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THE KLEIN-FOCK EQUATION, MINIMAL SURFACES AND GRAVITATIONAL FIELDS

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In our paper [1] we have developed a geometric method for reducing arbitrary Lagrangians of the form $L(x^1,\ldots,x^n, \varphi^1,\ldots, \varphi^m, \partial_i \varphi^p)$ depending on the finite number of scalar functions $\varphi^1 = \varphi^1(x^1,\ldots, x^n), \ldots, \varphi^m = \varphi^m(x^1,\ldots,x^n)$ and their first derivatives $\partial_i \varphi^p$ to uniform expressions of the first degree with respect to derivatives $a_i^\alpha = \partial y^\alpha / \partial u^i$ where $\{y^\alpha\} = \{x^1,\ldots,x^n, \varphi^1,\ldots\varphi^m\} = y$. The quantities

$$u^i = u^i (x^1,\ldots,x^n) \tag{1}$$

are admissible variables of integration in the action

$$S = k \int L (x, \varphi, \partial \varphi)\, d x^1 \ldots d x^n =$$

$$= k' \int \tilde{L} (y, a_i^\alpha)\, d u^1 \ldots d u^n , \tag{2}$$

where k is the dimensional constant. Scalar fields φ are given in space-time X_n. In relativistic theories X_n is Minkowski's space, and x are Cartesian coordinates in it. The Lagrangian function L is invariant with respect to the Poincare group.

Action (2) with a homogeneous Lagrangian defines the measure of n-dimensional surface in the areal space [2]. Let us denote this space by $M_N(x, \varphi) = M_{n+m}(x^1,\ldots, x^n, \varphi^1,\ldots, \varphi^m) = M_N(y)$ where y denote both "usual" coordinates x and field coordinates φ. It follows from the principle of least action that equations for the scalar fields φ in X_n coincide with equations for n-dimensional minimal surfaces in $M_N(x, \varphi)$. In arbitrary integration variables u in the integral (2) the minimal surface in M_N can be written as

$$y^\alpha = y^\alpha (u^1,\ldots, u^n) , \tag{3}$$

and in special integration variables x it is written as

$$y^1 = x^1 , \ldots , y^n = x^n ,$$

$$y^{n+1} = \varphi^1 (x) , \ldots , y^{n+m} = \varphi^m(x) . \tag{4}$$

The action for the linear scalar field in the Minkowski space X_n with the metric

$$d s^2 = \eta_{ik}\, d x^i\, d x^k =$$

$$- (c \; d t)^2 + (d x^2)^2 + \ldots + (d x^n)^2 , \qquad (5)$$

where $ct = x^1$, is chosen in the form [3]

$$S = k \int \{ - \tfrac{1}{2} [\eta^{ik}\, \partial_i\, \varphi\, \partial_k\, \varphi + m^2\, \varphi^2]\}\, d x^1 \ldots d x^n , \qquad (6)$$

where k is the dimensional constant, η^{ik} is the tensor inverse to the metric Minkowski tensor from metric (5), $m = m_o . c / \hbar$. From the principle of least action for (6) there follows the Klein-Fock equation

$$\eta^{ik}\, \partial^2_{ik}\, \varphi - m^2\, \varphi = - c^{-2}\, \partial^2\, \varphi \,/\, \partial\, t^2 + \Delta\, \varphi - m^2\, \varphi = 0 \; . \qquad (7)$$

The functional (6) determines all basic properties of the classical linear scalar field.

The dimensional constant k equalizes the dimension of the left- and right-hand sides of eq.(6). The dimension of the action S is known and the dimension of k is determined by the field dimension φ. If the quantity φ is dimensionless, the dimension of k equals [g cm^{4-n} s^{-1}]. The action (6) can be written in the form

$$S = k' \int \{- \tfrac{1}{2}\, [\eta^{ik}\, \partial_i (b\, \varphi)\, \partial_k (b\, \varphi) + m^2\, (b\, \varphi)^2]\} \; d x^1 \ldots d x^n , \qquad (8)$$

where b is the dimensional constant chosen so that the quantity b φ has dimension of cm, k' is a new constant equal to $k' = k / b^2$. Then, we assume that $k' = 1$. Under this condition the dimension of the variable b φ coincides with the dimension of the coordinates $x^1, \ldots , x^n$ and it may be thought of as one more coordinate $y^{n+1} = x^{n+1} = b\, \varphi$ in some areal space M_{n+1} which is known to have the measure of n-dimensional surfaces to be determined by the integral (8).

