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PHASE STRUCTURE OF $(\phi^4)_3$ FIELD THEORY AT FINITE TEMPERATURE

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1 Introduction

We will investigate the phase structure of the following quantum field models:

$$L(x) = -\frac{1}{2}\varphi(x)\left(\partial_\mu\partial^\mu + m^2\right)\varphi(x) - \frac{g}{4}\varphi^4(x)\ , \tag{1.1}$$

$$L(x) = -\frac{1}{2}\varphi(x)\left(\partial_\mu\partial^\mu - \frac{1}{2}m^2\right)\varphi(x) - \frac{g}{4}\varphi^4(x)\ , \tag{1.2}$$

in space-time R^3 at finite temperature T. Here $x = (\mathbf{x}, t), \mathbf{x} = (x_1, x_2)$.

The Lagrangians (1.1) and (1.2) describe a one-component scalar field and these Lagrangians are invariant under transformation $\varphi \to -\varphi$. The parameters m and g are positive.

If the dimensionless parameters

$$G = \frac{g}{2\pi m} \qquad \theta = \frac{T}{m}$$

are small enough, the Lagrangian (1.1) describes in quantum theory the symmetric under $\varphi \to -\varphi$ interaction, while the Lagrangian (1.2) corresponds to the situation of spontaneous symmetry breaking.

Above stated models are the simple but nontrivial objects for investigation of dynamical symmetry reconstruction. This phenomenon is realized in many profound four-dimensional quantum field theories (QFT) (see, *e.g.* [1]). Very physical approach to the problem is provided by variational estimation of an effective potential [2]. Meanwhile, usefulness of variational approach in QFT is restricted by some problems (see Feunman's paper in [3]). The Hamiltonians of the models (1.1), (1.2) are not the well-defined operators in the Hilbert space for $d > 2$ due to the higher order ultraviolet (UV) divergences. As a result, variational estimations with the help of the trial wave functionals is not defined either [4]. Another problem arises from impossibility of controlling the approximation accuracy directly within variational calculations [5].

Here we continue the investigations, undertaken in [6]-[8]. The phase structure of various scalar field models at zero temperature was considered in [6, 7]. Thermal effects in two-dimensional systems (1.1), (1.2) was investigated in [8].

The essence of our approach consists in combination of two powerful methods of QFT: canonical transformations and renormalization group. The idea of such a combination originates from the fundamental properties of the local QFT: existence of nonequivalent representations of canonical commutation relations (CCR) and UV divergences (e.g., see [9] and references therein). From a physical viewpoint, nonequivalent representations mean that the ground state of the QFT system is not unique. At the same time, vacuum instability originates from the radiative corrections to the physical parameters of the system. Renormalization (R) means actually taking into account the leading radiative corrections. Hence, the R-structure of the theory should contain the main (at least qualitative) information about its vacuum structure (see also [10]).

According to this intuitive motivation, our starting points are

- the phases appear in QFT as nonequivalent CCR representations,
- the renormalization structure of the theory contains basic information about its phase structure.

It is well known how to construct an appropriate QFT, if the coupling constant G is small enough and temperature is equal to zero. The standard canonical quantization in representation, given by the Fock space of scalar particles with renormalized mass m, should be performed. Having this in our mind, we want to know what is our system for other values of G and θ? We formulate the problem as follows:

what representation of CCR is suitable for different values of G and θ and what physical picture corresponds to this representation?

The apparatus of thermo field dynamics (TFD) [11, 12] provides a natural way to take into account the thermal effects within canonical quantization approach.

Detailed description of the method we use can be found in [6]-[8]. Briefly it can be formulated as follows. By virtue of canonical transformations a set of representations is introduced in such a way that the Hamiltonian has the "correct" form in any of this representations. It means that

$$H = H_0 + H_I + H_{ct} + VE .$$

Here H_0 is the standard free Hamiltonian. The interaction Hamiltonian H_I contains the field operators in degree more than two. The counterterm operator H_{ct} is defined by H_0 and H_I and corresponds to the same R-scheme in all representations (for detales see [7]). We will use zero momentum R-scheme in this paper. The quantity E relates to the free energy density F like

$$F = E - TS ,$$

where S is the entropy density. Then we select representation with the lowest free energy and smallest effective coupling constant

$$G_{eff} = \frac{g}{2\pi M(G,\theta)} .$$

The demands of the correct form of the total Hamiltonian and the criterion of the weak effective coupling relate to the conventional scattering picture of QFT. The Hamiltonian H_0 describes the noninteracting asymptotical fields. The Hamiltonian H_I describes the scattering of the particles and it should not contain quadratic and linear terms because they do not lead to any nontrivial scattering but only redefine

the parameters of the free Hamiltonian. Conventional perturbation expansion for scattering amplitudes becomes reasonable only if the effective (perturbation) coupling constant is small enough. Thus we consider the representation as suitable if the total Hamiltonian has the correct form and the effective coupling is weak. Moreover, we can use G_{eff} to control an accuracy of approximation. At the same time, energy E does not contribute to the S-matrix elements and it has not important physical meaning.

Our calculations in [6]-[8] show that both criteria give the same result. That are the reasons why we will not calculate energy E in this paper. Our analysis is based on the comparison of the effective coupling constants for different CCR representations.

We can summarize our main conclusions as follows. There are two symmetric (S) phases and one phase with broken symmetry (BS) for both models (1.1),(1.2) (see Figs.2,3). At the small coupling constant G system (1.1) is symmetric for any temperature and the symmetry is restored in system (1.2) at high temperature. Such a picture agrees with a motivation, arising from the perturbative calculation of the effective potential [1]. The phase structure of the models is more complicated at large G, but in any case both systems are in the S-phase, if the temperature or the coupling constant is large enough. Comparison with the two-dimensional case shows a crucial influence of higher order renormalization on the phase structure of the models. The method we use provides a procedure for determination the dependence of the mass on temperature and coupling constant. In other words, we obtain the Hamiltonian for doing the perturbative calculations with G_{eff} as the small perturbation parameter. This procedure is accurate only outside the critical region.

To our knowledge, nobody investigated the models (1.1), (1.2) in R^3 at nonvanishing temperature. A comparison of the results of different approaches (including our one) for zero temperature one can find in [6, 8]. Thermal effects in the two-dimensional model (1.2) are considered in [13].

