

СООБЩЕНИЯ
ОБЪЕДИНЕННОГО
ИНСТИТУТА
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ
ДУБНА

E2-92-281

A.Z.Dubničková, S.Dubnička, P.Striženec

NEW EVALUATION OF HADRONIC CONTRIBUTIONS
TO THE ANOMALOUS MAGNETIC MOMENT
OF CHARGED LEPTONS

1992

1 Introduction

Since the charged leptons obey the Dirac equation, their magnetic moments are related to the spin by

$$\mu_\ell = g_\ell \frac{e}{2m_\ell \cdot c} \cdot \frac{\hbar}{2}, \quad (1)$$

where $g_\ell = 2$ theoretically. The g -factor reflects the point-like nature of the Dirac equation and deviations from this expected value are of great interest since they would indicate a substructure of the lepton under consideration. However, even in the absence of any intrinsic structure the existing in nature interactions modify the g -factor because of the emission and absorption of virtual photons (electromagnetic effects), intermediate vector bosons (weak interaction effects) and the vacuum polarization into virtual hadronic states (strong interaction effects).

It has become conventional to describe the modification of the g -factor by the magnetic anomaly defined by

$$a_\ell \equiv \frac{g_\ell - 2}{2} = a_\ell^{(1)} \cdot \frac{\alpha}{\pi} + (a_\ell^{(2)QED} + a_\ell^{(2)had}) \cdot \left(\frac{\alpha}{\pi}\right)^2 + a_\ell^{(2)weak} + O\left(\frac{\alpha}{\pi}\right)^3, \quad (2)$$

where $\alpha \approx 1/137$ is a fine structure constant.

In this paper we present a new more precise evaluation of the lowest-order hadronic vacuum-polarization contribution $a_\ell^{(2)had}$ ($\ell = e, \mu, \tau$) given by the Feynman diagram presented in Fig. 1. Reasons why it is worth while to realize this program are as follows.

There is under way at Brookhaven National Laboratory a new $g - 2$ muon experiment [1] expected to be performed with an accuracy of

$$\Delta a_\mu^{exp} = \pm 0.05 \times 10^{-8}, \quad (3)$$

which could provide a good test of the standard electroweak model by assuming that the hadronic part can be determined sufficiently precisely.

Because of precise QED calculations up to four loops [2], yielding

$$a_\mu^{QED} = (116584.80 \pm 0.03) \times 10^{-8} \quad (4)$$

and the one loop weak interaction contribution evaluation [3] as

$$a_\mu^{(2)weak} = 0.181 \times 10^{-8}, \quad (5)$$

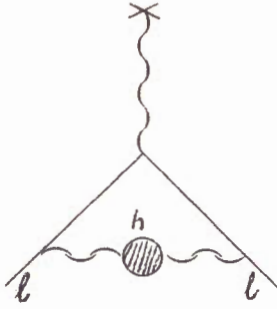


Fig. 1. The lowest-order hadronic vacuum polarization contribution to the anomalous magnetic moment of charged leptons $l = e, \mu, \tau$.

as well as highly accurate measurement [4] of the latest experimental value

$$a_{\mu}^{\text{exp}} = (116592.30 \pm 0.84) \times 10^{-8} \quad (6)$$

the lowest order hadronic vacuum-polarization contribution $a_{\mu}^{(2)had}$ (dominating among all other strong interaction contributions) is by far not negligible. The recent evaluations [2],[5],[6] show

$$a_{\mu}^{(2)had} = (7.070 \pm 0.060 \pm 0.170) \times 10^{-8} \quad (\text{KNO})[2] \quad (7)$$

$$a_{\mu}^{(2)had} = (7.100 \pm 0.105 \pm 0.049) \times 10^{-8} \quad (\text{CLY})[5] \quad (8)$$

$$a_{\mu}^{(2)had} = (7.052 \pm 0.060 \pm 0.046) \times 10^{-8} \quad (\text{MD})[6] \quad (9)$$

that are about eight times of the experimental uncertainty in (6). From the same results it is straightforward to see that the most precise up to now evaluation of $a_{\mu}^{(2)had}$ is achieved by Martinovič and Dubnička[6] with a total error to be about half as much as the one loop weak interaction contribution (5), while total errors of two previous evaluations [2],[5] are just comparable with (5).

