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A.A.Shanenko, E.P.Yukalova, V.I.Yukalov

STATISTICAL APPROACH TO DECONFINEMENT IN PURE GAUGE MODELS

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1. Introduction

As follows from the analysis of studies of the thermodynamics of gluon systems, a more complete investigation of peculiarities of deconfinement requires a more close interplay of the lattice description [1-9] and statistical models [5,10]. However, this is a complicated problem as there is no such statistical model that would be consistent both with the $SU(2)$ group- and with the $SU(3)$ group-lattice data.

Indeed, the approaches based on the Baacke method [10] and bag theory [11,12] predict that the deconfinement in $SU(2)$ and $SU(3)$ systems is a first - order phase transition, whereas the lattice calculations support the second - order deconfinement in the $SU(2)$ theory [5]. Besides, the above class of statistical models for the transition heat $\Delta\epsilon$ produces the relation $\Delta\epsilon \simeq \epsilon_{SB}(\theta_{dec})$, where ϵ_{SB} is the energy density of the ideal gluon gas and θ_{dec} is the deconfinement temperature, whereas lattice studies of an $SU(3)$ system give much smaller value, $\Delta\epsilon \simeq \epsilon_{SB}(\theta_{dec})/4$ [9]. Results of the phenomenological model [5] in which the free energy is a sum of two terms describing low - momentum massive modes of the gluon field and high - momentum massless gluons are in good agreement with lattice data for the $SU(2)$ theory. However, since the model predicts second-order deconfinement, this approach does not describe the $SU(3)$ system relative to first - order deconfinement (see refs. [4,9]).

In this paper, we present a statistical model of the deconfinement in $SU(2)$ and $SU(3)$ systems whose results are in good agreement with the lattice calculations. Basic points of the proposed approach are as follows:

First, possible coexistence is considered of nonseparated phases of glueballs and gluon plasma. Concentrations of the phases in our model are determined from the conditions of thermodynamic advantage. Note also that unlike stratified Gibbs mixtures, our phases coexist in a homogeneous system. The importance of such heterophase mixtures is demonstrated in a series of papers devoted to general problems [13-16] and to applications, in particular, to studies of quark - hadron matter at high baryon densities [17-22].

Second, we take into account the interaction between plasma gluons and their interaction with glueballs in the effective-field approximation.

The paper is organized as follows: In sect.2, we consider a simplified approach, based on the restriction rules in differentiation, as a first approximation that gives a qualitative behaviour of the mixture and satisfactory estimates. The results illustrated with figures are presented. In sect.3, we expound a more involved approach to the mixture using correcting functions and allowing a good agreement with lattice predictions. The corresponding results are also supplied with illustrations.

Some preliminary results of our approach have been announced in refs. [23,24].

2. Model with restriction rules

2.1. Fundamentals of method

As said in the Introduction, a key point of our approach is the possibility of simultaneous production of glueballs and unbound gluons from vacuum, the phases of glueballs and gluon plasma being not separated in space. Using the total density of gluons in the system,

$$\rho = \rho_g + \sum_{nj} n\rho_{nj}, \tag{1}$$

where ρ_g is the density of quasifree gluons, ρ_{nj} is the density of glueballs of sort nj consisting of n bound gluons, we can determine the concentrations of phases in the mixture

$$w_g \equiv \frac{\rho_g}{\rho}, \qquad w_G \equiv \frac{1}{\rho}\sum_{nj} n\rho_{nj}, \tag{2}$$

$$w_g + w_G = 1. \tag{3}$$

In formulae (2) and (3), the index g stands for gluon plasma; whereas the index G, for glueballs.

The density of unbound gluons in our model is given by the expression

$$\rho_g = \frac{\xi_g}{2\pi^2}\int\limits_0^{+\infty} k^2\left[\exp\left(\frac{\omega(k,\rho)-\mu_g}{\theta}\right)-1\right]^{-1} dk, \tag{4}$$

whereas the gluon spectrum in the mixture is defined by

$$\omega(k,\rho) = k + \frac{B}{\rho}. \tag{5}$$

In expression (4), ξ_g represents the number of internal degrees of freedom of gluons, $\xi_g = 6$ ($SU(2)$) or $\xi_g = 16$ ($SU(3)$) ; μ_g is the chemical potential of plasma gluons. The choice of the gluon spectrum, eq. (5), is based on the following: The energy of gluon plasma in the simplest case of the MIT model is defined by the expression [11]

$$E = \sum_k n_g(k)\cdot k + BV, \tag{6}$$

where $n_g(k)$ is the distribution of gluons over momentum; V is the volume occupied by plasma; B is a constant called the pressure of QCD vacuum. If the term BV is interpreted as the energy of gluon interaction in plasma, then the interaction energy per one gluon is clearly equal to BV/N ,where N is the number of gluons in plasma. Therefore, using the condition

$$\sum_k n_g(k) = N,$$

we may add the given quantity to the spectrum of gluons in plasma and rewrite the plasma energy in the form

$$E = \sum_k n_g(k)\left(k + \frac{BV}{N}\right). \tag{7}$$

Taking the gluon spectrum in the mixture of gluons and glueballs by analogy with (6) we arrive at the formula (5). Note that the alternative variant of choice $\omega = k + B/\rho_g$ corresponds to a less stable state of the mixture, as it may be verified. Also we should stress that the computation of the quark plasma energy with the spectrum of the form (5) gives results in satisfactory agreement with the fermion sector of the energy of the $SU(3)$ system with an isodoublet of light quarks above the deconfinement point [25].

As seen, the spectrum of unbound gluons in the mixture $\omega(k,\rho)$ depends on the total gluon density that contains both ρ_g and ρ_{nj}

for $\forall n,j$. Consequently, $\omega(k,\rho)$ defined by (5) describes not only the interaction of quasifree gluons with each other but also their interaction with glueballs; the latter being of course sensitive to the gluon plasma in the system. However, at a given step of our consideration we will assume that glueballs are strongly bound states of gluons and their energy in the equilibrium mixture slightly depends on gluon plasma.

To describe the interaction of hadrons with each other, let us first apply the Van der Waals method, as it is done by many authors [10,17,26]. In this case the following equality

$$\rho_{nj} = \left(1 - \sum_{mi} v_{mi}\rho_{mi}\right) \widetilde{\rho}_{nj} \tag{8}$$

is valid, where

$$\widetilde{\rho}_{nj} = \frac{\xi_{nj}}{2\pi^2} \int_0^{+\infty} k^2 \left[\exp\left(\frac{\sqrt{k^2 + M_{nj}^2} - \mu_{nj}}{\theta}\right) - 1\right]^{-1} dk. \tag{9}$$

Here v_{nj} is the volume of the particle core; M_{nj} is the glueball mass; ξ_{nj} is the number of internal degrees of freedom of a glueball; μ_{nj} is the chemical potential of nj glueballs. Upon a simple algebra we obtain the relation

$$\rho_{nj} = \widetilde{\rho}_{nj} \left(1 + \sum_{mi} v_{mi}\widetilde{\rho}_{mi}\right)^{-1}. \tag{10}$$

Since the main contribution to thermodynamic characteristics of the glueball phase comes from particles with the minimal masses, we will consider only the low-lying glueballs with the following properties [27,28]:

$$\xi_{20} = 6, \quad M_{20} = 960 Mev; \quad \xi_{21} = 6, \quad M_{21} = 1290 Mev;$$

$$\xi_{22} = 6, \; M_{22} = 1590 Mev; \quad \xi_{30} = 11; \quad M_{30} = 1460 Mev;$$

$$\xi_{31} = 36, \quad M_{31} = 1800 Mev$$

This choice is argued as follows: The lightest glueball of the above set has a mass about $1000 Mev$, which is in good agreement with estimates for the $SU(2)$ system [5,29]. In the $SU(3)$ case this value falls into the

range of lattice predictions [30,33] and agrees with model estimates [34]. Besides, if an unusual narrow resonance 0^{++} recently discovered [35] in the mass spectrum of a pion pair produced in proton-proton collisions is considered to be a scalar glueball, the experimental estimate of the mass gap of glueballs in the $SU(3)$ case amounts to $975 \pm 16 Mev$.