2 Hamiltonian φ^4 in R^3

2.1 Hamiltonian at zero temperature

It is convenient to deal with the following Lagrangian density:

$$L(x) = \frac{1}{2}\varphi(x)\left(\partial_\mu\partial^\mu - m^2\right)\varphi(x) - g_3\varphi^3(x) - \frac{g_4}{4}\varphi^4(x) , \qquad (2.1)$$

where g_3 and g_4 are the coupling constants.

The quantized Hamiltonian for the Lagrangian (2.1) has the form:

$$\mathrm{H} = \mathrm{H}_0 + \mathrm{H}_I + \mathrm{H}_{ct},$$

$$\mathrm{H}_0[\varphi,\pi] = \frac{1}{2}\int_V d\mathbf{x} : \left[\pi^2(\mathbf{x}) + (\nabla\varphi(\mathbf{x}))^2 + m^2\varphi^2(\mathbf{x})\right] : ,$$

$$\mathrm{H}_I[\varphi,\pi] \;=\; \int\limits_V d\mathbf{x} : \left[\frac{1}{4}g_4\varphi^4(\mathbf{x}) + g_3\varphi^3(\mathbf{x})\right] : , \qquad (2.2)$$

where

$$\begin{aligned}
\varphi(\mathbf{x}) &= \int \frac{d\mathbf{k}}{2\pi}\frac{1}{\sqrt{2\omega}}\left[a(\mathbf{k})e^{i\mathbf{kx}} + a^+(\mathbf{k})e^{-i\mathbf{kx}}\right] , \\
\pi(\mathbf{x}) &= \frac{1}{i}\int \frac{d\mathbf{k}}{2\pi}\sqrt{\frac{\omega}{2}}\left[a(\mathbf{k})e^{i\mathbf{kx}} - a^+(\mathbf{k})e^{-i\mathbf{kx}}\right] , \qquad (2.3)\\
\omega(\mathbf{k}) &= \sqrt{\mathbf{k}^2+m^2} , \quad [a(\mathbf{k}), a^+(\mathbf{k}')] = \delta(\mathbf{k}-\mathbf{k}') .
\end{aligned}$$

The fields φ, π are the canonical variables and they obey the commutation relations

$$[\varphi(\mathbf{x}), \pi(\mathbf{y})] = i\delta(\mathbf{x}-\mathbf{y}) .$$

Creation and annihilation operators a^+, a act on the Fock space with vacuum vector defined as

$$a(\mathbf{k})|o\rangle = 0 \quad \forall \mathbf{k} . \qquad (2.4)$$

An integration in Eq.(2.2) is performed over a large "volume" V. All operators in Eq.(2.2) are taken in the form of normal product in respect to the vacuum (2.4). The theory contains ultraviolet divergences, but it is super-renormalizable, i. e., it has only a finite number of divergent Feynman diagrams, representing in Fig.1 (it should be remained that we will not deal with the vacuum diagrams). In order to remove the divergences we should introduce in the Hamiltonian an operator H_{ct} containing counter-terms which cancel these divergences in perturbation calculations. The operator H_{ct} for the zero momentum R-scheme looks like

$$\begin{aligned}
\mathrm{H}_{ct}(m) &= \int d\mathbf{x} : \left[\frac{1}{2}A_0(m)\varphi^2(\mathbf{x}) + C_0(m)\varphi(\mathbf{x})\right] : , \qquad (2.5)\\
A_0(m) &= 3!g_4^2\Sigma_0(m) , \\
C_0(m) &= 3!g_3g_4\Sigma_0(m) .
\end{aligned}$$

Here and below index '0' denotes zero temperature.

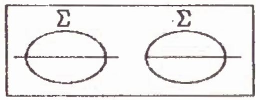

Fig.1 Divergent diagrams in R^3

These counter-terms can be computed, for example, in the Euclidean metrix where the propagator of the field takes the form:

$$\Delta_0(x,m) = \int \frac{d^3k}{(2\pi)^3}\frac{e^{-ikx}}{k^2+m^2} = \frac{1}{4\pi}\frac{e^{-mr}}{r} ,$$

where $x = (\mathbf{x}, x_3 = it)$, $r = \sqrt{\mathbf{x}^2 + x_3^2}$, $k^2 = \mathbf{k}^2 + k_3^2$.

We have

$$\Sigma_0(m) \;=\; reg\int d^3x\Delta_0^3(x,m) = \frac{1}{(4\pi)^2}reg\int_0^\infty \frac{ds}{s}e^{-3ms} . \qquad (2.6)$$

Here an appropriate regularization has to be introduced. We can now say that the theory (2.1) is defined, i. e., the S-matrix

$$S = T\exp\left\{-i\int dt\,[H_I(t) + H_{ct}(t)]\right\}$$

is finite in each order of perturbation expansion.

3 Thermal effects

3.1 Canonical transformation

Our aim is to realize what is the system (2.1) for different values of the coupling constants g_3, g_4 and temperature T. We will use the method of canonical transformations.

For the first step let us perform the canonical transformation:

$$\pi(\mathbf{x}) \to \pi_t(\mathbf{x}) , \quad \varphi(\mathbf{x}) \to \varphi_t(\mathbf{x}) + B , \qquad (3.1)$$

where the fields φ_t and π_t have the form (2.3) but their mass is equal to

$$M = m\cdot t .$$

The constant B has a meaning of vacuum condensate. The transformation (3.1) can be represented in terms of creation and annihilation operators in the following way [9]:

$$a(\mathbf{k}) \to a(\mathbf{k},t) - 2\pi mB\delta(\mathbf{k}) = U_2^{-1}(t)U_1^{-1}(B)a(\mathbf{k})U_1(B)U_2(t) ,$$

where

$$\begin{aligned}
U_1(B) &= \exp\left\{-2\pi mB\int d\mathbf{k}\delta(\mathbf{k})[a(\mathbf{k}) - a^+(\mathbf{k})]\right\} , \\
U_2(t) &= \exp\left\{\frac{1}{2}\int d\mathbf{k}\lambda(\mathbf{k},t)[a(-\mathbf{k})a(\mathbf{k}) - a^+(\mathbf{k})a^+(-\mathbf{k})]\right\} . \qquad (3.2)
\end{aligned}$$

