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N = 2 SUPER W_3 ALGEBRA AND N = 2 BOUSSINESQ EQUATIONS

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1 Introduction

Since time immemorial, symmetry principles in physics have played a decisive role in unfolding the secrets of nature. Recent upsurge of interest in the study of so-called W-symmetry [1] has been spurred by the existence of many diverse relationships between this symmetry and CFTs for $c > 1$ integrable systems in 1+1 dimensions, matrix models, topological gravity, Toda field theories and various kinds of integrable hierarchies etc.[2–8]. All these developments and corresponding understandings are primarily aimed to provide a consistent quantum theory of gravitation. One of the most popular condidate for such a theory is the superstring theory[9] where applications of two-dimensional super conformal field theories (SCFTs) and corresponding superalgebras have met with remarkable success. It is expected, therefore, that super W-algebras, which are generalization of SCFTs to higher spins, would play a pivotal role in the understanding of quantum theory of gravitation and non-critical string theories[10].

During the last couple of years, a substantial progress has been made to supersymmetrize W-algebras [11-14]. In particular, $N = 2$ classical [15] and quantum [16] super W_3 algebras have attracted a great deal of interest because of the rich-structure contained in the corresponding lower spin $N = 2$ super conformal algebras which play a prominent role in the context of compactification scheme of $N = 2$ strings to the Calabi-Yau manifold [17,18], coupling of $2D$ gravity to the minimal matter and topological gravity etc[19.20]. Furthermore, in addition to constituting the building blocks for space-time string vacua[21], $N = 2$ SCFTs seem to provide a consistent quantum theory of self dual gravity in four dimensions[22] which, in turn, has some veiled glimpses of relationships with twistor and harmonic superspace[23], quantization of integrable systems[24] and W_∞ algebras [22]. It is ,therefore, hoped that higher spin $N = 2$ super W_3 algebra would be very richly endowed with many interesting features which would have profound implications in the context of W-gravity and W-strings and other areas of research. Towards this end in mind, this algebra has been studied through super Toda field theories[25], super Lax pair formulation[26], superfield realization (and corresponding Hamiltonian flow))[27] and Polyakov's "soldering" procedure[15].

One of the most outstanding problem in the context of (super)W-algebras is to obtain the corresponding (super)integrable equations and to provide a geometrical origin of these equations and various hierarchies associated with them. The well-known non-linear realization (NLR) method[28] coupled with the ideas of inverse Higgs effect (IHE)[29] and covariant reduction procedure (CRP)[30] provides a framework where such kind of problems can be dealt with convincingly. To understand the geometry underlying a symmetry (super)algebra under NLR scheme, the key concept is to construct an infinite dimensional coset (super)space for a suitably chosen stability (super)subalgebra. In terms of the one-differential Cartan form associated with the coset element, all the geometrical quantities of interest such as: metric, curvature, torsion, complex structure etc. of the group (super)manifold are determined. The application of the CRP selects a lower dimensional geodesic (super)manifold from the starting infinite dimensional coset (super)manifold which is characterized by (super)integrable equations such as: (super)KdV, (super)Liouville, (super)Sine-Gordon etc. These eqautions are dynamical equations satisfied by finite number of essential (super)goldstone fields. These fields find a transparent geometrical meaning because they are identified with (super)coordinates of the coset manifold. For instance, the group transformations on them correspond to the isometries of the group (super) manifold.

A deeper understanding of the geometry underlying various super W-algebras will be useful in W-strings and W-gravity. The purpose of our paper is to study $N = 2$ classical super W_3-algebra in the framework of NLR method. We demonstrate that the application of IHE and CRP leads to the choice of a geodesic super manifold which is characterized by only four chiral (super) fields obeying $N = 2$ super Boussinesq equations on this manifold. A scalar field realization is also possible for this geodesic super manifold which is related to the super chiral realization by super Miura maps. Thus, a geometric meaning emerges in the sense that the $N = 2$ super Boussinesq equations are conditions on the essential goldstone fields which parametrize the embedding of the geodesic super manifold in the infinite dimensional starting manifold. In addition to our previous works [30,31,32] on NLRs, this work is another step in the direction to our main goal of providing a geometrical basis for all two-dimensional integrable systems which are very intimately connected with W_N algebras.

The organization of this paper is as follows. In the next Section we recapitulate the essential ingredient of our work [32] on NLR of $N = 0$ W_3 algebra which would be useful in our discussion of the NLR of $N = 2$ super W_3 algebra. This would be followed by derivations of all the OPE's needed for the NLR, firstly in the language of superfield realization and corresponding SOPE's in Section 3. Then, we list all the (anti)commutation relations essential for our purpose in Appendix 1. Section 4 is devoted for the thorough discussion of NLR of $N = 2$ super W_3 where we pin point various subtelties involved in the choice of stability super subalgebra, coset superspace construction, one -differential Cartan form, and IHE . This is followed by the derivation of $N = 2$ Boussinesq equations for the superchiral fields by applications of CRP under the NLR scheme in Section 5. In this Section, we also establish the precise agreement of these equations with the ones deduced by the application of the Hamiltonian formulation [27]. We make concluding remarks in the last Section.

2 Preliminaries: non-linear realization of W_3 symmetry

In this Section we discuss, in a nut-shell, the key points of the NLR of classical W_3 algebra obtained in ref.[32] which would be useful, in what follows,for our elaborate discussion of classical $N = 2$ super-W_3 algebra in the framework of NLR.

The basic trick invoked in refs.[31,32] is to obtain a linear infinite dimensional (W_3^∞) algebra in which all the composite higher spin generators of the standard W_3 algebra are treated as independent generators. For instance, the spin-4 generator $\Lambda_m^{(0)} = -\frac{8}{c}\sum_m L_{n-m}^{(0)} L_m^{(0)}$ appearing in the standard W_3 algebra [1]:

$$
\begin{aligned}
[L_n^{(0)}, L_m^{(0)}] &= (n-m)L_{n+m}^{(0)} + \frac{c}{12}(n^3 - n)\delta_{n+m,0} \\
[L_n^{(0)}, W_m^{(0)}] &= (2n-m)W_{n+m}^{(0)} \\
[W_n^{(0)}, W_m^{(0)}] &= 16(n-m)\Lambda_{n+m}^{(0)} - \frac{8(n-m)}{3}\left[(n+m)^2 - \frac{5}{2}nm - 4\right] L_{n+m}^{(0)} - \\
&\quad -\frac{c}{9}(n^2-4)(n^2-1)n\delta_{n+m,0} \quad ,
\end{aligned} \tag{1}
$$

together with other higher spin composite generators $J_n^{(0)\,S}$ $(s = 5, 6, 7, \ldots)$ constitute W_3^∞ algebra as given below:

$$
W_3^\infty = \{L_n^{(0)},\ W_n^{(0)},\ \Lambda_n^{(0)},\ \ldots J_n^{(0)\,S},\ \ldots\}\,. \tag{2}
$$

