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V.A.Slobodenyuk

ON CALCULATION OF EVOLUTION OPERATOR KERNEL OF SCHRÖDINGER EQUATION

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1 INTRODUCTION

The problem of going beyond the scope of perturbation theory in the quantum theory is now one of the most important problems. There are a lot of different approaches for this problem. Some authors try to sum the series of perturbation theory [1,2] using the information about its asymptotic behavior [3,4]. Others build up the expansions not connected with the coupling constant. This expansions are derived from the action, in which the contributions are redistributed between the free and interaction parts [5,6]. Besides, variational methods are extensively used [7-9]. But they do not allow to estimate the precision of the result obtained in intrinsic terms. To overcome this shortcoming, Sissakyan and Solovtsov are developing variational perturbation theory [10] (in this paper one can find a more complete review of references). Nevertheless, the problem of nonperturbative calculations is not yet solved.

We here propose a new method of computations in quantum theory that does not require the coupling constant to be small.

Evolution of a quantum-mechanical system is determined by the evolution operator $U(t',t)=\exp\{-iH(t'-t)/\hbar\}$, where $H=p^2/2m+V(q)$ is the Hamiltonian. Calculating the evolution operator kernel or Green function (let us consider the one-dimensional case for the sake of simplicity) $\langle q',t'|q,t\rangle = \langle q'|U(t',t)|q\rangle$ in the path integral formalism [11] one starts with the expression for the kernel for $\Delta t=t'-t$ known with a precision of the first order in Δt:

$$\langle q',t+\Delta t|q,t\rangle \simeq \left(\frac{-im}{2\pi\hbar\Delta t}\right)^{1/2} \exp\left\{ i\,\frac{m(q'-q)^2}{2\hbar\Delta t} - i\,\frac{\Delta t}{\hbar}\,V(q',q)\right\}. \quad (1)$$

Here $V(q',q)$ is the expression derived from the potential by symmetrization in q', q. For finite time intervals the kernel can be obtained as a limit

$$\langle q',t'|q,t\rangle=\lim_{n\to\infty}\int\langle q'=q_n,t'=t_n|q_{n-1},t_{n-1}\rangle dq_{n-1}\times$$

$$\times\langle q_{n-1},t_{n-1}|q_{n-2},t_{n-2}\rangle dq_{n-2}\ldots dq_1\langle q_1,t_1|q_0=q,t_0=t\rangle. \quad (2)$$

This is the path integral. Nevertheless, we may do not produce the functional integration, but seek the kernel either directly from the equation

$$\langle q'',t''|q,t\rangle=\int\langle q'',t''|q',t'\rangle dq'\langle q',t'|q,t\rangle, \quad (3)$$

which is called the Smoluchowski equation in the statistical physics, or from the Schrödinger equation, which really is a consequence of (3). It is possible to use for this purpose the representation of kind (1), in which, besides the term of the first order in Δt, all further terms of the expansion in the powers of Δt are present. This way for solution of the evolution equation is close to the space of small time method (which is a version of the asymptotic method of functional parameters) developed by Cherepennikov for the problems of classical mechanics in [12,13] (see also references therein).

The expansion in Δt was used in [14]. But there was in fact considered the harmonic oscillator and the lowest correction terms only were calculated.

The method suggested can be easily transferred to quantum field theory. This leads to a new version of the lattice field theory that in spirit is similar to the Hamiltonian approach developed by Barnes et al. [15-17].

The article is organized as follows. In Section 2 the method of solution the quantum mechanical problems is described. The anharmonic oscillator is considered here as an example. In Section 3 a possible generalization to quantum field theory is examined.

2 QUANTUM MECHANICS

2.1 Square Lagrangians

For problems with square Lagrangians equation (3) may be solved by the substitution

$$\langle q',t'|q,t\rangle = \left(\frac{-im}{2\pi\hbar\Delta t}\right)^{1/2}\exp\left\{ i\,\frac{m(q'-q)^2}{2\hbar\Delta t} - \sum_{n=1}^{\infty}\sum_{k,l}(i\Delta t)^n q'^k q^l d^n_{kl}\right\}. \quad (4)$$

Here the only coefficients d^n_{kl} are different from zero for which $k+l\le 2$. Calculating the Gaussian integral in the right-hand side of (3) we get the system of equations for d^n_{kl}. The representation (4) becomes fully determined after solving this system.