U_1-transformation shifts the field φ at the constant B. The transformation U_2 can be represented in the form

$$\begin{aligned}
a(\mathbf{k},t) &= a(\mathbf{k})\cosh(\lambda) - a^+(-\mathbf{k})\sinh(\lambda) \\
a^+(\mathbf{k},t) &= a^+(\mathbf{k})\cosh(\lambda) - a(-\mathbf{k})\sinh(\lambda) \qquad (3.3)
\end{aligned}$$

with the inverse transformation given by

$$a(\mathbf{k}) = a(\mathbf{k},t)\cosh(\lambda) + a^+(-\mathbf{k},t)\sinh(\lambda)$$
$$a^+(\mathbf{k}) = a^+(\mathbf{k},t)\cosh(\lambda) + a(-\mathbf{k},t)\sinh(\lambda)\ . \qquad (3.4)$$

If the parameter λ is chosen like

$$\lambda(\mathbf{k},t) = \frac{1}{2}\ln\left(\frac{\omega(\mathbf{k})}{\omega(\mathbf{k},t)}\right)\ ,\quad \omega(\mathbf{k},t) = \sqrt{\mathbf{k}^2 + m^2t^2}\ ,$$

then using Eqs.(3.4) we get representation for the fields φ_t, π_t with the mass M in terms of the operators $a(\mathbf{k},t), a^+(\mathbf{k},t)$. These operators act on the Fock space with the vacuum vector defined by the relations

$$|o(t,B)\rangle\rangle = U_2^{-1}(t)U_1^{-1}(B)|o\rangle\ ,\quad a(\mathbf{k},t)|o(t,B)\rangle\rangle = 0\ \ \forall\mathbf{k}\ . \qquad (3.5)$$

For $B \neq 0$ and $t \neq 1$ representations of CCR defined by Eqs.(2.4),(3.5) are unitary nonequivalent. Although the operators such as (3.2) are only defined in finite volume, they are so useful, for example, in determining the forms of transformations (3.3) that we will freely use them in the following discussion with the implicit understanding of the space cutoff. It is only important for us that such operators determine the canonical transformations.

Then we express the Hamiltonian (2.2) in the new canonical variables, go to the normal ordering of the operators $a(\mathbf{k},t), a^+(\mathbf{k},t)$ and introduce counter-terms, determined by the new representation of the free Hamiltonian within the same zero-momentum R-scheme. As a result we get the expression:

$$\mathrm{H} = \mathrm{H}_0' + \mathrm{H}_I' + \mathrm{H}_{ct}' + \mathrm{H}_1\ ,$$

where

$$\mathrm{H}_0' = \frac{1}{2}\int_V d\mathbf{x} : \left[\pi_t^2(\mathbf{x}) + (\nabla\varphi_t(\mathbf{x}))^2 + M^2\varphi_t^2(\mathbf{x})\right] :\ ,$$
$$\mathrm{H}_I' = \int_V d\mathbf{x} : \left[\frac{1}{4}h_4\varphi_t^4(\mathbf{x}) + h_3\varphi_t^3(\mathbf{x})\right] :\ , \qquad (3.6)$$
$$h_3 = g_3 + g_4B\ ,\quad h_4 = g_4\ .$$

The operator H_{ct}' is given by Eqs.(2.5),(2.6), where we should substitute:

$$\varphi \to \varphi_t\ ,\quad m \to M\ ,\quad g_3 \to h_3\ ,\quad g_4 \to h_4\ .$$

The operator H_1 looks like

$$\mathrm{H}_1 = \int_V d\mathbf{x} : \left[\frac{1}{2}R(t,B)\varphi_t^2(\mathbf{x}) + P(t,B)\varphi_t(\mathbf{x})\right] :\ ,$$
$$R = m^2 - M^2 + 3g_4\left(B^2 - D_0\right) + 6g_3B + 6g_4^2\left(\Sigma_0(m) - \Sigma_0(M)\right)\ ,$$
$$P = m^2B + g_4\left(B^3 - 3BD_0\right) + 3g_3\left(B^2 - D_0\right) + \qquad (3.7)$$
$$6g_4\left(g_3 + g_4B\right)\left(\Sigma_0(m) - \Sigma_0(M)\right)\ ,$$

where

$$D_0(t) = \Delta_0(0,m) - \Delta_0(0,M) = \int\frac{d\mathbf{k}}{(2\pi)^2}\left[\frac{1}{\omega(\mathbf{k})} - \frac{1}{\omega(\mathbf{k},t)}\right] = \frac{m}{4\pi}(t-1)\ .$$

Using Eqs.(2.6) one can get

$$\Sigma_0(m) - \Sigma_0(M) = \frac{1}{(4\pi)^2}\ln t\ . \qquad (3.8)$$

The next step should be made to introduce the temperature in our consideration.

3.2 Thermo field dynamics

The detailed description of TFD formalism can be found for example in [11, 12]. The principal point consists in the doubling of the field variables at nonzero temperature. In our case it means that we should introduce the following operators

$$\alpha(\mathbf{k}) = a(\mathbf{k},t)\otimes\mathbf{1}\ ,\quad \tilde\alpha(\mathbf{k}) = \mathbf{1}\otimes a(\mathbf{k},t)\ , \qquad (3.9)$$

which satisfy CCR:

$$[\alpha(\mathbf{k}),\alpha^+(\mathbf{k}')] = \delta(\mathbf{k}-\mathbf{k}')\ ,\quad [\tilde\alpha(\mathbf{k}),\tilde\alpha^+(\mathbf{k}')] = \delta(\mathbf{k}-\mathbf{k}')\ ,$$

and act on the Fock space with vacuum vector given by

$$|o(t,B)\rangle\rangle\rangle = |o(t,B)\rangle\rangle\otimes|o(t,B)\rangle\rangle\ . \qquad (3.10)$$

