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THE SMALL-x BEHAVIOUR OF PARTON DISTRIBUTION FUNCTIONS

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1. Introduction

Recently the small-x behaviour of the structure functions (SF) of deep inelastic scattering (DIS) was considered in connection with possibility of experimental studies on new powerful colliders *HERA* [1] and *LEP*LHC* [2]. Analysis of SF gives main information about the behaviour of parton (quark and gluon) distributions (PD) of nucleon. The knowledge of PD is a basis of the study of other processes.

At present both the SF and PD cannot be calculated analytically but their Q^2-evolution (for large Q^2,where perturbation theory (PT) is applied) only. Perturbatively, both the leading order (LO) and next-to-leading order (NLO) of Q^2-evolution are obtained from Gribov-Lipatov-Altarelli-Parisi (GLAP) equation [3]

$$\frac{d}{d\ln(Q^2/\mu^2)} f_a(x,Q^2) = P(x,\alpha)_{ab} * f_b(x,Q^2), \qquad (1)$$

where $f_a(x,Q^2)$ is the distribution function of parton a (i.e. q and g for quark and gluon, respectively), the symbol * denotes a convolution integral in the fractional momentum variable for the parton involved and the kernel[1)],

$$P(x,\alpha) = \alpha\left[P^{(0)}(x) + \alpha P^{(1)}(X) + \ldots\right]$$

is obtained from PT.

GLAP equation is based on summing large logarithms of Q^2 in any orders of PT. However in *HERA* region ($10^{-4}<x<10^{-2}$) the summation of large up $1/x$ logarithms is important also. This problem was solved in the papers [4], where Lipatov-Kuraev-Fadin-Balitsky (LKFB) equation was obtained. Resently (see[5]) both GLAP and LKFB equations were solved numerically. It was showed that these equations give similar results in large region of Q^2 and small x:

[1)]Hereafter contrary to the standard one we use the coupling $\alpha(Q^2) = \alpha_s(Q^2)/4\pi$.

$10^{-4}<x<10^{-2}$ and $10\ GeV^2<Q^2<10^3 GeV^2$.

Hence, the standard PD parametrizations (see recent review [6]), which are based on GLAP equation, can be used in this region.

2. Q^2- evolution of PD moments

In the present paper we analyse sea quarks and gluon distributions at small-x in LO and NLO of PT. We reproduce analytically the quality of the change of the small-x PD behaviour discovered in the paper [7]. We give the simple values of LO and NLO parts of the kernels $P(x,\alpha)_{ab}$ (or rather, the anomalous dimensions (AD) of Wilson operators) of eq.(1), which lead to this change.
We use our knowledge of the Q^2-evolution of the PD moments[2)]

$$M_a(n,Q^2) = \int_0^1 dx\ x^{n-1}\ f_a(x,Q^2).$$

Actually, the GLAP equation for the moments (hereafter we use the following definition {a,b} = {q,g})

$$\frac{d}{d\ln(Q^2/\mu^2)} M_a(n,Q^2) = \gamma_{ab}(n,\alpha)\ M_b(n,Q^2) \qquad (1a)$$

with

$$\gamma_{ab}(n,\alpha) = \alpha\Big[\gamma^{(0)}_{ab}(n) + \alpha\ \gamma^{(1)}_{ab}(n)\Big]$$

can be solved analytically by a diagonalization of AD matrix $\gamma(n,\alpha)$ (see, for example, [8])[3)]

$$M_a(n,Q^2) = \sum_{i=\pm} M_{a,i}(n,Q^2).$$

Multiplicatively renormalizable parts $M_{a,i}(n,Q^2)$ of the PD moments have the following form

[2)] We use PD multiplied by x.
[3)] We consider the singlet PD only, as the contribution of nonsinglet one is small when $x\to 0$.