Using the relation

$$\frac{v_{20}}{v_{nj}} = \frac{M_{20}}{M_{nj}}, \tag{11}$$

following from the bag theory [11] when the radius of the particle core is assumed to be proportional to the bag radius, we may reduce the number of model parameters to two, v_{20} and B .

Characteristics of the equilibrium mixture of gluons and glueballs (concerning equilibrium of a heterophase system see ref. [36]) are determined from the condition of local extremum of the free energy

$$(dF)_{\theta,V} = \frac{\partial F}{\partial N_g} dN_g + \sum_{nj} \frac{\partial F}{\partial N_{nj}} dN_{nj} = 0, \tag{12}$$

where N_g and N_{nj} are, respectively, numbers of plasma gluons and glueballs of sort nj . Since the relations

$$\frac{\partial F}{\partial N_g} = \mu_g, \qquad \frac{\partial F}{\partial N_{nj}} = \mu_{nj}$$

are valid, and the increments dN_g and dN_{nj} are independent, we obtain from (12) the following equality

$$\mu_g = \mu_{nj} = 0. \tag{13}$$

Solving the system of equations (1),(4),(5),(8),(9),(13) we determine the functions $\rho(\theta), w_g(\theta)$ and $w_G(\theta)$ for various sets of the parameters v_{20} and B . These functions are used to determine other thermodynamical characteristics of the system, specifically, the grand thermodynamical potential

$$\Omega(\theta, V, \mu_g, \{\mu_G\}) = \frac{\theta \xi_g V}{2\pi^2} \int_0^{+\infty} k^2 \ln\left[1 - \exp\left(-\frac{\omega(k,\rho) - \mu_g}{\theta}\right)\right] dk +$$

$$+\sum_{nj}\frac{\theta\xi_{nj}V_f}{2\pi^2}\int_0^{+\infty}k^2\ln\left[1-\exp\left(-\frac{\sqrt{k^2+M_{nj}^2}-\mu_{nj}}{\theta}\right)\right]dk, \qquad (14)$$

where the volume V_f free for motion of clusters is expressed by

$$V_f = V\left(1-\sum_{nj}v_{nj}N_{nj}\right),$$

and $\{\mu_G\}$ is a set of glueball chemical potentials. However, owing to the first approximation being quite rough, inaccurate use of the above potential may lead to a number of inconsistencies. In particular, there arises ambiguity in the definition of the pressure, because

$$\frac{\partial\Omega}{\partial V}\neq\frac{\Omega}{V}.$$

Besides,

$$\frac{\partial\Omega}{\partial\mu_g}\neq\frac{\xi_g V}{2\pi^2}\int_0^{+\infty}k^2\left[\exp\left(\frac{\omega(k,\rho)-\mu_g}{\theta}\right)-1\right]^{-1}dk,$$

$$\frac{\partial\Omega}{\partial\mu_{nj}}\neq\frac{\xi_{nj}V_f}{2\pi^2}\int_0^{+\infty}k^2\left[\exp\left(\frac{\sqrt{k^2+M_{nj}^2}-\mu_{nj}}{\theta}\right)-1\right]^{-1}dk.$$

These difficulties appear because the definition of the grand thermodynamic potential does not include additional terms, the correcting functions, whose meaning will be explained in the next section. A reasonable approximation of characteristics of the glueball mixture can also be found without corrections that highly complicate the computations. To this end, it is necessary to employ the restriction rules in the course of differentiation, which is made in the following way: We introduce an auxiliary function

$$\Omega\left(\theta,V,\mu_g,\{\mu_G\},X,Y\right)=$$

$$=\frac{\theta\xi_g V}{2\pi^2}\int_0^{+\infty}k^2\ln\left[1-\exp\left(-\frac{k+X-\mu_g}{\theta}\right)\right]dk+$$

$$+\sum_{nj}\frac{\theta\xi_{nj}VY}{2\pi^2}\int_0^{+\infty}k^2\ln\left[1-\exp\left(-\frac{\sqrt{k^2+M_{nj}^2}-\mu_{nj}}{\theta}\right)\right]dk, \qquad (15)$$

and define the differential of the grand thermodynamic potential by the equation

$$d\Omega=\lim_{X\to B/\rho}\ \lim_{Y\to V_f/V}d\,\widetilde{\Omega}, \qquad (16)$$

where the differential $d\,\Omega$ is taken at $X,Y=constant$. Now all the ambiguities in defining thermodynamic characteristics vanish,

$$p=-\frac{\partial\Omega}{\partial V}=-\frac{\Omega}{V},$$

$$\rho_g=\frac{1}{V}\frac{\partial\Omega}{\partial\mu_g},\qquad \rho_{nj}=\frac{1}{V}\frac{\partial\Omega}{\partial\mu_{nj}}. \qquad (17)$$

and the volume density of the energy of the system acquires the normal form,

$$\epsilon=\Omega+\mu_g N_g+\sum_{nj}\mu_{nj}N_{nj}-\theta\frac{\partial\Omega}{\partial\theta}. \qquad (18)$$

It is to be noted that the restriction rules are frequently used in differentiating the effective thermodynamic potentials. As a rule, the effective thermodynamic potentials include either the temperature- and density-dependent spectra of particles or the free volume dependent on their number, as in the case when the sizes of particles are taken into account. In these approaches that do not use correcting functions, differentiation with respect to the above-mentioned characteristics is not performed in order to prevent inconsistencies.

2.2. Results of consideration of model with restriction rules

Numerical analysis of the model with restriction rules provides the following results. For $SU(2)$ and $SU(3)$ mixtures with any set of the parameters v_{20} and B in a certain temperature region dependent on the parameters the functions ϵ/ϵ_{SB}, p/p_{SB} and w_g get sharply increasing while w_G gets decreasing (p_{SB} is the pressure of the ideal gas of gluons). The behaviour of the functions ϵ/ϵ_{SB} and w_G for the $SU(3)$

mixture at $B^{1/4} = 235 Mev$ and various values of $r_{20} = (3v_{20}/4\pi)^{1/3}$ is drawn in Figs. 1 and 2. When v_{20} increases and B is constant, the growth of ϵ/ϵ_{SB}, p/p_{SB} and w_g and decrease of w_G become still more sharp. And finally, when $v_{20} = v_{20}^{(c)}(B)$, these functions acquire a point of inflection, at which their derivatives with respect to temperature become infinite. As is seen, when $v_{20} < v_{20}^{(c)}(B)$, deconfinement in a mixture of gluon plasma and hadrons is a continuous transition, whereas deconfinement for $v_{20} = v_{20}^{(c)}(B)$ is a second-order phase transition, according to the Erenfest classification. Specific heat of the $SU(3)$ mixture at $B^{1/4} = 235 Mev$ and $r_{20} = 0.8 fm \approx r_{20}^{(c)}(235)$ is shown in Fig.3. When $v_{20} > v_{20}^{(c)}(B)$ thermodynamic characteristics acquire a typical loop (see Fig.1) testifying to a first-order phase transition in the system.