The temperature dependent operators $\alpha(\mathbf{k},\beta)$, $\tilde\alpha(\mathbf{k},\beta)$ ($\beta = 1/T$) are introduced by the canonical transformation of the form

$$\alpha(\mathbf{k},\beta) = \alpha(\mathbf{k})\cosh(\xi) - \tilde\alpha^+(\mathbf{k})\sinh(\xi)$$
$$\tilde\alpha(\mathbf{k},\beta) = \tilde\alpha(\mathbf{k})\cosh(\xi) - \alpha^+(\mathbf{k})\sinh(\xi)\ , \qquad (3.11)$$

or the same in the operator form

$$\alpha(\mathbf{k},\beta) = U^{-1}(\beta)\alpha(\mathbf{k})U(\beta)\ ,\quad \tilde\alpha(\mathbf{k},\beta) = U^{-1}(\beta)\tilde\alpha(\mathbf{k})U(\beta)\ ,$$
$$U(\beta) = \exp\left\{\int d\mathbf{k}\xi(\mathbf{k},\beta)[\tilde\alpha(\mathbf{k})\alpha(\mathbf{k}) - \alpha^+(\mathbf{k})\tilde\alpha^+(\mathbf{k})]\right\}\ . \qquad (3.12)$$

The operators $\alpha(\mathbf{k},\beta)$, $\tilde\alpha(\mathbf{k},\beta)$ act on the temperature dependent Fock space with the vacuum vector

$$|o(\beta,t,B)\rangle\rangle = U^{-1}(\beta)|o(t,B)\rangle\rangle\rangle\ , \qquad (3.13)$$

moreover

$$\alpha(\mathbf{k},\beta)|o(\beta,t,B)\rangle\rangle = \tilde\alpha(\mathbf{k},\beta)|o(\beta,t,B)\rangle\rangle = 0\ \ \forall\mathbf{k},\beta\ .$$

The parameter $\xi(\mathbf{k},\beta)$ is defined by the condition:

$$\sinh^2(\xi) = n(\omega(\mathbf{k},t)) = [\exp\{\beta\omega(\mathbf{k},t)\} - 1]^{-1} \ .$$

In accordance with Eq.(3.9) the fields (Φ, Π) and $(\tilde{\Phi}, \tilde{\Pi})$ defined on the Fock space with vacuum (3.10) are introduced. The fields (Φ, Π) have the form of Eq.(2.3), where one should make the substitution

$$\omega(\mathbf{k}) \to \omega(\mathbf{k},t) \ , \quad a(\mathbf{k}) \to \alpha(\mathbf{k}) \ , \quad a^+(\mathbf{k}) \to \alpha^+(\mathbf{k}) \ .$$

The expressions for conjugated variables $\tilde{\Phi}$, $\tilde{\Pi}$ are obtained from Φ, Π by means of substitution $(\alpha, \ \alpha^+) \to (\tilde{\alpha}, \ \tilde{\alpha}^+)$.

To define the field variables on the Fock space with thermal vacuum (3.13) one can use the inversed transformation (3.11) and express the operators Φ, Π, $\tilde{\Phi}$, $\tilde{\Pi}$ in terms of the temperature dependent operators $\alpha(\mathbf{k},\beta)$, $\alpha^+(\mathbf{k},\beta)$, $\tilde{\alpha}(\mathbf{k},\beta)$, $\tilde{\alpha}^+(\mathbf{k},\beta)$.

The total Hamiltonian in TFD-formalizm looks like

$$\hat{H} = H - \tilde{H} \ ,$$

where

$$H = \mathrm{H} \otimes \mathbf{1} \ , \quad \tilde{H} = \mathbf{1} \otimes \mathrm{H} \ .$$

The Hamiltonian H in representation associated with thermal vacuum (3.13) takes the form:

$$H = H''_0 + H''_I + H''_{ct} + H'_1 \ ,$$

$$H''_0 = \frac{1}{2}\int_V d\mathbf{x} : \left[\Pi^2(\mathbf{x}) + (\nabla\Phi(\mathbf{x}))^2 + M^2\Phi^2(\mathbf{x})\right] : \ ,$$

$$H''_I = \int_V d\mathbf{x} : \left[\frac{1}{4}h_4\Phi^4(\mathbf{x}) + h_3\Phi^3(\mathbf{x})\right] : \ , \tag{3.14}$$

$$h_3 = g_3 + g_4 B \ , \quad h_4 = g_4 \ .$$

The symbol of normal product in these formulas relates to the temperature dependent operators $\alpha(\mathbf{k},\beta)$, $\alpha^+(\mathbf{k},\beta)$. The counter-term operator H''_{ct} in the representation associated with the thermal vacuum takes the form (see Appendix)

$$H''_{ct}(M,\beta) = \int d\mathbf{x} : \left[\frac{1}{2}A(M)\Phi^2(\mathbf{x}) + C(M)\Phi(\mathbf{x})\right] : \ , \tag{3.15}$$

temperature dependent functions A, C has the form:

$$A(M) = 3!g_4^2\Sigma(M) \ , \quad C(M) = 3!g_3g_4\Sigma(M) \ ,$$
$$\Sigma(M) = \Sigma_0(M) + 3\Sigma_\beta(M) + 3\Sigma_{\beta\beta}(M) \ ,$$
$$\Sigma_\beta(M) = \frac{1}{2(2\pi)^2}\sigma_\beta(t,\theta) \ , \quad \Sigma_{\beta\beta}(M) = \frac{1}{2(2\pi)^2}\sigma_{\beta\beta}(t,\theta) \ ,$$

$$\sigma_\beta(t,\theta) = -\ln 3 \cdot \frac{\theta}{t}\ln\left(1 - \exp\left\{-\frac{t}{\theta}\right\}\right) \ , \tag{3.16}$$

$$\sigma_{\beta\beta}(t,\theta) = \int_1^\infty\int_1^\infty dx\, dy \left[\exp\left\{\frac{xt}{\theta}\right\} - 1\right]^{-1}\left[\exp\left\{\frac{yt}{\theta}\right\} - 1\right]^{-1} \times\left[\frac{1}{\sqrt{4(x^2+y^2+xy)-3}} + \frac{1}{\sqrt{4(x^2+y^2-xy)-3}}\right] \ . \tag{3.17}$$

The operator H'_1 takes the form

$$H'_1 = \int_V d\mathbf{x} : \left[\frac{1}{2}R(t,B,\beta)\Phi^2(\mathbf{x}) + P(t,B,\beta)\Phi(\mathbf{x})\right] : \ ,$$

$$R = m^2 - M^2 + 3g_4\left(B^2 - D\right) + 6g_3B + 6g_4^2\left(\Sigma_0(m) - \Sigma(M)\right),$$
$$P = m^2B + g_4\left(B^3 - 3BD\right) + 3g_3\left(B^2 - D\right) + 6g_4\left(g_3 + g_4B\right)\left(\Sigma_0(m) - \Sigma(M)\right) \ , \tag{3.18}$$

where (see Appendix)

$$D(t,\beta) = \frac{m}{4\pi}\left[t - 1 + 2\theta\ln\left(1 - \exp\left\{-\frac{t}{\theta}\right\}\right)\right] \ . \tag{3.19}$$

The Hamiltonian $\tilde{H}$ is constructed according to the tilde substitution rule $\tilde{H} = H^*[\tilde{\Phi}, \tilde{\Pi}]$.