One of the most important subalgebra of (2.2), which plays a crucial role for the NLR, contains all spin-s ($s > 2$) generators with indices ranging from $-(s-1)$ to $+\infty$. To extract geometry behind this algebra in the framework of NLR, the key ingredient is to construct an infinite dimensional coset space for a suitably chosen stability subgroup. Supplemented with the ideas of inverse higgs effect (IHE) and covariant reduction procedure (CRP), this formalism leads to the emergence of various integrable equations which characterize a lower dimensional geodesic manifold extracted out of the infinite dimensional starting coset manifold. The application of the CRP provides dynamics to the essential goldstone fields on the reduced manifold. As argued in section 1, the integrable equations in 1+1 dimension are very intimately connected with the W_N algebras. Thus, study of these algebras in the framework of NLR would lead to the derivation of new integrable equations and shed light on the geometrical origin of these equations which will be useful in the understanding of $2D$ gravity theories and non-critical string theories. In ref.[32], various (modified)Boussinesq equations have been deduced on account of different choices of the stability subalgebras and application of CRP in the framework of NLR. For instance, for the following choice of the stability subalgebra($\mathcal{H}$), containing minimum set of generators of W_3:

$$
\mathcal{H} = \left\{W_{-1}^{(0)},\ \Lambda_{-3}^{(0)}, \Lambda_n^{-2},\ \ldots J_{-S+1}^{(0)\,S},\ \ldots\right\}. \tag{3}
$$

the infinite dimensional coset space element g is parametrized in terms of coordinates and infinite tower of goldstone fields $(u_0, v_0, u_1, v_1, u_2, v_2, u, v, \psi_n, \xi_m)$ as follows:

$$
g = e^{tW_{-2}} e^{xL_{-1}} e^{uL_2} e^{vW_3} e^{\sum_{n\geq 3}\psi_n L_n} e^{\sum_{n\geq 4}\xi_n W_n} e^{u_1 L_1} e^{v_1 W_1} e^{v_2 W_2} e^{u_0 L_0} e^{v_0 W_0} \quad . \tag{4}
$$

where the fictitious "time" coordinate t is linked with t-translation generator W_{-2} as space coordinate x is associated with the linear momentum operator L_{-1}. The t- and x-directions on the coset manifold are entirely independent of each-other because W_{-2} commutes with L_{-1}. The most fundamental geometrical quantity in the NLR scheme is the one-differential Cartan form $\Omega = g^{-1}dg$ which can be expressed explicitly as sum of spin-s ($s \geq 2$) generators carrying indices ranging from $-(s-1)$ to $+\infty$. The application of IHE leads to the following expressions for the higher spin goldstone fields in terms of the essential goldstone fields u_0 and v_0:

$$
u_1 = \frac{u_0'}{2} \quad , \quad v_1 = \frac{v_0'}{3} \quad , \quad v_2 = \frac{1}{12}(v_0'' + u_0' v_0') \quad , \quad u = \frac{1}{6}\left[u_0'' + \frac{1}{2}(u_0')^2 + \frac{8}{3}(v_0')^2\right]
$$

$$
v = \frac{1}{5}\left[\frac{1}{12}v_0''' - \frac{1}{12}u_0'' v_0' + \frac{1}{4}u_0' v_0'' + \frac{1}{6}(u_0')^2 v_0' - \frac{8}{27}(v_0')^3\right] \quad , \tag{5}
$$

where prime stands for the derivative w.r.t. space coordinate x. On the other hand, application of the CRP produces the dynamical equations for the essential goldstone fields u_0 and v_0 given below:

$$
\dot{u}_0 = -\frac{16}{3}[v_0'' + 2u_0' v_0'] \quad , \quad \dot{v}_0 = u_0'' - (u_0')^2 + \frac{16}{3}(v_0')^2 \tag{6}
$$

where dot denotes derivative w.r.t. the t-coordinate. It is obvious from equations (2.3) and (2.5) that, whereas the coset manifold is parametrized by infinite number of goldstone fields, the reduced manifold is characterized by only two essential goldstone fields. In the physical and geometrical language, this amounts to logical choice of a two-dimensional geodesic manifold from the infinite dimensional coset manifold which provides, in turn, dynamics to the goldstone fields u_0 and v_0. Mathematically it can be expressed through following zero-curvature representation:

$$\Omega \rightarrow \Omega_{red} = (e^{-2u_0}dt)\, W^{(0)}_{-2} + e^{-u_0}\left[\left(dx + \frac{16}{3}v_0' dt\right) Cosh(4v_0) + (4u_0' dt)\, Sinh(4v_0)\right] L^{(0)}_{-1} - \frac{e^{-u_0}}{2}\left[\left(dx + \frac{16}{3}v_0' dt\right) Sinh(4v_0) + (4u_0' dt)\, Cosh(4v_0)\right] W^{(0)}_{-1} \quad (7)$$

It is straightforward to check that the Maurer-Cartan equation $d^{ext}\Omega_{red} = \Omega_{red} \wedge \Omega_{red}$ leads to the emergence of equation (2.5). It is worthwhile to lay stress here that equation (2.7) is not unique. It can also be expressed in terms of the spin-1 goldstone fields u_1 and v_1 as well as in terms of spin-2 u and spin-3 v fields which obey (modified)Boussinesq equations [32]. All these equations can be related by so-called generalized Miura-maps.

We shall follow the bare essentials of this Section in more detail in the following Sections.

3 $N = 2$ super W_3 algebra

In this section, we briefly recapitulate essential features of N=2 supercurrent formulation [27] of the classical N=2 super W_3 algebra and list all OPE's required for the NLR of this algebra through coset superspace construction. We also pin point an important OPE which is missed in ref.[15] but is very much essential for the proof of (graded) Jacobi identity.

