объединенный
институт
ядерных
исследований
дубна

E2-92-304

A.A.Bel'kov, D.Ebert[1], A.V.Emelyanenko[2]

GLUON CONDENSATION AND MODELLING OF QUARK CONFINEMENT IN QCD-MOTIVATED NAMBU - JONA - LASINIO MODEL

Submitted to "Nuclear Physics A"

[1]Institut für Elementarteilchenphysik, Humboldt-Universität, Invalidenstraße 110, O-1040 Berlin, Germany
[2]Institute for High Energy Physics, Moscow Region, 142284 Protvino, Russia

1992

The renewal of interest in extended Nambu-Jona-Lasinio (NJL) models happens basically due to the fact that they share a lot of conceptually important features with low-energy QCD [1-8]. Thus, they incorporate all relevant symmetries of the quark flavor dynamics of QCD including explicit symmetry breaking terms due to quark masses. The most important moment is that they offer a simple scheme to study spontaneous breakdown of chiral symmetry and its manifestation in hadron physics. This concerns, in particular, the transition of current quarks into constituent ones due to the appearance of a nonvanishing quark condensate, the emergence of light composite pseudoscalar Nambu-Goldstone bosons as well as of heavier composite vector and axial-vector mesons.

In view of these successes, one should, however, not forget that NJL models are incomplete since they do not contain a reliable confinement mechanism. In the present paper we consider a new type of a QCD-motivated NJL-model including gluon condensation as well as a nonperturbative gluon mass which provides a possible machanism of quark confinement. In the first step the dynamical running quark masses are determined from the Schwinger-Dyson (SD) equation taking into account a specific form of a nonperturbative gluon propagator. Using this solution as input, we describe the masses and decay constants of low-lying σ, π, ρ and a_1 mesons from the Bethe-Salpeter (BS) equation. Finally, on this basis the fundamental parameters of the gluon propagator are extracted to obtain predictions for the dynamical gluon mass (equivalently, the gluon condensate) and the low-energy value of $g^2/(4\pi)$.

The starting point of our considerations is the following (Euclidean) generating functional describing an effective form of truncated QCD

$$Z = \int Dq\, D\bar{q}\; e^{-S(q,\bar{q})} ,$$

$$S(q,\bar{q}) = \int dx\; \bar{q}(x)(\hat{\partial}+m_0)q(x) + S_{int},$$

where

$$S_{int} = \frac{1}{2}g^2 \iint dx\, dy\; G^{ab}_{\mu\nu}(x-y)\; \bar{q}(x)\gamma^{\mu}\frac{\lambda^a_c}{2}q(x)\; \bar{q}(y)\gamma^{\nu}\frac{\lambda^b_c}{2}q(y) \tag{1}$$

is the part of the effective action corresponding to quark interaction via gluon exchange with the nonperturbative gluon propagator $g^2G^{ab}_{\mu\nu}(x-y)=\delta^{ab}\delta_{\mu\nu}G(x-y)$ taken in the Feynman gauge.

The corresponding gluon propagator is determined by the path-integral average of the gluon field $G^a_\mu(x)$,

$$g^2G^{ab}_{\mu\nu}(x-y)=\langle g^2G^a_\mu(x)G^b_\nu(y)\rangle \; . \tag{2}$$

Next, the gluon field $G^a_\mu(x)$ is assumed to be decomposed into a constant condensate field $\mathbb{G}^a_\mu$ and the quantum fluctuations $g^a_\mu(x)$ arround it[1],

$$G^a_\mu(x) \;=\; \mathbb{G}^a_\mu \;+\; g^a_\mu(x) \; . \tag{3}$$

Taking into account eq. (3) and $\langle g^a_\mu(x)\rangle=0$, one obtains the decomposition of the nonperturbative gluon propagator into two parts:

$$g^2G^{ab}_{\mu\nu}(x-y)=\langle g^2\mathbb{G}^a_\mu\mathbb{G}^b_\nu\rangle \;+\; \langle g^2g^a_\mu(x)g^b_\nu(y)\rangle \; . \tag{4}$$

Note that, taking into consideration the transformation of the integration measure $DG^a_\mu=D\mathbb{G}^a_\mu\cdot Dg^a_\mu$ and the normalization factor $[\int DG^a_\mu e^{-S}]^{-1}$ appearing in (2), the average of the condensate field in (4) is just given by integrating the constant field $\mathbb{G}^a_\mu$ over all directions in color and Lorentz space.