In this integral the Lagrangian

$$L = - \frac{1}{2}\, [\eta^{ik}\, \partial_i\, (b\, \varphi)\, \partial_k\, (b\, \varphi) + m^2\, (b\, \varphi)^2] \qquad (9)$$

is to be reduced to a homogeneous form [1]. The construction of the space M_{n+1} and determination of its metric or other properties are the main problems of the areal geometry [2]. The form of the Lagrangian (9) shows that it does not determine in M_{n+1} a nondegenerate metric tensor.

Further, we assume that the nonlinear scalar field is defined by an action of the Born-Infeld type [4] - [7]

$$S_1 = \int \{ \sqrt{1 + 2 L} - 1\}\, d x^1 \ldots d x^n \qquad (10)$$

or

$$S_2 = \int \{1 - \sqrt{1 - 2 L} \}\, d x^1 \ldots d x^n \qquad (11)$$

where L equals (9). Both expressions (10) and (11) are equivalent. In the limit of weak fields ($b \longrightarrow 0$) they both turn into the starting expression (8).

The term

$$V = \int d x^1 \ldots d x^n$$

in expressions (10) and (11) will be further neglected as it does not influence the properties of the scalar field which are studied in the present paper. So from the action (10-11) we pass to the action

$$S = \int \sqrt{1 \pm 2 L}\; d x^1 \ldots d x^n . \qquad (12)$$

Denote

$$- g = 1 \pm 2 L . \qquad (13)$$

In the nonlinear Born-Infeld electrodynamics fields are considered for which $- g > 0$. Identifying the action (12) with the measure of the n-dimensional surface in M_{n+1} we can equivalently consider two cases $g > 0$ and $g < 0$. Specific is the case $g = 0$. In the Minkowski space with the metric (5) surfaces of that type are called spacelike, timelike and degenerate, respectively, [8] - [12] .

The measure of the surface (3) in the pseudo-Riemann space with the metric

$$d s^2 = h_{\alpha\beta} \, d y^\alpha \, d y^\beta , \qquad 1 < \alpha, \beta < n + 1 , \qquad (14)$$

has the form

$$S = \int \sqrt{\varepsilon \frac{1}{n!} h_{\alpha_1\beta_1} \ldots h_{\alpha_n\beta_n} J^{\alpha_1 \ldots \alpha_n} J^{\beta_1 \ldots \beta_n}} \, d x^1 \ldots d x^n$$

$$= \int \sqrt{\varepsilon \det (g_{ik})} \, d x^1 \ldots d x^n = \int \sqrt{|g|} \, d x^1 \ldots d x^n , \qquad (15)$$

where $\varepsilon = \pm 1$.

The tensor

$$g_{ik} = h_{\alpha\beta} (\partial y^\alpha / \partial x^i) (\partial y^\beta / \partial x^k) \qquad (16)$$

defines on the surface (3), imbedded in M_{n+1} , the induced metric

$$d s^2 = g_{ik} \, d x^i \, d x^k . \qquad (17)$$

The quantities J with complex indices in eq.(15) are minors of the nth order in the rectangular matrix $T= (\partial y^\alpha / \partial x^i)$. In the pseudo-Riemann spaces with the metric form (14) whose signature coincides with that of the form (5) for the timelike surface $\varepsilon = -1$ and for the spacelike surface $\varepsilon = 1$.

Taking into account the above-said we can finally write down the action of the Born-Infeld type [5, 6] for the scalar field in X_n as

$$S = \int \sqrt{|1 \pm \{\eta^{ik} \partial_i (b \varphi) \partial_k (b \varphi) + m^2 b^2 \varphi^2\}|} \, d x^1 \ldots d x^n . \qquad (18)$$

To express the actions (8) and (18) in the homogeneous form one should multiply their integrands by $J^{1 \ldots n}$ and change $\partial_i (b \varphi)$ by ξ_i where

$$\xi_i = J^{1 \ldots i \ldots n} / J^{1 \ldots n} , \qquad (19)$$

$J^{1 \ldots n}$ is a nonzero left extreme minor of the rectangular matrix T of an order of $n (n + 1)$, and $J^{1 \ldots i \ldots n}$ is derived from the previous minor by changing in it the ith column by the last column of the matrix T .