In this manner a number of well-known expressions for the kernel [11] was rederived: for the particle in the constant external field, for the free harmonic oscillator, for the harmonic oscillator in the time-depending homogeneous external field, for the charge in the constant homogeneous magnetic field. Consideration of the oscillator can be produced both in the coordinate and in the holomorphic representations. For the latter we should use, instead of (3), the equation for the kernel $U(a^*,a)$ (notations are standard)

$$U(a''^*,a,t''-t)=\int\frac{da'^*da'}{2\pi i}\,e^{-a'^*a'}\,U(a''^*,a',t''-t')U(a'^*,a,t'-t). \quad (5)$$

But in this way we cannot solve (3) (or (5)) for non-square Lagrangians because in this case the integral in the right-hand side of the equation cannot be calculated explicitly, and the term by term integration of an expansion for the exponential leads to a divergent series.

2.2 Quantum mechanics in one dimension

Let us seek the solution of (3) in the form (4). Here and further we will use the notation

$$D(\Delta t,q',q)=\sum_{n=1}^{\infty}(i\Delta t)^n P_n(q',q)=\sum_{n=1}^{\infty}\sum_{k,l}(i\Delta t)^n q'^k q^l d^n_{kl} \qquad (6)$$

The function P_1 should satisfy the condition $P_1(q,q)=V(q)/\hbar$ (we consider that $V(q)$ does not depend on the time explicitly). The problem is to find all functions $P_n(q',q)$ or, which is the same, function $D(\Delta t,q',q)$ using (3) and knowing the potential $V(q)$.

Let us consider (4) when $V(q)=0$, i.e. the free particle. The kernel is singular when $\Delta t\to 0$. This singularity may be expressed through the δ-function and its derivatives

$$\langle q'',t''|q',t'\rangle=\left(\frac{-im}{2\pi\hbar\Delta t}\right)^{1/2}\exp\left\{i\,\frac{m(q''-q')^2}{2\hbar\Delta t}\right\}=$$

$$=\delta(q''-q')+\sum_{n=1}^{\infty}\frac{(i\Delta t)^n}{n!}\left(\frac{\hbar}{2m}\right)^n\frac{\partial^{2n}}{\partial q'^{2n}}\delta(q''-q'). \qquad (7)$$

Substituting the kernel $\langle q'',t''|q',t'\rangle$ in the form (7) into (3) we can obtain after some mathematics the Schrödinger equation

$$\frac{1}{i}\frac{\partial}{\partial t}\langle q',t'|q,t\rangle=\frac{\hbar}{2m}\frac{\partial^2}{\partial q'^2}\langle q',t'|q,t\rangle-\frac{V(q')}{\hbar}\langle q',t'|q,t\rangle. \qquad (8)$$

Let us seek the solution of (8) in the form (4). For P_n we get the sequence of equations

$$P_1(q',q)+(q'-q)\frac{\partial P_1(q',q)}{\partial q'}=P_1(q',q'), \qquad (9)$$

$$nP_n(q',q)+(q'-q)\frac{\partial P_n(q',q)}{\partial q'}=$$

$$=\frac{\hbar}{2m}\frac{\partial^2 P_{n-1}(q',q)}{\partial q'^2}-\frac{\hbar}{2m}\sum_{n'=1}^{n-2}\frac{\partial P_{n'}(q',q)}{\partial q'}\frac{\partial P_{n-n'-1}(q',q)}{\partial q'}. \qquad (10)$$

Equation (9) with the condition $P_1(q,q)=V(q)/\hbar$ determines unambiguously the way of symmetrization of the potential in q', q. Equations (10) allow us to obtain recurrently any P_n. To do this we should expand P_n in powers of q' and q (see (6)). Then we have for the coefficients d^n_{kl} (which are symmetric in k, l)

$$d^1_{k,l}=\frac{\delta_{l0}}{k+1}\sum_{k'=0}^{k}d^1_{k',k-k'}+d^1_{k+1,l-1}, \qquad (11)$$

$$d^n_{k,l}=\frac{1}{n+k}\Bigg[(k+1)d^n_{k+1,l-1}+\frac{\hbar}{2m}(k+1)(k+2)d^{n-1}_{k+2,l}-$$

$$-\frac{\hbar}{2m}\sum_{n'=1}^{n-2}\sum_{k'=1}^{k+1}\sum_{l'=0}^{l}k'(k-k'+2)d^{n'}_{k',l'}d^{n-n'-1}_{k-k'+2,l-l'}\Bigg]. \qquad (12)$$

