fig.2. Values of $\varepsilon/\varepsilon_{SB}$ and P/P_{SB} versus temperature for the quarkless SU(3) system: the solid lines show our results as compared with the data of lattice simulations taken from Refs. 13—15 (○ and △) and from Ref.19 (□ and ◇)

$\varepsilon/\varepsilon_{SB}$ and P/P_{SB} in the case of the SU(2) system with $\alpha = 0.62$, $C^{1/(3\alpha+1)} = 175$ MeV, and $\Phi_2 = 5\cdot10^{-3}$ MeV^{-2} are shown in Fig.1. The deconfinement occurs as a second order phase transition at $T_{dec} = 210$ MeV, were the specific heat has the chatacteristic singularity. Figure 2 displays our calculations for the internal energy and pressure in the case of the SU(3) system with $\alpha = 0.62$, $C^{1/(3\alpha+1)} = 225$ MeV, and $\Phi_2 = 2\cdot10^{-3}$ MeV^{-2}. Here the deconfinement is a first order phase transition, with the latent heat $\Delta\varepsilon/\varepsilon_{SB} \approx 0.23$, occurring at the temperature $T_{dec} = 225$ MeV. In both cases our results coincide with the lattice-simulation data [12-15] within the accuracy of the latter being about 10%. Below T_{dec} the system is almost complete in a clustered state of glueballs, and above T_{dec} it predominantly consists of unbound gluons. The existence of clustered states as well as of unbound ones around the deconfinement temperature is also in agreement with the results of lattice simulations [28].

4. CLUSTERING QUARK-HADRON MATTER

Consider now the SU(3) system with quarks, when the total density of the matter is

$$\rho = \rho_g + \sum_a \rho_a + \sum_{nj} n\rho_{nj}; \tag{16}$$

here ρ_g is the gluon density; ρ_a is a density of unbound quarks of a kind $a = u, \bar{u}, d, \bar{d}, \ldots.$; ρ_{nj} is a density of the quark clusters.

The mean-field potential of unbound particles is again taken in the form (9). By analogy with (11), we find [36,39], using the energy-momentum conservation laws, the interaction potentials of multiquark clusters

$$\Phi_{nj,\,mi}(r) = \frac{nm}{9}\,\Phi_{33}(r) \tag{17}$$

expressed through the nucleon-nucleon interaction potential.

Following the general way, described in the previous sections, of constructing correct cluster Hamiltonians satisfying the conditions of thermodynamic correctenss (4) and (6), we come to the Hamiltonian

$$H_c = \sum_\sigma \int \overset{+}{\psi}_g(\vec{r},\sigma)\left(\sqrt{-\nabla^2} + C/\rho^\alpha\right)\psi_g(\vec{r},\sigma)\,d\vec{r} +$$
$$+ \sum_a \sum_s \int \overset{+}{\psi}_a(\vec{r},s)\left(\sqrt{-\nabla^2 + m_a^2} + C/\rho^\alpha\right)\psi_a(\vec{r},s)\,d\vec{r} +$$
$$+ \sum_{nj} \sum_s \int \overset{+}{\psi}_{nj}(\vec{r},s)\left(\sqrt{-\nabla^2 + M_{nj}^2} + U_{nj}\right)\times$$
$$\times\,\psi_{nj}(\vec{r},s)\,d\vec{r} - BV, \tag{18}$$

in which $\psi_g(\vec{r},\sigma)$ is a gluon field operator, $\psi_a(\vec{r},s)$ is a quark field operator, and $\psi_{nj}(\vec{r},s)$ is a cluster field operator; the following expressions are true:

$$U_{nj} = \frac{nC}{\rho^\alpha} - \frac{nC}{\left(\rho - \rho_g - \sum_a \rho_a\right)^\alpha} +$$
$$+ \frac{n}{9}\left(\rho - \rho_g - \sum_a \rho_a\right)\Phi_3, \tag{19}$$

$$B = -\frac{\alpha C}{1-\alpha}\rho^{1-\alpha} + \frac{\alpha C}{1-\alpha}\left(\rho - \rho_g - \sum_a \rho_a\right)^{1-\alpha} +$$
$$+ \frac{\Phi_3}{18}\left(\rho - \rho_g - \sum_a \rho_a\right)^2,$$

where

$$\Phi_3 \equiv \int \Phi_{33}(r)\,d\vec{r}. \tag{20}$$

In the Hamiltonian (18) we consider the light quarks: u, $\bar{u}$, d and $\bar{d}$ with the physical masses $m_a = 7$ MeV and the number of quantum states $\xi_a = 6$. The strange and other heavier quarks are thermodynamically less important. Owing to the same fact that only the lightest clusters give the main contribution to thermodynamic characteristics [32,33], the following lightest hadrons are included into the Hamiltonian (18): π-mesons, K^+- and K^--mesons, K^0- and $\bar{K}^0$-mesons, η-mesons, ρ-mesons and ω-mesons. The masses M_{nj} and the numbers of spin-isospin states ξ_{nj} for these two-quark clusters are

$$M_{20} = 140 \text{ MeV}, \quad \xi_{20} = 3 \quad (\pi-\text{meson}),$$

$$M_{21} = 494 \text{ MeV}, \quad \xi_{21} = 2 \quad (K^+ - \text{ and } K^- - \text{meson}),$$

$$M_{22} = 498 \text{ MeV}, \quad \xi_{22} = 4 \quad (K^0 - \text{ and } \bar{K}^0 - \text{meson}),$$