In the framework of N=2 supercurrent realization of the classical N=2 super W_3 algebra, the spin-1 supercurrent $J(z,\theta,\bar\theta)$ and spin-2 supercurrent $T(z,\theta,\bar\theta)$ include all the basic currents of N=2 super W_3 algebra. In fact, as it turns out, the two N=2 supermultiplets with conformal spins $(1,3/2,3/2,2)$ and $(2,5/2,5/2,3)$ are the components of above cited supercurrents. Furthermore, it has been explicitely demonstrated in this work that three SOPE's of these supercurrents contain all the OPE's of ref.[15]. These three SOPE's are required so as to make neat conformity with the key ingredients of N=2 superconformal algebra. For instance the first SOPE between two $J(z,\theta,\bar\theta)$ results in the standard superconformal algebra. The next SOPE between $J(z,\theta,\bar\theta)$ and $T(z,\theta,\bar\theta)$ leads to the emergence of $T(z,\theta,\bar\theta)$ as primary superfield w.r.t. N=2 superconformal algebra and it carries zero $U(1)$ charge. The final SOPE between two $T(z,\theta,\bar\theta)$ is obtained due to the requirement of the closure of the N=2 superconformal algebra. As a consequence of the above requirements, following redefinitions of the currents belonging to supermultiplets $(1,3/2,3/2,2)$ and $(2,5/2,5/2,3)$ emerge[27]:

$$\begin{aligned}
J = 4J_0 \quad &, \quad T = T_0 + 4\widetilde{T}_0 - \frac{128}{c}J_0^2 \\
\bar{G} = \bar{G}_0 \quad &, \quad \bar{U} = \frac{3}{4}\bar{U}_0 - \frac{64}{c}J_0\bar{G}_0 \\
G = -G_0 \quad &, \quad U = -\frac{3}{4}U_0 + \frac{64}{c}J_0G_0 \\
T = T_0 + \widetilde{T}_0 \quad &, \quad W = \frac{3}{4}W_0 + \frac{32}{c}\left(T_0 + 4\widetilde{T}_0 - \frac{128}{c}J_0^2\right)J_0 + \frac{40}{c}G_0\bar{G}_0 \,.
\end{aligned} \quad (1)$$

where currents with subscript "0" obey OPE's of ref. [15].

The consistency with N=2 superconformal algebra, obtained at the supercurrent level, is also manifested at the component level of the supercurrents. For example, following OPE's:

$$\begin{aligned}
T(z)\widetilde{T}(x) &= \frac{2\widetilde{T}(x)}{(z-x)^2} + \frac{\widetilde{T}'(x)}{z-x} \\
J(z)\widetilde{T}(x) &= 0
\end{aligned} \quad (2)$$

demonstarate that $\widetilde{T}$ is a spin two primary field w.r.t. T and it carries zero $U(1)$ charge. All the OPE's amongst redefined currents can be read off from SOPE's of ref.[27] (see Appendix).

Now few remarks about above OPE's vis-a-vis OPE's of ref.[15] are in order. Firstly, the fermionic supercharges G and $\bar{G}$ are nilpotent. They can be, therefore, used as BRST charge to construct "twisted" algebra which is useful in the context of two-dimensional topological gravity [33]. The other fermionic generators U and $\bar{U}$ are not nilpotent because the OPE between two U's and $\bar{U}$'s are as follows:

$$U(z)U(x) = \frac{60GU - 32G\partial G}{c(z-x)} \quad , \quad \bar{U}(z)\bar{U}(x) = \frac{-60\bar{G}\bar{U} - 32\bar{G}\partial\bar{G}}{c(z-x)} \quad (3)$$

Secondly, as a consequence of supercurrent formalism adopted in ref.[27], all the OPE's are consistent with the (graded) Jacobi identity for arbitrary non-zero value of the central charge c. However, currents of ref.[15] do not respect Jacobi identity unless OPE corresponding to (3.3) is taken into account. For instance, it will be seen that the following graded Jacobi identity:

$$[L^{(0)}_m, \{G^{(0)}_r, U^{(0)}_s\}] + \text{graded cyclic} \quad (4)$$

is not equal to zero unless following OPE's are included in ref[15]; namely:

$$U_0(z)U_0(x) = 64\frac{G_0U_0 - \frac{8}{3}G_0\partial G_0}{c(z-x)} \quad \text{and} \quad \bar{U}_0(z)\bar{U}_0(x) = -64\frac{\bar{G}_0\bar{U}_0 + \frac{8}{3}\bar{G}_0\partial\bar{G}_0}{c(z-x)} \quad (5)$$

The OPE's listed in above equations and corresponding (anti)commutation relations would be essential for the study of N=2 super W_3 algebra in the framework of NLR method which we discuss in the next section.

4 Non-linear realization of N=2 super W_3

In this Section, following the prescription briefly outlined in Section 2, we would obtain the linear infinite dimensional N=2 super W_3 (W_3^∞) algebra from the standard non-linear N=2 W_3 algebra and discuss various subtleties involved in the choice of the stability super subalgebra ($\mathcal{H}$) and construction of infinite dimensional coset superspace which would finally culminate into the NLR of N=2 super W_3 algebra.

4.1 From N=2 super W_3 to N=2 super W_3^∞

For the application of the NLR method to the non-linear classical N=2 super W_3 algebra, it is of utmost importance to firstly obtain a linear infinite dimensional N=2 super W_3^∞ algebra [31,32]. Such a superalgebra (sW_3^∞) can be obtained by treating all the higher spin composite generators of standard N=2 super W_3 algebra listed in the Appendix and discussed in the previous Section as independent generators of the algebra. This linear algebra (sW_3^∞) contains following generators:

$$sW_3^\infty = \left\{J_n, G_r, \bar{G}_r, T_n, \widetilde{T}_n, U_r, \bar{U}_r, W_n, \Psi_r, \bar{\Psi}_r, \phi_n, \ldots, \Psi_r^h, \bar{\Psi}_r^h, \phi_n^h, \ldots\right\} \qquad (1)$$

where the basic generators ($J, G, \bar{G}, T, \widetilde{T}, U, \bar{U}$ and W) form N=2 supermultiplet of N=2 super W_3 algebra and composite fields ($\Psi_r, \bar{\Psi}_r, \phi_n, \ldots$) are higher spin composite fields with conformal spins $(2, 5/2, 5, 2, 3, 7/2, 7/2, 4, 4, 9/2, 3, 7/2, 7/2, 4)$ and ($\Psi_r^h, \bar{\Psi}_r^h, \phi_n^h, \ldots$) stand for still higher spin composite fields of the sW_3^∞ algebra.

One of the interesting subalgebra of sW_3^∞ which is useful for the NLR method is the one in which all spin- h ($h \geq 1$) generators of sW_3^∞ possess indices varying from $-(h-1)$ to $+\infty$. The reflection symmetry: $n \to (-n)$ and $r \to (-r)$ present in sW_3^∞ guarantees existence of an infinite dimensional wedge type [34] of algebra $sW_{3\wedge}^\infty$ which contains all spin-h generators ($h \geq 1$) with indices varying from $-(h-1)$ to $+(h-1)$. One of the factor algebra of $sW_{3\wedge}^\infty$ relevant for our purposes is as follows:

$$osp(3|2) \sim sW_{3\wedge}^\infty / \left\{\Psi_r, \bar{\Psi}_r, \phi_n, \ldots\right\} \qquad (2)$$

It would be worthwhile to mention here that, in general, generators of the conformal spin 2, 3 and 5/2 present in the N=2 supermultiplets can be modified by adding composite generators of the same conformal spins such as: $J^2, (TJ, \widetilde{T}J, J^3, G\bar{G})$ and $(JG, J\bar{G})$ with an arbitrary non-zero parameter without changing the structure of the factor subalgebra (4.2). Usefulness of this statement would become more lucid and transparent when we shall discuss super-Boussinesq equations for the superchiral fields in Section 5.