In the next sections of this paper we show that owing to

- inclusion into the analysis of additional data on some exclusive processes (K^+K^- [7],[17], $K^0\bar{K}^0$ [8], 3π [9], $\omega\pi^0$ [9] and 4π [9])
- application of more accomplished models [10] for a description of the pion and kaon electromagnetic structure

- a more correct calculation of the external covariance matrix in an estimation of errors in 2π and $K\bar{K}$ channels
- application of the revised QCD formula [11],[12] for $R = \sigma_{tot}(e^+e^- \rightarrow had) / \sigma_{tot}(e^+e^- \rightarrow \mu^+\mu^-)$ with electroweak corrections [13] in order to evaluate a contribution to $a_{\ell}^{(2)had}$ from the high-energy region in a proper way

we are able to diminish the error $\Delta a_{\mu}^{(2)had}$ to be approximately four times as small as the one loop weak interaction contribution (5) and thus it becomes comparable with accuracy (3) expected in the new $g-2$ muon experiment at Brookhaven National Laboratory.

Since the procedure of evaluation of $a_e^{(2)had}$ and $a_{\tau}^{(2)had}$ is very similar to the evaluation of $a_{\mu}^{(2)had}$, we obtain new results on the latter quantities too.

The next section is devoted to a short review of the formalism used in the evaluation of $a_{\ell}^{(2)had}$ ($\ell = e, \mu, \tau$). Our practical treatment of $a_{\ell}^{(2)had}$ and the corresponding error analysis is given in Sect. 3, while conclusions and summary are given in Sect. 4.

2 The lowest-order hadronic vacuum-polarization contribution to a_{ℓ}

All evaluations of $a_{\ell}^{(2)had}$ ($\ell = e, \mu, \tau$) are based on the integral representation [14]

$$a_{\ell}^{(2)had} = \frac{1}{4\pi^3} \int_{4m_{\pi}^2}^{\infty} \sigma^h(s) K_{\ell}(s) ds \quad (10)$$

where $\sigma^h(s)$ stands for the total cross section $\sigma(e^+e^- \rightarrow had)$ and

$$K_{\ell}(s) = \int_0^1 \frac{x^2(1-x)}{x^2 + (1-x)s/m_{\ell}^2} ds. \quad (11)$$

The integral (11) can be calculated explicitly and its final form depends on the mass of lepton under consideration in relation to the pion mass m_{π} .

Since $m_e, m_\mu < m_\pi$, then for the electron and muon one obtains

$$K_{e,\mu} = \left(\frac{1}{2} - A\right) - \left(A - \frac{A^2}{2}\right) \ln A - \frac{A(A^2 - 4A + 2)}{2\sqrt{A^2 - 4A}} \ln \frac{[(2 - A) - \sqrt{a^2 - 4A}][-A + \sqrt{a^2 - 4A}]}{[(2 - A) + \sqrt{a^2 - 4A}][-A - \sqrt{a^2 - 4A}]} \quad (12)$$

for all s from the range $4m_\pi^2 < s < \infty$, where $A = s/m_{e,\mu}^2$.

On the other hand, as $m_\tau > m_\pi$, for the τ -lepton the explicit form of the integral (11) depends on the relation of s to $4m_\tau^2$.

If $4m_\pi^2 < s < 4m_\tau^2$, then

$$K_\tau(s) = \left(\frac{1}{2} - A\right) - \left(A - \frac{A^2}{2}\right) \ln A - \frac{A(A^2 - 4A + 2)}{\sqrt{4A - A^2}} \left[\arctan \frac{2 - A}{\sqrt{4A - A^2}} - \arctan \frac{-A}{\sqrt{4A - A^2}} \right] \quad (13)$$

and for all $s > 4m_\tau^2$ the $K_\tau(s)$ has an identical form with (12), where now $A = s/m_\tau^2$.

In this place we would like to note that equation (7) in ref.[6] and expression (A.3) in the second paper of ref.[2], even the explicit formula in ref.[15], are not completely correct, however they do not exert influence upon the final result remarkably.

So, the most important component in the evaluation of $a_\ell^{(2)had}$ is a rich experimental information on the total cross section $\sigma^h(s)$ from threshold to high energies employed either in the form of experimental points with errors or in the form of models used in a perfect description of the cross section $\sigma^h(s)$. Sometimes both of these approaches are taken to be complementary.

From the practical point of view it is advantageous to divide the integral in (10) into the low-energy ($4m_\pi < s < s_0$) and high-energy parts and to write it in the form

$$a_\ell^{(2)had} = \frac{1}{4m_\pi^3} \left\{ \int_{4m_\pi^2}^{s_0} \left[\sum_F \sigma^h(e^+e^- \rightarrow F) \right] K_\ell(s) ds + \int_{s_0}^{\infty} R(e^+e^- \rightarrow had) \sigma_{tot}(e^+e^- \rightarrow \mu^+\mu^-) K_\ell(s) ds \right\}, \quad (14)$$

where with the aim of a demonstration of an independence of final results of the choice of the value of s_0 , we left the latter to vary in the range $2\text{GeV}^2 < s_0 < 4\text{GeV}^2$. The sum in (14) goes over all known exclusive final states.