$$M_{a,i}(n,Q^2) = \sum_{b=q,g} \xi^i_{ab}(n)\ M_a(n,Q^2), \qquad (2)$$

where for $a \neq b$

$$\xi^{\pm}_{aa}(n) = \left(1+ \frac{\gamma^{(0)}_{aa}(n) - \gamma^{(0)}_{bb}(n)}{\gamma_n^{\pm} - \gamma_n^{\mp}}\right)$$

$$\text{and}\quad \xi^{\pm}_{ab}(n) = \frac{\gamma^{(0)}_{ab}(n)}{\gamma_n^{\pm} - \gamma_n^{\mp}} \qquad (2a)$$

with $\xi_{aa}^{\pm}(n) = 1- \xi_{aa}^{\mp}(n)$, $\xi_{aa}^{\pm}(n) = \xi_{bb}^{\mp}(n)$ and

$$\gamma_n^{\pm}= \Big(\gamma^{(0)}_{qq}(n)+ \gamma^{(0)}_{gg}(n) \pm \Big[(\gamma^{(0)}_{qq}(n)- \gamma^{(0)}_{gg}(n))^2 + 4\gamma^{(0)}_{qg}(n)\gamma^{(0)}_{gq}(n)\Big]^{1/2}\Big)/2.$$

Its Q^2-evolution can be represented in the form [8]

$$M^{(p)}_{a,i}(n,Q^2)/M^{(p)}_{a,i}(n,Q_0^2) = \Big(\rho_p(Q^2,Q_0^2)\Big)^{d_n^i}\Big\{1 + C^{(p)}_{a,i}(n,Q^2,Q_0^2)\Big\} \qquad (3)$$

(p=0 for LO and p=1 for NLO, respectively).
where

$$C^{(0)}_{a,i}(n,Q^2,Q_0^2) = 0 \quad \text{and}$$

$$C^{(1)}_{a,\pm}(n,Q^2,Q_0^2)= \alpha_1(Q_0^2)\ \Big\{ Z_{n,\pm}\Big(\rho_1(Q^2,Q_0^2)-1\Big) + K^{(a)}_{n,\pm}\Big(\Big[\rho_1(Q^2,Q_0^2)-1\Big]^{(d_n^{\pm}-d_n^{\mp})} - \rho_1(Q^2,Q_0^2)\Big)\Big\} \qquad (3a)$$

with $\rho_p(Q^2,Q_0^2) \equiv \alpha_p(Q^2)/\alpha_p(Q_0^2)$ and $d_n^i = \gamma_n^i/(2\beta_0)$

and also $\alpha_p(Q^2) = \dfrac{1}{\beta_0\ \ln(Q^2/\Lambda_p^2)}\ \{1- L_p(Q^2/\Lambda_p^2)\}$

with $L_0(t)=0$ and $L_1(t) = (\beta_1/\beta_0^2)\ \dfrac{\ln\ln(t)}{\ln(t)}$.

The NLO coefficients of eq.(3a) have the following form (for simplisity the index n is omitted here)

$$Z_{\pm} = (\gamma^{\pm\pm} - \gamma^{\pm}\beta_1/\beta_0)/2\beta_0$$
$$K^{(q)}_{n,\pm} = \frac{\gamma^{\pm\mp}}{\gamma^{\pm} - \gamma^{\mp}}, \qquad K^{(g)}_{n,\pm} = K^{(q)}_{n,\pm}\frac{\xi^{\mp}_{gq}}{\xi^{\mp}_{qq}} \tag{3b}$$

where

$$\gamma^{ii} = \sum_{a,b=q,g} \xi^{i}_{ab}\,\gamma^{(1)}_{ba} \qquad (\{i,j\} = \{+,-\})$$
$$\gamma^{ij} = \sum_{a=q,g} \xi^{i}_{ag}\,\gamma^{(1)}_{ga} - \xi^{j}_{qq}\,\gamma^{(1)}_{qq} - (\xi^{j}_{gq} + \xi^{j}_{qq}/\xi^{j}_{qg})\,\gamma^{(1)}_{qg} \tag{3c}$$

for $i \neq j$.

Here $\beta_0=(11C_A-4T_F)/3$ and $\beta_1=(34C_A^2-20C_AT_F-12C_FT_F)/3$ are the first two coefficients of α-expansion of Gell-Mann-Low function.

3. Small-x behaviour of parton distributions

Assuming a Regge-like behaviour of the PD for $x\to 0$, we can obtain for them the following form (see[9-12])[4]

$$f_a(x,Q^2) = x^{1-\lambda}\, M_a(\lambda,Q^2) + O(1), \tag{4}$$

where λ is the intercept of a pomeron. The value of λ determines the PD form for $x\to 0$. The "conventional" choice is $\lambda = 1$. It leads to nonsingular behaviour (see B_0 fit from ref.[13]) of PD when $x\to 0$. Another value $\lambda\equiv 1+\delta \cong 3/2>1$ has been obtained in the papers [4] as the sum of leading powers of $\ln(1/x)$ in all orders of PT. We note, that for the latter choice there is the unlimited increase of PD, which leads to a conflict with unitarity, i.e. too rapid $s\propto Q^2/x$ dependence

[4] We use the PD parametrization for all x in the following form $\propto x^{1-\lambda}(1-x)^{\nu}(1-\bar{\gamma}x)$.

of high energy cross sections (see B_- fit from ref.[13]) violates the Froissart bound [14].