It is interesting that if $v_{20} = 0$, then a basic role in the mixture at asymptotically high temperatures belongs to glueballs, and the quantity ϵ/ϵ_{SB} is much larger than unity; whereas for any $v_{20} > 0$, when $\theta \to \infty$, we have

$$w_g \to 1, \ \epsilon/\epsilon_{SB} \to 1, \ p/p_{SB} \to 1; \ \theta \to \infty.$$

For the $SU(2)$ mixture our results are in the best agreement with lattice data if $B^{1/4} \approx 165 Mev$ and $r_{20} \approx 1 fm$, and deconfinement in the mixture is either a second-order phase transition, or a close to a second-order transition continuous crossover with a high maximum of specific heat at the transition point. So, in Figs. 4 and 5 we present the results of the consideration of the model with restriction rules for the $SU(2)$ system at $B^{1/4} = 165 Mev$ and $r_{20} = 1.2 fm$ ($\theta_{dec} \approx 210 Mev$). Note that both in $SU(2)$ and $SU(3)$ variants the agreement for ϵ/ϵ_{SB} with lattice calculations is good, but for p/p_{SB} it is worse since in the temperature region $\theta_{dec} \div 2\theta_{dec}$ the pressure in the model is twice that in the lattice calculations.

For the $SU(3)$ mixture our results are in good agreement with lattice predictions at $B^{1/4} \approx 170 \div 270 Mev$ and $r_{20} \approx 0.4 \div 1 fm$. The range of the values of parameters depends on large inaccuracy in estimation of the temperature of deconfinement on the basis of lattice studies due to difficulties in transition from the lattice units to physical

units [6,9] If, for instance, $B^{1/4} = 210 Mev$, then $r_{20} \approx 0.66 fm$ and a first-order phase transition occurs at the point $\theta_{dec} \approx 215 Mev$; but when $B^{1/4} = 235 Mev$, then $r_{20} \approx 0.82 fm$ and a first - order phase transition occurs at the temperature $\theta_{dec} \approx 240 Mev$. In Fig. 6 we plot the function $(\epsilon + p)/(\epsilon_{SB} + p_{SB})$ calculated within our model (a solid line, $B^{1/4} = 220 Mev, r_{20} = 0.72 fm, \theta_{dec} \approx 225 Mev$) and by the lattice simulations (points, the data for the lattice $24^3 \times 6$) taken from ref.[4]).

3. The model with correcting functions

To simplify the presentation of the object of this section, we first consider particular cases of the mixture of gluon plasma and glueballs: in subsect.3.1 we discuss the thermodynamic correction in the case of gluon plasma; in subsect. 3.2 we analyze the mixture of glueballs; and finally, in subsect. 3.3, based on these results, we formulate the complete model with corrections.

3.1. Gluon plasma

As follows from the study of the gluon-glueball mixture within the model with the restriction rules, the spectrum of gluons in plasma is well approximated by the function

$$\omega(k, \rho_g) = k + \frac{C}{\rho_g^{\alpha}}, \quad (\alpha \sim 1), \tag{19}$$

where C is a constant of the dimension $[Mev]^{3\alpha+1}$. To construct an approach consistent with thermodynamics for describing a system of unbound gluons with the spectrum (19), we should write the Hamiltonian of the system in the form

$$H^{(g)} = \sum_{\vec{k},\sigma} \omega(k, \rho_g) \overset{+}{a}_g(\vec{k}, \sigma) a_g(\vec{k}, \sigma) + \mathcal{U}^{(g)}(\theta, \rho_g) V, \tag{20}$$

where $\overset{+}{a}_g(\vec{k}, \sigma)$; $(a_g(\vec{k}, \sigma))$ are operators of creation (annihilation) of a gluon with momentum $\vec{k}$ in a quantum state σ obeying the Bose commutation relations. The term $\mathcal{U}^{(g)}(\theta, \rho_g)$ in (20) appears because the effective Hamiltonian in the mean-field approximation is always a sum of the operator and numerical parts, the form of the correcting function being dictated by the rules of statistical mechanics [36]. The gluon plasma with the Hamiltonian (20) in the thermodynamic limit possesses the following free energy

$$F^{(g)}(\theta, V, N_g) = \frac{\xi_g \theta V}{2\pi^2} \int_0^{+\infty} k^2 \ln\left[1 - \exp(-\frac{\omega(k,\rho_g) - \mu_g}{\theta})\right] dk +$$

$$+\mathcal{U}^{(g)}(\theta, \rho_g)V + \mu_g N_g, \tag{21}$$

and the chemical potential μ_g is given by the equation

$$\rho_g = \frac{\xi_g}{2\pi^2} \int_0^{+\infty} k^2 \left[\exp\left(\frac{\omega(k,\rho_g) - \mu_g}{\theta}\right) - 1\right]^{-1} dk. \tag{22}$$

Owing to the relation

$$\frac{\partial F^{(g)}}{\partial N_g} = \mu_g,$$

we can, by differentiating (21) with respect to the variable N_g , get the equality

$$\frac{\partial F^{(g)}}{\partial N_g} = \mu_g + \frac{\partial \mathcal{U}^{(g)}}{\partial \rho_g} - \frac{\alpha C}{\rho_g^\alpha}. \tag{23}$$

From (23) and the respective expression for $\partial F^{(g)}/\partial N_g$ we derive the following equation for the correcting function

$$\frac{\partial \mathcal{U}^{(g)}}{\partial \rho_g} = \frac{\alpha C}{\rho_g^\alpha}$$

with the solution

$$\mathcal{U}^{(g)}(\theta, \rho_g) = \frac{\alpha}{1-\alpha} C \rho_g^{1-\alpha} + \lambda_1(\theta), \qquad \alpha \neq 1;$$

$$\mathcal{U}^{(g)}(\theta, \rho_g) = C \ln\left(\frac{\rho_g}{\lambda_2(\theta)}\right), \qquad \alpha = 1; \tag{24}$$

where $\lambda_1(\theta)$ and $\lambda_2(\theta)$ are some functions of the temperature, the first being of dimension of the energy volume density and the second of the volume density of particles. To determine the temperature dependence of $\lambda_1(\theta)$ and $\lambda_2(\theta)$, we make use of the well-known identity

$$< H^{(g)} >= F^{(g)} - \theta \frac{\partial F^{(g)}}{\partial \theta}, \tag{25}$$

where

$$< \ldots >= Tr(\ldots \exp(-H^{(g)}/\theta))/Tr(\exp(-H^{(g)}/\theta)),$$

and the right-hand side looks as follows

$$F^{(g)} - \theta \frac{\partial F^{(g)}}{\partial \theta} = \frac{\xi_g V}{2\pi^2} \int_0^{+\infty} k^2 \omega(k,\rho_g) \left[\exp\left(\frac{\omega(k,\rho_g) - \mu_g}{\theta}\right) - 1\right]^{-1} dk +$$

$$+\mathcal{U}^{(g)}(\theta, \rho_g)V - \theta \frac{\partial \mathcal{U}^{(g)}}{\partial \theta} V.$$

Respectively, the left-hand side is given by

$$< H^{(g)} >= \sum_{\vec{k},\sigma} \omega(k,\rho_g) < \overset{+}{a}_g(\vec{k},\sigma) a_g(\vec{k},\sigma) > + \mathcal{U}^{(g)}(\theta, \rho_g)V.$$

For $N_g \to \infty$, $V \to \infty$ and $\rho_g = const$

$$< H^{(g)} > \to \frac{\xi_g V}{2\pi^2} \int_0^{+\infty} k^2 \omega(k,\rho_g) \left[\exp\left(\frac{\omega(k,\rho_g) - \mu_g}{\theta}\right) - 1\right]^{-1} dk +$$

$$+\mathcal{U}^{(g)}(\theta, \rho_g)V.$$

Consequently, we may conclude that

$$\frac{d\lambda_1}{d\theta} = \frac{d\lambda_2}{d\theta} = 0.$$

Therefore, for the correcting function $\mathcal{U}^{(g)}$ we have the following equalities

$$\mathcal{U}^{(g)} = \frac{\alpha}{1-\alpha} C \rho_g^{1-\alpha} + \lambda_1, \quad \lambda_1 = const, \quad \alpha \neq 1;$$

$$\mathcal{U}^{(g)} = C\ln(\rho_g/\lambda_2), \quad \lambda_2 = const, \quad \alpha = 1. \tag{26}$$