The variation of s_0 was enabled by the existence of an overlap of the data on the total cross sections of exclusive processes $e^+e^- \rightarrow F$ and the data on $R = \sigma_{tot}(e^+e^- \rightarrow had) / \sigma_{tot}(e^+e^- \rightarrow \mu^+\mu^-)$. Finally it provides a reliability test of an evaluation of (14).

We note before a presentation of the numerical results, that such a variation of s_0 gives, in the most interesting from the experimental point of view case of the muon, the change of the central value no more than half of the estimated total error $\Delta a_\mu^{(2)had}$.

The central value given by (10) can be determined in two ways. The function $K_\ell(s)$ can be taken in the explicit form (12) or (13), then we carry out a numerical integration only over one integral from $4m_\pi^2$ to ∞ , or $K_\ell(s)$ is taken in the integral form (11), then we carry out a numerical integration over two integrals simultaneously. Practically we have verified that both ways lead to the same results if sufficient precision in numerical integration is attained.

In order to achieve realistic and quantitative error estimates, we have included in our analysis the uncertainties coming from the experimental input as well as the ones induced by the models used for the cross section $\sigma^h(s)$. Then the total error $\Delta a_\ell^{(2)had}$ consists of three parts as follows

$$\Delta a_\ell^{(2)had} = \sqrt{[\Delta a_\ell^{(2)had}(\text{stat})]^2 + [\Delta a_\ell^{(2)had}(\text{syst})]^2 + [\Delta a_\ell^{(2)had}(\text{mod})]^2} \quad (15)$$

where $\Delta a_\ell^{(2)had}(\text{stat})$ is a statistical error, $\Delta a_\ell^{(2)had}(\text{syst})$ is a systematic error and $\Delta a_\ell^{(2)had}(\text{mod})$ is the model error defined as a deviation of the value of a contribution obtained by integration over experimental points using the trapezoidal rule and by the integration based on the model parametrization. All $\Delta a_\ell^{(2)had}(\text{stat})$, $\Delta a_\ell^{(2)had}(\text{syst})$ and $\Delta a_\ell^{(2)had}(\text{mod})$ consist of sums of the corresponding errors evaluated in all exclusive channels, including also the high energy region contribution determined by R . Here we meet two types of situations. First, $\sigma^h(s)$ (or R) is given by data with errors and simultaneously we have a well founded model for a description of the latter. In this case we evaluate the error to be compound of statistical and systematic errors, and independently we

evaluate the model error. Second, $\sigma^h(s)$ is given only by data with errors i.e. there is no satisfactory model able to describe the latter. In this case we evaluate only statistical and systematic errors to be compound into so-called experimental errors.

3 Numerical evaluation of contributions from exclusive channels and high energy region

First, we concentrate on the evaluation of contributions to $a_\ell^{(2)had}$ from the low-energy region given by the first integral in (14), where the sum contains total cross sections of exclusive processes as follows

$$\begin{aligned} \sum_F \sigma^h(e^+e^- \rightarrow F) &= \sigma_{tot}(e^+e^- \rightarrow \pi^+\pi^-) + \sigma_{tot}(e^+e^- \rightarrow K^+K^-) + \\ &+ \sigma_{tot}(e^+e^- \rightarrow K^0\bar{K}^0) + \sigma_{tot}(e^+e^- \rightarrow \pi^+\pi^-\pi^0) + \\ &+ \sigma_{tot}(e^+e^- \rightarrow \omega\pi^0) + \sigma_{tot}(e^+e^- \rightarrow \pi^+\pi^-2\pi^0) + \quad (16) \\ &+ \sigma_{tot}(e^+e^- \rightarrow 2\pi^+2\pi^-) + \sigma_{tot}(e^+e^- \rightarrow 2\pi^+2\pi^-\pi^0) + \\ &+ \sigma_{tot}(e^+e^- \rightarrow 3\pi^+3\pi^-). \end{aligned}$$

All other known exclusive channels are not considered here because they give negligible contributions on the level of estimated errors. The latter is caused by two reasons. First, the cross sections of channels like $K^0K^\pm\pi^\pm$, $K^+K^-\pi^0$, $K^+K^-\pi^+\pi^-$ are of order 1-3 nb. Second, as can be seen from Eq.(11), $K_\ell(s)$ behaves like $\approx m_\ell^2/s$ for $s \gg m_\ell^2$, suppressing in this way the contributions from the higher energy region.