The PD behaviour for the value $\delta=1/2$ was analysed earlier (see [10,11]). We use $\delta\geq 0$ following [7] - the paper, which stimulated this invistigation. We note also that this choice agrees to the present experimental data for pp and $\bar{p}p$ total cross-sections (see [15,16])[5] and model of Landshoff and Nachmann pomeron [17] with exchange of the pair of a nonperturbative gluons, yielding $\delta=0.086$.

The values of moments $M_a(\lambda,Q^2)$ are singular when $\lambda=1$ (see Appendix). To avoid troubles we can rewrite the eq.(4) in the form

$$f_a(x,Q^2) = x^{1-\lambda}\, M_a(\bar{\lambda},Q^2) + O(x), \tag{5}$$

where we include the terms $\propto O(1)$ exactly. The new value of $\bar{\lambda}\equiv 1+\varepsilon$ depends on both $\lambda\equiv 1+\delta$ and x variables

$$1/\varepsilon = \left[1 - \frac{\Gamma(1+\nu)\Gamma(1-\delta)}{\Gamma(1+\nu-\delta)}\, x^{\delta}\right]/\delta$$

and is regular when $\delta\to 0$

$$1/\varepsilon = \ln(1/x) - \Big(\Psi(1+\nu) - \Psi(1)\Big).$$

Here $\Gamma(1+\nu)$ and $\Psi(1+\nu)$ are Gamma- and Eulier functions, respectively.

1. Let us give the analysis of Q^2-evolution of PD in the LO of PT. The values of AD[6] in the LO

$$\gamma^{(0)}_{qq} = O(\varepsilon), \qquad \gamma^{(0)}_{qg} = -(16/3)T_F(1 - 13\varepsilon/12)$$
$$\gamma^{(0)}_{gq} = -8C_F(1/\varepsilon - 3/4), \qquad \gamma^{(0)}_{gg} = -8C_A(1/\varepsilon - 11/12) + (16/3)T_F$$

lead to the following meanings of the parameters of eq.s (3) and (4)

[5] In ref.[16] one shows that the high energy $\bar{p}p$ data have a linear $\ln s$ behaviour.

[6] Hereafter symbol $\bar{\lambda}$ is omitted in moments with "number" $\bar{\lambda}$.

$$f_q^{+}= (2T_F/3C_A)\varepsilon\left[(C_F/C_A)f_q + f_g\right], \qquad f_q^{-}= f_q - f_q^{+}$$

$$f_g^{-}= (C_F/C_A)\left[(2T_F/3C_A)\varepsilon f_g - f_q\right], \qquad f_g^{+}= f_g - f_g^{-}$$

$$\gamma^{+}= -8C_A(1/\varepsilon - 11/12) +(16/3)T_F(C_A-C_F)/C_A, \quad \gamma^{-} = (16/3)T_FC_F/C_A.$$

As one can see, the well-know rapid growth of the PD is given by "+" component.

Let us begin our analysis with the gluon distribution. Indeed, a gluon distribution is much larger than the quark one for small x (see[7]). Hence, the gluonic part of eqs.(3)-(5) can be represented approximatelly as follows

$$f_g^{(0)}(x,Q^2) \approx \left\{f_g^{(0)}(x,Q_0^2) + (C_F/C_A)f_q^{(0)}(x,Q_0^2)\right\}\left[\rho_0(Q^2,Q_0^2)\right]^{d^+}$$

$$\approx f_g^{(0)}(x,Q_0^2)\left[\rho_0(Q^2,Q_0^2)\right]^{d^+}. \qquad (8)$$

The situation for quark distribution is more complicated

$$f_q^{(0)}(x,Q^2)=f_g^{(0)}(x,Q_0^2)(2T_F/3C_A)\varepsilon\left\{\left[\rho_0(Q^2,Q_0^2)\right]^{d^+}-\left[\rho_0(Q^2,Q_0^2)\right]^{d^-}\right\}$$

$$+f_q^{(0)}(x,Q_0^2)\left\{\left[\rho_0(Q^2,Q_0^2)\right]^{d^-}-(2T_FC_F/3C_A^2)\varepsilon\left[\rho_0(Q^2,Q_0^2)\right]^{d^+}\right\}.$$