Now let us formulate the basic problem of the present paper. It is required that the metric tensor $h_{\alpha\beta}$ of the areal space $M_{n+1}(x, \varphi)$ be found by the given action (18) for the nonlinear scalar field and its physical treatment be given.

The following steps are needed to solve the problem stated: 1) equating expressions (15) and (18) to form the system of equations for determining components $h_{\alpha\beta}$; 2) to solve this system, as far as possible; 3) to study the obtained solutions.

First, we will solve the above-stated problem for the scalar field in the two-dimensional Minkowski space with the metric

$$d s^2 = - c^2 d t^2 + d x^2 = - (d x^1)^2 + (d x^2)^2 . \qquad (20)$$

The action (18) in this case takes the form

$$S = \iint \sqrt{\left|1 \pm \left[- \left(\frac{\partial(b\varphi)}{\partial(ct)} \right)^2 + \left(\frac{\partial(b\varphi)}{\partial x} \right)^2 + m^2 b^2 \varphi^2 \right]\right|} \, c \, d t \, d x . \qquad (21)$$

In the homogeneous form eq.(21) is written as

$$S = \iint \sqrt{|(J^{12})^2 \pm [-(J^{32})^2 + (J^{13})^2 + (m b \varphi J^{12})^2]|} \, c \, d t \, d x . \qquad (22)$$

We assume that the coordinates $x^1 = ct$, $x^2 = x$, and $x^3 = b \varphi$ in the space $M_3 (x, \varphi)$ are orthogonal. Then, the metric form of this space is

$$d s^2 = h_{11} (d x^1)^2 + h_{22} (d x^2)^2 + h_{33} (d x^3)^2 , \qquad (23)$$

and formula (15) is

$$S = \iint \sqrt{|h_{11}\, h_{22}\, (J^{12})^2 + h_{22}\, h_{33}\, (J^{32})^2 + h_{33}\, h_{11}\, (J^{13})^2|}\; d\, x^1\, d\, x^2. \qquad (24)$$

Equation for the surface with the measure (24) has the form

$$x^1 = c\, t\,, \quad x^2 = x\,, \quad x^3 = b\, \varphi\, (x^1,\, x^2)\,. \qquad (25)$$

Comparing (22) with (24) we get the following algebraic systems of equations:

for the upper sign in (18)

$$\begin{aligned} &1)\; 1 + (m\, b\, \varphi)^2 = -\, h_{11}\, h_{22}\,, \\ &-1 = -\, h_{22}\, h_{33}\,, \quad 1 = -\, h_{33}\, h_{11}\,, \\ &2)\; 1 + (m\, b\, \varphi)^2 = +\, h_{11}\, h_{22}\,, \\ &-1 = +\, h_{22}\, h_{33}\,, \quad 1 = +\, h_{33}\, h_{11}\,, \end{aligned} \qquad (26)$$

for the lower sign in (18)

$$\begin{aligned} &1)\; 1 - (m\, b\, \varphi)^2 = -\, h_{11}\, h_{22}\,, \\ &1 = -\, h_{22}\, h_{33}\,, \quad -1 = -\, h_{33}\, h_{11}\,, \\ &2)\; 1 - (m\, b\, \varphi)^2 = +\, h_{11}\, h_{22}\,, \\ &1 = +\, h_{22}\, h_{33}\,, \quad -1 = +\, h_{33}\, h_{11}\,. \end{aligned} \qquad (27)$$