Now we treat each considered channel in (16) in detail

2π : According to the preceding remark it will be the contribution of the process $e^+e^- \rightarrow \pi^+\pi^-$ which will dominate in $a_\ell^{(2)had}$ for all (see Tables 1-3 below) $\ell = e, \mu$ and τ . Its cross section is given by

$$\sigma^{2\pi}(s) = \frac{\pi\alpha^2\beta_\pi^3}{3s} |F_\pi^{I=1}(s) + Re^{i\phi} \frac{m_\omega^2}{m_\omega^2 - s - im_\omega\Gamma_\omega}|^2 \quad (17)$$

where $\beta_\pi = (1 - 4m_\pi^2/s)^{1/2}$ is the velocity of an outgoing pion in the c.m. system and R and ϕ are the $\rho - \omega$ interference amplitude and phase, respectively. A diminishing of estimated errors in

comparison with previous evaluations [6] was achieved by a more accomplished unitary and analytic VMD model [10] of $F_\pi^{I=1}(s)$ with three excited states $\rho'(1450)$, $\rho''(1700)$, $\rho'''(2150)$ of the $\rho(770)$ meson which reproduces all existing pion electromagnetic (e. m.) form factor (ff) data more precisely (see model errors in Tables 1-3.) It is obtained by the incorporation of a two-cut approximation of the correct pion e.m. ff analytic properties into the standard VMD model defined by the relation

$$F_\pi^{I=1}(s) = \sum_{v=\rho,\rho',\rho'',\rho'''} \frac{m_v^2}{m_v^2 - s} \frac{f_{v\pi\pi}}{f_v}. \quad (18)$$

Practically, the latter is performed [10] by application of the non-linear transformation

$$s = s_0 - \frac{4(s_1 - s_0)}{(1/W - W)^2} \quad (19)$$

to (18) with subsequent correct incorporation of the nonzero values of vector-meson widths, at the same time taking into account the relations

$$(m_\rho^2 - \Gamma_\rho^2/4) < s_1; \quad (m_{\rho'}^2 - \Gamma_{\rho'}^2/4) > s_1 \quad (20)$$

$$(m_{\rho''}^2 - \Gamma_{\rho''}^2/4) > s_1; \quad (m_{\rho'''}^2 - \Gamma_{\rho'''}^2/4) > s_1$$

found in a fitting procedure, which all together lead finally to the unitary and analytic VMD model of pion e.m. structure

$$\begin{aligned} F_\pi^{I=1}(s) &= \left(\frac{1 - W^2}{1 - W_N^2} \right)^2 \quad (21) \\ &\left[\frac{(W_N - W_\rho)(W_N - W_\rho^*)(W_N - 1/W_\rho)(W_N - 1/W_\rho^*)}{(W - W_\rho)(W - W_\rho^*)(W - 1/W_\rho)(W - 1/W_\rho^*)} (f_{\rho\pi\pi}/f_\rho) + \right. \\ &\left. + \sum_{v=\rho',\rho'',\rho'''} \frac{(W_N - W_v)(W_N - W_v^*)(W_N + W_v)(W_N + W_v^*)}{(W - W_v)(W - W_v^*)(W + W_v)(W + W_v^*)} (f_{v\pi\pi}/f_v) \right]. \end{aligned}$$

The parameters in (19) $s_0 = 4m_\pi^2$ and $s_1 \approx 1\text{GeV}^2$ (in a fit of the data s_1 was left to be free parameter) are square-root branch points

generating a four-sheeted Riemann surface in s -variable on which the model (21) is defined.

A perfect reproduction of all existing data on the pion e.m. ff by means of (21), where also the isospin violating $\omega \rightarrow \pi^+\pi^-$ decay contribution (the so-called $\rho-\omega$ interference effect) was taken into account by means of (17) with R as an additional free parameter and the phase to be [16]

$$\phi = \arctan \frac{m_\rho \Gamma_\rho}{(m_\rho^2 - m_\omega^2)} \quad (22)$$

is presented in Fig.2.