The value of $\varepsilon(1/\rho)^{1/\varepsilon}$ is singular for $\varepsilon\rightarrow 0$ and $\rho<1$ and gives the basic contribution. Hence, the quark distribution for $Q^2>Q_0^2$ has the simple form

$$f_q^{(0)}(x,Q^2)=(2T_F/3C_A)\varepsilon\left\{f_g^{(0)}(x,Q_0^2)-(C_F/C_A)f_q^{(0)}(x,Q_0^2)\right\}$$

$$\left[\rho_0(Q^2,Q_0^2)\right]^{d^+} \approx (2T_F/3C_A)\varepsilon\ f_g^{(0)}(x,Q^2), \qquad (7)$$

as $\gamma^{+}\ll 0$ and $\gamma^{-}>0$ for small x. Thus, for small x the forms of quark and gluon distributions are close. This is confirmed numerically by the analysis of the paper [7].

2. For simplicity in the NLO analysis we confine ourselves here by the basic (i.e. $\propto 1/\varepsilon$) contributions only. The full contributions of the NLO coefficients can be found in the Appendix.

The values of the NLO anomalous dimensions

$$\gamma_{qq}^{(1)}= -8C_FT_F(40/9)/\varepsilon + O(1),$$

$$\gamma_{qg}^{(1)}= -8C_AT_F(40/9)/\varepsilon + O(1)$$

$$\gamma_{gq}^{(1)}= -8C_F(C_A- 40/9T_F)/\varepsilon + O(1),$$

$$\gamma_{gg}^{(1)}= (16/3)T_F(23C_A/3- 8C_F)/\varepsilon + O(1)$$

lead to the following meanings of the "+" and "-" AD (see [8])

$$\gamma^{--}= \gamma_{qq}^{(1)} - (C_F/C_A)\gamma_{qg}^{(1)} + O(1) = O(1), \qquad \gamma^{-+}= O(1)$$

$$\gamma^{+-}= \gamma_{gg}^{(1)} - \gamma_{qq}^{(1)} - \left[(3C_A/2T_F)(1/\varepsilon +1/6) - 1\right] \gamma_{qg}^{(1)} + O(1) =$$

$$-(3C_A/2T_F)/\varepsilon\ \gamma_{qg}^{(1)} + O(1/\varepsilon) = (160/3)(C_A^2/\varepsilon^2) + O(1/\varepsilon) \qquad (8)$$

$$\gamma^{++}= \gamma_{gg}^{(1)} + (C_F/C_A)\gamma_{qg}^{(1)} + O(1) = (16/9)T_F(23C_A-26C_F)/\varepsilon + O(1)$$

and the NLO parameters of eq.(3b)

$$K_{-}^{(q)}= O(\varepsilon), \qquad K_{-}^{(g)}= O(1)$$

$$K_{+}^{(q)}= -(20C_A/3\varepsilon) + O(1), \qquad K_{+}^{(g)}= (40/9)(C_FT_F/C_A) + O(\varepsilon).$$

The addition of NLO corrections changes the eqs. (6) and (7) (we take into account only the new variables, including terms $\propto 1/\varepsilon$):

$$f_g^{(1)}(x,Q^2)=\left\{f_g^{(1)}(x,Q_0^2)+(C_F/C_A)f_q^{(1)}(x,Q_0^2)\right\}\left[\rho_1(Q^2,Q_0^2)\right]^{d^+}$$

$$\left\{1 + [\alpha_1(Q^2)-\alpha_1(Q_0{}^2)]\ Z_+\right\} \qquad (9a)$$

$$f_q^{(1)}(x,Q^2)=(2T_F/3C_A)\varepsilon\left\{f_g^{(1)}(x,Q_0^2)-(C_F/C_A)f_q^{(1)}(x,Q_0^2)\right\} \quad (9b)$$

$$\left[\rho_1(Q^2,Q_0^2)\right]^{d^+}\left\{1+[\alpha_1(Q^2)-\alpha_1(Q_0^2)]\,Z_+-\alpha_1(Q^2)K_+^{(q)}\right\}.$$