To complete this subsection, we make the following comments. When we consider a system of particles with the spectrum

$$\omega(k,\theta,\rho) = \epsilon(k) + \mathcal{U}_{spec}(\theta,\rho),$$

where $\epsilon(k)$ is the kinetic part of the spectrum, $\mathcal{U}_{spec}(\theta,\rho)$ is the energy of interaction of a particle with an effective mean field, and θ and ρ are, respectively, the temperature and density of particles, then a thermodynamically consistent approach requires the following effective Hamiltonian

$$H_{eff} = \sum_{\vec{k},\alpha} \omega(k,\theta,\rho) \overset{+}{a} (\vec{k},\alpha) a(\vec{k},\alpha) + \mathcal{U}_{corr}(\theta,\rho)V,$$

in which $\overset{+}{a} (\vec{k},\alpha)$ $(a(\vec{k},\alpha))$ is the operator of creation (annihilation) of a particle of any statistics with momentum $\vec{k}$ and in a quantum state α; V is the volume occupied by the system. The functions $\mathcal{U}_{spec}(\theta,\rho)$ and $\mathcal{U}_{corr}(\theta,\rho)$ are connected by the system of differential equations

$$\rho\frac{\partial \mathcal{U}_{spec}}{\partial \rho} + \frac{\partial \mathcal{U}_{corr}}{\partial \theta} = 0,$$

$$\rho\frac{\partial \mathcal{U}_{spec}}{\partial \theta} + \frac{\partial \mathcal{U}_{corr}}{\partial \theta} = 0. \tag{27}$$

The first approximation, as is discussed above, can here be found with the use of the Hamiltonian

$$H_{eff} = \sum_{\vec{k},\alpha} \omega(k,\theta,\rho) \overset{+}{a} (\vec{k},\alpha) a(\vec{k},\alpha),$$

together with the restriction rules in the course of differentiating.

3.2. Mixture of glueballs

The effective Hamiltonian of a mixture of glueballs of different sorts (the limiting case of the mixture of plasma with glueballs when $\rho_g = 0$) is

written in the form

$$H^{(G)} = \sum_{nj}\sum_{\vec{k},s}\left(\sqrt{k^2 + M_{nj}^2} + \mathcal{U}_{nj}^{(G)}(\{\rho_G\})\right) \overset{+}{a}_{nj} (\vec{k},s) a_{nj}(\vec{k},s) +$$

$$+ \mathcal{U}^{(G)}(\{\rho_G\})V. \tag{28}$$

where $\overset{+}{a}_{nj} (\vec{k},s)$ and $a_{nj}(\vec{k},s)$ are operators of creation and annihilation of a glueball of sort nj with momentum $\vec{k}$ and in a spin state s satisfying the Bose commutation relations; $\mathcal{U}_{nj}^{(G)}$ is the energy of interaction of a hadron of sort nj with other glueballs; $\{\rho_G\}$ stands for the set of densities of glueballs; $\mathcal{U}^{(G)}$ is a correcting function. Upon standard computations we have expounded above we arrive at the system of equations

$$\sum_{nj} \rho_{nj} \frac{\partial \mathcal{U}_{nj}^{(G)}}{\partial \rho_{mi}} + \frac{\partial \mathcal{U}^{(G)}}{\partial \rho_{mi}} = 0, \quad (\forall mi),$$

$$\sum_{nj} \rho_{nj} \frac{\partial \mathcal{U}_{nj}^{(G)}}{\partial \theta} + \frac{\partial \mathcal{U}^{(G)}}{\partial \theta} = 0. \tag{29}$$

While constructing a thermodynamically consistent model it is reasonable to give up a rough Van der Waals approach to glueballs and use an approach of the Hartree type. Note that the hadron spectrum given in a Hartree approximation provides good results in the description of the high-temperature gas of hadrons produced from the vacuum [37]. In our case it is convenient to employ the following approximation

$$\mathcal{U}_{nj}^{(G)} = A_{nj}(\rho - \rho_g), \tag{30}$$

where $\rho - \rho_g$ is the density of gluons in glueballs,

$$\rho - \rho_g = \sum_{nj} n\rho_{nj}.$$

and A_{nj} is a constant of dimension Mev^{-2}. To get certain results from (29) and (30), we take two arbitrary sorts of glueballs nj and mi. It is obviously that

$$\frac{\partial \mathcal{U}^{(G)}}{\partial \rho_{nj}} = -nA_{nj}\rho_{nj} - \sum_{lp\neq nj} nA_{lp}\rho_{lp}.$$

This equation can be transformed to

$$\frac{\partial \mathcal{U}^{(G)}}{\partial \rho_{nj}} = -A_{nj}(\rho - \rho_g) - \sum_{lp \neq nj} \rho_{lp}(A_{lp}n - lA_{nj}), \tag{31}$$

integration of which gives

$$\mathcal{U}^{(G)} = -\frac{A_{nj}}{2n}(\rho - \rho_g)^2 - \sum_{lp \neq nj} \rho_{nj}\rho_{lp}(A_{lp}n - lA_{nj}) + $$
$$+ \varphi(\{\rho_G^*\}), \tag{32}$$

where $\{\rho_G^*\}$ denotes the set of glueball densities without the variable ρ_{nj} . Differentiating (32) with respect to ρ_{mi} we arrive at the equality

$$\frac{\partial \mathcal{U}^{(G)}}{\partial \rho_{mi}} = -\frac{A_{nj}m}{n}(\rho - \rho_g) - \rho_{nj}(A_{mi}n - mA_{nj}) + \frac{\partial \varphi}{\partial \rho_{mi}}.$$

On the other hand, from (29) we have

$$\frac{\partial \mathcal{U}^{(G)}}{\partial \rho_{mi}} = -mA_{mi}\rho_{mi} - \sum_{lp \neq mi} mA_{lp}\rho_{lp},$$

and , hence ,

$$\frac{\partial \varphi}{\partial \rho_{mi}} = (\rho - \rho_g)(\frac{m}{n}A_{nj} - A_{mi}) + \rho_{nj}(A_{mi}n - mA_{nj}) -$$
$$- \sum_{lp \neq mi} \rho_{lp}(A_{lp}m - lA_{mi}).$$

Since the function $\varphi(\{\rho_G^*\}$ should be independent of ρ_{nj} , we obtain the relation

$$nA_{mi} = mA_{nj}, \quad (\forall n, m). \tag{33}$$

Using the notation $\Phi \equiv A_{20}$, from eq.(32) and relation (33) we get the following result

$$\mathcal{U}_{nj}^{(G)} = \frac{n}{2}\Phi(\rho - \rho_g),$$

$$\mathcal{U}^{(G)} = -\frac{\Phi}{4}(\rho - \rho_g)^2 + \lambda_3, \quad \lambda_3 = const, \tag{34}$$

where λ_3 is to be put zero because for $\Phi \to 0$ the mixture of interacting glueballs should transform into the ideal gas of glueballs.

