сообщения
объединенного
института
ядерных
исследований
дубна

E2-92-276

G.Petrov*

IMPULSIVE MOVING MIRROR MODEL AND IMPULSIVE DIFFERENTIAL EQUATIONS IN BANACH SPACE

*Institute of Nuclear Research and Nuclear Energy, Sofia, Bulgaria

1992

1 Introduction

In 1948 Casimir [1] showed that when two uncharged plane conductors are placed parallel to each other in vacuum there is an attractive force between them proportional to the inverse forth power of their separation. Introducing boundaries, we change the energy of the electromagnetic vacuum. The dependence of the vacuum state energy on the distance between the capacitor plane is used to calculate the Casimir force. The question considered is whether the motion of one of the mirrors with constant velocity v may change appreciably the structure of the vacuum state and, consequently, the Casimir force.

Another field theoretical approach for studing the properties of the vacuum starts from analysis of the behaviour of local field quantities in Minkowski space with uniformly moving mirrors. The advantage of the local definition is that it permits a different point of view and a deeper anderstanding of the nature of vacuum energy [2,3,4].

The key problem of quantum field theory in arbitrary Riemannian space is that, in general, there is no preferred choice of time-like vector field with respect to which positive and negative frequencies are defined. Different choices lead generally to different Fock spaces and, in particular, to different vacuum states. In the usual discussions one treats the solution of the wave equation as an operator; but there is an implicit dependence in the definition of the local operator algebra upon the timelike vector field. This is reflec ted in the non-trivial Bogoliubov transformation relating the field operators arising from different choices of the "vacuum" [5].

We formulate the problem of moving mirrors, possibly with suddenly change of velocity v in $t = t_n$, n = 0,1,2,..., upon which quantum fields satisfy Dirichlet boundary conditions, with the associated Casimir effect, in a func tional Schrödinger picture.

In the Schrödinger picture one must constract a Hamiltonian which propagates the wave-fanctional (or local operators) on the time slices defined by a time like vector field. One selects a family of space-like surfaces, a foliation, $\{\Sigma_t\}$, where the usual tools of differential geometry fail to apply because the space under study is not a "good" manifold (space of leaves of foliation, $\{\Sigma_t\}$), each foil labelled by the "time" parameter, $t_n < t \leq t_{n+1}$, n = 0,1,... and locally orthogonal, $\Psi(\varphi, t)$, of "snap-

shot"(t-independent) field configuration on Σ_t. ϕ is the fundamental variable, analogus to $\vec{x}$ in ordinary single particle quantum mechanics, and the probability of finding configuration ϕ_1 on the slice labelled by t_1 is $|\Psi(\phi_1, t_1)|^2$.

Consider a real massles scalar field $\phi(t,x)$ on two-dimensional Minkowski space with coordinates (t,x) bounded by a mirror (actualy a path) at $x = \dot{x}_n.t$, $t_n < t \leq t_{n+1}, n = 0, 1, ...$, $\dot{x}_n = dx/dt|\ t = t_n$. The field is required to fulfiel Dirichlet boundary conditions at the location of of the mirror.

$$\phi(t, \dot{x}_n.t) = 0,\ t_n < t \leq t_{n+1},\ n = 0, 1, ... \qquad (1)$$

assumed that this change of the path takes place by jumps.

The ϕ equation of motion is simply.

$$(\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2})\phi(t,x) = 0. \qquad (2)$$

$t_n < t \leq t_{n+1}$, n = 0,1,...

If we quantize the field in the Schrödinger picture with Σ_t chosen as the $t_n = t = const$ lines, for the basis on Σ_t, we can use

$$\phi_\omega(x,t) = \sqrt{2/\pi}\sin(\omega(x - \dot{x}_n.t)). \qquad (3)$$

$\omega > 0$, $t_n < t \leq t_{n+1}$, n = 0,1,...,[for t_n and x_n see (2,3,4)], since the set $[\sin\omega x]$, $\omega > 0$, is complete for $x > 0$.