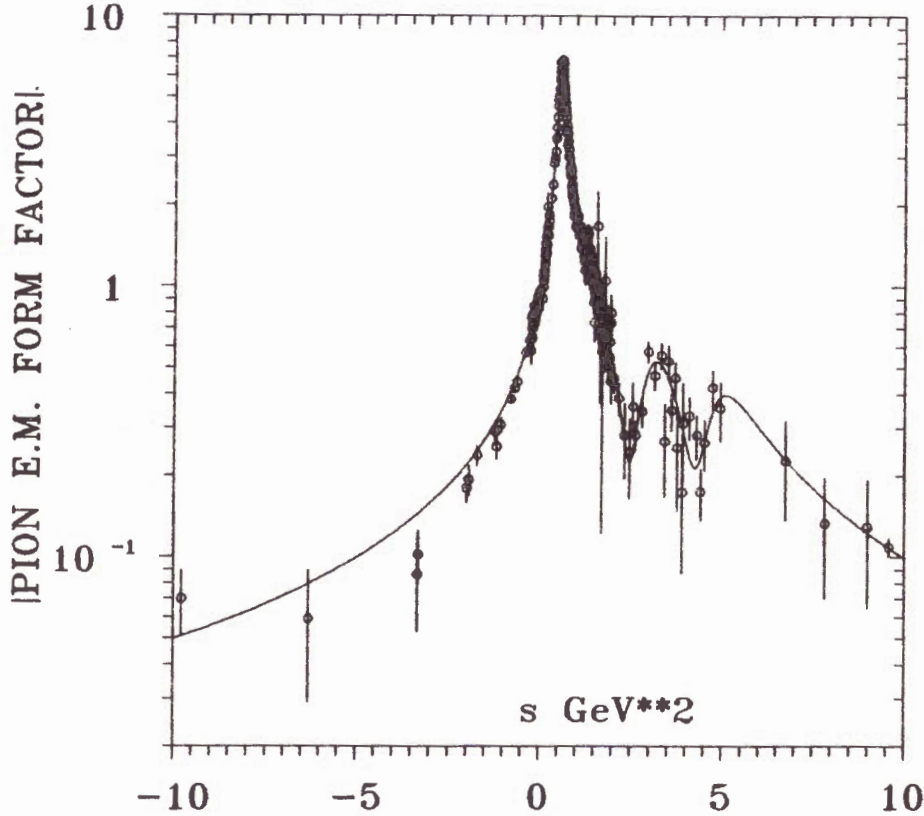


Fig. 2. The most accomplished up to now reproduction of all existing pion e.m. ff data by means of the unitary and analytic VMD model (20).

Another diminishing of estimated errors in this case in comparison with previous evaluation [6] was achieved by a more profound calculation of the external covariance matrix C_{ij} (as given by the HESSE subroutine of the MINUIT program with UP adjusted to the number of fitted parameters) determining the experimental error to be given by

$$\sigma^2 = \sum_{i,j} C_{ij} D_i D_j, \quad (23)$$

where $D_i = \partial a_l / \partial A_i$ and A_i are fitted parameters. The parameters A_i have no upper and lower limits in finding the definitive minimum. The latter leads to C_{ij} , which gives smaller error (23) in comparison with a contrary approach.

The evaluated 2π channel contributions to $A_l^{(2)had}$ for $l = e, \mu$ and τ are numerically presented in the first lines of Tables 1-3.

| Channel | Central value $\times 10^{-12}$ | Exp. error $\times 10^{-15}$ | Model error $\times 10^{-15}$ |
|---|------------------------------------|---------------------------------|----------------------------------|
| $\pi^+\pi^-$ | 1.3610 | 5.614 | 0.31 |
| K^+K^- | 0.0593 | 2.386 | 1.42 |
| $K_S^0 K_L^0$ | 0.0325 | 1.592 | 1.42 |
| $\pi^+\pi^-\pi^0$ | 0.1350 | 6.769 | - |
| $\omega\pi^0$ | 0.0187 | 1.719 | - |
| $\pi^+\pi^-\pi^0\pi^0$ | 0.0336 | 5.080 | - |
| $\pi^+\pi^-\pi^+\pi^-$ | 0.0147 | 1.534 | - |
| 5π | 0.0008 | 0.247 | - |
| 6π | 0.00003 | 0.064 | - |
| QCD 1.reg. $2 \text{ GeV}^2 \leq s \leq 9.61 \text{ GeV}^2$ | 0.0633 | } 0.974 | 0.002 |
| QCD 2.reg. $20.2 \text{ GeV}^2 \leq s \leq 81.0 \text{ GeV}^2$ | 0.0208 | | 0.003 |
| QCD 3.reg. $s \geq 196.0 \text{ GeV}^2$ | 0.0248 | | 0.004 |
| $c\bar{c}$ reg. $9.61 \text{ GeV}^2 \leq s \leq 20.2 \text{ GeV}^2$ | 0.0217 | 0.581 | - |
| $b\bar{b}$ reg. $81.0 \text{ GeV}^2 \leq s \leq 196.0 \text{ GeV}^2$ | 0.0045 | 0.025 | - |
| Resonances | 0.0197 | 1.147 | - |