The free energy of a system of glueballs with the Hamiltonian (28) and specifications (34) in the thermodynamic limit looks as follows:

$$F^{(G)}(\theta, V, \{N_G\}) = \sum_{nj} \mu_{nj}N_{nj} - \frac{\Phi}{4}(\rho - \rho_g)^2 V +$$

$$+ \sum_{nj} \frac{\theta \xi_{nj} V}{2\pi^2} \int_0^{+\infty} k^2 \ln\left[1 - \exp\left(-\frac{\omega_{nj}(k, \{\rho_G\}) - \mu_{nj}}{\theta}\right)\right] dk, \tag{35}$$

where

$$\omega_{nj}(k, \{\rho_G\}).== \sqrt{k^2 + M_{nj}^2} + n\frac{\Phi}{2}(\rho - \rho_g), \tag{36}$$

and $\{N_G\}$ is the set of numbers of glueballs, N_{nj} is the number of glueballs of the sort nj . The chemical potentials μ_{nj} are determined by the equation

$$\rho_{nj} = \frac{\xi_{nj}}{2\pi^2} \int_0^{+\infty} k^2 \left[\exp\left(\frac{\omega_{nj}(k, \rho_G) - \mu_{nj}}{\theta}\right) - 1\right]^{-1} dk. \tag{37}$$

To complete this subsection, we note that for any effective Hamiltonian of a system of particles of several sorts having the form

$$H_{eff} = \sum_i \sum_{\vec{k},\alpha} (\epsilon_i(k) + \mathcal{U}_i(\theta, \{\rho\})) \overset{+}{a}_i (\vec{k}, \alpha) a_i(\vec{k}, \alpha) + \mathcal{U}(\theta, \{\rho\})V, \tag{38}$$

the following relations

$$\sum_i \rho_i \frac{\partial \mathcal{U}_i}{\partial \rho_j} + \frac{\partial \mathcal{U}}{\partial \rho_j} = 0, \quad (\forall j);$$

$$\sum_i \rho_i \frac{\partial \mathcal{U}_i}{\partial \theta} + \frac{\partial \mathcal{U}}{\partial \theta} = 0 \tag{39}$$

are valid. In (37) $\overset{+}{a}_i (\vec{k}, \alpha)$ and $a_i(\vec{k}, \alpha)$ are operators of creation and annihilation of particles of the sort i ; $\epsilon_i(k)$ and $\mathcal{U}$ are, respectively, the kinetic part of the spectrum of particles of the sort i and their interaction energy with an effective mean field; $\{\rho\}$ is the set of particle densities.

3.3. Mixture of glueballs and gluon plasma

The mixture of gluon plasma and glueballs is described by the following Hamiltonian

$$H^{(g,G)} = \sum_{nj} \sum_{\vec{k},s} [\omega(k, \{\rho_G\}) + \mathcal{U}_{nj}(\theta, \rho_g, \{\rho_G\})] \overset{+}{a}_{nj} (\vec{k}, s) a_{nj}(\vec{k}, s) +$$

$$+ \sum_{\vec{k},\sigma} \omega(k, \rho) \overset{+}{a}_g (\vec{k}, \sigma) a_g(\vec{k}, \sigma) +$$

$$+ \left(\mathcal{U}(\theta; \rho_g, \{\rho_G\}) - \frac{\Phi}{4}(\rho - \rho_g)^2 \right) V, \quad (40)$$

where ω_{nj} has been defined in subsection 3.2, and

$$\omega(k, \rho) = k + \frac{C}{\rho^\alpha}. \quad (41)$$

In the expression (40) $\overset{+}{a}_g (\ldots)$, $(a_g(\ldots))$ is the operator of creation (annihilation) of a unbound gluon; $\overset{+}{a}_{nj} (\ldots)$, $(a_{nj}(\ldots))$ are the glueball operators of creation (annihilation); the term $\mathcal{U}_{nj}(\theta, \rho_g, \{\rho_G\})$ in the glueball spectrum and the term $\mathcal{U}(\theta, \rho_g, \{\rho_G\})$ in the correcting function are stipulated by the presence of unbound gluons in the system.

From expressions(39)-(41) we can derive the relations

$$\sum_{mi} \rho_{mi} \frac{\partial \mathcal{U}_{mi}}{\partial \rho_g} + \frac{\partial \mathcal{U}}{\partial \rho_g} = \alpha C \frac{\rho_g}{\rho^{\alpha+1}};$$

$$\sum_{mi} \rho_{mi} \frac{\partial \mathcal{U}_{mi}}{\partial \rho_{nj}} + \frac{\partial \mathcal{U}}{\partial \rho_{nj}} = \alpha n C \frac{\rho_g}{\rho^{\alpha+1}}, \quad (\forall nj);$$

$$\sum_{mi} \rho_{mi} \frac{\partial \mathcal{U}_{mi}}{\partial \theta} + \frac{\partial \mathcal{U}}{\partial \theta} = 0. \quad (42)$$

which give the following solutions:

a) if $\alpha \neq 1$, then

$$\mathcal{U}_{nj} = n \frac{C}{\rho^\alpha} + \frac{\partial \eta}{\partial \rho_{nj}},$$

$$\mathcal{U} = \frac{\alpha}{1-\alpha} C \rho^{1-\alpha} + \eta - \sum_{nj} \rho_{nj} \frac{\partial \eta}{\partial \rho_{nj}}, \quad (43)$$

where η is a function of the temperature and densities of glueballs;

b) if $\alpha = 1$, then

$$\mathcal{U}_{nj} = n \frac{C}{\rho} + \frac{\partial \eta_1}{\rho_{nj}},$$

$$\mathcal{U} = C \ln\left(\frac{\rho}{\lambda_4}\right) + \eta_1 - \sum_{mi} \frac{\partial \eta_1}{\partial \rho_{mi}} \rho_{mi}, \quad (44)$$

where λ_4 is a constant of the dimension of the particle density and η_1 is a function of the temperature and glueball densities.

Further consideration will require the free energy of the mixture of gluon plasma and glueballs which has in the thermodynamic limit the following form

$$F^{(g,G)}(\theta, V, N_g, \{N_G\}) = \frac{\xi_g \theta V}{2\pi^2} \int_0^{+\infty} k^2 \ln\left[1 - \exp\left(-\frac{\omega(k,\rho) - \mu_g}{\theta}\right)\right] dk +$$

$$\sum_{nj} \frac{\xi_{nj} \theta V}{2\pi^2} \int_0^{+\infty} k^2 \ln\left[1 - \exp\left(-\frac{\omega_{nj}(k, \{\rho_G\}) + \mathcal{U}_{nj} - \mu_{nj}}{\theta}\right)\right] dk +$$

$$+ \left(\mathcal{U} - \frac{\Phi}{4}(\rho - \rho_g)^2\right) V + \mu_g N_g + \sum_{nj} \mu_{nj} N_{nj}. \quad (45)$$

In this case the chemical potentials obey the relations

$$\rho_g = \frac{\xi_g}{2\pi^2} \int_0^{+\infty} k^2 \left[\exp\left(\frac{\omega(k,\rho) - \mu_g}{\theta}\right) - 1\right]^{-1} dk, \quad (46)$$

$$\rho_{nj} = \frac{\xi_{nj}}{2\pi^2} \int_0^{+\infty} k^2 \left[\exp\left(\frac{\omega_{nj}(k, \{\rho_G\}) + \mathcal{U}_{nj} - \mu_{nj}}{\theta}\right) - 1\right]^{-1} dk. \quad (47)$$

When $\theta, V, \{N_G\}$ are fixed and $\rho_g \to 0$, the free energy of the plasma - glueball mixture should tend to the free energy of the gas of glueballs

(35); and μ_{nj} calculated from (47), to the values of chemical potentials determined from (36). At fixed $\theta, V, \{N_G\}$ and $\rho_g \to 0$ we have:

1) $\mu_g \to -\infty$, which follows from (46);

2) $\frac{\xi_g \theta V}{2\pi^2} \int\limits_0^{+\infty} k^2 \ln \left[1 - \exp\left(-\frac{\omega(k,\rho)-\mu_g}{\theta}\right)\right] dk \to 0;$

3) $\mu_g N_g \to 0$, which also follows from (46);