Because of the time-dependent conditions the ϕ_ω's are necessarily time-dependent too. Introducing a new spatial coordinate x' = x $-\dot{x}_n.t$, $t_n < t \leq t_{n+1}$, we can get a time independent boundary conditions, but this changes the Minkowski metric to:

$$ds^2 = (1 - \dot{x}_n)dt^2 - 2\dot{x}_n dx'dt - dx'^2 \qquad (4)$$

and the field equation to:

$$[\frac{\partial^2}{\partial t^2} - 2\dot{x}_n\frac{\partial}{\partial t}.\frac{\partial}{\partial x'} - \ddot{x}_n\frac{\partial}{\partial x'} - (1 - \dot{x}_n^2).\frac{\partial^2}{\partial x'^2}]\phi(t,x') = 0,\ t_n < t \leq t_{n+1}. \qquad (5)$$

For the case $\ddot{x} = 0$, the solutions of eq.(5) are separable as

$$\phi = \phi_\omega(x')\exp(-i\omega t).$$

Assuming this implies that the foliatian $t_n = const$ is separable and in this case the Schrödinger Hamiltonian decuples into a set of harmonic oscillator Hamiltonian, possibly with time-dependent frequencies. In this case it is fairly easy to study, for instance, cosmological particle creation. Making a Gaussian product ansatz for the ground state wave-functional, the Schrödinger equation becomes an ordinary differential equation for its covariance. Once this is solved, it is straightforward to obtain particle number expectation values [10].

Identifying t_n with the cosmic time, t, and $\Sigma_s = \Sigma_t$ with the section $t_n = s = t = const$, it is obvious that for every t the surface Σ_t is the euclidean space R. Therefore we can use the same basis on each hypersurface. Foliations of a given space for which this is possible we call separable. In general an arbitrary foliation is non-separable.

The purpose of this paper is two-fold. First, we set up a general formalism for field quantization on non-separable foliations and in this case the im pulsive Schrödinger Hamiltonian decouples into a set of oscillators interacting with a "impulsive effect".

Second, we apply this formalism to the impulsive moving mirror model for different foliations, such that will be relevant to the discussion of Casimir effect.

In section /2/ we show that a free field theory on a non-separable foliation does not lead to a set of harmonic oscilators, but rather to a set of oscillators interacting with a "impulsive functional gauge connection" [6]. In section /3/ we discuss the impulsive moving mirror model with a $t_n = s = t = const$ foliation as an example.

We consider the invariant wave equation with non-geometrically simple characteristics, but are defined on Lorentzian manifolds having natural classes of decompositions into time x space factors, invariant under large subgroups of the Lorentzian isometry groups. It is natural to take advantage of this circumstance by regording an unknown wave function such as $\phi(t,x)$ as a vector-valued function $\phi(t,.)$ of t whose values are functions on space. This leads to a reformulation of the original linear wave equation as an ordinary differential equation for a Banach-space-valued function that is relatively transparent in structure and frequently facilitates a treatment ofissues that are global in space.

The standpoint is thus an extension to the impulsive-linear case of the Cauchy problem in the linear case [7,8,9].

2 Covariant impulsive Schrödinger picture for non-separable foliations

A.We briefly repeat the main ingradients of the Schrödinger picture [10].

Given a spacetime manifold, we fix a coordinate system, $x^\mu(\mu = 0,1,...,d)$ and choose a family $\{\Sigma_s\}$ of spacelike hypersurfaces label led by a parameter s, $t_n < s \leq t_{n+1}$. On Σ_s we choose coordinate ξ^i (i = 1,...,d). The embedding of the hypersurfaces into spacetime is defined by a set of equations

$x^\mu = x^\mu\ (s,\xi^i)$.

Then the timelike vector-field $\partial x^\mu/\partial s$ is normal to Σ_s.

The volume element of the surface reads:

$$d^d\sigma_\mu = \sqrt{-g}\epsilon_{\mu\nu_1...\nu_d}\frac{\partial x^{\nu_1}}{\partial\xi^1}\cdots\frac{\partial x^{\nu_d}}{\partial\xi^d}d^d\xi \equiv \frac{D\sigma_\mu}{D\xi}d^d\xi. \tag{6}$$

It is invariant under change of the coordinates ξ^i on $\Sigma_s, t_n < s \leq t_{n+1}$. n = 0,1,...