From symmetry considerations the first term arising from the nonvanishing gluon condensate can be written in the form

$$\langle g^2\mathbb{G}^a_\mu\mathbb{G}^b_\nu\rangle \;=\; \frac{\delta^{ab}\delta_{\mu\nu}}{32}\langle g^2\mathbb{G}^2\rangle \; .$$

Moreover, the second term in (4) will be approximated in the low-energy region by a term $\delta^{ab}\delta_{\mu\nu}\delta^{(4)}(x-y)\cdot g^2/m^2_G$ in the spirit of the usual NJL-model. Such an approximation is suggested by the fact that for the shifted gluon field $g^a_\mu(x)$ there might arise a dynamical gluon mass connected with the gluon condensate by the

[1] Due to the fact that local gauge transformations mix $\mathbb{G}^a_\mu$ and $g^a_\mu(x)$ the separation (3) becomes possible only in a fixed gauge.

relation[2]

$$m^2_G \;=\; \frac{15}{32}\langle g^2\mathbb{G}^2\rangle \; . \tag{5}$$

The above motivations lead us to consider in Euclidean momentum space the following simple low-energy model for an effective nonperturbative gluon propagator corresponding to the decomposition (4)

$$G(q) \;=\; [\mu\delta^{(4)}(q) \;+\; G\theta(\Lambda^2-q^2)] \; . \tag{6}$$

Here the δ-function contribution arises from the constant gluon condensate. The second term emerges from the low-energy massive gluon propagator where the θ-function corresponds to the usual regularization of the NJL-model with a momentum cutoff Λ. The constants μ and G are related to the unknown nonperturbative dynamics and can be expressed through the QCD coupling constant g and the dynamical gluon mass m_G as follows:

$$\mu \;=\; \frac{16\pi^4}{15}\, m^2_G \; , \qquad G \;=\; \frac{g^2}{m^2_G} \; . \tag{7}$$

In this paper the constants μ and G will be treated as free parameters to be fixed from the spectroscopy of mesons.

After Fierz transformation the action (1) becomes

$$S_{int} \;=\; -\,\frac{1}{2}\iint dx\; dy\; G(x-y)\; \bar{q}(x)\frac{M^\theta}{2}q(y)\; \bar{q}(y)\frac{M^\theta}{2}q(x), \tag{8}$$

where M^θ are tensor products of Dirac, flavor and color matrices of the type

$$\{1, i\gamma^5, \frac{i}{\sqrt{2}}\gamma^\mu, \frac{i}{\sqrt{2}}\gamma^5\gamma^\mu\}^D \; \{\tfrac{1}{2}\tau^a\}^F \; \{\tfrac{4}{3}1\}^C \; .$$

Here we consider the $SU(2)_F$ flavor group ($a=0,1,2,3$; $\tau^0\equiv1$) with τ^a being Pauli matrices, and we have restricted ourselves to color-singlet $q\bar{q}$ contributions.

After introducting in a standard way [10-12] scalar (S), pseudoscalar (P), vector (V) and axial-vector (A) bilocal

[2] For a discussion of this issue we refer to ref.[9]. We slightly differ from this work by the factor 15/32 in front of eq. (5) instead of their value 1/4.

collective meson fields, the effective action (8) becomes bilinear in quark fields and the integration over quark degrees of freedom may be performed to obtain a bilocal effective action for collective fields. Defining the vacuum configuration of fields to minimize the bilocal action, we obtain SD-type equations which have the following form for translation-invariant solutions:

$$S^0_{a=0}(x-y) = \frac{8}{9}G(x-y)\mathrm{tr}\left(D(x-y)\frac{\tau^{a=0}}{2}\right) ,$$

$$V^0_{\mu a=0}(x-y) = \frac{4}{9}G(x-y)\mathrm{tr}\left(D(x-y)i\gamma_\mu\frac{\tau^{a=0}}{2}\right) , \quad (9)$$

and $P^0_a=A^0_{\mu a}=0$ by parity-conservation of the vacuum. Here $D(x-y)$ is the propagator for u- and d-quarks moving in the background of composite meson fields parametrized in momentum space by the relation

$$\left(D^0(q)\right)^{-1} = i\hat{q} + \frac{1}{2}\hat{V}^0_0(q) + m_0 + \frac{1}{2}S^0_0(q) \equiv id(q)\hat{q} + m(q) . \quad (10)$$

The "running" quark mass $m(q)$ and the function $d(q)$ are defined as solutions of the equations

$$m(p) = m_0 + \frac{16}{3}\int\frac{d^4q}{(2\pi)^4}\, G(p-q)\frac{m(q)}{q^2d^2(q)+m^2(q)} ,$$

$$[d(p)-1]p_\mu = \frac{8}{3}\int\frac{d^4q}{(2\pi)^4}\, G(p-q)\frac{d(q)q_\mu}{q^2d^2(q)+m^2(q)} . \quad (11)$$

following by inserting the ansatz (10) into (9). The "running" constituent quark mass can be defined as $m_{con}(q)\equiv m(q)/d(q)$.