4.2 Stability super subalgebra

As discussed and enunciated in Section 2, the basic concept for the understanding of the underlying geometry behind a symmetry group G is to construct an appropriate infinite dimensional coset space for suitably chosen stability subgroup H in the framework of NLR method [28]. The appropriate stability super subalgebra ($s\mathcal{H}$), contained in sW_3^∞, consists of following generators for the NLR of this algebra:

$$\begin{aligned} s\mathcal{H} \;=\; & \Big\{W_0, L_0, \widetilde{L}_0, J_0, U_{-3/2}, \bar{U}_{-3/2}, G_{-1/2} + U_{-1/2}, \bar{G}_{-1/2} - \bar{U}_{-1/2}, \\ & L_{-1} - \widetilde{L}_{-1}, W_{-1} + \frac{3}{2}L_{-1}, \text{Higher spin}\Big\} \end{aligned} \qquad (3)$$

As is evident from the expression for the sW_3^∞ (4.1), we are dealing here with an infinite dimensional coset superspace unlike the case of finite dimensional cosets for the NLR of finite dimensional linear algebras (see,e.g.[30]) In the case under consideration, it turns out to be handy to put all higher spin composite (super)generators in the stability super subalgebra [31,32].

It will be noticed that only appropriate combinations of generators quoted in equation (4.3) form the closed super subalgebra together with the higher spin composite generators.

Independently, ($G_{-1/2}, \bar{G}_{-1/2}, U_{-1/2}, \bar{U}_{-1/2}, W_{-1}, L_{-1}$) do not constitute the super subalgebra $s\mathcal{H}$. Furthermore, the choice (4.3) includes maximal set of generators of sW_3^∞ which lead to the closure of the super subalgebra $s\mathcal{H}$. It is interesting to point out that choice of only one combination between $W_{-3} \pm \frac{3}{2}L_{-1}$ and $W_{-3} \pm \frac{3}{2}\widetilde{L}_{-1}$ in super subalgebra $s\mathcal{H}$ is a bit subtle and requires physical reasoning involving ideas of supersymmetry. This is made lucid and clear in the next subsection where we pick out the combination $W_{-3} \pm \frac{3}{2}\widetilde{L}_{-1}$ as the appropriate generator belonging to the super subalgebra $s\mathcal{H}$.

4.3 Construction of the coset superspace

For the NLR of the classical N=2 super sW_3^∞ algebra with stability super subalgebra (4.3), the element g of the coset supermanifold is parametrized as follows:

$$\begin{aligned} g \;=\; & e^{tW_{-2}} e^{xL_{-1}} e^{\theta G_{-1/2} + \bar{\theta}\bar{G}_{-1/2}} e^{\chi G_{1/2} + \bar{\chi}\bar{G}_{1/2}} e^{uL_1} e^{\tilde{u}\tilde{L}_1} e^{\phi J_1} e^{vW_1} e^{\mu G_{3/2} + \bar{\mu}\bar{G}_{3/2}} e^{\nu U_{3/2} + \bar{\nu}\bar{U}_{3/2}} \\ & e^{u_2 L_2} e^{\tilde{u}_2 \tilde{L}_2} e^{\phi_2 J_2} e^{v_2 W_2} e^{\mu_1 G_{5/2} + \bar{\mu}_1 \bar{G}_{5/2}} e^{\nu_1 U_{5/2} + \bar{\nu}_1 \bar{U}_{5/2}} e^{\xi U_{1/2} + \bar{\xi}\bar{U}_{1/2}} e^{u_3 L_3} e^{\tilde{u}_3 \tilde{L}_3} \ldots \end{aligned} \qquad (4)$$

where the time "t" coordinate with dimension $(cm)^2$ is associated with t- translator generator W_{-2} as space coordinate x and supersymmetric coordinates θ and $\bar{\theta}$ are associated with the linear momentum generator L_{-1} and supersymmetric generators $G_{-1/2}$ and $\bar{G}_{-1/2}$. Furthermore, the infinite tower of supergoldstone fields ($\chi, \bar{\chi}, \xi, \bar{\xi}, u, \tilde{u}\phi, v, u_1, \tilde{u}_1, \ldots$) with various integral and half integral spins, together with above translation operators, fully parametrize the coset supermanifold (4.4).

We offer some comments here for the choice of the group factors in (4.4) to make the physical picture transparent. Firstly, it is elementary to check that the t-translation operator W_{-2} commutes with all the other translation operators such as : L_{-1}, $G_{-1/2}$ and $\bar{G}_{-1/2}$. On the infinite dimensional coset supermanifold, physically it amounts to the fact that the translation along t-direction is entirely independent of the rest of the translations along x, θ and $\bar{\theta}$ directions on the coset supermanifold. In addition to the W_{-2}, the linear momentum generator L_{-1} also commutes with $G_{-1/2}$ and $\bar{G}_{-1/2}$. Physically, it establishes the fact that the translation along space direction is also independent of the translation along t, θ and $\bar{\theta}$ directions. However, the anticommutation relation between $G_{-1/2}$ and $\bar{G}_{-1/2}$ leads to the emergence of L_{-1} on the r.h.s. Physically, it transpires to the fact that two-consecutive supersymmetric translations are equivalent to a space translation on the coset supermanifold. All these comments finally amount to the fulfillment of all the requirements of SUSY on the coset supermanifold (4.4)

The consistency requirements of SUSY on the coset supermanifold also leads to the choice of combination $W_{-1} + \frac{3}{2}L_{-1}$ from the pair of generators $W_{-1} \pm \frac{3}{2}L_{-1}$ and $W_{-1} \pm \frac{3}{2}\widetilde{L}_{-1}$ present in the stability super subalgebra (4.3). This is due to the fact that L_{-1} is common between $L_{-1} - \widetilde{L}_{-1}$ and $W_{-1} + \frac{3}{2}L_{-1}$ and therefore, considering the economy of operators on the coset supermanifold, one can choose only L_{-1} and associate with it the space coordinate on the manifold. However, the decisive role, in the favour of the choice $W_{-1} + \frac{3}{2}L_{-1}$ is played by the ideas of the SUSY because the anticommutator between $G_{-1/2}$ and $\bar{G}_{-1/2}$ also leads to the emergence of L_{-1} on the r.h.s.