Now consider a scalar field $\phi(s,\xi) \equiv \phi(x^\mu(s,\xi))$ with action S and energy momentum tensor $T_{\mu\nu}$. The theory is quantized by definig the canonical momentum as:

$$\pi =\mid g\mid^{-1/2}\frac{\partial x^\mu}{\partial s}\frac{\delta S}{\delta(\partial^\mu\phi)}, t_n < s \leq t_{n+1}, n = 0,1,... \tag{7}$$

and requiring the following commutation relations on each hypersurface:

$$[\phi(s,\xi),\pi(s,\xi')] = i\delta_\Sigma(\xi,\xi'), t_n < s \leq t_{n+1}, n = 0,1,... \tag{8}$$

Here the δ-function

$$\delta_\Sigma(\xi,\xi') =\mid \frac{\partial x^\mu}{\partial s}\frac{\partial x_\mu}{\partial s}\mid^{1/2}\mid\frac{D\sigma^\nu}{D\xi}\frac{D\sigma_\nu}{D\xi}\mid^{-1/2}\prod_{i=1}^{d}\delta(\xi^i - \xi'^i) \tag{9}$$

it is invariant under both $s \to s'(s)$ transformations and ξ- reparametrazation on Σ_s. The commutation relation eq.(8) is fulfilled if on each Σ_s the operator ϕ is represented by the multiplication with an s-independent function $\phi(\xi)$ and the momentum by:

$$\pi(\xi) = -i\mid\frac{\partial x^\mu}{\partial s}\frac{\partial x_\mu}{\partial s}\mid^{1/2}\mid\frac{D\sigma^\nu}{D\xi}\frac{D\sigma_\nu}{D\xi}\mid^{-1/2}\frac{\delta}{\delta\phi(\xi)}. \tag{10}$$

The functional derivative is to be anderstood in a d-dimensional sense.

The evolution of the wave-functional $\Psi(\phi(\xi);s)$ is governed by the Schrö dinger equation:

$$
\begin{aligned}
H\Psi(\phi(\xi);s) &= \int d^d\sigma^\mu \frac{\partial x^\nu}{\partial s} T_{\mu\nu}(\phi(\xi),\pi(\xi))\Psi(\phi(\xi),s) \\
&= id/ds\Psi(\phi(\xi),s),
\end{aligned}
$$

$$t_n < s \leq t_{n+1},\ n = 0,1,\ldots \tag{11}$$

The Hamiltonian H is s-independent only if the spacetime under consideration allows $\frac{\partial x^\mu}{\partial s}, t_n < s \leq t_{n+1}$, n = 0,1,... to chosen as a timelike Killing vector field. The Schödinger equation, eq.(11), is form invariant under change of the spacetime coordinates x^μ, changes of the surface coordinates ξ^i and under reparametraza tions $s \to s'(s)$ of the evolution parameter. It is in this sense that the formalism is covariant, however. Different choices of $\{\Sigma_s\}$ lead to different quantum theoris.

We consider now the case of non-separable foliations. By definition this means that it is impossible to use the same system $\{\phi_\omega(\xi)\}$ on all hypersurface Σ_s. One has to introduce a family $\{\phi_\omega(\xi,s)\}$ of complete system, one for each surface. The time evolution of $\Psi[\{a_\omega\},s]$ consists of two pieces then: there is also an explicit time dependence due to the fact that the basis functions are s-dependent. Therefore the Hamiltonian no longer has the simple form of eq. (11). We can study the time evolution of Ψ including the effect of the s-dependence of ϕ_ω. For definiteness we consider a minimally coupled, real scalar field with the action:

$$S = 1/2\int_{t_n}^{t_{n+1}} dt \int d^dx\sqrt{-g}[g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - m^2\phi^2]. \tag{12}$$

Let $\phi = \phi(x^\mu)$ be an arbitrary field configuration defined on the entire spacetime manifold. Using the relation $x^\mu = x^\mu(s,\xi^i)$ defining the embeding, we write ϕ as function of s and ξ^i: $\phi(s,\xi^i) \equiv \phi(x^\mu(s,\xi^i)), t_n < s \leq t_{n+1}$, n = 0,1,...