Then let us expand the bilocal effective action over the fluctuations $p_a(x,y)=P_a(x,y)$, $a^\mu_a(x,y)=A^\mu_a(x,y)$ and $s_a(x,y)=S_a(x,y)-\delta_{a0}S^0_0(x-y)$, $v^\mu_a(x,y)=V^\mu_a(x,y)-\delta_{a0}V^{0\mu}_0(x-y)$ of the collective fields around their vacuum expectation values. Varying the quadratic part of the bilocal action over the fluctuations s_a, p_a, v^μ_a, a^μ_a (a=0,1,2,3) and neglecting (p-a)-mixing, for simplicity, one gets the following BS-type equations for the corresponding bound state vertex functions (fig.1)

$$\begin{pmatrix} s_a(q_2,q_1) \\ p_a(q_2,q_1)\end{pmatrix} = \frac{16}{3}\int\frac{d^4q}{(2\pi)^4}\, G(q)\mathcal{B}^{(\pm)}(q_1-q,q_2-q)\begin{pmatrix} s_a(q_2-q,q_1-q) \\ p_a(q_2-q,q_1-q)\end{pmatrix}, \quad (12)$$

$$\begin{pmatrix} v^\mu_a(q_2,q_1) \\ a^\mu_a(q_2,q_1)\end{pmatrix} = \frac{8}{3}\int\frac{d^4q}{(2\pi)^4}\, G(q)\mathcal{D}^{(\pm)\mu\nu}(q_1-q,q_2-q)\begin{pmatrix} v_{\nu a}(q_2-q,q_1-q) \\ a_{\nu a}(q_2-q,q_1-q)\end{pmatrix} \quad (13)$$

where

$$\mathcal{B}^{(\pm)}(q_1,q_2) = [-(q_1\cdot q_2)b(q_1)b(q_2)\pm c(q_1)c(q_2)],$$

$$\mathcal{D}^{(\pm)\mu\nu}(q_1,q_2) = (q_1^\mu q_2^\nu+q_1^\nu q_2^\mu)b(q_1)b(q_2) + \delta^{\mu\nu}[(q_1\cdot q_2)b(q_1)b(q_2)\pm c(q_1)c(q_2)]$$

(the indices "±" refer to the upper or lower part of the vertex components) and

$$c(q) = \frac{-m(q)}{q^2d^2(q)+m^2(q)} , \qquad b(q) = \frac{-d(q)}{q^2d^2(q)+m^2(q)} .$$

Let us introduce the relative and absolute momenta $q=\frac{1}{2}(q_1+q_2)$ and $Q=(q_1-q_2)$, of the $q\bar{q}$-system, respectively, and expand the function $\mathcal{B}^{(\pm)}(q_1-q,q_2-q)\equiv\mathcal{B}^{(\pm)}(Q,q)$ in eq.(12) over Q near $Q^2=0$ taking into account only the terms up to the second order. Using a factorizable ansatz

$$\begin{pmatrix} s_a(q_2,q_1) \\ p_a(q_2,q_1)\end{pmatrix} = \begin{pmatrix} s_a(q,Q) \\ p_a(q,Q)\end{pmatrix} \equiv \begin{pmatrix} s_a(q)S(Q) \\ p_a(q)P(Q)\end{pmatrix}.$$

one gets for $Q^2=-M^2$ the following mass functional [13] for composite scalar and pseudoscalar mesons

$$M^2_\pm = \frac{1}{f^2_\pm}\left[-24\int\frac{d^4q}{(2\pi)^4}I_0^{(\pm)}(q)\begin{pmatrix} p^2_a(q) \\ s^2_a(q)\end{pmatrix} + \frac{9}{2}\int d^4y\left[G(y)\right]^{-1}\begin{pmatrix} p^2_a(y) \\ s^2_a(y)\end{pmatrix}\right] ,$$

$$f^2_\pm = 6\int\frac{d^4q}{(2\pi)^4}\left[I_1^{(\pm)}(q) - q^2I_2^{(\pm)}(q)\right]\begin{pmatrix} p^2_a(q) \\ s^2_a(q)\end{pmatrix}. \quad (14)$$

Here

$$I_0^{(\pm)} = (q^2d^2 \pm m^2)/\tilde{Q}^2 , \qquad I_1^{(\pm)}=(-d^2 \pm \mathcal{M} - 2\tilde{Q}DI_0^{(\pm)})/\tilde{Q}^2 ,$$

$$I_2^{(\pm)} = [\pm\tilde{\mathcal{M}} - (2\tilde{Q}F - D^2)I_0^{(\pm)}]/\tilde{Q}^2 ,$$

with $D=d^2+2mm'$, $\tilde{Q}=q^2d^2+m^2$, $F=mm''+m'^2$, $\mathcal{M}=2mm'$, $\tilde{\mathcal{M}}=2mm''-m'^2$ and

$m' \equiv \frac{d}{dq^2}m$ etc. It is not difficult to see that this mass functional is minimized by the solutions of the BS-equations.