To define the parameters t, B we demand that

$$H'_1 = 0 \ \Leftrightarrow \ \begin{cases} R(t,B,\beta) = 0 \\ P(t,B,\beta) = 0 \ . \end{cases} \tag{3.20}$$

This requirement provides correct form of the Hamiltonian H (see Introduction) . From a physical viewpoint, it means that H describes the scalar particles with the mass M, which depends both on the coupling constants and temperature. This dependence is determined by Eqs.(3.20).

It is convenient to introduce the following dimensionless quantities

$$G_4 = \frac{g_4}{2\pi m} \ , \quad G_3 = \frac{g_3}{m\sqrt{4\pi m}} \ , \quad b = B\sqrt{\frac{4\pi}{m}} \ . \tag{3.21}$$

Using definitions (3.21) we represent Eqs.(3.18) in the form:

$$-\frac{1}{2}t^2 + \frac{1}{2} + \frac{3}{4}G_4\left(b^2 - d(t,\theta)\right) + 3G_3b + \frac{3}{4}G_4^2\left(\ln t - 6\sigma_\beta(t,\theta) - 6\sigma_{\beta\beta}(t,\theta)\right) = 0 \tag{3.22}$$
$$b + \frac{1}{2}G_4b\left(b^2 - 3d(t,\theta)\right) + 3G_3\left(b^2 - d(t,\theta)\right) + 3G_4\left(G_3 + \frac{G_4}{2}b\right)\left(\ln t - 6\sigma_\beta(t,\theta) - 6\sigma_{\beta\beta}(t,\theta)\right) = 0 \ ,$$

where

$$d(t,\theta) = t - 1 + 2\theta \ln\left(1 - \exp\left\{-\frac{t}{\theta}\right\}\right) . \tag{3.23}$$

Different solutions of these equations describe the possible phases of the system.

4 Phase structure

4.1 Symmetric model

Here we consider phase structure of the model with symmetric Lagrangian (1.1). This case corresponds to the choice

$$G_4 = G , \quad G_3 = 0 . \tag{4.1}$$

Using Eqs.(3.22) we obtain the equations for the symmetric model:

$$t^2 - 1 - \frac{3}{2}G\left(b^2 - d(t,\theta)\right) - \frac{3}{2}G_4^2\left(\ln t - 6\sigma_\beta(t,\theta) - 6\sigma_{\beta\beta}(t,\theta)\right) = 0 , \tag{4.2}$$
$$b\left[1 + \frac{1}{2}G\left(b^2 - 3d(t,\theta)\right) + \frac{3}{2}G^2\left(\ln t - 6\sigma_\beta(t,\theta) - 6\sigma_{\beta\beta}(t,\theta)\right)\right] = 0 .$$

Using the first equation one can get for the second one two solutions:

$$b = 0 \text{ (symmetric)} , \quad b^2 = \frac{t^2}{G} \text{ (nonsymmetric)} .$$

Let us consider the symmetric phase.

S-phase

Using Eqs.(4.2),(3.23) and (3.16) we obtain the following equation for t

$$2t^2 + 3Gt - 2 - 3G - 3G^2 \ln t + 18G^2\sigma_{\beta\beta}(t,\theta) \tag{4.3}$$
$$+6G\left(\theta - 3\ln 3 \cdot G\frac{\theta}{t}\right)\ln\left(1 - \exp\left\{-\frac{t}{\theta}\right\}\right) = 0 ,$$

where the function $\sigma_{\beta\beta}$ is given by Eq.(3.17). Eq.(4.3) has two solutions in the regions S_1, S_2 in Fig.2. Solutions are absent in the region BS.

High temperature asymptotics are determined by the linear over G term. This becomes clear, if one notices that (see Eq.(3.17)

$$\sigma_{\beta\beta}(t,\theta) \stackrel{\theta \gg t \gg 1}{\longrightarrow} C \cdot \frac{\theta^2}{t^2} + o\left(\frac{\theta}{t}\ln\frac{\theta}{t}\right) , \tag{4.4}$$
$$C = \int_1^\infty \int_1^\infty \frac{dx\, dy}{xy}\left[\frac{1}{\sqrt{4(x^2+y^2+xy)-3}} + \frac{1}{\sqrt{4(x^2+y^2-xy)-3}}\right]$$

and

$$\theta \gg \frac{\theta}{t} , \quad \theta \ln\theta \gg \frac{\theta^2}{t^2} .$$

Asymptotical behavior of the mass $M = m \cdot t$ and effective coupling constant $G_{eff} = G/t$ have the form

$$t \stackrel{\theta \gg G}{\longrightarrow} \sqrt{3G\theta\ln\theta} , \quad t \stackrel{G \gg \theta}{\longrightarrow} \sqrt{\frac{3}{2}G^2 \ln G} ,$$
$$G_{eff} \stackrel{\theta \gg G}{\longrightarrow} \sqrt{\frac{G}{3\theta\ln\theta}} \ll 1 , \quad G_{eff} \stackrel{G \gg \theta}{\longrightarrow} \sqrt{\frac{2}{3\ln G}} \ll 1 . \tag{4.5}$$

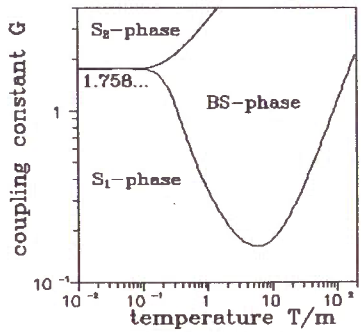

Fig.2 Phase diagram for model (1.1) in R^3

To make our description more detailed let us consider the case $\theta = 0$, which is described by the equation

$$2t^2 + 3Gt - 3G^2 \ln t - 2 - 3G = 0 .$$

We have here two solutions: $t_1(G,0) \equiv 1$ corresponding to the initial representation (2.2) and $t_2(G,0)$, moreover

$$t_2 < 1 \text{ if } G < G_c , \quad t_2 \geq 1 \text{ if } G \geq G_c ,$$

where

$$G_c = \frac{1}{2}\left(1 + \sqrt{\frac{19}{3}}\right) = 1.758... .$$

Free energy for the solution t_2 is negative for $G > G_c$, while it is equal to zero for t_1 [6] . Therefore both criteria show, that the point G_c corresponds to the phase transition from one S-phase to another. In order to be convinced of this statement let us consider the function $t_2(G,0)$ in the limit $G \to 0$. We obtain

$$t_2(G,0) \stackrel{G\to 0}{\longrightarrow} \exp\left\{-\frac{2}{3G}\right\} .$$