Table 1: Partial contributions to the $a_l^{(2)had}$

| Channel | Central value $\times 10^{-11}$ | Exp. error $\times 10^{-11}$ | Model error $\times 10^{-11}$ |
|---|------------------------------------|---------------------------------|----------------------------------|
| $\pi^+\pi^-$ | 4942.16 | 20.59 | 0.86 |
| K^+K^- | 232.53 | 9.34 | 5.54 |
| $K_S^0 K_L^0$ | 127.31 | 6.23 | 5.54 |
| $\pi^+\pi^-\pi^0$ | 512.36 | 25.57 | - |
| $\omega\pi^0$ | 74.15 | 6.74 | - |
| $\pi^+\pi^-\pi^0\pi^0$ | 133.82 | 20.23 | - |
| $\pi^+\pi^-\pi^+\pi^-$ | 59.99 | 10.45 | - |
| 5π | 3.17 | 0.99 | - |
| 6π | 1.30 | 0.44 | - |
| QCD 1.reg. $2 \text{ GeV}^2 \leq s \leq 9.61 \text{ GeV}^2$ | 604.78 | } 2.00 | 8.22 |
| QCD 2.reg. $20.2 \text{ GeV}^2 \leq s \leq 81.0 \text{ GeV}^2$ | 89.98 | | 11.76 |
| QCD 3.reg. $s \geq 196.0 \text{ GeV}^2$ | 10.70 | | 1.68 |
| $c\bar{c}$ reg. $9.61 \text{ GeV}^2 \leq s \leq 20.2 \text{ GeV}^2$ | 92.48 | 2.40 | - |
| $b\bar{b}$ reg. $81.0 \text{ GeV}^2 \leq s \leq 196.0 \text{ GeV}^2$ | 19.40 | 0.22 | - |
| Resonances | 83.03 | 4.83 | - |

Table 2: Partial contributions to the $a_\mu^{(2)had}$

$2K$: In these channels we have found older data of Novosibirsk on the $e^+e^- \rightarrow K^+K^-$ and $e^+e^- \rightarrow K^0\bar{K}^0$ total cross sections in the ϕ region measured in arbitrary units. Since the value of the latter in both charged and neutral cases is known at the maximum of the peak we were able to scale all other experimental points.

There are also new DM2 data [17] on the $e^+e^- \rightarrow K^+K^-$ total cross section which have been taken into account in the analysis of all existing data by means of the unitary and analytic VMID model of the e.m. structure of kaons unlike the paper of ref.[6]. Of course a more correct calculation of the internal covariance matrix C_{ij} , like in the 2π case was carried out.

As a result (see Table 2), smaller values of $K\bar{K}$ contributions to $a_\mu^{(2)had}$ in comparison with [6] were obtained, however, with a remarkable diminishing of experimental errors.

| Channel | Central value $\times 10^{-8}$ | Exp. error $\times 10^{-8}$ | Model error $\times 10^{-8}$ |
|---|-----------------------------------|--------------------------------|---------------------------------|
| $\pi^+\pi^-$ | 170.10 | 0.73 | 0.38 |
| K^+K^- | 12.68 | 0.50 | 0.84 |
| $K_S^0 K_L^0$ | 6.80 | 0.33 | 0.84 |
| $\pi^+\pi^-\pi^0$ | 21.84 | 1.08 | - |
| $\omega\pi^0$ | 4.62 | 0.42 | - |
| $\pi^+\pi^-\pi^0\pi^0$ | 8.62 | 1.30 | - |
| $\pi^+\pi^-\pi^+\pi^-$ | 3.93 | 1.01 | - |
| 5π | 0.21 | 0.06 | - |
| 6π | 0.11 | 0.04 | - |
| QCD 1.reg. $2 \text{ GeV}^2 \leq s \leq 9.61 \text{ GeV}^2$ | 59.72 | } 0.40 | 1.04 |
| QCD 2.reg. $20.2 \text{ GeV}^2 \leq s \leq 81.0 \text{ GeV}^2$ | 17.72 | | 1.84 |
| QCD 3.reg. $s \geq 196.0 \text{ GeV}^2$ | 2.79 | | 0.12 |
| $c\bar{c}$ reg. $9.61 \text{ GeV}^2 \leq s \leq 20.2 \text{ GeV}^2$ | 14.66 | 0.36 | - |
| $b\bar{b}$ reg. $81.0 \text{ GeV}^2 \leq s \leq 196.0 \text{ GeV}^2$ | 4.64 | 0.05 | - |
| Resonances | 12.54 | 0.71 | - |

Table 3: Partial contributions to the $a_\mu^{(2)had}$

$\pi^+\pi^-\pi^0$:

Unlike the previous calculations in ref. [6], new data on $\sigma_{tot}(e^+e^- \rightarrow 3\pi)$ were published [9]. As a consequence the resonant model formula in [6] is no more applicable. Therefore a contribution of 3π channel to $a_\mu^{(2)had}$ was evaluated only numerically by means of the trapezoidal integration over experimental points. One can see immediately from Tables 1 - 3 that the 3π channel contribution is in magnitude on the second place.