In the same way the following ansatz

$$\begin{pmatrix} v_{\mu a}(q_2,q_1) \\ a_{\mu a}(q_2,q_1)\end{pmatrix} = \begin{pmatrix} v_{\mu a}(q,Q)\\ a_{\mu a}(q,Q)\end{pmatrix} \equiv (Q_\mu Q_\nu - Q^2\delta_{\mu\nu})\begin{pmatrix} v^\nu_a(q)V(Q)\\ a^\nu_a(q)A(Q)\end{pmatrix},$$

which is transversal in momentum Q_μ leads from eq.(13) to mass functionals for vector and axial-vector composite mesons:

$$M^2_\pm = \frac{1}{f^2_\pm}\left[-24\int\frac{d^4q}{(2\pi)^4}J_0^{(\pm)}(q)\begin{pmatrix}v^2_{\mu a}(q)\\ a^2_{\mu a}(q)\end{pmatrix} + 9\int d^4y\left[G(y)\right]^{-1}\begin{pmatrix}v^2_{\mu a}(y)\\ a^2_{\mu a}(y)\end{pmatrix}\right],$$

$$f^2_\pm = 6\int\frac{d^4q}{(2\pi)^4}\left[J_1^{(\pm)}(q) - q^2J_2^{(\pm)}(q)\right]\begin{pmatrix}v^2_{\mu a}(q)\\ a^2_{\mu a}(q)\end{pmatrix}. \qquad (15)$$

Here

$$J_0^{(\pm)} = \left(\frac{q^2}{2}d^2 \pm m^2\right)/\tilde{Q}^2, \quad J_1^{(\pm)} = \left(-d^2 \pm \mathcal{M} - 2\tilde{Q}DJ_0^{(\pm)}\right)/\tilde{Q}^2,$$

$$J_2^{(\pm)} = \left[\pm\tilde{\mathcal{M}} - (2\tilde{Q}F - D^2)\tilde{J}_0^{(\pm)}\right]/\tilde{Q}^2$$

with $\tilde{J}_0^{(\pm)} = \left(\frac{2}{3}q^2d^2 \pm m^2\right)/\tilde{Q}^2$.

The substitution of the model gluon propagator (6) into eqs.(11) leads to the system of equations

$$m(s) = \tilde{m} + \frac{\nu^2 m(s)}{sd^2(s)+m^2(s)}, \quad 2(d(s)-1) = \frac{\nu^2 d(s)}{sd^2(s)+m^2(s)}, \qquad (16)$$

where for convenience we introduce the notations

$$3\pi^4\nu^2 \equiv \mu, \quad 3\pi^4\tau^2 \equiv G; \qquad (17)$$

$$\tilde{m} = m_0 + \tau^2(2\pi)^4\mathcal{M}(\Lambda), \quad \mathcal{M}(\Lambda) = \int_0^{\Lambda^2}\frac{s\cdot ds}{(4\pi)^2}\,\frac{m(s)}{sd^2(s)+m^2(s)} \qquad (18)$$

and $s=q^2$.

Using the trial ansatz for vertex functions[3)]

$$\{s(q),p(q),v(q),a(q)\} \approx m(q) - m_0 \qquad (19)$$

one gets from eqs (14) and (15) the following formulae for meson masses:

$$m^2_\pi = \frac{m_0}{f^2_\pi}\int_0^{\Lambda^2}\frac{s\cdot ds}{(4\pi)^2}\,\frac{m(s)-m_0}{sd^2(s)+m^2(s)},$$

$$m^2_\sigma = \frac{m_0}{f^2_\sigma}\int_0^{\Lambda^2}\frac{s\cdot ds}{(4\pi)^2}\,\frac{[m(s)-m_0][2m^3(s)+m_0(sd^2(s)-m^2(s))]}{[sd^2(s)+m^2(s)]^2},$$

$$m^2_\rho = \frac{m_0}{f^2_\rho}\int_0^{\Lambda^2}\frac{s\cdot ds}{(4\pi)^2}\,\frac{[m(s)-m_0]\left[m(s)\left(\frac{3}{2}sd^2(s)+m(s)\right)+m_0\left(\frac{1}{2}sd^2(s)+m^2(s)\right)\right]}{[sd^2(s)+m^2(s)]^2},$$

$$m^2_{a_1} = \frac{m_0}{f^2_{a_1}}\int_0^{\Lambda^2}\frac{s\cdot ds}{(4\pi)^2}\,\frac{[m(s)-m_0]\left[m(s)\left(\frac{3}{2}sd^2(s)+3m(s)\right)+m_0\left(\frac{1}{2}sd^2(s)-m^2(s)\right)\right]}{[sd^2(s)+m^2(s)]^2}. \qquad (20)$$