Of two systems (26) the second has no solutions and the first has two solutions. The form (23) for these solutions becomes

$$d\, s^2 = +\sqrt{H}\; (-\, c^2\, d\, t^2 + d\, x^2) + b^2\, d\, \varphi^2 / \sqrt{H} \qquad (28)$$

$$d\, s^2 = -\sqrt{H}\; (-\, c^2\, d\, t^2 + d\, x^2) - b^2\, d\, \varphi^2 / \sqrt{H}\,, \qquad (29)$$

where

$$H = 1 + (m\, b\, \varphi)^2\,. \qquad (30)$$

The forms (28) and (29) have opposite signatures and spaces of that type become identical. We assume that the space M_3 $(x,\ \varphi)$ has the form (28). Note that this form is determined for all values of the coordinates, $b\ \varphi$ being the space

coordinate. The signature of the form (28) coincides with the signature of the form (5) at $n = 3$. Consequently, the metric tensor, given by that form, can describe the gravitational field. The scalar curvature determined by that tensor equals

$$R = m^2\, \{4 + (m\, b\, \varphi)^2\} / 2\, H^{3/2}\,. \qquad (31)$$

The Einstein tensor components determined by it equal

$$G_{11} = (m\, c)^2 / 2\, H\,, \qquad G_{22} = -\, m^2 / 2\, H\,,$$

$$G_{33} = -\, m^4\, b^2\, \varphi^2 / 4\, H^2. \qquad (32)$$

Under a specific choice of units the tensor $G_{\alpha\beta}$ equals the energy-momentum tensor $T_{\alpha\beta}$ of the matter generating the gravitational field [12]. In our case, the scalar field serves as such a matter.

It is to be noted that the metric (28) is conformally flat. In the limiting case of a weak scalar field (when $b \longrightarrow 0$) the metric (28) is degenerate; however, neither the scalar curvature (31) nor the Einstein tensor (32) vanish. To study gravitational fields with the degenerate metric tensor it is efficient to use both the relativistic theory of gravitation [13] and the theory of gravitation with two connections [14].

Now let us consider massless fields. At $m = 0$ the space M_3 with the metric (28) becomes flat and the action (22) defines in it the two-dimensional surface. In this case, the nonlinear equation for the scalar field

$$\begin{aligned} -\, \varphi_{tt}\, (1 + b^2\, \varphi_x\, \varphi_x) + 2\, b^2\, \varphi_{tx}\, \varphi_t\, \varphi_x\, + \\ +\, \varphi_{xx}\, (1 - b^2\, \varphi_t\, \varphi_t) = 0 \end{aligned} \qquad (33)$$

coincides with the equation of minimal surfaces in the Minkowski space M_3 . Here the light velocity equals unity. At $b = 0$ equation (33) turns into the d'Alembert equation. The scalar field with the action (22) and eq.(33) has first been studied in papers [6, 7] not only on the classical level but also on the quantum one.

Now we pass to the pair of systems (27). At $(m\, b\, \varphi)^2 < 1$ the second system has no solutions whereas the first has two solutions, one corresponding to the fundamental form

$$d\,s^2 = \sqrt{K}\,(-c\ \ d\,t^2 + d\,x^2) - b^2\, d\,\varphi^2 / \sqrt{K}\ , \qquad (34)$$

where

$$K = 1 - (m\, b\, \varphi)^2\ .$$

On the contrary, at $(m\, b\, \varphi)^2 > 1$ the first system has no solutions and the second has two solutions, one corresponding to the fundamental form

$$d\,s^2 = \sqrt{-K}\,(-c^2\, d\,t^2 + d\,x^2) + b^2\, d\,\varphi^2 / \sqrt{-K}\ , \qquad (35)$$

The second solution gives the form with the opposite signature. Note that in this case the quantity b cannot tend to zero.

Let us assume that the space M_3 at $(m\, b\, \varphi)^2 < 1$ has the metric (34) and at $(m\, b\, \varphi)^2 > 1$ has the metric (35). There is a critical value of the field $(m\, b\, \varphi)^2 = 1$ at which the metric tensor has a singularity. Passing over the critical value the coordinate $x^3 = b\, \varphi$ changes its meaning: from the "timelike" in (34) it turns into the "spacelike" one in (35).