Following the paper [7] consider the change of Q^2-evolution of PD with NLO corrections. We think, analogously to paper [7], that the PD in the LO and ones in the NLO are equal for $Q^2=Q_0^2$. The LO coupling constant $\alpha_0(Q_0^2)$ and NLO one $\alpha_1(Q_0^2)$ can be connected by two different ways:

a) $\alpha_0(Q_0^2) = \alpha_1(Q_0^2)$, (10)

as it was done in the paper [7], and

b) $\Lambda_0 = \Lambda_1$, (11b)

that is close to the relation discovered by BCDMS group (see [18] and review [19]).

Let us begin with the gluon distribution. The ratio of LO&NLO contribution and LO one depends only on coefficients calculated in PT and has the following form

$$f_g^{(1)}(x,Q^2)/f_g^{(0)}(x,Q^2)=\left[\rho_{10}(Q^2,Q_0^2)\right]^{d^+}\left\{1+[\alpha_1(Q^2)-\alpha_1(Q_0^2)]\,Z_+\right\}, \quad (12)$$

where $\rho_{10}(Q^2,Q_0^2) = \rho_1(Q^2,Q_0^2)/\rho_0(Q^2,Q_0^2)$.

The latter term in r.h.s. of the eq.(12) reduces the LO contribution as $Z_+>0$ and $\alpha_p(Q^2)<\alpha_p(Q_0^2)$ for $Q^2>Q_0^2$. The former term gives distinct contributions in each of two cases mentioned above.

In case (a) the value $\rho_{10}(Q^2,Q_0^2)$ has the following form

$$\rho_{10}(Q^2,Q_0^2) = \alpha_1(Q^2)/\alpha_0(Q^2).$$

Expanding the coupling constants $\alpha_k(Q^2)$ up to ones $\alpha_k(Q_0^2)$ $(k=0,1)$, we get

$$\rho_{10}(Q^2,Q_0^2) = \frac{\alpha_1(Q_0^2)}{\{1+\alpha_1(Q_0^2)\ln(Q^2/Q_0^2)[\beta_0+\beta_1\alpha_1(Q_0^2)]\}}\cdot\frac{\{1+\beta_0\alpha_0(Q_0^2)\ln(Q^2/Q_0^2)\}}{\alpha_0(Q_0^2)}.$$

Using the eq.(10), we obtain that

$$\rho_{10}(Q^2,Q_0^2)-1 = \frac{-\beta_1\alpha_1^2(Q_0^2)\ln(Q^2/Q_0^2)}{\{1+\alpha_1(Q_0^2)\ln(Q^2/Q_0^2)[\beta_0+\beta_1\alpha_1(Q_0^2)]\}} = -\beta_1\alpha_1(Q_0^2)\alpha_1(Q^2)\ln(Q^2,Q_0^2)$$

is negative for $Q^2>Q_0^2$. Hence the former term increases the LO contribution as $d^+>0$. Thus, the full NLO contribution reduces the LO growth of gluon distribution at small x (see paper [7]) weakly.

In case (b) we get for value $\rho_{10}(Q^2,Q_0^2)$:

$$\rho_{10}(Q^2,Q_0^2) = \frac{\ln(Q_0^2/\Lambda_1^2)}{\ln(Q^2/\Lambda_1^2)}\,\frac{[1-L(Q^2/\Lambda_1^2]}{[1-L(Q_0^2/\Lambda_1^2]}\,\frac{\ln(Q^2/\Lambda_0^2)}{\ln(Q_0^2/\Lambda_0^2)}.$$

From the eq.(11) we obtain, that

$$\rho_{10}(Q^2,Q_0^2) -1 = L(Q_0^2/\Lambda_1^2) - L(Q^2/\Lambda_1^2)$$

is positive for $Q^2>Q_0^2$. Hence, the former term decreases the LO contribution as $d^+>0$. The full NLO contribution reduces the LO growth of gluon distribution at small x strongly (see [19] and references of it)

Analysis of Q^2-evolution of quark distributions in the LO&NLO of PT can be given analogously. The difference of quarks and gluon Q^2-evolutions, which has been discovered numerically in the paper [7], is given by the term "$-\alpha_1(Q^2)K_+^{(q)}$" from eq.(9b). The value of $K_+^{(q)}$ is large and negative. Hence, the second term of eq.(9b) can lead to enforce the growth of the quark distribution. In case (a) both the former and the latter terms of eq.(9b) lead to increase of quark distribution. Such an increase at $x=10^{-4}$ is about 30% (see paper [7]). In case (b) these terms give contributions of opposite sign and can lead to weak dampening of LO growth of the quark distribution (see [19]).