Here

$$f^2_{\pi/\sigma} = 6\int_0^{\Lambda^2}\frac{s\cdot ds}{(4\pi)^2}\left[I_1^{(\pm)}(s)-sI_2^{(\pm)}(s)\right][m(s)-m_0]^2,$$

$$f^2_{\rho/a_1} = 6\int_0^{\Lambda^2}\frac{s\cdot ds}{(4\pi)^2}\left[J_1^{(\pm)}(s)-sJ_2^{(\pm)}(s)\right][m(s)-m_0]^2 \qquad (21)$$

are meson decay constants.

The ansatz (19) allows us to reproduce the relations between masses and decay coupling constants of mesons obtained in ref.[1] within the usual NJL-model:

[3)] Such an ansatz follows for pseudoscalar mesons from Ward identities. As we shall see it makes also sense for the other mesons (comp. discussions around eq. (22)).

$$m_\sigma^2 \approx m_\pi^2 + 4M^2(\Lambda), \quad f_\sigma^2 \approx f_\pi^2; \quad m_{a_1}^2 \approx m_\rho^2 + 6M^2(\Lambda), \quad f_\rho^2 \approx \frac{2}{3} f_\pi^2, \tag{22}$$

where $M^2(\Lambda) = \frac{12}{f_{\pi_0}^2} \int^{\Lambda^2} \frac{s \cdot ds}{(4\pi)^2} \left[\frac{m^2(s)}{sd^2(s)+m^2(s)} \right]^2$. In the limit of chiral symmetry ($m_0=0$) the relation

$$m_{a_1}^2 f_{a_1}^2 - m_\rho^2 f_\rho^2 = m_\sigma^2 f_\sigma^2 .$$

follows from eqs.(20) and (21).

The SD equations (16) can be solved using an iteration method with free parameters m_0, ν, τ and Λ being fixed from the experimental values of masses m_π=135 MeV, m_ρ=770 MeV, m_{a_1} = (1260 ± 30) MeV and the decay constant f_π=93 MeV.[4)] These results are shown in table 1 for various fixed values of the cutoff Λ. In the limiting case $\nu=0$, corresponding to the usual NJL-model without gluon condensate, a satisfactory fit of experimental masses and the decay constant f_π can be achieved for $\Lambda\approx$800 MeV. In the general case, for $\nu\neq0$, the fit allows the bigger value of the mass m_{a_1} and the larger predicted value for m_σ. The best fit of the experimental parameters can be achieved for the value $\Lambda\approx$700 MeV. Note that a smaller value of the cutoff $\Lambda\approx$700 MeV was also obtained in the modified NJL-model [14] where the gluon condensate contribution was, however, treated differently as a perturbation. The numerical values for the quark condensate $\langle\bar{q}q\rangle = -12\mathcal{M}(\Lambda)$ and the "constituent" quark mass $m_{con}(s=0)$ are also presented in table 1.

---

4) The present SU(2) model essentially serves us to demonstrate the interplay of gluon condensation and quark confinement providing us with a consistent pattern of meson properties. Clearly, the predictive power of this type of models is essentially increased for SU(3) flavor symmetry where on the basis of the fitted model parameters, the masses and decay constants of four low-lying meson octets are determined.

We should note that in the discussed model the mass of the current quark proved to be larger than its standard value $m_0\approx$5MeV. At the same time the additional δ-function term in the gluon propagator (6) provides roughly the standard magnitude of the constituent quark mass (see table 1) and increases the axial-vector mass m_{a_1} up to its experimental value. The scalar meson mass m_σ also becomes increased for 100 MeV in average.

It is worth mentioning that the simplified expression for the gluon propagator given by $G(q)\sim\delta^{(4)}(q)$ alone proved to be not adequate to describe the mass of ρ-mesons (compare also ref. [15]). The results of ref.[12] have also demonstrated that simultaneous description of the masses m_ρ and m_{a_1} based on the use of a simple δ-function-type gluon propagator together with an ansatz of type (19) for meson wave functions, is problematic. Evidently, the main reason for this is the absence of a contribution to the quark condensate proportional to $\mathcal{M}(\Lambda)$ in eq.(18) which is related to the additional NJL-like contribution to the gluon propagator (6). This fact just allows one to reproduce the well-known NJL-model relations for the masses m_π, m_σ, m_ρ, m_{a_1} and to get a correct simultaneous fit of their experimental values using now "running" quark masses m(s).