Above arguments can be compared and contrasted with the choice $W_{-1} \pm \frac{3}{2}\widetilde{L}_{-1}$ in the stability subalgebra (4.3). It is obvious in this case that L_{-1} is common between $L_{-1} - \widetilde{L}_{-1}$ and $W_{-1} + \frac{3}{2}L_{-1}$ and, therefore, space coordinate x can be associated with it on the coset supermanifold. However, the anticommutator between supercharges does not lead to the emergence of this operator on the r.h.s. Thus, space coordinate cannot be associated with it because the basic principle of SUSY is violated on the coset supermanifold. Thus, consistency with the fundamental ideas of SUSY rules out the inclusion of combination $W_{-1} + \frac{3}{2}\tilde{L}_{-1}$ in the stability subalgebra (4.3).

4.4 Non-linear realization

The most fundamental geometrical quantity which determines curvature, torsion and complex structure etc. of a group manifold in the framework of NLR method is the one-differential covariant Cartan form (Ω). For the element g of the coset supermanifold (4.4), this is introduced as follows:

$$\begin{aligned}\Omega \equiv g^{-1}dg &= \sum_{n=-2}^{\infty} \omega_n W_n + \sum_{n=-1}^{\infty} l_n L_n + \sum_{n=-1}^{\infty} \tilde{l}_n \tilde{L}_n \sum_{r=-1/2}^{\infty} g_r G_r + \sum_{r=-1/2}^{\infty} \bar{g}_r \bar{G}_r + \\ &+ \sum_{r=-3/2}^{\infty} f_r U_r + \sum_{r=-3/2}^{\infty} \bar{f}_r \bar{U}_r + \sum_{n=0}^{\infty} j_n J_n + \text{Higher spin contribution} \qquad (5)\end{aligned}$$

where the (super)parameter forms $\omega_n, l_n, \tilde{l}_n, j_n, g_r, \bar{g}_r, f_r, \bar{f}_r, \ldots$ correspond to the generators belonging to the N=2 supermultiplets and higher spin generators of (4.4). For the application of the IHE and CRP, it is essential that all the (super)forms associated with coset generators transform homogeneously. First few (super)forms which depend only on the differentials on the super coordinates are as follows:

$$\begin{aligned}\omega_{-2} = dt \quad &, \quad g_{-1/2} = d\theta + \left(dx - \frac{\bar{\theta}d\theta + \theta d\bar{\theta}}{2}\right)\chi \\ l_{-1} = dx - \frac{\theta d\bar{\theta} + \bar{\theta}d\theta}{2} \quad &, \quad f_{-1/2} = 2\xi\left(dx - \frac{\bar{\theta}d\theta + \theta d\bar{\theta}}{2}\right) \qquad (6) \\ \tilde{l}_{-1} = 0 \quad &, \quad \text{etc.}\end{aligned}$$

It is worthwhile to mention that relations obtained in the above equation furnish combination(s) of (super)differentials which change covariantly under the transformations corresponding to the group under consideration. For instance, equation (4.6) provides a covariant combination ($d\tilde{x}$) which is expressed in terms of the differentials on the super coordinates as follows:

$$d\tilde{x} = dx - \frac{\bar{\theta}d\theta + \theta d\bar{\theta}}{2} \qquad (7)$$

It will be seen, in what follows, that all the (super)forms and super covariant derivatives ($\mathcal{D}, \bar{\mathcal{D}}$) would be expressed in terms of (4.7). For instance, the some of the fermionic forms which depend on the diffrentials on the (super) coordinates and (super)goldstone fields are as follows:

$$\begin{aligned}g_{1/2} &= d\chi + \left(u\chi - \frac{1}{4}\phi\chi - 2\xi\bar{\xi}\chi - 2\mu - 6v\xi - 4\tilde{u}\xi\right)d\tilde{x} - \\ &- \left(u - \frac{1}{4}\phi - 2\xi\bar{\xi} - \frac{1}{2}\chi\bar{\chi}\right)d\theta + S(G)dt \qquad (8)\end{aligned}$$

$$\begin{aligned}f_{1/2} &= d\xi + \left(u\xi + \frac{1}{2}\chi\bar{\chi}\xi + 3v\xi + \frac{1}{4}\phi\xi + 2\tilde{u}\xi + \chi\xi\bar{\xi} + \tilde{u}\chi - \frac{3}{2}v\chi - 3\eta\right)d\tilde{x} - \\ &- \left(\tilde{u} - \frac{3}{2}v + \xi\bar{\xi} + \bar{\chi}\xi\right)d\theta + S(U)dt\end{aligned}$$

We lay emphasis here on the fact that computation of above expressions are straightforward but tedious. In particular, the expression for $S(U)$ and $S(G)$ are very complicated because there are many contributions to it. In this case, even the composite fields listed in the Appendix and discussed in Section 2 also contribute. However, these expressions have to be computed in order to obtain N=2 super-Boussinesq equations for the super chiral fields $\xi, \bar{\xi}, \chi$ and $\bar{\chi}$ which we discuss in the next Section. Furthermore, it will be noticed, in what follows, that these super chiral fields are the essential goldstone fields in terms of which higher spin (super) goldstone fields would be expressed by application of IHE. Finally, we would like to point out that for the other (super)goldstone fields the expressions corresponding to $S(U)$ and $S(G)$ would not be required for the Boussinesq equations because these fields can be expressed in terms of the superchiral fields $\xi, \bar{\xi}, \chi$ and $\bar{\chi}$.

The expressions for the other forms are very-very long which can not be written here because of constraints and logistic reasons imposed by the journal.

5 N=2 Super Boussinesq equations and hamiltonian formulation

In this Section, we shall derive equations of motion for the essential goldstone fields in the framework of the NLR method and establish their complete agreement with the Hamilton's equations obtained by the application of the Hamiltonian formulation of ref.[27]. To avoid square roots and various complicated fractions, we take the value of central charge c to be 8 in this Section. However, all the equations can be derived for arbitrary non-zero value of the central charge.