The polynomial potential $V(q)$ determines non zero coefficients d^1_{k0}, which allows us to find all d^1_{kl} from (11) and then subsequently all d^n_{kl} from (12). That determines the function $D(\Delta t,q',q)$ as a series in powers of its arguments.

It may occur that it is convenient for technical reasons to seek the solution of the Schrödinger equation (8) in the form

$$\langle q',t'|q,t\rangle=\left(\frac{-im}{2\pi\hbar\Delta t}\right)^{1/2}\exp\left\{i\,\frac{m(q'-q)^2}{2\hbar\Delta t}\right\}\times$$

$$\times\left\{1-\sum_{n=1}^{\infty}(i\Delta t)^n Y_n(q',q)\right\}. \qquad (13)$$

Here $Y_1(q',q)=P_1(q',q)$. Then we get for Y_n $(n>1)$

$$nY_n(q',q)+(q'-q)\frac{\partial Y_n(q',q)}{\partial q'}=$$

$$= -Y_1(q',q')Y_{n-1}(q',q) + \frac{\hbar}{2m}\frac{\partial^2 Y_{n-1}(q',q)}{\partial q'^2}. \tag{14}$$

If Y_n is represented by the expansion

$$Y_n(q',q) = \sum_{k,l} q'^k q^l w^n_{kl}, \tag{15}$$

then for w^n_{kl} we have $(n>1)$

$$w^n_{k,l} = \frac{1}{n+k}\Big[(k+1)w^n_{k+1,l-1} + \frac{\hbar}{2m}(k+1)(k+2)w^{n-1}_{k+2,l} -$$

$$- \sum_{k',l'} w^1_{k',l'}\, w^{n-1}_{k-k'-1',l}\Big]. \tag{16}$$

The sequence of equations (16) is really linear in w^n_{kl} because the coefficients w^1_{kl} are given initially.

2.3 Anharmonic oscillator

Let us apply the equations obtained to the anharmonic oscillator with the potential

$$V(x) = \frac{m\omega^2 x^2}{2} + \tilde{\alpha}x^4. \tag{17}$$

We introduce dimensionless variables $q=x/\sqrt{\hbar/m\omega}$, $T=\omega(t'-t)$, $\alpha=\tilde{\alpha}\hbar/m^2\omega^3$. In this case the solution of (11) gives

$$d^1_{20}=d^1_{02}=d^1_{11}=1/6, \qquad d^1_{40}=d^1_{04}=d^1_{31}=d^1_{13}=d^1_{22}=\alpha/5.$$

It is obvious that the coefficients d^n_{kl} are polynomial functions of α. They can be represented as

$$d^n_{kl} = \sum_m \alpha^m \kappa^n_{mkl},$$

where κ^n_{mkl} are numbers, and D can be treated as a function of

four arguments T, α, q', q,:

$$D(T,q',q,\alpha) = \sum_{n=1}^{\infty}\sum_{m,k,l}(iT)^n\alpha^m q'^k q^l \kappa^n_{mkl}. \tag{18}$$

It is easy to obtain the equations for κ^n_{mkl} from (12)

$$\kappa^n_{m,k,l} = \frac{1}{n+k}\Big[(k+1)\kappa^n_{m,k+1,l-1} + \frac{1}{2}(k+1)(k+2)\kappa^{n-1}_{m,k+2,l} -$$

$$- \frac{1}{2}\sum_{n'=1}^{n-2}\sum_{m'=0}^{m}\sum_{k'=1}^{k+1}\sum_{l'=0}^{l} k'(k-k'+2)\kappa^{n'}_{m',k',l'}\kappa^{n-n'-1}_{m-m',k-k'+2,l-l'}\Big]. \tag{19}$$

Initial values are $\kappa^1_{020}=\kappa^1_{011}=1/6$, $\kappa^1_{140}=\kappa^1_{131}=\kappa^1_{122}=1/5$. Calculation of the coefficients κ^n_{mkl} with the help of (19) allows us to determine the function $D(T,q',q,\alpha)$ in the region of convergence of the series (19).