Usually, after Fierz transformation of the gluon exchange only one coupling constant G_1 does appear in the effective four-quark Lagrangian of QCD-motivated NJL-models. Then one has to introduce by hand another coupling constant G_2 in order to describe simultaneously the masses of π- (G_1) and ρ-mesons (G_2) [1]. In the last paper a gauge-invariant regularization was used for quark loop calculations so that quark loops do not contribute into the ρ-meson mass. The mass term then emerges only from the quadratic field terms arising in the bosonization procedure from the Gaussian path-integral. Let us emphasize that the present approach does contain the same number of coupling constants (μ, G instead of G_1, G_2) as ref.[1].

On the other hand, the choice of a gauge-invariant regularization for calculating quark loop diagrams emitting ρ-mesons is not obligatory in general because the ρ-meson is massive. For instance, in nonlocal quark models [12,13] the vertex functions of mesons are claimed to regularize themselves all loop integrals which leads to a finite contribution of quark loops to the ρ meson mass. To simulate this situation in the present extended QCD-motivated NJL-model we have calculated the corresponding quark loops without requiring gauge invariance of the regularization procedure. The used momentum cutoff can be understood to model, in some sense, the effect of form-factor functions.

Fig.2 shows the solutions $m(q^2)$ and $d(q^2)$ of the SD equations (16) in the extended QCD-motivated model with the gluon propagator (6), analytically continued into Minkowski space. The values Λ=700 MeV, m_0=14 MeV, ν=360 MeV and τ^{-1}=1965 MeV were used. Due to eqs. (16) the denominator of the quark propagator satisfies the relation

$$-q^2d^2(q^2) + m^2(q^2) = \frac{\nu^2 m(q^2)}{m(q^2)-\tilde{m}} > 0$$

which in Minkowski space obviously guarantees the absence of a pole of the quark propagator for the solutions $m(q^2)$ and $d(q^2)$. Such a property is generally believed to be a possible realization of quark confinement.

It is a well known fact that in the standard NJL-model with constant quark masses the quark loop diagram for $\rho\longrightarrow\rho$ transitions contains an imaginary part $\mathrm{Im}\Pi_{\rho\rho}\sim|\Gamma_{\rho\to\bar{q}q}|^2$ which contributes into the ρ-meson propagator. Thus, the decay $\rho\longrightarrow\bar{q}q$ into free quarks would be allowed, in contrast to the idea of quark confinement. Clearly, in our model the quark loop for $\rho\longrightarrow\rho$ transition does not contain such an imaginary part because the quark propagator cannot be put on the mass shell. In consequence, $\rho\longrightarrow\bar{q}q$ decay becomes forbidden. An additional argument is that now ρ-meson decay also becomes impossible due to purely kinematical reasons because $2m_{con}(q^2)$>1GeV for $q^2\sim m_\rho^2$ in Minkowski space. This simply follows from the behaviour of the running quark mass $m(q^2)$ and the function $d(q^2)$ shown in fig.2.

The parameters ν=360 MeV and τ^{-1}=1965 MeV, fixed together with Λ and m_0 from the experimental values of meson masses and the decay constant f_π, lead to the following estimates for the dynamical gluon mass and the QCD coupling constant g (see eqs.(7) and (17)):

$$m_G^2 = \frac{45}{16}\nu^2 = (603\ \mathrm{MeV})^2, \qquad \frac{g^2}{4\pi} = \frac{135\pi^3}{64}\nu^2\tau^2 = 2.19\ .$$

This can be compared with the estimate

$$m_G^2 = [(806 \pm 275)\mathrm{MeV}]^2$$

obtained from the value of the gluon condensate

$$<\frac{g^2}{4\pi^2}(\mathbb{G}^a_{\mu\nu})^2> = [(410 \pm 80)\mathrm{MeV}]^4$$

taken from $e^+e^-\longrightarrow$hadrons [16], using eq.(5) and

$$<g^2\mathbb{G}^2> = \left[\frac{32\pi^2}{2}<\frac{g^2}{4\pi^2}(\mathbb{G}^a_{\mu\nu})^2>\right]^{1/2} = -(1.72 \pm 0.68)\mathrm{GeV}^2$$

where $\mathbb{G}^a_{\mu\nu}$ is the field strength tensor of the gluon condensate.