To obtain the super Boussinesq equations under NLR scheme, it is of utmost importance to apply the idea of CRP which primarily amounts to the imposition of infinite number of (super)covariant constraints on the (super)Cartan forms ($\Omega = 0$). As a consequence of CRP, two-types of (super) constraints emerge: kinematical (or algebraic) and dynamical. The former constraints, christened as IHE [29], furnish expressions for the higher spin (super) goldstone fields in terms of the essential (super)fields and (super) derivatives on them. On the other hand, the latter constraints lead to the dynamical equations of motion for the essential goldstone fields. For instance, due to the IHE, following expressions emerge for the first few goldstone fields in terms of the essential spin 1/2 fields:

$$\begin{aligned}u = \frac{\mathcal{D}\chi + \bar{\mathcal{D}}\bar{\chi}}{2}, \qquad \varphi = 2(\bar{\mathcal{D}}\bar{\chi} - \mathcal{D}\chi - 4\xi\bar{\xi} - \chi\bar{\chi}), \\ \tilde{u} = \frac{1}{2}(\mathcal{D}\xi - \bar{\mathcal{D}}\bar{\xi} + \chi\bar{\xi} - \bar{\chi}\xi), \quad v = \tfrac{1}{3}(2\xi\bar{\xi} + \bar{\chi}\xi + \chi\bar{\xi} - \mathcal{D}\xi - \bar{\mathcal{D}}\xi), \qquad (1) \\ \text{etc}\ldots\end{aligned}$$

Furthermore, these spin 1/2 fields are chiral superfields because, as a result of application of IHE, we obtain following equations for them:

$$\mathcal{D}\bar{\chi} = \bar{\mathcal{D}}\chi = \mathcal{D}\bar{\xi} = \bar{\mathcal{D}}\xi = 0,$$

$$\mathcal{D}\chi = u - \frac{\varphi}{4} - 2\xi\bar{\xi} - \frac{\chi\bar{\chi}}{2}, \qquad (2)$$
$$\mathcal{D}\xi = \tilde{u} - \frac{3}{2}v + \xi\bar{\xi} + \bar{\chi}\xi.$$

where supercovariant derivatives and present in equations (5.1) and (5.2) are defined through following total derivative on a given arbitrary superfield:

$$d\Psi = \frac{\partial\Psi}{\partial\tilde{x}}d\tilde{x} + \mathcal{D}\Psi d\theta + \bar{\mathcal{D}}\Psi d\bar{\theta} + \frac{\partial\Psi}{\partial t}dt \qquad (3)$$

with,

$$\mathcal{D} = \frac{\partial}{\partial\theta} - \frac{\bar{\theta}}{2}\frac{\partial}{\partial\tilde{x}}, \quad \bar{\mathcal{D}} = \frac{\partial}{\partial\bar{\theta}} - \frac{\theta}{2}\frac{\partial}{\partial\tilde{x}},$$
$$\left\{\mathcal{D}, \bar{\mathcal{D}}\right\} = -\frac{\partial}{\partial\tilde{x}}, \quad \mathcal{D}^2 = \bar{\mathcal{D}}^2 = 0. \qquad (4)$$

It is worth mentioning that there are very complicated expressions for the other higher spin (super)goldstone fields which are essential for the determination of the equations of motions for the chiral superfields in the framework of NLR. However,these are not quoted here because of space- restrictions. In fact, the determination of all the above higher spin goldstone fields in terms of essential goldstone fields requires computation of all the forms upto spin 3/2 corresponding to $G_{3/2}$ and $U_{3/2}$. On similar logic, equations analogous to (5.2) for higher spin (super)goldstone fields etc. are not quoted here.

As mentioned earlier, the dynamics of the chiral superfields emerges by the application of CRP [30]. This is mathematically expressed as follows:

$$g^{-1}dg \in \mathcal{G} \longrightarrow g_{red}^{-1}dg_{red} \in \widetilde{\mathcal{G}} \qquad (5)$$

where $\mathcal{G}$ is the infinite dimensional super algebra (sW_3^{∞}), and is some algebra which contains stability super subalgebra $s\mathcal{H}$ given by equation (4.3). In particular, in our case the the reduced super algebra $\widetilde{calG}$ contains all the generators of the super algebra osp(3/2) of equation (4.2) plus higher spin composite generators.

Before we derive equation of motion for the essential superchiral fields, we offer some comments on the remark made at the end of the subsection (4.1). It can be easily seen that the consistent with the requirements of SUSY, there is another generator which can be treated as the t-translator operator in the infinite dimensional coset superspace (4.4). This operator is as follows:

$$W_{-2} + \alpha(TJ - \frac{1}{4}G\bar{G}) \qquad (6)$$

It can be readily checked that this generator too, like W_{-2}, commutes with all the rest of the super translation generators present in the coset superspace (4.4). Furthermore, the factor algebra discussed in subsection (4.1) remains the same even if we change the initial generator W_{-2} of the coset superspace (4.4) by the generator of equation (5.6) with arbitrary non-zero parameter α.

The application of the CRP with the most general translation operator (5.6) leads to the following $N = 2$ super Boussinesq equations in the framework of NLR method:

$$\dot{\chi} = 2\partial^2\xi - \frac{\alpha}{4}\partial^2\chi - (5 - \frac{\alpha}{4})\left[\mathcal{D}\bar{\mathcal{D}} + \partial\right](\chi\xi\bar{\xi}) + \partial\left[2\bar{\mathcal{D}}\bar{\xi}\chi - \frac{\alpha}{2}\bar{\mathcal{D}}\bar{\chi}\chi - (3 + \frac{\alpha}{4})\bar{\mathcal{D}}\bar{\xi}\xi + 4\bar{\mathcal{D}}\bar{\chi}\xi\right]$$
$$+\partial\chi\left[2\mathcal{D}\xi + \frac{\alpha}{2}\mathcal{D}\chi\right] + \partial\xi\left[2\mathcal{D}\chi + 4\mathcal{D}\xi\right] \qquad (7)$$

and,

$$\dot{\xi} = -\partial^2\xi - 2\partial^2\chi + (5 - \frac{\alpha}{4})\left[\mathcal{D}\bar{\mathcal{D}} + \partial\right](\chi\bar{\chi}\xi) + \partial\left[2\bar{\mathcal{D}}\bar{\chi}\chi - 2\mathcal{D}\xi\xi - (3 + \frac{\alpha}{4})\bar{\mathcal{D}}\bar{\chi}\xi - 4\bar{\mathcal{D}}\bar{\xi}\chi\right]$$
$$+\partial\chi\left[4\mathcal{D}\chi + (3 + \frac{\alpha}{4})\mathcal{D}\xi\right] + \partial\xi\left[(3 + \frac{\alpha}{4})\mathcal{D}\chi - 2\bar{\mathcal{D}}\bar{\xi}\right]. \qquad (8)$$

The other equations for $\bar{\xi}$ and $\bar{\chi}$ can be obtained from the above equations by applying the same conjugation rules as that of the generators G and U in the Appendix.

It is important to note that above equations for ξ and χ emerge in the framework of NLR when the original coset superfields are scaled as : $\chi \longrightarrow \chi$ and $\xi \longrightarrow \xi/2$. This is required so as to make neat conformity with the $N = 2$ SCFTs as well as with the Hamiltonian formulation which we discuss in the following paragraphs. Furthermore, the $\alpha = 0$ limit yields the $N = 2$ Boussinesq equations for the choice of the element (4.4) for the coset superspace.