Convergence of the expansion (18) will be studied in detail in a subsequent paper. Now we just refer to the results of numerical estimates which show that the finite region of convergence in T with the radius r_T takes place at every fixed set of q', q, α. It is obvious that this region cannot be infinite, because even the series for $D(T,q',q,0)$, corresponding to the harmonic oscillator, has a finite convergence range in T which is equal to π.

So far as the kernel should be determined at every q', q and at every positive T, the problem arises to continue analytically the function D outside the region of convergence of the series (18).

For the harmonic oscillator that continuation can be made with the use of periodicity of the functions ctn(T) and 1/sin(T). In the case of the anharmonic oscillator we may hope that the information about periodical character of the classical solution (which is expressed through the Jacobi elliptic functions) will help us to produce the continuation.

Notice that it is easy to distinguish the terms corresponding to the quasi-classical approximation in the expression (6). It is enough for this to retrieve the dimension factors for the variables q', q, α.

Note that the representation (18) allows us not to restrict ourselves to a small coupling constant α. One can consider large α, but this diminishes the convergence range for the series in T.

2.4 Quantum mechanics in three-dimensional space

The method being developed can be easily generalized to the case of three-dimensional space. If the coordinates in this space are $\vec{q}=(q_1,q_2,q_3)$, then the general representation for the kernel is

$$\langle q',t'|q,t\rangle=\left(\frac{-im}{2\pi\hbar\Delta t}\right)^{3/2}\exp\left\{i\frac{m(\vec{q}'-\vec{q})^2}{2\hbar\Delta t}-D(\Delta t,\vec{q}',\vec{q})\right\}. \qquad (20)$$

The function D is represented by (6) in which now the coordinates are vectors and indexes k, l are multi-indexes $k=\{k_1,k_2,k_3\}$, $l=\{l_1,l_2,l_3\}$.

In analogy with the one-dimensional case we get the Schrödinger equation

$$\frac{1}{i}\frac{\partial}{\partial t'}\langle\vec{q}',t'|\vec{q},t\rangle=\frac{\hbar}{2m}\sum_{j=1}^{3}\frac{\partial^2}{\partial q_j'^2}\langle\vec{q}',t'|\vec{q},t\rangle-\frac{V(\vec{q}')}{\hbar}\langle\vec{q}',t'|\vec{q},t\rangle. \qquad (21)$$

When one seeks the solution of (21) in the form (20), then one gets the sequence of equations for P_n

$$P_1(\vec{q}',\vec{q})+\sum_{j=1}^{3}(q_j'-q_j)\frac{\partial P_1(\vec{q}',\vec{q})}{\partial q_j'}=P_1(\vec{q}',\vec{q}'), \qquad (22)$$

$$nP_n(\vec{q}',\vec{q})+\sum_{j=1}^{3}(q_j'-q_j)\frac{\partial P_n(\vec{q}',\vec{q})}{\partial q_j'}=$$

$$=\frac{\hbar}{2m}\sum_{j=1}^{3}\left[\frac{\partial^2 P_{n-1}(\vec{q}',\vec{q})}{\partial q_j'^2}-\sum_{n'=1}^{n-2}\frac{\partial P_{n'}(\vec{q}',\vec{q})}{\partial q_j'}\frac{\partial P_{n-n'-1}(\vec{q}',\vec{q})}{\partial q_j'}\right], \qquad (23)$$

which are a direct generalization of (9), (10). Then one can obtain from (22), (23) the equations for the coefficients d^n_{kl} generalizing (11), (12). For n=1 we have (in dimensionless units)

$$d^1_{k,l}=\frac{1}{\sum\limits_{i=1}^{3}k_i+1}\left[\delta_{l0}\sum_{k'}d^1_{k',k-k'}+\sum_{j=1}^{3}(k_j+1)d^1_{k+\delta_j,l-\delta_j}\right], \qquad (24)$$