Thus, we have demonstrated that the above new type of QCD-motivated NJL-model, based on the idea of gluon condensation and dynamical gluon mass, reproduces earlier realistic estimates of masses and decay constants of low-lying mesons. On the other hand, it is important to note, that all this is now achieved with a running quark mass which guarantees the absence of a pole in the quark propagator, generally interpreted as a signal of quark confinement. As a result, a well-known deficiency of the standard NJL-model, the possible decay $\rho\longrightarrow\bar{q}q$ into free quarks, is now cured. Finally, let us mention that there are another approaches to quark confinement based on infrared $1/q^4$ behaviour of the gluon propagator [17]. As it has been widely discussed in the literature, the confinement properties of the quark propagator in this case strongly depend on the infrared regularization chosen.

We thank M.K.Volkov (JINR Dubna) and E.Wieczorek (DESY-IfH) for fruitful discussions.

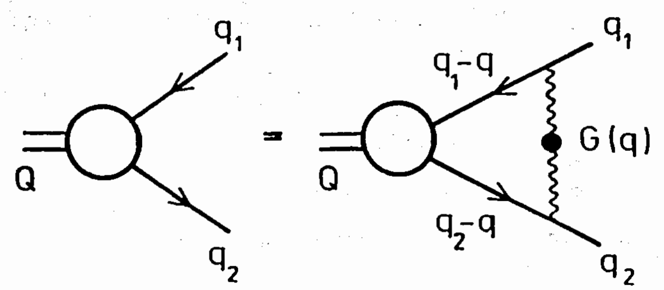

Fig.1. Bethe-Salpeter equation for quark-antiquark bound states.

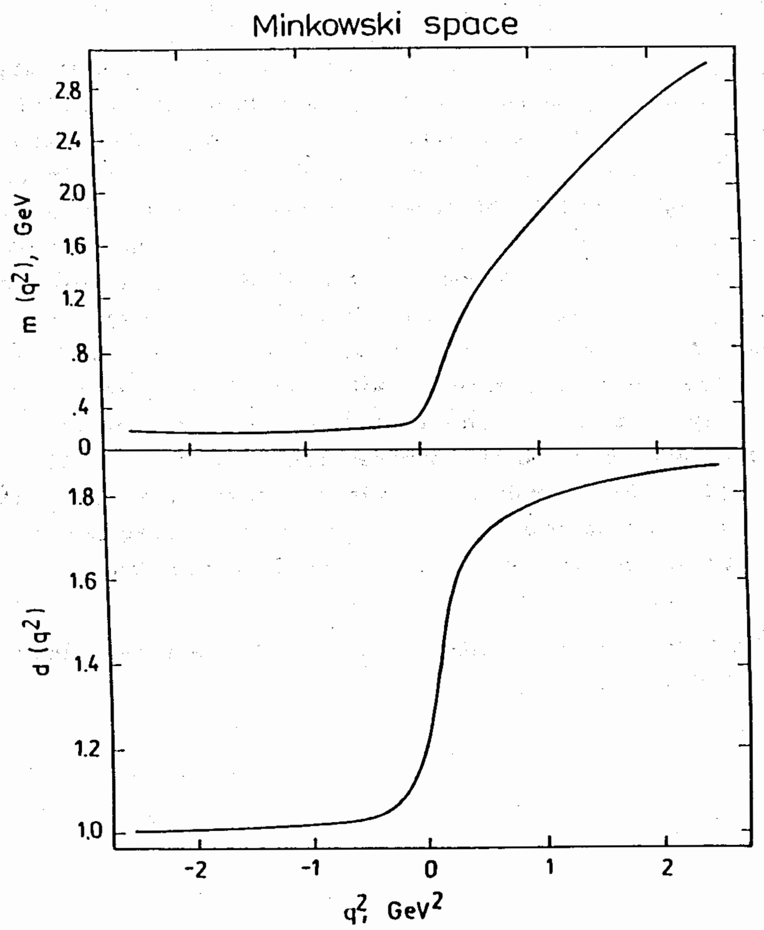

Fig.2. The behaviour of the solutions $m(q^2)$ and $d(q^2)$ of the SD equations (14) in Minkowski space.