Nonanalyticity of the function t_2 (the mass M) at the point $G = 0$ shows that the difference between t_1 and t_2 can not be reduced to the perturbative corrections calculated within initial representation (2.2). Therefore we conclude that at zero temperature there are two S-phases and a kind of phase transition without symmetry rearrangement takes place for $G = G_c$. Let us return to nonzero temperature case. Analysis of Eq.(4.3) shows that for $(\theta, G) \in S_1, S_2$ there are two solutions, moreover

$$t_2(G,\theta) < t_1(G,\theta) \not\equiv 1 \text{ if } (\theta, G) \in S_1$$

$$t_2(G,\theta) > t_1(G,\theta) \not\equiv 1 \text{ if } (\theta, G) \in S_2 .$$

According to our criterion of the smallest effective coupling constant we should put

$$t(G,\theta) = \begin{cases} t_1(G,\theta), \text{ if } (\theta,G) \in S_1 \\ t_2(G,\theta), \text{ if } (\theta,G) \in S_2 . \end{cases} \quad (4.6)$$

We will note that at nonzero temperature neither t_1 nor t_2 correspond to the initial representation (2.2).

BS-phase

We have the following equations in this case

$$\begin{aligned} &b^2 = \frac{t^2}{G} \\ &t^2 - 3Gt + 3G^2 \ln t - 18G^2\sigma_{\beta\beta}(t,\theta) \\ &-6G\left(\theta - 3\ln 3 \cdot G\frac{\theta}{t}\right) \ln\left(1 - \exp\left\{-\frac{t}{\theta}\right\}\right) = 0 , \end{aligned} \quad (4.7)$$

where we used Eqs.(4.2),(3.23) and (3.16).

The second Eq.(4.7) has unique solution for any (θ, G). One can check, that asymptotical solutions like

$$1 \ll t \ll \theta \quad \text{or} \quad t \gg \theta \qquad \text{for } \theta \gg 1$$

are absent. This means that at high temperature $G_{eff}(G,\theta)$ is smaller in the S-phase, then in the BS-phase. We conclude that the system is symmetric at high temperature. Numerical solution of Eqs.(4.3), (4.7) shows that the same situation takes place for any $(\theta, G) \in S_1, S_2$.

The result of our consideration can be represented in the form of the phase diagram given in Fig.2. The phase boundaries in Fig.2 correspond to the phase transitions of the first order, since the parameter of order

$$\sigma = \pm\frac{t(G,\theta)}{\sqrt{G}}$$

is discontinuous on the boundaries. Asymptotical behavior of the effective coupling constant (see Eq.(4.5)) shows that our approach is accurate outside the critical region. At the same time, our method becomes rough in the domain of phase transitions, hence phase boundaries in Fig.2 are determined very approximately.

In any case we can summarize our conclusions as follows.

- Symmetry breaking is absent in three dimensional model (1.1) $\forall\, \theta$, if $G \ll 1$.
- There are two S-phases and one BS-phase, transitions with symmetry rearrangement take place at intermediate values of G, θ (Fig.2).
- The system is in the S-phase, if the temperature θ or coupling constant G is large enough.
- Outside the critical regions $G_{eff} \ll 1$ and one can do usual perturbative calculations, using the Hamiltonians (3.14), (3.15)

4.2 The two-well potential

In this section we consider the phase structure of the model (1.2). Equations defining the parameters t and b in this case are obtained from Eqs.(3.22) by means of the substitution

$$G_4 = G\ , \quad G_3 = \frac{1}{2}\sqrt{G}\ .$$

As a result we obtain:

$$\begin{aligned} &t^2 - 1 - \frac{3}{2}G\left(b^2 - d(t,\theta)\right) - 3\sqrt{G}b - \frac{3}{2}G^2\left(\ln t - 6\sigma_\beta(t,\theta) - 6\sigma_{\beta\beta}(t,\theta)\right) = 0 \\ &2b + Gb\left(b^2 - 3d(t,\theta)\right) + 3\sqrt{G}\left(b^2 - d(t,\theta)\right) \\ &\qquad +3G\sqrt{G}\left(1 + \sqrt{G}b\right)\left(\ln t - 6\sigma_\beta(t,\theta) - 6\sigma_{\beta\beta}(t,\theta)\right) = 0 . \end{aligned} \quad (4.8)$$

Two solutions for b follow from Eqs.(4.8):

$$b = -\frac{1}{\sqrt{G}} \text{ (symmetric)}, \quad b = -\frac{1}{\sqrt{G}} \pm \frac{t}{\sqrt{G}} \text{ (nonsymmetric)} .$$

Using the nonsymmetric solution for b we get the following equation

$$\begin{aligned} &t^2 - 3Gt - 1 + 3G + 3G^2 \ln t - 18G^2\sigma_{\beta\beta}(t,\theta) \\ &-6G\left(\theta - 3\ln 3 \cdot G\frac{\theta}{t}\right) \ln\left(1 - \exp\left\{-\frac{t}{\theta}\right\}\right) = 0 , \end{aligned} \quad (4.9)$$

while for the symmetric case ($b = -1/\sqrt{G}$) we obtain

$$\begin{aligned} &2t^2 + 3Gt + 1 - 3G - 3G^2 \ln t + 18G^2\sigma_{\beta\beta}(t,\theta) \\ &+6G\left(\theta - 3\ln 3 \cdot G\frac{\theta}{t}\right) \ln\left(1 - \exp\left\{-\frac{t}{\theta}\right\}\right) = 0 , \end{aligned} \quad (4.10)$$

where we used Eqs.(3.23) and (3.16).