and for n>1 –

$$d^n_{k,l}=\frac{1}{\sum\limits_{i=1}^{3}k_i+n}\sum_{j=1}^{3}\left[(k_j+1)d^n_{k+\delta_j,l-\delta_j}+\frac{1}{2}(k_j+1)(k_j+2)d^{n-1}_{k+2\delta_j,l}-\right.$$

$$\left.-\frac{1}{2}\sum_{n'=1}^{n-2}\sum_{k'}\sum_{l'}k_j'(k_j-k_j'+2)d^{n'}_{k',l'}\,d^{n-n'-1}_{k-k'+2\delta_j,l-l'}\right]. \qquad (25)$$

Here k', l', like k, l, are multi-indexes and summation runs over all components k_i', l_i', for which the coefficients d differ from zero. The symbol δ_{l0} is treated as $\delta_{l_1 0}\delta_{l_2 0}\delta_{l_3 0}$, and the indexes of kind $k+m\delta_j$ mean here and further that the j-th component is equal to k_j+m; and the i-th one ($i\neq j$) to k_i. The sequence of equations (24), (25) allows us to find any terms in the expansion (6) starting from the initial coefficients d^1_{kl} and in this way to determine the function D and the kernel (20) in the range of convergence of the series.

3 QUANTUM FIELD THEORY

In the field theory the components of fields play a role of generalized coordinates. For the definiteness we will consider the real scalar field $\varphi(x)$. Let us divide the three-dimensional space into small elements of volume ΔV_a. Here $a=\{a_1,a_2,a_3\}$ is the multi-index numbering the elements of division. We will consider that the field may be represented by the countable set of functions of the time $\varphi_a(t)=\varphi(\vec{x}_a,t)$.

Introduce the state vectors $|\varphi(\vec{x})>$ satisfying at the moment of time t the equation

$$\hat{\varphi}(\vec{x})|\varphi(\vec{x})>=\varphi(\vec{x})|\varphi(\vec{x})>,$$

where $\hat{\varphi}(\vec{x})$ is the field operator. If we will consider the set of φ_a so as the components of the radius-vector in the case of quantum mechanics, then the formalism developed for quantum mechanics may be easily transferred to the field theory.

The kernel can be sought in the form

$$<\varphi',t'|\varphi,t>=\prod_a\left(\frac{-i\Delta V_a}{2\pi\Delta t}\right)^{1/2}\exp\left\{\frac{i}{2\Delta t}\sum_a \Delta V_a(\varphi'_a-\varphi_a)^2 - \right.$$
$$\left. -\sum_{n=1}^{\infty}(i\Delta t)^n P_n(\varphi',\varphi)\right\}. \qquad (26)$$

Here for the sake of brevity we denoted $\varphi_a\equiv\varphi_a(t)$, $\varphi'_a\equiv\varphi_a(t')$. The symbol φ means the complete set of $\{\varphi_a\}$, and the system of units in which $c=\hbar=1$ is used. The analogue of the Schrödinger equation (21)

$$\frac{1}{i}\frac{\partial}{\partial t'}<\varphi',t'|\varphi,t>=\frac{1}{2}\sum_a\frac{1}{\Delta V_a}\frac{\partial^2}{\partial\varphi'^2_a}<\varphi',t'|\varphi,t>-P_1(\varphi',\varphi')<\varphi',t'|\varphi,t> \qquad (27)$$

in the same way as in Section 2 can be derived from the equation

$$<\varphi'',t''|\varphi,t>=\int<\varphi'',t''|\varphi',t'>\prod_a d\varphi'_a<\varphi',t'|\varphi,t>. \qquad (28)$$

The sequence of the equations for P_n is: for n=1

$$P_1(\varphi',\varphi)+\sum_a(\varphi'_a-\varphi_a)\frac{\partial P_1(\varphi',\varphi)}{\partial\varphi'_a}=P_1(\varphi',\varphi'), \qquad (29)$$

for n>1

$$nP_n(\varphi',\varphi)+\sum_a(\varphi'_a-\varphi_a)\frac{\partial P_n(\varphi',\varphi)}{\partial\varphi'_a}=$$

$$=\frac{1}{2}\sum_a\frac{1}{\Delta V_a}\left[\frac{\partial^2 P_{n-1}(\varphi',\varphi)}{\partial\varphi'^2_a}-\sum_{n'=1}^{n-2}\frac{\partial P_{n'}(\varphi',\varphi)}{\partial\varphi'_a}\frac{\partial P_{n-n'-1}(\varphi',\varphi)}{\partial\varphi'_a}\right]. \qquad (30)$$

We will not write an equations analogous to (24), (25). They can be obtained in obvious way.