$\omega\pi^0$:

This channel was not taken into account explicitly in ref. [6], because at that time it was contained in global data on $\pi^+\pi^-\pi^0\pi^0$ channel. Now separate data on $\sigma_{tot}(e^+e^- \rightarrow \omega\pi^0)$ appeared [9] and corrected data on $\sigma_{tot}(e^+e^- \rightarrow \pi^+\pi^-\pi^0\pi^0)$ were published [9]. The contribution of $\omega\pi^0$ channel to $a_\mu^{(2)had}$ was evaluated also by the trapezoidal integration directly over experimental points.

$4\pi, 5\pi, 6\pi$:

All these channels were completed by new data and contributions of them to $a_l^{(2)had}$ were evaluated by the numerical integration. Results are presented in Tables 1 - 3.

Now we are left with high energy contributions to $a_l^{(2)had}$. They are obtained by means of the second integral in (14). For parametrization of $R(e^+e^- \rightarrow had)$ we use the corrected QCD formula up to the third order of α_s , where the coefficient of the third power of α_s is changed from the value 64 used in ref. [6] to the value (-12) (see refs. [11],[12]). Moreover, the electroweak corrections are taken into account in the sense of the Marshall analysis [13], but in our case all electroweak parameters are fixed at the table values and only the scale parameter of QCD $\Lambda_{\overline{MS}}$ is left to be a free parameter.

The whole high energy region is divided into three regions, the natural boundaries of which are thresholds of the creation of $q\bar{q}$ pairs as it is shown in Tables 1 -3. In the description of data on $R(e^+e^- \rightarrow had)$ all resonances are formally disregarded and their contributions to $a_l^{(2)had}$ were evaluated separately. Then all three regions like in ref. [6],[13] are described by the QCD corrected formula of $R(s)$ with the number of quarks $n_f = 3, n_f = 4$ and $n_f = 5$, respectively. Every region is thus characterized by a different value of $\Lambda_{\overline{MS}}^{(i)}$ $i = 3, 4, 5$. Then the scale parameter of QCD $\Lambda_{\overline{MS}}$ is identified with $\Lambda_{\overline{MS}}^{(5)}$, that upon our fitting procedure takes the value

$$\Lambda_{\overline{MS}} = 398 \pm 45 \text{ MeV.} \quad (24)$$

The contributions of all resonances existing up to now in the region described by $R(s)$ were evaluated like in ref. [6] in the narrow-width approximation.

The summation of all contributions in Tables 1-3 give the following final values

$$a_e^{(2)had} = (1.810 \pm 0.011 \pm 0.002) \times 10^{-12} \quad (25)$$

$$a_\mu^{(2)had} = (6.986 \pm 0.042 \pm 0.016) \times 10^{-8} \quad (26)$$

$$a_\tau^{(2)had} = (3.436 \pm 0.024 \pm 0.024) \times 10^{-6} \quad (27)$$

They are the most precise up to now determined values of the lowest-order hadronic vacuum polarization contributions to the anomalous magnetic moment of charged leptons.

4 Conclusions and summary

By using the standard procedure of evaluation of the lowest order hadronic vacuum-polarization contribution $a_l^{(2)had}$ to the anomalous magnetic moment of leptons based on the integral representation (10) we have diminished total errors of the latter for all charged leptons. This was achieved due to inclusion of additional data on some exclusive processes, application of more accomplished models for the description of the pion and kaon electromagnetic structure, a more correct calculation of the external covariance matrix as given by the HESSE subroutine of the MINUIT program and application of the revised value of the coefficient of the third power of QCD formula for $R(s)$.

Comparing (25) with the total value of the anomalous magnetic moment of the electron [18]

$$a_e = (1159652140 \pm 28) \times 10^{-12} \quad (28)$$

we come to the conclusion that our precision given by the errors in (25) is not at the centre of interest.

A different situation is with the case of the τ -lepton, where only a rough estimate [19]

$$a_\tau^{(2)had} = (3.6 \pm 0.3 \pm 0.1) \times 10^{-6} \quad (29)$$

was carried out in comparison with our results (27) up to now.

In the very actual case of the μ -meson we have achieved diminishing of the error $\Delta a_\mu^{(2)had}$ to be four times as small (see (26)) as the one loop weak interaction contribution (5) and it becomes comparable with accuracy (3) expected in the new $g - 2$ muon experiment to be under way at Brookhaven National Laboratory [1].

One of us (A.Z.D.) would like to thank Prof. M.P.Rekalo for very fruitful discussions.

References

- [1] V. Hughes and T. Kinoshita, Comm. Nucl. Part. Phys. **14**, 341(1985).
- [2] T. Kinoshita, B. Nizic and Y. Okamoto, Phys.Rev.Lett. **52**, 717(1984) and Phys.Rev. **D31**, 2108(1985).