4. Summary

In the paper [7] GLAP equation (1) has been analysed. As one has shown, the LO kernel functions $P^{(0)}_{ab}(x)$ and NLO ones $P^{(1)}_{ab}(x)$ have the form

$$P^{(0)}_{qq}(x)=C_F\,\frac{1+x^2}{1-x}+\dots \qquad P^{(1)}_{qq}(x)=2T_FC_F\,\frac{20}{9x}+\dots$$

$$P^{(0)}_{qg}(x)=2T_F[x^2+(1-x)^2] \qquad P^{(1)}_{qg}(x)=2T_FC_A\,\frac{20}{9x}+\dots$$

$$P^{(0)}_{gq}(x)=C_F[1+(1-x)^2]/x \qquad P^{(1)}_{gq}(x)=2T_FC_F(-\frac{20}{9x})+C_FC_A/x+\dots$$

$$P^{(0)}_{gg}(x)=2C_A[\frac{1}{x}+\frac{1}{1-x}]+\dots \qquad P^{(1)}_{gg}(x)=2T_F[-\frac{20}{9}C_A+\frac{2}{3}C_F]/x+\dots$$

and lead to the following consequences:

1) The NLO kernel functions $P^{(1)}_{qq}(x)$ and $P^{(1)}_{qg}(x)$ contain the singular $1/x$ terms whereas the corresponding LO kernel functions $P^{(0)}_{qq}(x)$ and $P^{(0)}_{qg}(x)$ do not. Hence, the evolution of the quarks distributions at small x will be completely dominated by the NLO kernel rather than LO one. The NLO kernel functions $P^{(1)}_{qq}(x)$ and $P^{(1)}_{qg}(x)$ have the positive $1/x$ terms and lead to growth of the quark distributions at small x.

2) The $1/x$ terms in $P^{(1)}_{gg}(x)$ and $P^{(1)}_{gq}(x)$ are large in magnitude and opposite in sign with respect to the corresponding terms in the LO kernel functions. Thus, the well-know rapid growth of gluon distribution at small x seen in the usual LO calculations will be dampened by the inclusion of NLO terms.

The numerical analysis of eq.(1), which has been given also in the paper [7], confirmed the above conclusions.

It is well-know (see [8]),however, that only product $\gamma^{(0)}_{qg}\gamma^{(0)}_{gq}$

but not absolute values of AD $\gamma^{(0)}_{qg}$ and $\gamma^{(0)}_{gq}$ (and the kernel functions $P^{(0)}_{qg}(x)$ and $P^{(0)}_{gq}(x)$, respectively) is essential for the physical quantities. So we can change the values of the LO AD keeping product $\gamma^{(0)}_{qg}\gamma^{(0)}_{gq}$ unchanged. For simple example, we make the replace

$$\gamma_{qg}^{(0)} \rightarrowtail -\gamma_{qg}^{(0)}, \qquad \gamma_{gq}^{(0)} \rightarrowtail -\gamma_{gq}^{(0)}$$

and the same for corresponding NLO AD (and LO and NLO kernel functions $P^{(1)}_{qg}(x)$ and $P^{(1)}_{gq}(x)$). All above results of our analysis do not change, while the singular part of the new kernel function $P^{(1)}_{qg}(x)$ becomes negative and the one of $P^{(1)}_{gq}(x)$ does positive. Hence, the simple conclusions given in the paper [7] (and considered above),which do not take into account the connection between gluons and quarks in GLAP equation, are not quite correct.