S-phase
There are two solutions of Eq.(4.10) in the regions S_1, S_2 in Fig.3. Solutions are absent in the region BS. Asymptotical behavior of the mass $M = m \cdot t$ and effective coupling constant $G_{eff} = G/t$ is the same one, which is given in Eqs.(4.5).

BS-phase
Eq.(4.9) has unique solution for all (G, θ). Analysis of the asymptotical behavior and numerical solution of Eqs.(4.10), (4.9) and comparison of the effective coupling constants lead to the phase diagram represented in Fig.3. Phase boundary for $G \ll 1$ is in agreement with that one, which is expected from the perturbative calculation of an effective potential [1].

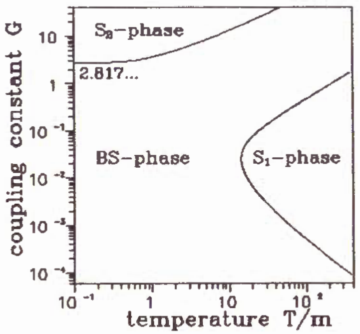

Fig.3 Phase diagram for model (1.2) in R^3

Our description is accurate outside the critical regions (see Eq.(4.5)). At the same time, phase boundary is determined very approximately, because the coupling constants are large enough in the domain of the phase transitions .

The parameter of order is discontinuous at the boundary, therefore the phase transitions are of the first order. We will stress that such a result can not be considered as well established in our approach.

Summary of the results obtained in this section is as follows.

- Symmetry is restored in the system (1.2), if the temperature or coupling constant is large enough.
- There is the phase transitions between BS- and S-phases. Phase boundary is shown in Fig.3.
- The method we used provides a procedure for determination the temperature dependence of the mass. This procedure is accurate outside the critical region, where one can do perturbative calculations using the Hamiltonians (3.14), (3.15). (This item is valid for both considered models.)

4.3 The systems in R^2 and R^3

We can now compare the phase structure of the models (1.1), (1.2) in the space-time R^3 and R^2 (see [8, 13]). The phase diagrams for the two-dimensional models are shown in Figs.4,5. One can see that the behavior of the systems in G-direction is completely different from that one in three-dimensional case (see Figs.2,3). We have BS-phase in the space-time R^2 and S-phase in R^3 for $G \gg 1$ irrespective of the symmetry of initial Lagrangians (1.1), (1.2).

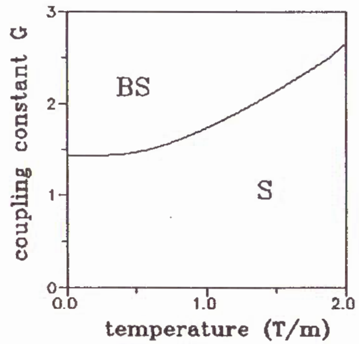

Fig.4 Phase diagram for model (1.1) in R^2

We can say that the different UV-behavior in R^2 and R^3 leads to the different phase structure. At the same time the behavior in the θ-direction is qualitatively the same both for R^2 and R^3. The systems are symmetric, if the temperature is high enough.

Appendix

Here we calculate divergent diagrams at finite temperature. Their temperature independent terms are given by Eq.(2.6). Therefore we should calculate only temperature dependent contributions. The propagator for the field Φ has the following form [12]

$$\tilde{\Delta}(k, M) = \tilde{\Delta}_0(k, M) + \tilde{\Delta}_\beta(k, M) ,$$
$$\tilde{\Delta}_0(k, M) = \frac{1}{k^2 - M^2 + i\epsilon} ,$$
$$\tilde{\Delta}_\beta(k, M) = -2\pi i \delta(k^2 - M^2) n(|k_0|) ,$$
$$n(|k_0|) = (\exp\{\beta |k_0|\} - 1)^{-1} .$$

First of all, let us calculate normal ordering divergent diagram

$$\Delta(0,M) = \Delta_0(0,M) + \Delta_\beta(0,M)\ ,$$

where the function $\Delta_0(0,M)$ is temperature independent. For the second term we have

$$\Delta_\beta(0,M) = \int \frac{d^3k}{(2\pi)^2}\delta(k^2 - M^2)n(|k_0|) =$$

$$\frac{1}{2\pi}\int_M^\infty dk_0\,(\exp\{\beta k_0\} - 1)^{-1} = \frac{T}{2\pi}\ln\left(1 - \exp\left\{-\frac{M}{T}\right\}\right)\ .$$

This expression contributes to Eqs.(3.19) and (3.23).

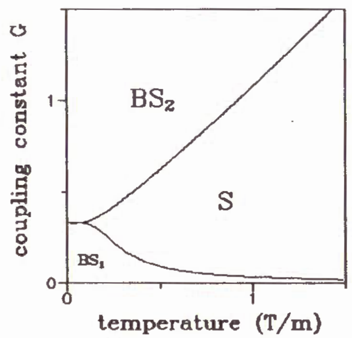

Fig.5 Phase diagram for model (1.2) in R^2

The function $\Sigma(M)$, corresponding to the diagrams in Fig.1, is given by the following expression (external momentum is equal to zero):

$$\Sigma(M) = i^2\iint \frac{d^3pd^3k}{(2\pi)^6}\tilde\Delta(p,M)\tilde\Delta(k,M)\tilde\Delta(k+p,M)$$

$$= \Sigma_0(M) + 3\Sigma_\beta(M) + 3\Sigma_{\beta\beta}(M) + \Sigma_{\beta\beta\beta}(M)\ .$$

The term Σ_0 is temperature independent, it is defined in Eq.(2.6). The other terms look like

$$\Sigma_\beta(M) = i^2\iint \frac{d^3pd^3k}{(2\pi)^6}\tilde\Delta_\beta(p,M)\tilde\Delta_0(k,M)\tilde\Delta_0(k+p,M)\ ,$$

$$\Sigma_{\beta\beta}(M) = i^2\iint \frac{d^3pd^3k}{(2\pi)^6}\tilde\Delta_\beta(p,M)\tilde\Delta_\beta(k,M)\tilde\Delta_0(k+p,M)\ ,$$

$$\Sigma_{\beta\beta\beta}(M) = i^2\iint \frac{d^3pd^3k}{(2\pi)^6}\tilde\Delta_\beta(p,M)\tilde\Delta_\beta(k,M)\tilde\Delta_\beta(k+p,M)\ .$$