- [3] K.B. Samuel and M.A. Samuel, Canadian J.Phys. **68**, 1359(1990).
- [4] J. Biley et al., Nucl.Phys. **B150**, 1(1979).
- [5] J.A. Casas, C. López and F.J. Ynduráin, Phys.Rev **D32**, 736(1985).
- [6] Ľ. Martinovič and S. Dubnička, Phys.Rev. **D42**, 884(1990).
- [7] V.E. Balakin et al, Phys.Lett. **34B**, 328(1971).
- [8] A.D. Bukin et al, Sov.J.Nucl.Phys. **27(4)**, 516(1978).
- [9] D. Bisello et al, Preprint LAL 90-35, ORSAY (1990) and to be published in the Z.Phys. **C** (1992); S.I.Dolinsky et al, Phys.Lett. **174B**, 453(1986); N.Albrecht et al, Phys.Lett. **185B**, 223(1987).
- [10] M.E. Biagini, S. Dubnička, E. Etim and P. Kolář, Nuovo Cimento **A104**, 363(1991).
- [11] S.G. Gorishny, A.L. Kataev and S.A. Larin, Phys.Lett. **B259**, 144(1991).
- [12] L.R. Surguladze and M.A. Samuel, Phys.Rev.Lett. **66**, 560(1991).
- [13] R. Marshall, Z.Phys. **C43**, 595 and 607(1989).
- [14] B.E. Lautrup, A.Peterman and E.de Rafael, Phys.Rep. **3C**, 193(1972).
- [15] S.J. Brodsky and E.de Rafael, Phys.Rev. **168**, 1620(1968).
- [16] S.A.Coon, P.McNamee and M.D.Scadron, Nucl.Phys. **A287**, 381(1977).
- [17] D.Bisello et al, Z.Phys. **C39**, 13(1988).
- [18] T.Kinoshita, in Quantum Electrodynamics, World Scientific Pub.Comp., Singapore(1990), p.218.
- [19] M.A.Samuel, G.Li and R.Mendel, Phys.Rev.Lett. **67**, 668 (1991).

Received by Publishing Department
on June 30, 1992.

WILL YOU FILL BLANK SPACES IN YOUR LIBRARY?

You can receive by post the books listed below. Prices — in US \$, including the packing and registered postage.

| | | |
|----------------|--|-------|
| D13-85-793 | Proceedings of the XII International Symposium on Nuclear Electronics, Dubna, 1985. | 14.00 |
| D1,2-86-668 | Proceedings of the VIII International Seminar on High Energy Physics Problems, Dubna, 1986 (2 volumes) | 23.00 |
| D3,4,17-86-747 | Proceedings of the V International School on Neutron Physics. Alushta, 1986. | 25.00 |
| D9-87-105 | Proceedings of the X All-Union Conference on Charged Particle Accelerators. Dubna, 1986 (2 volumes) | 25.00 |
| D7-87-68 | Proceedings of the International School-Seminar on Heavy Ion Physics. Dubna, 1986. | 25.00 |
| D2-87-123 | Proceedings of the Conference "Renormalization Group-86". Dubna, 1986. | 12.00 |
| D2-87-798 | Proceedings of the VIII International Conference on the Problems of Quantum Field Theory. Alushta, 1987. | 10.00 |
| D14-87-799 | Proceedings of the International Symposium on Muon and Pion Interactions with Matter. Dubna, 1987. | 13.00 |
| D17-88-95 | Proceedings of the IV International Symposium on Selected Topics in Statistical Mechanics. Dubna, 1987. | 14.00 |
| E1,2-88-426 | Proceedings of the 1987 JINR-CERN School of Physics. Varna, Bulgaria, 1987. | 14.00 |
| D14-88-833 | Proceedings of the International Workshop on Modern Trends in Activation Analysis in JINR. Dubna, 1988 | 8.00 |
| D13-88-938 | Proceedings of the XIII International Symposium on Nuclear Electronics. Varna, 1988 | 13.00 |
| D17-88-681 | Proceedings of the International Meeting "Mechanisms of High-T _c Superconductivity". Dubna, 1988. | 10.00 |
| D9-89-52 | Proceedings of the XI All-Union Conference on Charged Particle Accelerators. Dubna, 1988 (2 volumes) | 30.00 |
| E2-89-525 | Proceedings of the Seminar "Physics of e ⁺ e ⁻ Interactions". Dubna, 1988. | 10.00 |
| D9-89-801 | Proceedings of the International School-Seminar on Heavy Ion Physics. Dubna, 1989. | 19.00 |
| D19-90-457 | Proceedings of the Workshop on DNA Repair on Mutagenesis Induced by Radiation. Dubna, 1990. | 15.00 |