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RADIATION PERTURBATION THEORY IN GRAVITY AND QUANTUM UNIVERSE AS A HYDROGEN ATOM

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1 Statement of problem

There is an opinion [1] that the Faddeev-Popov (FP) functional integral in the theory of gravity

$$Z^{FP} = \int Dg_{\mu\nu}\delta(f_\nu(g_{\mu\nu}))\Delta_{FP} exp[iW[g_{\mu\nu}]] \quad (1)$$

$$W[g_{\mu\nu}] = \int d^Dx[-\frac{1}{2\kappa^2}\,{}^DR(g_{\mu\nu})\sqrt{-g}];\ D = n+1 \quad (2)$$

cannot describe homogeneous cosmological models.

In this paper we show that there is a choice of a metric $g_{\mu\nu}$ and of the gauge $f_\mu(g_{\mu\nu}) = 0$ for which the FP integral (1) leads to a definite quantum version of the Friedmann-Robertson-Worker model and discuss the physical consequence of this version.

2 The choice of a metric and gauges

Let us choose the ADM metric [2] (which is used for the canonical quantization) with the factorization of the "scale-space variable" [3] $a(x) = exp\mu(x)$; and the conformal-invariant "graviton": h_{ij}, ($\det h = 1$)

$$(ds)^2 = g_{\mu\nu}dx^\mu dx^\nu = \alpha^2(dx^0)^2 - a^2h_{ij}(dx^i - \beta^i dx^0)(dx^j - \beta^j dx^0); \quad (3)$$

$$\sqrt{-g} = \alpha a^n$$

Taking into account the expansion of the n-dimensional space curvature $R(a^2h) = R(a^2) + R(h)/a^2$ we can represent the action (2) for the metric (3) in the first order time derivative formalism

$$W = \int d^Dx[P\partial_0\mu + K^{lk}\partial_0 h_{lk} - \alpha\mathcal{H} + \beta^k\mathcal{P}_k] + W_\Sigma, \quad (4)$$

where

$$\mathcal{H} = a^n[-\frac{1}{2}\kappa_n^2\frac{P^2}{a^{2n}} - \frac{1}{2\kappa^2}R(a^2) + T_0^0(h)]; \quad (5)$$

$$T_0^0(h) = \frac{2\kappa^2K^{ij}K_{ij}}{a^{2n}} - \frac{1}{2\kappa^2a^2}R(h);\ \kappa_n^2 = \kappa^2/n(n-1);$$

$$\mathcal{P}_k = \frac{a^n}{n}\partial_k(\frac{P}{a^n}) + 2\partial_i K^i_j - K^{ij}\partial_k h_{ij}; \quad (6)$$

$$W_\Sigma = \int d^Dx\left\{-\partial_k[\frac{a^{n-2}}{\kappa^2}\partial^k\alpha - \beta^k\frac{P}{n-1}] - \frac{\partial_0 P}{n-1}\right\}. \quad (7)$$

$\mathcal{H}$ and $\mathcal{P}_k$ are the density of the Hamiltonian and momentum which form the primary constraints

$$\mathcal{H} = 0;\ \mathcal{P}_k = 0; \tag{8}$$

W_Σ is the surface integral (which is usually neglected). We see that in the metric (3) the total energy density (5) splits into the negative energy of the scale dynamics (μ) and the positive energy of the gravitons (h_{ij}). This fact gives a possibility to use the analogy with the simplest system, invariant with respect to the time reparametrization, relativistic particle [4], where the Hamiltonian has two roles of the primary constraint (2) and of the generator of evolution with respect to the physical, nonholonomic (invariant) time

$$d\tau = \alpha dx^0 \tag{9}$$

In the gauge $\beta_k = 0$ one can easily be convinced that the Hamiltonian of the theory (2): $H = \int d^n x \alpha \mathcal{H}$ also plays the role of the evolution generator for the quantum scale $\mu(x)$; $P = \frac{1}{i}\delta/\delta\mu(x)$ with respect to the time (9). The Heisenberg equations

$$\frac{1}{\alpha}\partial_0\mu = \partial_\tau\mu = i\frac{1}{\alpha}\left[\hat{H}, \mu(x)\right] = -\frac{\kappa_n^2 P}{a^n}; \tag{10}$$

$$\partial_\tau P = -a^n\left[n\frac{\kappa_n^2 P^2}{2a^{2n}} - \frac{(n-2)}{2\kappa^2}R(a^2h) - n\frac{2\kappa_n^2 K^2}{a^{2n}} + \frac{(n-1)}{\kappa^2 a^n \alpha}\partial_k(a^{n-2}\partial^k\alpha)\right] \tag{11}$$

completely coincide with the Einstein classical equation for $\mu(x)$. We cannot see this "scale" dynamics on the level of the FP integral (1) as the action (4) on the constraints (8), $W = \int d^D x[P[\mu]\partial_0\mu]$, depends on this dynamics only on the time boundaries of the finite interval of the physical time (9)