$$M_{23} = 549 \text{ MeV}, \quad \xi_{23} = 1 \quad (\eta-\text{meson}),$$

$$M_{24} = 765 \text{ MeV}, \quad \xi_{24} = 9 \quad (\rho-\text{meson}),$$

$$M_{25} = 784 \text{ MeV}, \quad \xi_{25} = 3 \quad (\omega-\text{meson}).$$

Taking for the nucleon-nucleon potential $\Phi_{33}(r)$ the Bonn potential [45] averaged over spin and isospin states [33,39], we get for the parameter (20) the value $\Phi_3 = 4.1 \cdot 10^{-5}$ MeV^{-2}. The number of gluon quantum states for the SU(3) system considered is $\xi_g = 16$. In order that eliminating quarks, we could return to the quarkless SU(3) system of the previous section, we put here the same parameters $\alpha = 0.62$ and $C^{1/(3\alpha+1)} = 225$ MeV as in Sec.3.

In this way, all parameters of the Hamiltonian (18) are fixed. Calculating the corresponding thermodynamic characteristics, we find that the deconfinement transition now becomes a continuous crossover in the vicinity of $T_{dec} = 150$ MeV. The relative internal energy $\varepsilon/\varepsilon_{SB}$ and pressure P/P_{SB} are shown in Fig.3. Thus, the inclusion of quarks with physical masses into the quarkless SU(3) system lowers the transition temperature by about 75 MeV and changes the first order phase transition of the quarkless system to a continuous crossover. These results are in striking agreement with the lattice simulation data displaying the same qualitative and quantitative change of the behaviour when passing from the quarkless SU(3) system [13-15] to that containing physical quarks [16-18].

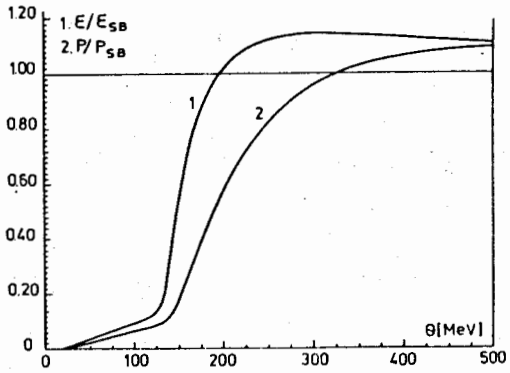

Fig.3. Relative internal energy $\varepsilon/\varepsilon_{SB}$ and pressure P/P_{SB} versus temperature for the SU(3) system with physical quarks. As we have checked, the limits $\varepsilon/\varepsilon_{SB} \to 1$ and $P/P_{SB} \to 1$ hold as temperature tends to infinity

5. CONCLUSION

Constructing effective cluster Hamiltonians, it is necessary to take into account the conditions of thermodynamic correctness (4) following from the principle of thermodynamic equivalence (3). This allows us to find correct forms of the cluster Hamiltonians for which all thermodynamic relations are true.

The application of this approach to the clustering matter of quarks and gluons shows that at low temperatures these particles are clustered into hadrons and at high temperatures the matter transforms into the plasma of unbound quarks and gluons. The deconfinement-confinement transition is nothing but the declustering-clustering transformation.

Within the framework of the same statistical model of hot clustering matter one can describe all three known types of the deconfinement transition: the second order phase transition in the quarkless SU(2) system, the first order phase transition in the quarkless SU(3) system, and a continuous crossover in the SU(3) system with physical quarks at zero baryon density. The behaviour of all thermodynamic chracteristics is in quantitative agreement with the available lattice data of Monte — Carlo simulation techniques.

The system with the non-zero baryon number density can also be treated by using this approach. Some predictions for high baryon densities have been published elsewhere [39]. These predictions cannot be compared with the corresponding lattice data as far as an accurate lattice simulation at finite baryon density is yet an unsolved problem.

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Шаненко А.А., Юкалова Е.П., Юкалов В.И. E2-92-378
Статистическое описание горячей и плотной кластеризующейся материи

Построена статистическая модель, трактующая адроны как кластеры, состоящие из элементарных объектов, глюонов и кварков. Модель основана на эффективном кластерном гамильтониане. Сформулирован принцип термодинамической эквивалентности гамильтонианов, позволяющий корректно описывать термодинамические свойства системы с эффективным гамильтонианом. Этот принцип приводит к соотношениям, названным условиями термодинамической корректности, которые контролируют выбор кластерных гамильтонианов. Кластеризация, при низкой температуре и плотности, партонов в адроны соответствует конфайнменту, тогда как обратный процесс распада кластеров соответствует деконфайнменту. Для проверки правильности модели проведены вычисления в области нулевой барионной плотности, для которой известны аккуратные компьютерные расчеты на решетке. Результаты нашей модели находятся в прекрасном согласии с решеточными данными.

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Shanenko A.A., Yukalova E.P., Yukalov V.I. E2-92-378
Statistical Description of Hot and Dense Clustering Matter

A statistical model is constructed by treating hadrons as clusters of elementary objects, gluons and quarks. The basis of the model is an effective cluster Hamiltonian. The principle of thermodynamic equivalence for Hamiltonians is formulated, which makes it possible to describe all the thermodynamics of the system with an effective Hamiltonian in a completely correct way. This principle yields restrictions, called the conditions of thermodynamic correctness, controlling the choice of the cluster Hamiltonians. The clustering, at low temperature and density, of partons into hadrons means the confinement while the inverse process of the cluster disintegration corresponds to the deconfinement. To check the accuracy of the model, detailed calculations for the region of zero baryon density have been made for which computer simulations on the lattice are known. Our results are found to be in a beautiful agreement with the lattice numerical data.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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