Let us consider, for example, the scalar field with the Lagrangian

$$\mathcal{L}(x)=\frac{1}{2}\partial_\mu\varphi\,\partial^\mu\varphi-\frac{1}{2}m^2\varphi^2-\frac{\alpha}{4!}\varphi^4, \qquad (31)$$

which determines the function

$$P_1(\varphi,\varphi)=\sum_a\Delta V_a\left\{\frac{1}{2}\sum_{l=1}^{3}\left(\frac{\varphi_a-\varphi_{a-\delta_l}}{\Delta x_{al}}\right)^2+\frac{1}{2}m^2\varphi_a^2+\frac{\alpha}{4!}\varphi_a^4\right\}. \qquad (32)$$

Here we denoted $\Delta x_{al}=x_a-x_{a-\delta_l}$. Solving (29) we will find

$$P_1(\varphi',\varphi)=\sum_a\Delta V_a\left\{\frac{1}{6}\left[\sum_{l=1}^{3}\frac{1}{(\Delta x_{al})^2}+\sum_{l=1}^{3}\frac{1}{(\Delta x_{a+\delta_l,l})^2}\frac{\Delta V_{a+\delta_l}}{\Delta V_a}\right]\times\right.$$
$$\times\left(\varphi'^2_a+\varphi'_a\varphi_a+\varphi_a^2\right)-$$

$$- \frac{1}{6} \sum_{l=1}^{3} \frac{1}{(\Delta x_{al})^2} \left(2\varphi'_a\varphi'_{a-\delta_l} + 2\varphi_a\varphi_{a-\delta_l} + \varphi'_a\varphi_{a-\delta_l} + \varphi'_{a-\delta_l}\varphi_a\right) +$$

$$+ \frac{1}{6}m^2\left(\varphi'^2_a + \varphi'_a\varphi_a + \varphi^2_a\right) + \frac{\alpha}{5!}\left(\varphi'^4_a + \varphi'^3_a\varphi_a + \varphi'^2_a\varphi^2_a + \varphi'_a\varphi^3_a + \varphi^4_a\right)\Bigg\}. \quad (33)$$

For n=2 the solution of (30) is

$$P_2(\varphi', \varphi) = \sum_a \left\{ \frac{1}{12} \sum_{l=1}^{3} \frac{1}{(\Delta x_{al})^2} \left(1 + \frac{\Delta V_a}{\Delta V_{a-\delta_l}}\right) + \right.$$

$$\left. + \frac{m^2}{12} + \frac{\alpha}{5!}\left(3\varphi'^2_a - \varphi'_a\varphi_a + 3\varphi^2_a\right)\right\}. \quad (34)$$

The technique described is really a variant of the lattice field theory. It is close to the lattice Hamiltonian field theory approach developed in [15-17]. In that approach the state functional is sought from the Schrödinger equation discretized in space and time variables. Besides, the space of discrete values of field φ_a at every point of the ordinary three-dimensional space is considered, but our method requires to discretize only the three-dimensional space. Initial and final field configurations should be given as boundary conditions. The time evolution is described by (26), where the fields at intermediate moments of time do not arise at all.

Nevertheless, the scheme suggested (like any other founded on equation (27)) has one unpleasant feature. Let us discuss it. We use the notation a for the characteristic size of a cell of the three-dimensional lattice. The structure of the expression (33) is such that it is possible to perform the limit $a \to 0$ in it, and $P_1(\varphi', \varphi)$ will be represented as the integral over the three-dimensional volume. However (34) and the expressions for subsequent P_n do not have this property yet. The functions P_n do not allow us to perform this limit

and cannot be represented as an integral over volume. The magnitudes of P_n depend on the choice of division of three-dimensional space into the cells ΔV. Because the expressions for P_n are singular as $a \to 0$, one cannot say about any limit magnitudes of the functions P_n.