$$W(0|T) = W(\mu(T,x)) \tag{12}$$

The gauge revealing the boundary scale dynamics (10), (11) is

$$\mu(x) = \mu(T, x^i) \tag{13}$$

For "gravitons" we can choose the space harmonical gauge [3]

$$\partial_i h^{ik} = 0. \tag{14}$$

3 The calculation of the FP functional integral

These two constraints (13), (14) lead to the FP integral (1) for the Green function of the time evolution of the system in the finite time interval $(0, T)$

$$Z^R(0|T) = \int D(h_{ij}, \alpha, \beta_k, \mu) DPDK^{ij} \times \prod_x \delta(\mu - \mu(T, x^i))\delta(\partial_i h^{ik})\Delta_{FP} exp\{iW(0|T)\} \tag{15}$$

$$\Delta_{FP} = \int d\bar{\eta}^\mu d\eta^\nu exp\left\{-i\int_{0,T} d^D x\left[\bar{\eta}^0\left(\frac{\kappa_n^2 P}{a^n}\eta^0 + \frac{\partial_k(a^n\eta^k)}{na^n}\right) + +\partial^l\bar{\eta}^k\left(\frac{\kappa^2 4K_{lk}}{a^n}\eta^0 + 2\eta_{(l,k)}\right)\right]\right\} \tag{16}$$

where Δ_{FP} is the FP determinant, $\eta_{(l,k)} = (1/2)(\eta_{l,k} + \eta_{k,l})$ is the covariant derivative in the metric h_{ij}. The scale $\mu(T,x)$ is defined from the solution of eqs.(8), (10), (11) on the boundary of the time interval where one gives the "in" and "out" physical states. We consider here one of such "in" physical states the Absolute Vacuum (AV) as the state without particles (gravitons) with positive energy which is characterized by the vacuum energy density: $T_0^0(h) =< T_0^0 >$ as the function from the scale.

The primary constraints (8) on the AV state: $\mathcal{H}| >_{AV} = 0; \mathcal{P}_k| >_{AV} = 0$ have the form

$$P_F(a_F) = \pm\frac{a_F^n}{\kappa_n}[2 < T_0^0 > -\frac{1}{\kappa^2}R(a_F^2)]^{1/2};\ \partial_k(\frac{P_F}{a_F^n}) = 0 \tag{17}$$

and describe one of the versions of the homogeneous Universe. As has been noted above, on the boundary of the physical time these constraints (17) should be complected by the scale evolution equation (10)

$$\partial_T\mu_F(T) = -\kappa_n^2\frac{P_F}{a_F^n} \Rightarrow T = -\int_0^{a_F(T)} da\frac{a^{n-1}}{\kappa_n^2 P_F(a)} \tag{18}$$

which coincides with the Friedmann equation.

We see that in the spectrum of physical excitations of the theory (2) there is a collective (global) excitation of the physical space as a whole which plays the role of the zero-mode of the operator of differentiation with respect to the space coordinate (∂_k) (17).

Let us extract all collective variables and their factors from the integral (15), (16) in the separate functional integral:

$$\prod_x DP = dP_G \prod_x DP_L;$$

$$\prod_x \delta(\mu(x) - \mu(T, x')) = \delta(\mu_G - \mu_F(T)) \prod_x \delta(\mu_L - \mu_L(T, x)) \quad (19)$$

$$W = W_G + W_L, W_G = \int d^D x(\partial_0 \mu P) - \partial_0 P/(n-1) \quad (20)$$

and neglect the influence of the local variables (radiation h_{ij}) on the zero-mode (like the radiation perturbation theory in QED description of a hydrogen atom).

Then, in the lowest order of the radiation perturbation theory we get the factorization of the integral (15) $Z^R \simeq Z^{Global} \otimes Z^{Local}$, where the global part has the form

$$Z^G(0|T) = \int d\mu_G dP_G \delta((P_G^2 - P_F^2(a))\frac{\kappa_n^2}{a_F^2}) \times$$
$$\delta(\mu_G - \mu_F(T)) exp\{iW_G\} \Delta_{FP}^G \quad (21)$$

$$\Delta_{FP}^G = \int d\bar{\eta}^0 d\eta^0 exp\Big(-i \int d^D x [\bar{\eta}^0 \frac{\kappa_n^2 P}{a^n} \eta^0]\Big) \quad (22)$$

and is easily calculated

$$Z^G(0|T) = \Psi_G(a(T)) = exp\Big\{iV_n [\int_0^{a_F(T)} d\mu'_F P_F(a'_F) - \frac{P_F(a)}{n-1}]\Big\} \quad (23)$$

where V_n is the dimensionless volume of the n-space, especially for the positive curvature space

$$R(a^2) = \frac{n(n-1)k}{a^2}; \; k = 1, V_n = 2\pi^{\frac{n+1}{2}}/\Gamma(\frac{n+1}{2}) \quad (24)$$

In accordance with two roles of the Hamiltonian: as the constraint (8) and as the evolution generator (10), the expression (23) has two interpretations as the stationary Wheeler-De Witt state $\Psi_G(a)$ and as the Green function of the evolution of the Universe in the AV state, respectively.