As one can see from our analysis, only the NLO AD $\gamma^{(1)}_{qg}$ and $\gamma^{(1)}_{gg}$ (or rather, the ratio $\gamma^{(1)}_{qg}/\gamma^{(0)}_{qg}$ (see r.h.s. of eq.(3c)) give the basic contribution (whilest we use "conventional" form of PD) to PD Q^2-evolution. The AD $\gamma^{(1)}_{gg}$ is the basic part of the value Z_+ and leads to decrease of the LO gluon distribution. The AD $\gamma^{(1)}_{qg}$ gives two basic contributions: decreases the value Z_+ and leads to large and positive value of $K^{(q)}_+$ and, hence, to increase of the LO quarks distributions. Thus, there is essential influence of AD $\gamma^{(1)}_{gg}$ and $\gamma^{(1)}_{qg}$ on a quark and gluon distributions, respectively. As for $\gamma^{(1)}_{gq}$ and $\gamma^{(1)}_{qq}$ their contribution vanishes as the AD $\gamma^{(1)}_{gq}$ is absent in eq.(8) and the basic (i.e.$\propto 1/\varepsilon$) contribution of AD $\gamma^{(1)}_{qq}$ is canceled by one of AD $\gamma^{(1)}_{qg}$.

The consequences 1) and 2) given above have been reproduced numerically only due to "good" numerical values of AD and "successful" choice of conventional $\gamma^{(0)}_{qg}$ and $\gamma^{(0)}_{gq}$. Indeed, if the AD $\gamma^{(1)}_{gg}$ were much larger its normal meaning, then both the LO gluon and LO quark distributions would be dampened by inclusion of NLO terms. On the other hand, if an inequality $\gamma^{(1)}_{qg}\geq 9\gamma^{(1)}_{gg}/4$ were hold then both PD might be enforced by additional NLO contribution.

We would like to pay attention also to the following fact. Rapid growth of the gluon distribution at small-x seen in the usual LO calculations is dampened strongly by inclusion of the NLO terms in case of close LO and NLO values of QCD parameter Λ. It's confirmed by present data. Such a dampening of the LO increase due to higher order effects competes with nonperturbative "saturation" effects (see [20]) and could replace (at least, partially) it.

In conclusion, we note also, that we obtained the simple form of the coefficients for "+" and "-" components of Q^2-evolution of PD moments using the new projectors $\xi^{\pm}_{ab}(n)$ (see eq.s (2) and (3)).

Appendix

The values of NLO AD have the form[7]

$$\gamma^{(1)}_{NS} = 8C_F(C_A-2C_F)\Big[2\zeta(3)-3\zeta(2)+13/4\Big] \cong (32/9)\Big[2\zeta(3)-3\zeta(2)+13/4\Big] \approx 2.558$$

$$\gamma^{(1)}_{qq} = \gamma^{(1)}_{NS} + 16C_F T_F\Big[-(20/9)/\delta + 317/54\Big] \cong \gamma^{(1)}_{NS} + (32/3)f\Big[-(20/9)/\delta + 317/54\Big] \approx (2560/27)\Big[-1/\delta + 2.669\Big]$$

$$\gamma^{(1)}_{qg} = -16T_F\Big[C_F+C_A\Big((20/9)/\delta - 67/9 + (4/3)\zeta(2)\Big)\Big] \cong -24f\Big[(20/9)/\delta -7+ (4/3)\zeta(2)\Big)\Big] \approx -160\Big[1/\delta - 2.163\Big]$$

$$\gamma^{(1)}_{gq} = -8C_F\Big[\Big[\Big(C_A+(40/9)T_F\Big)/\delta - C_A\Big(\zeta(3)-(37/3)\zeta(2)-4793/108\Big) - T_F\Big(1+(8/3)\zeta(2)\Big) + 2C_F\Big(3\zeta(2)-2\zeta(3)-9/2\Big)\Big] \cong -32\Big[\Big(1+(20/27)f\Big)/\delta - (25/9)\zeta(3) - (29/3)\zeta(2) - 5225/108 - (f/6)\Big(1+(8/3)\zeta(2)\Big)\Big] \approx (2816/27)\Big[1/\delta + 36.28\Big]$$

$$\gamma^{(1)}_{gg} = 16C_A^{\,2}\Big(4\zeta(3)+(11/3)\zeta(2)-773/108\Big) + 16T_F\Big[C_A\Big((23/9)/\delta-86/27\Big) - C_F\Big((2/3)/\delta-61/18\Big)\Big] \cong 16\Big[9\Big(4\zeta(3)+(11/3)\zeta(2)-773/108\Big) + f\Big((67/18)/\delta-455/108\Big)\Big] \approx (2144/9)\Big[1/\delta + 1.094\Big]$$