Note that the singularities appear even for a free field because of the singular structure of initial equation (27) (term ~ $1/\Delta V$!). Early it was pointed out in [16] that theory is singular as $a \to 0$, nevertheless, in that paper this difficulty was by-passed with the help of a not very rigorous mathematical trick.

The singular behavior of the functions P_n can be treated as artifact. Then we should tend to avoid it in some way. But it is possible to try to look at this fact from another viewpoint. Maybe, it is an indication of the necessity to work in the quantized space. Choice of the latter variant would mean that quantization of the field is closely connected with quantization of the space (and time?).

4 CONCLUSION

We suggest the method for calculation of Green functions in the theories with polynomial potentials as an expansion in powers of the time interval Δt. In fact, the perturbation theory in parameter Δt is being built. Here the coupling constant α should not be small, i.e. the so-called nonperturbative aspects are taken into account. The expansion parameter Δt should not be small too. Its maximal possible value is determined by the convergence radius of the series r_T that depends on the initial conditions and coupling constant. When it is necessary to calculate the kernel for $\Delta t > r_T$, then analytical continuation of the function D beyond the convergence range of the series should be done. Probably, it may be achieved by using information on the quasiclassical behavior.

Application of this method to quantum field theory leads to a new version of the lattice field theory in which only the three- dimensional space is discretized. Here singularity of the Schrödinger equation (and hence, its solutions) as the lattice cell size tends to zero becomes essential. Maybe, it is an artifact of the lattice Hamiltonian field theory, but maybe, it is an indication of the necessity to quantize the space.

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APPENDIX

Here the functions $P_n(q',q)$ for some initial numbers n are presented:

$$P_1 = \frac{1}{6}q'^2 + \frac{1}{6}q'q + \frac{1}{6}q^2 + \alpha\left(\frac{1}{5}q'^4 + \frac{1}{5}q'^3q + \frac{1}{5}q'^2q^2 + \frac{1}{5}q'q^3 + \frac{1}{5}q'^4\right),$$

$$P_2 = \frac{1}{12} + \alpha\left(\frac{3}{10}q'^2 + \frac{2}{5}q'q + \frac{3}{10}q^2\right),$$

$$P_3 = -\frac{1}{90}q'^2 - \frac{7}{360}q'q - \frac{1}{90}q^2 + \frac{\alpha}{10} -$$

$$-\alpha\left(\frac{4}{105}q'^4 + \frac{17}{210}q'^3q + \frac{2}{21}q'^2q^2 + \frac{17}{210}q'q^3 + \frac{4}{105}q^4\right) -$$

$$-\alpha^2\left(\frac{8}{225}q'^6 + \frac{13}{150}q'^5q + \frac{2}{15}q'^4q^2 + \frac{7}{45}q'^3q^3 + \right.$$

$$\left. + \frac{2}{15}q'^2q^4 + \frac{13}{150}q'q^5 + \frac{8}{225}q^6\right),$$

$$P_4 = -\frac{1}{360} - \alpha\left(\frac{1}{14}q'^2 + \frac{13}{105}q'q + \frac{1}{14}q^2\right) -$$

$$-\alpha^2\left(\frac{19}{150}q'^4 + \frac{22}{75}q'^3q + \frac{9}{25}q'^2q^2 + \frac{22}{75}q'q^3 + \frac{19}{150}q^4\right).$$

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Слободенюк В.А. E2-92-359

О вычислении ядра оператора эволюции уравнения Шредингера

Предлагается метод вычисления ядра оператора эволюции в виде ряда по степеням временного интервала Δt. Метод применим к задачам с полиномиальными потенциалами в квантовой механике и квантовой теории поля. Он позволяет учитывать непертурбативные (по константе связи) эффекты.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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On Calculation of Evolution Operator Kernel of Schrödinger Equation

The method for calculation of the evolution operator kernel as an expansion in powers of the time interval Δt is proposed. This method can be applied to the problems of quantum mechanics and quantum field theory with polynomial potentials. Nonperturbative (in coupling constant) effects can be considered in the framework of this approach.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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