4) μ_{nj} will tend to the solution of eq. (37) only under the condition

$$\mathcal{U}_{nj} \to 0, \quad (\rho_g \to 0);$$

5) Moreover, it is necessary that

$$\mathcal{U} \to 0, \quad (\rho_g \to 0).$$

As a result, using 4) we arrive at the equality

$$\frac{\partial \eta}{\partial \rho_{nj}} = -n \frac{C}{(\rho - \rho_g)^\alpha}.$$

for the case $\alpha \neq 1$, $(\alpha \sim 1)$, from which we get

$$\eta = -\frac{1}{1-\alpha} C (\rho - \rho_g)^{1-\alpha} + \eta_2(\theta), \tag{48}$$

where $\eta_2(\theta)$ is a function of the temperature. Now from formulae (48) and (43) under the condition 5) we may for $\alpha \neq 1$, $(\alpha \sim 1)$ derive the following equalities

$$\mathcal{U}_{nj} = n \frac{C}{\rho^\alpha} - n \frac{C}{(\rho - \rho_g)^\alpha},$$

$$\mathcal{U} = \frac{\alpha}{1-\alpha} C \rho^{1-\alpha} - \frac{\alpha}{1-\alpha} C (\rho - \rho_g)^{1-\alpha}. \tag{49}$$

Upon a similar procedure for $\alpha = 1$ we arrive at the relations

$$\mathcal{U}_{nj} = n \frac{C}{\rho} - n \frac{C}{\rho - \rho_g},$$

$$\mathcal{U} = C \ln \left(\frac{\rho}{\rho - \rho_g} \right). \tag{50}$$

Now it is to be verified that at fixed θ, V, N_g and when $N_{nj} \to 0$, $(\forall nj)$, the free energy of a mixture of the plasma and glueballs (45) tends to the free energy of the plasma (21). We will not overload the paper with cumbersome algebra and thus expound only basic results of our computations. It appears that the free energy is continuous at the point $\theta, V, N_g, \{N_G\}$, where $N_{nj} = 0$, $(\forall nj)$, only at $\alpha < 1$ and the constant λ_1 in formula (26) is to be taken zero. Consequently, feasible values of the coefficient α obey the relations

$$\alpha \sim 1, \quad \alpha < 1. \tag{51}$$

Note is to be made that parameter α is directly related to the degree of screening of interactions of gluons in plasma.

3.4. Discussion of results

Solving the systems of equations (1)-(3),(13),(46),(47),(49) for various sets of parameters Φ, C, α we get the following results. The behaviour of the mixture in the model with correcting functions is analogous to its behaviour in the model with the restriction rules. Indeed, investigating the situation, when the parameters C and α are fixed, we find at a sufficiently small Φ (both in the $SU(2)$ and $SU(3)$ cases) that the deconfinement is a continuous crossover. With increasing Φ the continuous crossover turns to a second-order phase transition according to the Erenfest classification and then to a first order phase transition. With further increase in Φ the weak first order transition $(\Delta\epsilon << \epsilon_{SB})$) becomes strong $(\Delta\epsilon \sim \epsilon_{SB})$.

However, in contradistinction to the model with restriction rules that fails to describe the pressure of $SU(2)$ and $SU(3)$ systems well, the energy and pressure of the mixture can now be described in agreement with the lattice data with an appropriate choice of parameters. It is to be noted that this result is achieved due to the influence of correction rather than due to increasing number of the parameters. In Fig. 7 the relative energy ϵ/ϵ_{SB} and pressure p/p_{SB} for the $SU(2)$ mixture are plotted computed by our method (a solid curve, $\alpha = 0.62$, $\Phi = 2.5 \cdot 10^{-3} Mev^{-2}$, $C^{1/(3\alpha+1)} = 175 Mev$) and with the use of lattices (circles and squares are drawn for the case $\theta_{dec} = 210 Mev$, data are from ref. [5]). For the $SU(2)$ system, the calculations within the model with correcting functions well approximate the lattice data provided that $\alpha \approx 0.6 \div 0.65$, $\Phi \approx 2.5 \cdot 10^{-3} Mev^{-2}$, $C^{1/(3\alpha+1)} \approx 175 Mev$, and in the mixture of gluon plasma and glueballs there is either a continuous phase transition accompanied by a high peak of specific heat (Fig. 8) or a second - order phase transition (according to Erenfest). It is interesting that in the $SU(3)$ case our results are in good agreement with the lattice results when $\alpha \approx 0.5 \div 0.65$. This interval includes also the range of α value for the $SU(2)$ mixture, therefore, it may happen that the parameter α is the same in magnitude both for the $SU(2)$ and $SU(3)$ cases. The acceptable values of C and Φ are in the regions

$$C^{1/(3\alpha+1)} \approx 175 \div 275 Mev, \quad \Phi \approx 5 \cdot 10^{-4} \div 2 \cdot 10^{-3} Mev^{-2}$$

and the temperature of a first - order phase transition varies within the range $\theta_{dec} \approx 225 \pm 50 Mev$, which corresponds to the lattice estimates [6,9]. In particular, in Fig.9 we show the functions ϵ/ϵ_{SB} and p/p_{SB} for the $SU(3)$ system (o , the lattice data for the energy from refs.[7,8]; $\triangle$, the lattice results for the pressure [7,8]; $\square$ and $\diamondsuit$, the data for the energy and the pressure from ref. [6], solid lines are predictions of our model at $\alpha = 0.62$, $\Phi = 10^{-3} Mev^{-2}$, $C^{1/(3\alpha+1)} = 225 Mev$). Note that the lattice results for the energy and pressure shown were obtained with corrections using the weak coupling expansion [5,6,38] and for the case $\theta_{dec} = 225 Mev$. Our results agree with the conclusions of the approach [5,6,38] that when $\theta > \theta_{dec}$, glueballs still play an important role in the system, as well, and for a continuous crossover this role being larger than for a first - order phase transition. What more, the influence of gluon plasma is significant when $\theta < \theta_{dec}$, though in a narrow interval of temperatures (Figs. 10 and 11). As is known, an unbound gluon cannot appear in vacuum when $\rho = 0$ owing to its energy being infinite, which also follows from the shape of the plasma - gluon spectrum in our model. However, in a medium consisting of glueballs $\rho \neq 0$, which makes the energy necessary for production of an unbound gluon finite. Therefore, it is no wonder that the gluon plasma influences the behaviour

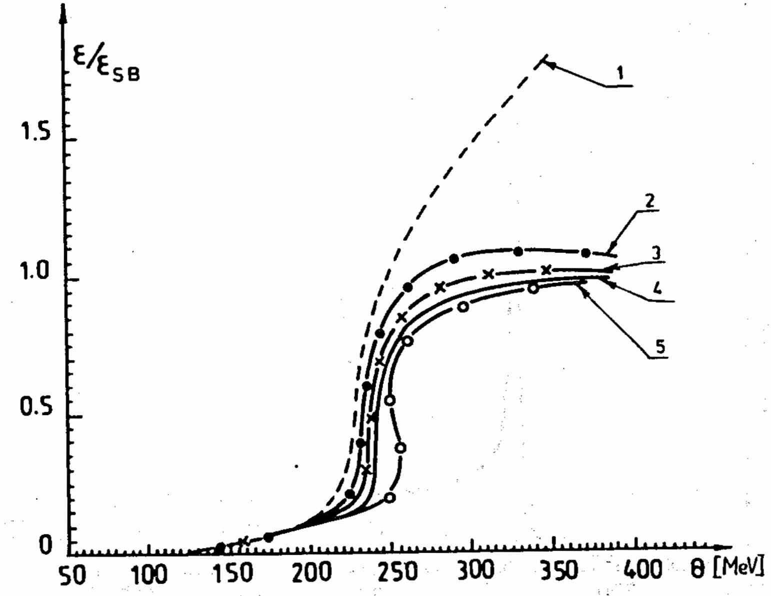

Fig.1 The dependence on temperature of ϵ/ϵ_{SB} for the $SU(3)$ mixture at $B^{1/4} = 235 Mev$ and different values of $r_{20} = (3v_{20}/4\pi)^{1/3}$: 1) $r_{20} = 0$; 2) $r_{20} = 0.5 fm$; 3) $r_{20} = 0.7 fm$; 4) $r_{20} = 0.8 fm$; 5) $r_{20} = 1 fm$.