It is discussed extensively in ref.[32] that spin-three currents lead to the definition of the second Hamiltonian structure. In a similar way, it has been demonstrated in ref.[27] that for the $N = 2$ super W_3 case the most general form of the Hamiltonian is as follows:

$$H = \int dZ(T + \alpha J^2) \qquad (9)$$

where $Z = (z, \theta, \bar{\theta})$ is the $N = 2$ super coordinates and the expressions for the $J(Z)$ and $T(Z)$ in terms of the chiral superfields for $c = 8$ are as follows:

$$J = \bar{\mathcal{D}}\bar{\chi} - \mathcal{D}\chi - \chi\bar{\chi} - \xi\bar{\xi} \qquad (10)$$

(5.10) and,

$$T = \partial(\mathcal{D}\xi - \bar{\mathcal{D}}\bar{\xi}) + 2\partial\xi\bar{\chi} + \partial\xi\bar{\xi} + \partial\bar{\xi}\xi - 2\partial\bar{\xi}\chi + \partial\chi\xi - \partial\bar{\chi}\xi - 5\xi\bar{\xi}\chi\bar{\chi}$$
$$+\bar{\mathcal{D}}\bar{\xi}\left[\bar{\mathcal{D}}\bar{\xi} - \mathcal{D}\chi - \bar{\mathcal{D}}\bar{\chi} + \chi\bar{\chi} - \xi\bar{\xi} + 2\bar{\xi}\chi - 4\xi\bar{\chi}\right] \qquad (11)$$
$$+\mathcal{D}\xi\left[\mathcal{D}\chi + \bar{\mathcal{D}}\bar{\chi} + \mathcal{D}\xi - 3\bar{\mathcal{D}}\bar{\xi} + 2\xi\bar{\chi} - 4\bar{\xi}\chi + \chi\bar{\chi} - \xi\bar{\xi}\right]$$
$$+\mathcal{D}\chi\left[\xi\bar{\chi} - 2\bar{\xi}\chi - \xi\bar{\xi}\right] + \bar{\mathcal{D}}\bar{\chi}\left[2\xi\bar{\chi} - \bar{\xi}\chi + \xi\bar{\xi}\right].$$

It is worthwhile to lay stress on the fact that the above expressions (5.10) and (5.11) for J and T are nothing other than the expressions for the goldstone fields ϕ and $\tilde{u}$ associated with the coset generators J_1 and $\tilde{L}_2$ in terms of essential chiral superfields modulo overall scaling factors, and the scaling of χ and ξ mentioned in the previous paragraph. Thus, the form of the second Hamiltonian structure emerges automatically in the framework of NLR method as the expressions for the appropriate higher spin goldstone fields in terms of the essential goldstone fields.

It will be also noticed that integrations over θ and $\bar{\theta}$ in equation (5.9) lead to the expression for the Hamiltonian in exactly the same form as that of equation (5.6). Thus, the origin of the expression (5.6) becomes transparent, taking into account the arguments of the superfield realization of $N = 2$ super W_3 of ref.[27]. It is now straightforward to check that the evolution equation for the Kac-Moody super currents χ and ξ with the Hamiltonian (5.9) are:

$$\dot{\chi} = [\chi, H] \qquad (12)$$

and,

$$\dot{\xi} = [\xi, H] \tag{13}$$

(5.13) which exactly match with the the equations (5.7) and (5.8) obtained in the framework of NLR if we take into account the the standard two-point functions for the spin-1/2 chiral superfields; namely:

$$< \chi(Z_1)\bar{\chi}(Z_2) >= \frac{1}{Z_{12}} + \frac{\theta_{12}\bar{\theta}_{12}}{2Z_{12}^2} \tag{14}$$

and,

$$< \xi(Z_1)\bar{\xi}(Z_2) >= \frac{1}{Z_{12}} + \frac{\theta_{12}\bar{\theta}_{12}}{2Z_{12}^2} \tag{15}$$

6 Conclusion

The key ingredient in our study of (super)W_3 symmetry in the framework of NLR method is to obtain a linear infinite dimensional algebra (sW_3^∞) from the standard non-linear (super)W_3 algebra by treating all composite generators of these algebras as independent generators [31,32]. The application of the well-known techniques of NLR method coupled with the ideas of IHE and CRP singles out a four dimensional geodesic super manifold (characterized by four chiral superfields) from the starting infinite dimensional coset superspace. The N=2 Boussinesq equations turn out to be the conditions on the super chiral fields leading to the embedding of the four dimensional geodesic manifold into the infinite dimensional coset super manifold. Thus, these integrable (Boussinesq) equations acquire a transparent geometrical meaning in the framework of NLR method.

By using the ideas of NLR method we have been able to provide geometrical meaning to some 2D integrable equations such as: (super)Liouville equations[30], Toda field equations[31] and (modified)Boussinesq equations[32]. We are sure that, on similar grounds, one can also obtain NLR of (super)conformal algebras which would lead to the emergence of (modified) KdV and super KdV equations. Our present work, in addition to our preveious works, provides hope to look for a common geometrical origin of all 2D integrable systems which, in turn, are very intimately connected with the W_N algebras.

At the moment, we do not see any principal difficulties in the extension of our work to other W_N symmetries associated with algebras $sl(N,R)$ as well as with the ones belonging to the other Cartan series such as $so(N)$ etc. We expect that there is one-to-one correspondence between NLR of (super)W_N^∞ and related two-dimensional Toda field theories. It would be interesting to provide geometrical basis for all the (super) Toda field theories in future.

Appendix

$$[J_n, G_r] = -\tfrac{1}{4}G_{n+r} \qquad [J_n, U_r] = -\tfrac{1}{4}U_{n+r}$$
$$\left[J_n, \widetilde{L}_m\right] = 0 \qquad [J_n, J_m] = \tfrac{nc}{64}\delta_{n+m,0}$$

$$[L_n, G_r] = \left(\frac{n}{2} - r\right) G_{n+r} \qquad [L_n, U_r] = \left(\frac{3n-2r}{2}\right) U_{n+r}$$
$$[L_n, W_m] = (2n - m)W_{n+m} \qquad [L_n, J_m] = -mJ_{n+m}$$

$$[L_n, L_m] = (n-m)L_{n+m} + \frac{3c}{48}\left(n^3 - n\right)\delta_{n+m,0}$$

$$\left[L_n, \widetilde{L}_m\right] = (n-m)\widetilde{L}_{n+m} \qquad [W_n, G_r] = \left(2r - \frac{n}{2}\right) U_{n+r}$$
$$[W_n, J_m] = -\frac{m}{2}\widetilde{L}_{n+m}$$