[7] The symbols $\cong$ and $\approx$ transform SU(N) group coefficients to QCD (i.e. for N=3) and QCD with f=4 ones, respectively.

and lead to the following "+" and "-" ones

$$\gamma^{--} = \gamma^{(1)}_{NS} + 16C_F T_F\Big[(4/3)\zeta(2) - 47/54 + C_F/C_A + (23T_F)/(9C_A) - (58C_F T_F/27C_A^{\,2})\Big] \cong \gamma^{(1)}_{NS} + (32/3)f\Big[(4/3)\zeta(2)-23/54+(389/1458)f\Big] \approx 123.5$$

$$\gamma^{-+} = (16C_F T_F/C_A^{\,2})\Big[C_A^{\,2}+(2/3)T_F(C_A-2C_F)\Big] \cong (32/81)f\Big[9+f/9\Big] \approx 14.92$$

$$\gamma^{+-} = (16C_A^{\,2}/\delta)\Big[(10/3\delta)+2\zeta(2) - 191/18 + (3C_F/2C_A)+T_F/(3C_A) + (14C_F T_F/9C_A^{\,2})\Big] \cong (144/\delta)\Big[(10/3\delta)+ 2\zeta(2) - 179/18 + (41/486)f\Big] \approx (1440/3\delta)\Big[1/\delta - 1.895\Big]$$

$$\gamma^{++} = 16\Big[C_A^{\,2}\Big(4\zeta(3)+(11/3)\zeta(2)-773/108\Big) + C_A T_F\Big((23/9)/\delta - 86/27\Big) - C_F T_F\Big((26/9)/\delta+(4/3)\zeta(2)-547/54\Big) - C_F^{\,2}T_F/C_A + (58C_F^{\,2}T_F^{\,2})/(27C_A^{\,2}) - (2C_F T_F^{\,2})/(3C_A)\Big] \cong 16\Big[9\Big(4\zeta(3)+(11/3)\zeta(2)-773/108\Big) + f\Big((103/54)/\delta+(8/9)\zeta(2)- 455/108\Big) + (70/2187)f^2\Big] \approx (1236/27)\Big[1/\delta + 8.082\Big]$$

and to the following values of NLO coefficients of equations (10):

$Z_- \approx 6.826$, $\qquad Z_+ \approx 11.617\Big[1/\delta + 1.022\Big]$

$K^{(q)}_- \approx 0.622\delta$, $\qquad K^{(g)}_- \cong -3\Big[1+f/81\Big] \approx -3.148$

$$K_+^{(q)} \cong -20\left[1/\delta+(3/5)\zeta(2)-179/60+(151/1620)f\right] \approx -20\left[1/\delta-1.623\right]$$

$$K_+^{(g)} \cong (80/27)f \approx 11.852$$

Note here, that the corrections $\propto O(1)$ to structures $\propto 1/\delta$ are small for all NLO coefficients. The exception is AD $\gamma_{gq}^{(1)}$. However, it is contained only with factor δ in all "+" and "-" parts of AD.

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Котиков А.В. E2-92-361

Поведение партонных функций распределений в области малых значений бьеркеновской переменной x

Дан анализ партонных распределений в области малых значений переменной Бьеркена x. Показано, каким образом Q^2-эволюция партонных распределений изменяется при включении неведущих поправок к аномальным размерностям операторов Вильсона. Для стандартного, т.е. несингулярного при $x \to 0$, выбора партонных распределений это изменение определяется только двумя (из четырех) аномальными размерностями $\gamma_{qg}^{/1/}(n)$ и $\gamma_{gg}^{/1/}(n)$. Приведены также более простые выражения, по сравнению со стандартными, для коэффициентов в уравнениях Q^2-зависимости моментов партонных распределений.

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The Small-x Behaviour of Parton Distribution Functions

The analysis of parton distributions for the small-x region is given. It is shown how the inclusion of the next-to-leading corrections to the anomalous dimensions of the Wilson operators changes the behaviour of Q^2-evolution of parton distributions. For "conventional" (nonsingular for $x \to 0$) choice of parton distributions this change is determined by the values of anomalous dimensions $\gamma_{qg}^{/1/}(n)$ and $\gamma_{gg}^{/1/}(n)$ only. We obtain also the new simple form for the coefficients of Q^2-evolution of parton distribution moments.

The investigation has been performed at the Particle Physics Laboratory JINR.

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