The Green function of the Universe (23) with the conformal vacuum energy density

$$< T_0^0 >= \frac{\varepsilon_0^R}{a^{n+1}} \quad (25)$$

for all three types of the space (24), $k = 0, \pm 1$ exactly coincides in form with the wave function of a relativistic particle in the rest frame

$$\Psi_G(a(T)) = exp\{\pm i m_c T_c(a|k)\} \quad (26)$$

where

$$T_c(a|k) = \frac{2}{n-1} f_k(A); \; A = [a^{(n-1)}/2\varepsilon_0^R \kappa_n^2]^{1/2} \quad (27)$$

$$f_0(A) = A; f_{+1}(A) = arcsin A; f_{-1}(A) = ln(A + \sqrt{1+A^2}) \quad (28)$$

is the conformal time:

$$dT = a_F dT_c, (0 < T_c(k = 0, -1) < \infty; \quad 0 < T_c(k = 1) < 2\pi),$$

$m_c = V_n \varepsilon_0^R$ is the conformal mass of the Universe. (For the real case $n = 3; k = 0, m_c \sim 10^{120}, T_c \sim 1$). The sign (-) corresponds to the creation and expansion of the Universe.

4 Problems of quantum cosmology

The cosmological consequences of these results are the new answers to the old problems of quantum cosmology.

1) The homogeneity of the Universe created from "nothing" is the consequence of the definition of "nothing" or of the Absolute Vacuum as the state without a particle with positive energy.

2) The cause of the Universe expansion is the new collective excitation of the physical space (of the type of the superfluid motion of quantum liquid) hidden in the boundary conditions. This excitation is induced by the density energy of the vacuum of all particles which can be created in future in the process of expansion.

3) Just this unobservable vacuum energy plays the role of the "hidden mass" of the plain Universe.

4) The Quantum Nature of the expansion could solve the problem of "horizon".

5 The status of the radiation gauge and relativistic covariance

In conclusion, we discuss the physical status of the radiation gauge in QED and gravity and its relativistic covariance.

As it has been shown in ref. [5], the radiation gauge in QED is physically favored and is based i) on the explicit solving of the Lagrange equations for unphysical components with zero momentum and ii) on the construction of the minimal set of physical variables on the surface, admitted by these equations, as the gauge invariant functionals from the initial fields with the nonlinear relativistic Heisenberg-Pauli [6] transformation. The last corresponds to the change of the time-axis of quantization or the change of a gauge.

We tried here to follow the internal logic of the construction of such a type of variables in QED for quantum gravity. According to this logic, the change of the time-axis on the "surface of admissible dynamics" should lead to the boost of the wave function of the Universe

$$exp\{-im_c T_c\} \Rightarrow exp\{-i\mathcal{P}_{(c)\mu} X_c^{\mu}\};, \quad \mathcal{P}_{(c)\mu}\mathcal{P}^{\mu}_{(c)} = m_c^2$$

likewise the transformation for the wave function of the hydrogen atom.

In both cases for asymptotical nonfree particles (in the atom) and for asymptotical nonplain space-time (in the Universe) the theorem of equivalence of different gauges does not work as this theorem is based on the change of asymptotical states. It is impossible to reproduce in any other gauge the result of construction of the wave function of the Universe in the lowest order in radiation, as it is impossible to reproduce the construction of the instantaneous wave function of the hydrogen atom and its relativistic covariant properties in the relativistic gauge which contains only the light-cone singularity propagators [5].

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Радиационная теория возмущений в гравитации и квантовая Вселенная как атом водорода

В квантовой теории гравитации (n+1)-мерного пространства-времени строится функциональный интеграл Фаддеева-Попова для радиационной теории возмущений. Показано, что этот интеграл в низшем порядке по радиации частиц с положительной энергией описывает квантовую версию однородной Вселенной Фридмана (подобно тому как низший порядок по радиации в КЭД описывает атом водорода). В этой версии расширение Вселенной выглядит как коллективное сверхтекучее движение квантового пространства, а плотность энергии вакуума играет роль "скрытой массы".

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Pervushin V.N. E2-92-322

Radiation Perturbation Theory in Gravity and Quantum Universe as a Hydrogen Atom

In quantum theory of gravity of the (n+1)-dimensional space-time the Faddeev-Popov functional integral is constructed for radiation perturbation theory. We show that this integral in the lowest order in the radiation of particles with positive energy describes the quantum version of the Friedmann homogeneous Universe (like the lowest radiation order in QED describes a hydrogen atom). In this version the Universe expansion looks as the collective superfluid motion of quantum space, and the vacuum energy density plays the role of the "hidden mass".

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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