Table 1. Model parameters Λ, m_0, τ, ν, constituent quark mass, quark condensate and masses and decay constants of mesons (all quantities are given in MeV)

	NJL-model limit				Gluon propagator (6)			
Λ	700.	800.	900.	1000.	700.	800.	900.	1000.
m_0	13.8	11.3	9.2	7.7	14.0	11.5	9.9	8.8
τ^{-1}	2004.	2380.	2739.	3086.	1965.	2358.	2725.	3077.
ν	0.	0.	0.	0.	360.	350.	630.	595.
$m_{con}(s=0)$	210.	184.	162.	152.	338.	311.	447.	419.
$\langle\bar{q}q\rangle^{1/3}$	-182.	-196.	-207.	-219.	-181.	-194.	-203.	-215.
f_π	93.	93.	93.	93.	93.	93.	93.	93.
m_π	135.	135.	135.	135.	135.	135.	135.	135.
f_σ	73.	78.	79.	81.	70.	75.	74.	76.
m_σ	424.	361.	315.	294.	553.	418.	453.	401.
f_ρ	81.	80.	78.	78.	82.	81.	81.	79.
m_ρ	729.	770.	810.	859.	770.	800.	874.	901.
f_{a1}	57.	61.	63.	64.	54.	59.	59.	60.
m_{a1}	1141.	1089.	1070.	1092.	1308.	1198.	1317.	1278.

References

[1] D.Ebert and M.K.Volkov, Yad. Fiz. 36(1982)1265; Z. Phys. **C16** (1983)205.
D.Ebert and H.Reinhardt, Nucl. Phys. **B271**(1986)188.

[2] A.Dhar, R.Shankar and S.R.Wadia, Phys. Rev. **D31**(1985)3256.

[3] V.Bernard and U.G.Meissner, Ann. Phys. **206**(1991)50.

[4] M.Wakamatsu and W.Weise, Z.Phys. **A331**(1988)173;

[5] T.Hatsuda, Phys. Lett. **206**(1988)385; ibid **B213**(1988)361.

[6] H.Reinhardt, Phys. Lett. **B244**(1990)316.

[7] R.D.Ball, Int. J. Mod. Phys. **A5**(1990)4391.

[8] T.Kunihiro, Nucl. Phys. **B351**(1991)593.

[9] L.S.Celenza and C.M.Shakin, Phys. Rev. **D34**(1986)1591; ibid **D35**(1987)2843.

[10] H.Kleinert, in: "Understanding the fundamental constituents of matter", ed. by A.Zichichi, Plenum Pub. Corp. (1978).

[11] D.Ebert and V.N.Pervushin, Teor. Mat. Fiz. **36**(1978)313;

[12] R.T.Cahill and C.D.Roberts, Phys. Rev. **D32**(1985)2419.
R.T.Cahill, C.D.Roberts and J.Praschifka, Phys. Rev. **D36** (1987)2804.

[13] J.Praschifka, R.T.Cahill and C.D.Roberts, Int. Journal Mod. Phys. **A4**(1989)4929.

[14] D.Ebert and M.K.Volkov, Phys. Lett. **B272**(1991)86.

[15] H.J.Munczek and A.M.Nemirovsky, Phys. Rev. **D28**(1983)181.

[16] R.A.Bertlmann et al., Z.Phys. **C39**(1988)231.

[17] H.Pagels, Phys.Rev. **D15**(1977)2991.
S.Mandelstam, Phys.Rev. **D20**(1979)3223.
M.Baker, J.S.Ball and F.Zachariasen, Nucl. Phys. **B186**(1981) 531.
B.A.Arbusov, Phys.Lett. **B125**(1983)497.
A.A.Slavnov, Theor. Math. Phys. **52**(1983)541 (Sov.J.).
V.Sh.Gogokhia, Phys.Rev. **D40**(1989)4157.

Received by Publishing Department
on July 15, 1992.

Бельков А.А., Эберт Д., Емельяненко А.В. E2-92-304
КХД-мотивированная модель
Намбу - Йона - Лазинио,
учитывающая глюонную конденсацию
и моделирование конфайнмента кварков

Рассмотрена возможность моделирования бесполюсного кваркового пропагатора, отвечающего конфайнменту кварков. Подход основан на непертурбативном глюонном пропагаторе, учитывающем глюонную конденсацию и динамическую массу глюона. При этом сохраняются свойства спонтанного нарушения киральной симметрии, обеспечивающего разумное описание свойств легких мезонов.

Работа выполнена в Лаборатории сверхвысоких энергий ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1992

Bel'kov A.A., Ebert D., Emelyanenko A.V. E2-92-304
Gluon Condensation and Modelling of
Quark Confinement in QCD-Motivated
Nambu - Jona - Lasinio Model

The possibility of modelling of a quark propagator without poles realizing quark confinement is considered on the basis of a nonperturbative gluon propagator including gluon condensation and a dynamical gluon mass. The property of spontaneous chiral symmetry breaking is retained providing us with a reasonable pattern of low-lying meson properties.

The investigation has been performed at the Particle Physics Laboratory, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1992