$$\left[\widetilde{L}_n, \widetilde{L}_m\right] = (n-m)\left[5\widetilde{L}_{n+m} - 4L_{n+m} + 16J^2_{n+m}\right] - \frac{c}{12}\left(n^3 - n\right)\delta_{n+m,0}$$

$$\left[\widetilde{L}_n, G_r\right] = U_{n+r} \qquad \{U_r, U_s\} = \frac{1}{c}(60GU - 32G\partial G)_{r+s}$$

$$\{U_r, \bar{U}_s\} = (s-r)\left[W_{r+s} + (2r^2 - 2s - 5)J_{r+s}\right] + \left(3s^2 + 3r^2 - 4rs - \frac{9}{2}\right)\left[L_{r+s} - \frac{1}{2}\widetilde{L}_{r+s}\right]$$
$$+\frac{c}{8}\left(r^2 - \frac{1}{4}\right)\left(r^2 - \frac{9}{4}\right)\delta_{r+s,0} + \left(\phi_4^{(0)}\right)_{r+s} + \left(r + \frac{3}{2}\right)\left[\phi_3^{(0)} - 16\left(r + \frac{1}{2}\right)J^2\right]_{r+s}$$

$$\left[W_n, \widetilde{L}_m\right] = (n-2m)W_{n+m} + m\left(m^2 - 1\right)J_{n+m} + \left(\phi_4^{(1)}\right)_{n+m} + (n+2)\left(\phi_3^{(1)}\right)_{n+m}$$
$$\left[\widetilde{L}_n, U_r\right] = (3n-2r)\left(U_{n+r} + nG_{n+r}\right) - \left(r^2 - \frac{9}{4}\right)G_{n+r} + \left(\psi_{\frac{7}{2}}^{(0)}\right)_{n+r} - 8(n+1)(JG)_{n+r}$$

$$[W_n, W_m] = -\frac{c}{16}(n^3 - n)(n^2 - 4)\delta_{n+m,0} + \left(2L_{n+m} - \frac{1}{2}\widetilde{L}_{n+m}\right)(n-m)\left(4 - n^2 - m^2 + \frac{mn}{2}\right)$$
$$+ (\phi_5)_{n+m} + (n+2)\left\{\phi_4^{(2)} + 4(+1)\left[2nJ^2 + 3\partial J^2\right]_{(n+r)}\right\}$$

$$[W_n, U_r] = \left(r^2 + \frac{n^2}{2} - nr - \frac{5}{4}\right)U_{n+r} + \left\{\frac{n^3}{2} - 2r^3 + \frac{3}{2}nr^2 - n^2r - \frac{19}{8}n + \frac{9}{2}r\right\}G_{n+r}$$
$$+ \left(\psi_{\frac{9}{2}}\right)_{n+r} + (n+2)\left[\left(\psi_{\frac{7}{2}}^{(1)}\right) - 4(n+1)(JG)\right]_{(n+r)}$$

Where all the composite

$$\begin{aligned}\phi_4^{(0)} &= 4\left(WJ - G\bar{U} - U\bar{G}\right) - 9\left(\widetilde{T} - T\right)T + \partial J\left(14\widetilde{T} - 10T - 12\partial J\right)\\ &\quad -4\partial\left(J^2\right) + 14J\partial\widetilde{T} - 20J\partial T - 9\partial G\bar{G} - 4G\partial\bar{G}\end{aligned}$$

$$\begin{aligned}\phi_3^{(0)} &= 28J\widetilde{T} - 16\partial\left(J^2\right) - 40JT - 13G\bar{G}\\ \phi_4^{(1)} &= 32\partial J\left(\widetilde{T} - T\right) + 12J\partial\widetilde{T} - 16J\partial T - 10\partial\left(G\bar{G}\right) - 5\left(U\bar{G} - G\bar{U}\right)\\ \phi_3^{(1)} &= 28J\widetilde{T} - 32JT - 15G\bar{G}\\ \phi_5 &= 7\left(\partial^2 G\bar{G} - G\partial^2\bar{G}\right) - 64J\partial^3 J + \partial\left[12JW - 16T^2 + 8T\widetilde{T} + \frac{11}{2}\left(G\bar{U} - U\bar{G}\right) + 48J\partial^2 J\right]\\ \phi_4^{(2)} &= 24\left[JW - 3(\partial J)^2 - J\partial^2 J\right] - 32T^2 + 16T\widetilde{T} + \frac{41}{6}\left(G\bar{U} + U\bar{G}\right) - \frac{92}{3}G\partial\bar{G} - \frac{308}{3}\partial G\bar{G}\end{aligned}$$

$$\begin{aligned}\psi_{\frac{9}{2}} = 5\left[WG - \frac{1}{9}(T + 2\widetilde{T})U - 4\partial^2 JG + \frac{26}{7}G\partial T\right] &+ 4\left[3J\partial U + 2J\partial^2 G + 3\partial J\partial G - 4(\partial J)^2\right]\\ &- \frac{93}{14}G\partial T_2 + 22\partial JU + \frac{1}{7}\left(158T - 64\widetilde{T}\right)\partial G\end{aligned}$$

$$\begin{aligned}\psi_{\frac{7}{2}}^{(1)} &= 26JU + 30J\partial G - 33\partial JG - 16J\partial J + \frac{71}{2}TG - \frac{27}{2}\widetilde{T}G\\ \psi_{\frac{7}{2}}^{(0)} &= 4JU - 2\partial JG - 4J\partial G + 9\left(\widetilde{T} - T\right)G\end{aligned}$$

All the other (anti) commutations and compositi filds can be calculated by following conjugatings rules

$$\begin{gathered}L^+ = -L, W^+ = W, J^+ = J, G^+ = i\bar{G},\\ \bar{G}^+ = iG, U^+ = -i\bar{U}, \bar{U}^+ = -iU, \widetilde{L}^+ = -\widetilde{L},\end{gathered}$$

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N = 2 супер W_3 алгебра и N = 2 уравнения Буссинеска

Получено N = 2 суперсимметричное уравнение Буссинеска чисто геометрическим путем из нелинейной реализации N = 2 супер W_3 алгебры. Для этого уравнения получена вторая гамильтонова структура и установлена связь с суперполевой реализацией N = 2 супер W_3.

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N = 2 Super W_3 Algebra and N = 2 Boussinesq Equations

We study N = 2 super W_3 algebra in the framework of nonlinear realization method and provide a geometric basis for the derivation of N = 2 super Boussinesq equation in terms of four spin – 1/2 chiral superfields. We obtain second Hamiltonian structure for these equations and establish the correspondence with the superfield realization of N = 2 super W_3.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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