These integrals are UV-finite. Let us calculate them. Σ_β can be represented in the form:

$$\Sigma_\beta(M) = i\int \frac{d^3p}{(2\pi)^3}\tilde\Delta_\beta(p,M)\Pi_0(p^2) = \Pi_0(M^2)\int \frac{d^3p}{(2\pi)^2}n(|p_0|)\delta(p^2 - M^2)\ ,$$

where

$$\Pi_0(p^2) = i\int \frac{d^3k}{(2\pi)^3}\tilde\Delta_0(k,M)\tilde\Delta_0(k+p,M)\ .$$

One can check that:

$$\Pi_0(M^2) = \frac{\ln 3}{8\pi M}\ .$$

Using the identity

$$\delta\left(p^2 - M^2\right) = \frac{\delta\left(\sqrt{p_0^2 - M^2} - |\mathbf{p}|\right)}{\sqrt{p_0^2 - M^2} + |\mathbf{p}|}$$

and integrating over the angular, $|\mathbf{p}|$ and p_0 we obtain:

$$\Sigma_\beta(M) = -\frac{\ln 3}{2(2\pi)^2}\frac{T}{M}\ln\left(1 - \exp\left\{-\frac{M}{T}\right\}\right)\ .$$

The next term $\Sigma_{\beta\beta}$ can be represented like

$$\Sigma_{\beta\beta}(M) = i\int \frac{d^3p}{(2\pi)^3}\tilde\Delta_\beta(p,M)\Pi_\beta(p_0,|\mathbf{p}|)\ ,$$

where

$$\Pi_\beta(p_0,|\mathbf{p}|) = i\int \frac{d^3k}{(2\pi)^3}\tilde\Delta_\beta(k,M)\tilde\Delta_0(k+p,M)\ .$$

After integration over $|\mathbf{p}|$ and angular we get:

$$\Sigma_{\beta\beta}(M) = \frac{1}{4\pi}\int_M^\infty dp_0 n(p_0)\left[\Pi_\beta\left(p_0,\sqrt{p_0^2 - M^2}\right) + \Pi_\beta\left(-p_0,\sqrt{p_0^2 - M^2}\right)\right]\ .$$

The function Π_β looks like

$$\Pi_\beta\left(p_0,\sqrt{p_0^2 - M^2}\right) = \frac{1}{(2\pi)^2}\int_0^\infty d|\mathbf{k}|\ |\mathbf{k}|\int_0^{2\pi} d\varphi\int_M^\infty dk_0\frac{n(k_0)\delta\left(|\mathbf{k}| - \sqrt{k_0^2 - M^2}\right)}{|\mathbf{k}| + \sqrt{k_0^2 - M^2}}$$

$$\times\left[\frac{1}{M^2 + 2k_0p_0 - 2\sqrt{p_0^2 - M^2}|\mathbf{k}|\cos\varphi} + \frac{1}{M^2 - 2k_0p_0 - 2\sqrt{p_0^2 - M^2}|\mathbf{k}|\cos\varphi}\right]\ ,$$

here $p_0 \geq M$. Integration over $|\mathbf{k}|$ and angular φ leads to the result:

$$\Pi_\beta\left(p_0,\sqrt{p_0^2-M^2}\right)=\frac{1}{4\pi M}\int\limits_M^\infty dk_0 n(k_0)$$
$$\times\left[\frac{1}{\sqrt{4\,(p_0^2+k_0^2+k_0p_0)-3M^2}}+\frac{1}{\sqrt{4\,(p_0^2+k_0^2-k_0p_0)-3M^2}}\right] .$$

At last, using this equation we obtain

$$\Sigma_{\beta\beta}(M)=\frac{1}{2(2\pi)^2 M}\int\limits_M^\infty\int\limits_M^\infty dk_0\; dp_0\; n(k_0)n(p_0)$$
$$\times\left[\frac{1}{\sqrt{4\,(p_0^2+k_0^2+k_0p_0)-3M^2}}+\frac{1}{\sqrt{4\,(p_0^2+k_0^2-k_0p_0)-3M^2}}\right] .$$

The last term $\Sigma_{\beta\beta\beta}$ can be written in the form:

$$\Sigma_{\beta\beta\beta}(M)=i\int\frac{d^3p}{(2\pi)^3}\tilde{\Delta}_\beta(p,M)\Pi_{\beta\beta}(p_0,|\mathbf{p}|)$$
$$=\frac{1}{4\pi}\int\limits_M^\infty dp_0 n(p_0)\left[\Pi_{\beta\beta}\left(p_0,\sqrt{p_0^2-M^2}\right)+\Pi_{\beta\beta}\left(-p_0,\sqrt{p_0^2-M^2}\right)\right] ,$$

where

$$\Pi_{\beta\beta}(p_0,\sqrt{p_0^2-M^2})=-i\int\frac{d^3k}{2\pi}n(|k_0|)n(|k_0+p_0|)\delta(k^2-M^2)\delta((k+p)^2-M^2)\;|_{p^2=M^2}$$
$$=-\frac{i}{2\pi}\int\limits_M^\infty dk_0\; n(k_0)\int\limits_0^{2\pi}d\varphi\left[\frac{n(k_0+p_0)}{M^2+2k_0p_0}\delta\left(1-2\frac{\sqrt{k_0^2-M^2}\sqrt{p_0^2-M^2}}{M^2+2k_0p_0}\cos\varphi\right)\right.$$
$$\left.+\frac{n(|k_0-p_0|)}{M^2-2k_0p_0}\delta\left(1-2\frac{\sqrt{k_0^2-M^2}\sqrt{p_0^2-M^2}}{M^2-2k_0p_0}\cos\varphi\right)\right] .$$

One can check that

$$0<2\frac{\sqrt{k_0^2-M^2}\sqrt{p_0^2-M^2}}{M^2+2k_0p_0}<1 ,$$
$$-1<2\frac{\sqrt{k_0^2-M^2}\sqrt{p_0^2-M^2}}{M^2-2k_0p_0}<0 ,\quad \forall k_0\in[M,\infty), p_0\in[M,\infty) ,$$

hence

$$\Pi_{\beta\beta}\left(p_0,\sqrt{p_0^2-M^2}\right)=0 .$$

As a consequence we obtain:

$$\Sigma_{\beta\beta\beta}(M)=0 .$$

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