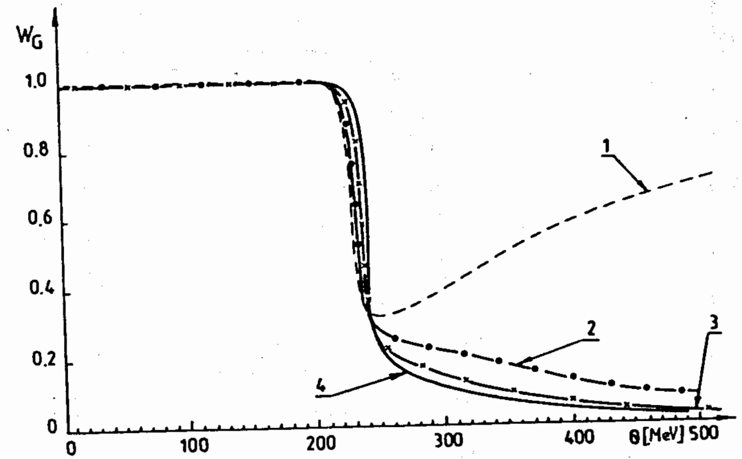

Fig.2 The concetration of the glueball phase w_G versus temperature in the $SU(3)$ mixture at $B^{1/4} = 235 Mev$ and different values of r_{20} : 1) $r_{20} = 0$; 2) $r_{20} = 0.5 fm$; 3) $r_{20} = 0.7 fm$; 4) $r_{20} = 0.8 fm$.

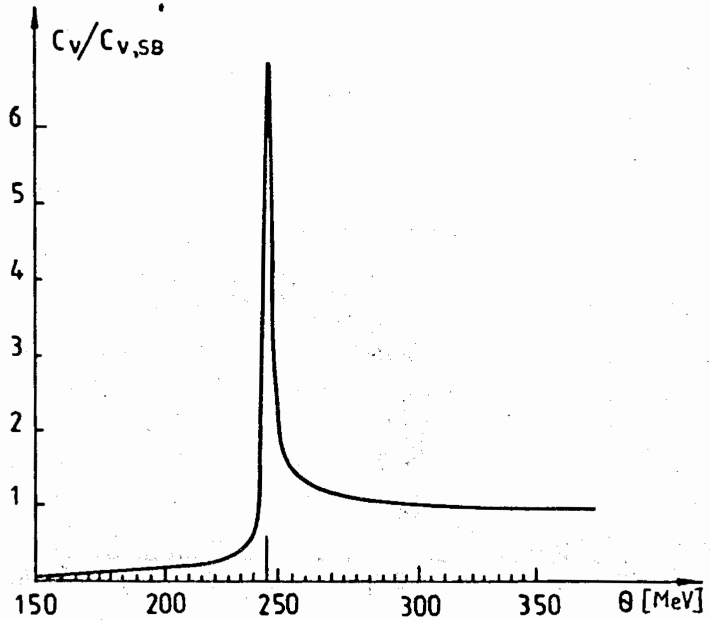

Fig.3 The relative heat capacity $c_v/c_{v,SB}$ versus temperature for the $SU(3)$ mixture ($c_{v,SB}$ is the heat capacity of the ideal gluon gas) at $B^{1/4} = 235Mev$, $r_{20} = 0.8fm \approx r_{20}^{(c)}(235)$.

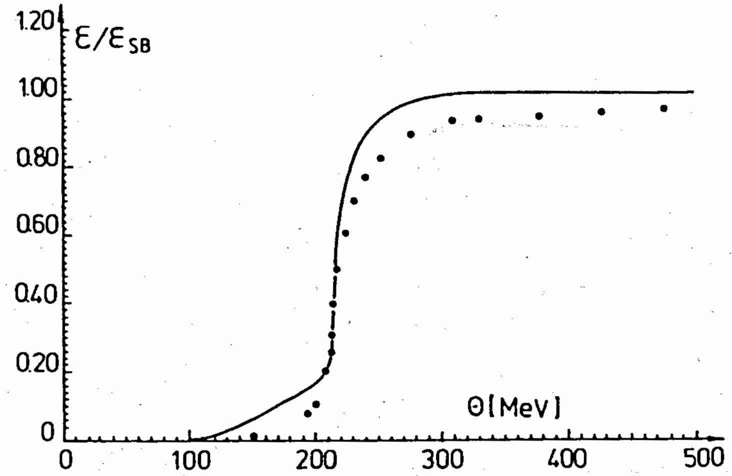

Fig.4 The relative energy ϵ/ϵ_{SB} versus temperature for the $SU(2)$ system: the solid line is for the model with restriction rules, $B^{1/4} = 165Mev$, $r_{20} = 1.2fm$; the points present the lattice data from the paper [5].

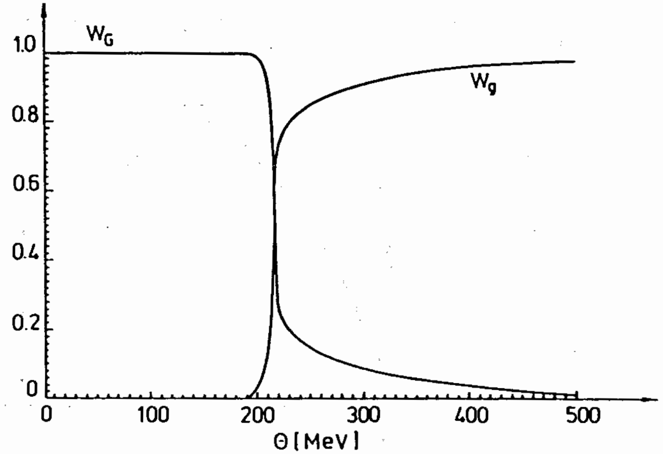

Fig.5 The concentrations w_g and w_G versus temperature for the $SU(2)$ mixture at $B^{1/4} = 165Mev$, $r_{20} = 1.2fm$.

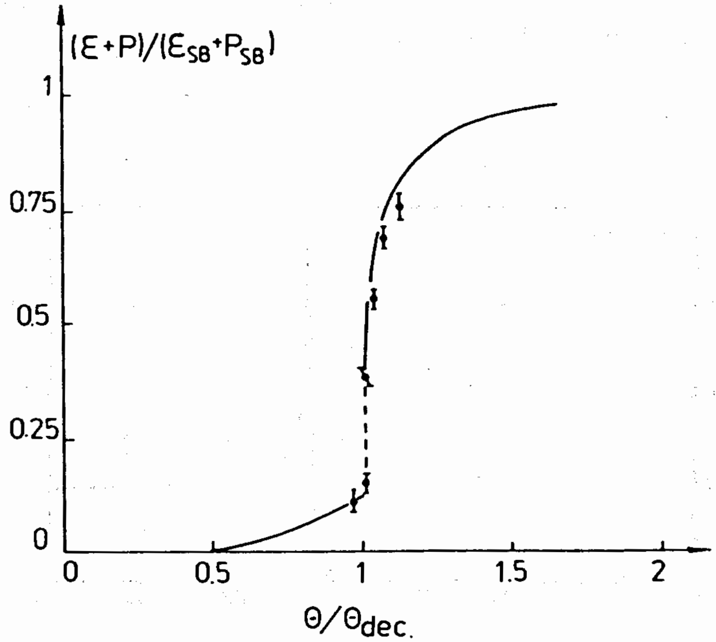

Fig.6 The dependence on temperature of the value $(\epsilon + p)/(\epsilon_{SB} + p_{SB})$ for the $SU(3)$ mixture: the solid line is for the model with restriction rules, $B^{1/4} = 220Mev$, $r_{20} = 0.72fm$; the points are the data for the lattice $24^3 \times 6$ from Ref.[37].