The scalar curvature determined by the metric (34) equals

$$R = m^2\, \{4 - (m\, b\, \varphi)^2\} / 2\, K^{3/2}.$$

The Einstein tensor components determined by it equal

$$G_{11} = (m\, c)^2 / 2\, K\ , \qquad G_{22} = -\, m^2 / 2\, K\ ,$$

$$G_{33} = -\, m^4\, b^2\, \varphi^2 / 4\, K^2\ . \qquad (36)$$

The scalar curvature determined by the metric (35) equals

$$R = m^2\, \{(m\, b\, \varphi)^2 - 4\} / 2\, (-K)^{3/2}.$$

The Einstein tensor components determined by it equal

$$G_{11} = -\, (m\, c)^2 / 2\, (-K)\ , \qquad G_{22} = m^2 / 2\, (-K)\ ,$$

$$G_{33} = -\, m^4\, b^2\, \varphi^2 / 4\, (-K)^2\ . \qquad (37)$$

It can be assumed that the tensors (36) and (37) describe the same gravitational field at different values of $(m\, b\, \varphi)^2$.

From the physical point of view, of much interest are the spaces $M_4\ (x,\ \varphi)$ and $M_5(x,\ \varphi)$ whose geometry is generated by scalar fields of the Born-Infeld type given in X_3 and in X_4. Let us write down for these spaces some formulae analogous to those for $M_3\ (x,\ \varphi)$.

For the space $M_4\ (x,\ \varphi)$ metric we have the solution

$$d\,s^2 = H^{1/3}\,(-c^2\, d\,t^2 + d\,x^2 + d\,y^2) + H^{-2/3}\, b^2\, d\,\varphi^2\ , \qquad (38)$$

analogous to (28), and the solution described by a pair of formulae

$$d\,s^2 = K^{1/3}\,(-c^2\, d\,t^2 + d\,x^2 + d\,y^2) - K^{-2/3}\, b^2\, d\,\varphi^2\ ,$$

$$(39)$$

$$d\,s^2 = (-K)^{1/3}(-c^2\, d\,t^2 + d\,x^2 + d\,y^2) + (-K)^{-2/3}\, b^2\, d\,\varphi^2,$$

analogous to (34) and (35).

At $m = 0$ the space $M_4\ (x,\ \varphi)$ with the metric (38) becomes the Minkowski space X_4. A massless scalar nonlinear field is described in it by the equation of three-dimensional minimal surfaces. The latter may be both timelike and spacelike.

For the space $M_5\ (x,\ \varphi)$ metric we have the solution

$$d\,s^2 = H^{1/4}(-c^2\, d\,t^2 + d\,x^2 + d\,y^2 + d\,z^2) + H^{-3/4}\, b^2\, d\,\varphi^2, \qquad (40)$$

analogous to (28), and the solution described by a pair of formulae

$$d s^2 = K^{1/4} (- c^2 d t^2 + d x^2 + d y^2 + d z^2) - K^{-3/4} b^2 d \varphi^2,$$

$$d s^2 = (-K)^{1/4} (- c^2 d t^2 + d x^2 + d y^2 + d z^2) + + (-K)^{-3/4} b^2 d \varphi^2 , \quad (41)$$

analogous to (34) and (35).

At m = 0 the space M_5 (x, φ) with the metric (40) becomes the Minkowski space X_5. A massless scalar nonlinear field is described in it by the equation of four-dimensional minimal surfaces. The latter may be both timelike and spacelike.

Let us write down also analogous results in the general case of the space M_{n+1} (x, φ) whose geometry is generated by a scalar field of the Born-Infeld type given in the Minkowski space X_n with the metric (5). We have the solution

$$d s^2 = H^{1/n} (\eta_{ik} d x^i d x^k + H^{-1} b^2 d \varphi^2) , \quad (42)$$

analogous to (28), and the solution described by a pair of formulae

$$d s^2 = K^{1/n} (\eta_{ik} d x^i d x^k - K^{1/n} b^2 d \varphi^2) ,$$
$$d s^2 = (- K)^{1/n} \{\eta_{ik} d x^i d x^k + (-K)^{-1} b^2 d \varphi^2\} , \quad (43)$$

analogous to (34) and (35). As is seen, these are conformally flat metrics.