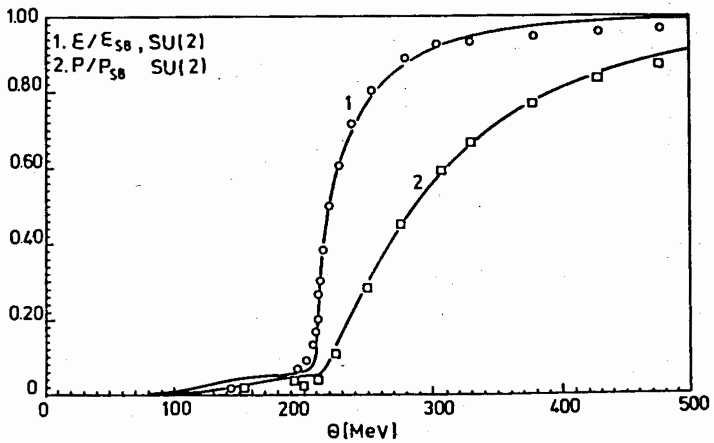

Fig.7 The behaviour of ϵ/ϵ_{SB} and p/p_{SB} versus temperature in the $SU(2)$ system: the solid lines correspond to the model with correcting functions at $\alpha = 0.62$, $\Phi = 2.5 \cdot 10^{-3} Mev^{-2}$, $C^{1/(3\alpha+1)} = 175 Mev$; o and ⊔ are lattice data from Ref.[5].

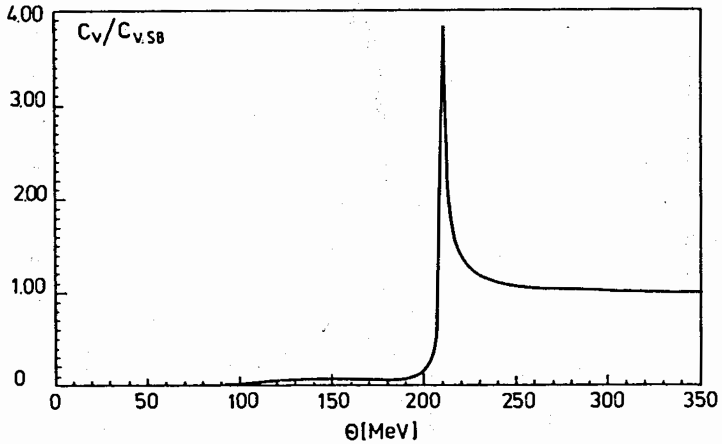

Fig.8 The quantity $c_v/c_{v,SB}$ versus temperature for the $SU(2)$ mixture in the model with correcting functions at $\alpha = 0.62$, $\Phi = 2.5 \cdot 10^{-3} Mev^{-2}$, $C^{1/(3\alpha+1)} = 175 Mev$.

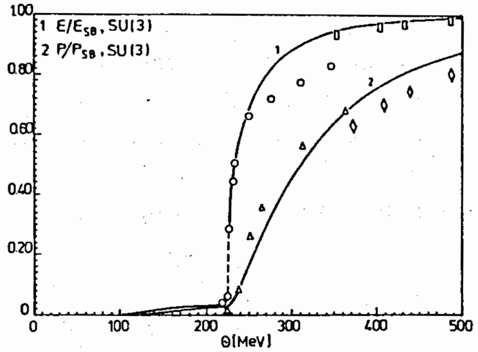

Fig.9 The quantities ϵ/ϵ_{SB} and p/p_{SB} versus temperatures for the $SU(3)$ mixture: the solid lines are for the model with correcting functions at $\alpha = 0.62$, $\Phi = 10^{-3} Mev$, $C^{1/(3\alpha+1)} = 225 Mev$; o and $\triangle$ - are lattice data from Ref.[8]; ⊔ and ◊ are lattice data from Ref.[6].

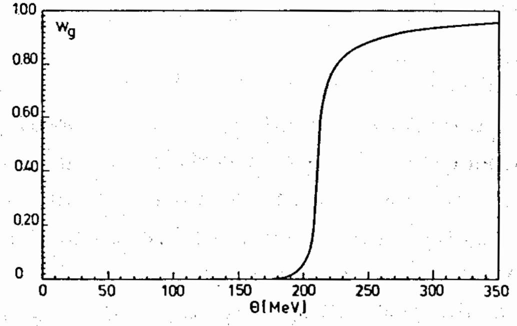

Fig.10 The concentration w_g versus temperature for the $SU(2)$ mixture at $\alpha = 0.62$, $\Phi = 2.5 \cdot 10^{-3} Mev^{-2}$, $C^{1/(3\alpha+1)} = 175 Mev$.

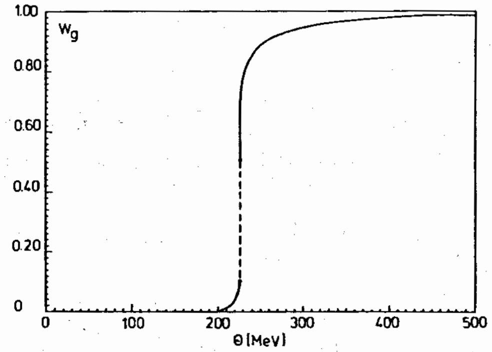

Fig.11 The concentration w_g versus temperature for the $SU(3)$ mixture at $\alpha = 0.62$, $\Phi = 10^{-3} Mev^{-2}$, $C^{1/(3\alpha+1)} = 225 Mev$.

of the system even below the transition temperature. Deconfinement in our approach is not the transition from a state when the system has no color objects to a state with color objects; it is rather the transition from a state dominated by colorless clusters to a state dominated by color objects (gluons).

Besides, we would like to note the following. The integral

$$\frac{\xi_g}{2\pi^2}\int_0^{+\infty} k^2 \ln\left[1-\exp\left(-\frac{k+C/\rho^\alpha}{\theta}\right)\right] dk,$$

can be rewritten by changing variables $k \to \tau = k + C/\rho^\alpha$ into the form

$$\frac{\xi_g}{2\pi^2}\int_k^{+\infty} \left(\tau-\frac{C}{\rho^\alpha}\right)^2 \ln\left[1-\exp\left(-\frac{\tau}{\theta}\right)\right] d\tau, \quad k=\frac{C}{\rho^\alpha}.$$

As ρ is a function of the temperature, the above construction is an integral with a cut-off momentum dependent on θ. Therefore, introduction of the cut - off momentum [5,6,38] to a certain extent simulates interaction of plasma gluons with a medium composed of gluons and glueballs. The palette of our results is more rich as compared to the model studied in [5,38] because particle interaction is more naturally described by determining the gluon and glueball spectra dependent on ρ_g and $\{\rho_G\}$ and coexistence of phases is taken into account in a way consistent with general rules of statistical mechanics [36].

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Шаненко А.А., Юкалова Е.П., Юкалов В.И. E2-92-329

Статистический подход к деконфайнменту в чисто калибровочных моделях

Предлагается новый статистический подход к рассмотрению деконфайнмента в SU(2) и SU(3) глюонных системах. Показывается, что сосуществование глюболов и глюонной плазмы термодинамически выгоднее, чем реализация системы в виде соответствующих чистых фаз. При этом учитывается взаимодействие плазмы и глюболов. Результаты исследования очень хорошо согласуются с решеточными предсказаниями.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1992

Shanenko A.A., Yukalova E.P., Yukalov V.I. E2-92-329

Statistical Approach to Deconfinement in Pure Gauge Models

We suggest a new statistical approach for considering deconfinement in SU(2) and SU(3) gluon systems. A mixture of coexisting glueballs and of the gluon plasma is shown to be thermodynamically more profitable than the corresponding pure phases. The interactions of gluons and glueballs are taken into account. Our results are in a very good agreement with numerical lattice calculations.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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