The scalar curvature determined by the metric (42) equals

$$R = m^2 \{n - 1 + (n + 1) H^{-1}\} / n H^{1/n}. \quad (44)$$

The Einstein tensor components determined by it equal

$$G_{11} = (m c)^2 \{n - 2 + (n + 2) H^{-1}\} (n - 1) / 2 n^2 ,$$

$$G_{22} = \ldots = G_{nn} = - m^2 \{n - 2 + (n + 2) H^{-1}\} (n - 1) / 2 n^2,$$

$$G_{n+1\ n+1} = (m b)^2 (1 - H) (n - 1) / 2 n H^2 . \quad (45)$$

Nondiagonal components $G_{\alpha\beta}$ are equal to zero. The Einstein tensor components determined by the metric (43) and the scalar curvature have an analogous form.

Assume that under the action of a strong scalar field a test particle moves along the geodesic of the space M_{n+1} (x, φ). The equations for the geodesics can be written in the form [15]

$$\frac{d x^\alpha}{d \tau} = p^\alpha , \qquad \frac{d p^\alpha}{d \tau} = - \Gamma^\alpha_{\mu\nu} p^\mu p^\nu , \quad (46)$$

where p^α is a particle momentum, $\Gamma^\alpha_{\mu\nu}$ is the Christoffel connection, and τ is the proper time of a particle determined in an appropriate way. The scalar square of momentum is the first integral of eqs.(46). Within the sign it equals the square of the rest mass M of a particle. In the case (42)

$$M^2 = H^{1/n} \{ M_0^2 - H^{-1} b^2 (\frac{d \varphi}{d \tau})^2\} , \quad (47)$$

where M_0^2 is the square of the rest mass of a particle free from the action of the scalar field

$$M_0^2 = - \eta_{ik} p^i p^k . \quad (48)$$

In this case, the equations for the geodesics are easily integrated as along with (47) there are n first integrals

$$H^{1/n} p^i = C^i . \quad (49)$$

The case (43) can be treated analogously.

Finally, we should like to note that at m = 0 nonlinear scalar fields describe extended relativistic objects of the type of strings and membranes in the (n + 1)-dimensional Minkowski space [16, 17].

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Уравнение Клейна–Фока, минимальные поверхности и гравитационные поля

Рассматривается непинейное скалярное поле типа Борна–Инфельда, линейным аналогом которого является скалярное поле, подчиненное уравнению Клейна–Фока. С помощью метода, разработанного в работе[1], действие для такого поля представляется в виде меры площади в римановом пространстве-времени, метрика которого подлежит определению. В этом пространстве-времени наряду с обычными координатами выступает полевая переменная φ, умноженная на размерную константу b. Показано, что если исходное скалярное поле является безмассовым, то нелинейное поле типа Борна–Инфельда, соответствующее ему, описывается уравнениями минимальных поверхностей в псевдоевклидовых пространствах с координатами $(ct, x, b\varphi)$. При этом координата $b\varphi$ может иметь как пространственный, так и временной характер. Если исходное поле является массивным, то соответствующее поле типа Борна–Инфельда описывается уравнениями минимальных поверхностей в искривленных пространствах с теми же координатами $(ct, x, b\varphi)$. Можно сказать, что массивные поля типа Борна–Инфельда являются протяженными источниками гравитационных полей.

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The Klein–Fock Equation, Minimal Surfaces and Gravitational Fields

A nonlinear scalar field of the Born–Infeld type is considered whose linear analog is the scalar field obeying the Klein–Fock equation. By the method developed in paper[1] the action for such a field is represented as a measure of the area in the Riemann space-time whose metric is to be determined. In this space-time, along with the usual coordinates there appears a field variable φ multiplied by the dimensional constant b. It is shown that if the initial scalar field is massless, a nonlinear field of the Born–Infeld type corresponding to it is described by the equations of minimal surfaces in pseudo-Euclidean spaces with the coordinates ct, x, $b\varphi$. The coordinate φ may be have both space-like and time-like nature. If the initial field is massive, the corresponding field of the Born–Infeld type is described by the equations of minimal surfaces in curved spaces with the same coordinates ct, x, $b\varphi$. We can say that massive fields of the Born–Infeld type are extended sources of the gravitational fields.

The investigation has been performed at the Laboratory of Nuclear Problems, JINR.

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