This function is expanded in terms of the complete sets $\{\phi_\omega(\xi,s)\}$ as:

$$\phi(s,\xi^i) = \sqrt{2/\pi}\int_0^\infty d\omega[a_\omega(s)\phi_\omega(\xi,s) + a^*_\omega(s)\phi^*_\omega(\xi,s)], t_n < s \leq t_{n+1}, \tag{13}$$

n=0,1,... Note that the basis vectors $\{\phi_\omega(\xi,s)\}$ are complete on each Σ_s, but this does not imply that they are complete when considered as one function $f_\omega(x^\mu) = \phi_\omega(\xi(x^\mu),s(x^\mu))$ on the spacetime manifold. This obvious from the extreme case of a separable foliation where ϕ_ω has no s-dependence at all. Thus, in the expansion eq. (13) the coefficients are generally required to be s-dependent.

We now express the action in terms of the a_ω's by inserting eq. (13) into eq. (12). One obtains:

$$
\begin{aligned}
S &= 1/2\int_{t_n}^{t_{n+1}} ds \int_0^\infty d\omega \int_0^\infty d\omega'[(\dot a_\omega,\dot a^*_\omega)\begin{pmatrix} T^{(1)}_{\omega\omega'} & T^{(2)}_{\omega\omega'} \\ T^{(2)}_{\omega\omega'} & T^{(1)*}_{\omega\omega'}\end{pmatrix}(\dot a_{\omega'},\dot a^*_{\omega'})^T - \\
&- (a_\omega,a^*_\omega)\begin{pmatrix} V^{(1)}_{\omega\omega'} & V^{(2)}_{\omega\omega'} \\ V^{(2)}_{\omega\omega'} & V^{(1)*}_{\omega\omega'}\end{pmatrix}(a_{\omega'},a^*_{\omega'})^T + \\
&+ (\dot a_\omega,\dot a^*_\omega)\begin{pmatrix} f^{(1)}_{\omega\omega'} & f^{(2)}_{\omega\omega'} \\ f^{(2)}_{\omega\omega'} & f^{(1)*}_{\omega\omega'}\end{pmatrix}(a_{\omega'},a^*_{\omega'})^T]
\end{aligned} \tag{14}
$$

where:

$$
\begin{aligned}
T^{(1)}_{\omega\omega'}(s) &= \int d^d\tau\dot g^{ss}\phi_\omega\phi_{\omega'} \\
T^{(2)}_{\omega\omega'}(s) &= \int d^d\tau\dot g^{ss}\phi_\omega\phi^*_{\omega'} \\
V^{(1)}_{\omega\omega'}(s) &= \int d^d\tau\{m^2\phi_\omega\phi_{\omega'} - \dot g^{ss}\dot\phi_\omega\dot\phi_{\omega'} - 2\dot g^{sj}\dot\phi_\omega\partial_j\phi_{\omega'} - \dot g^{ij}\partial_i\phi_\omega\partial_j\phi_{\omega'}\} \\
V^{(2)}_{\omega\omega'}(s) &= \int d^d\tau\{m^2\phi_\omega\phi^*_{\omega'} - \dot g^{ss}\dot\phi_\omega\phi^*_{\omega'} - 2\dot g^{sj}\dot\phi_\omega\partial_j\phi^*_{\omega'} - \dot g^{ij}\partial_i\phi_\omega\partial_j\phi^*_{\omega'}\} \\
f^{(1)}_{\omega\omega'}(s) &= \int d^d\tau\{\tilde g^{ss}\phi_\omega\dot\phi_{\omega'} + \dot g^{sj}\phi_\omega\partial_j\phi_{\omega'}\} \\
f^{(2)}_{\omega\omega'}(s) &= \int d^d\tau\{\tilde g^{ss}\phi_\omega\dot\phi^*_{\omega'} + \dot g^{sj}\phi_\omega\partial_j\phi^*_{\omega'}\}
\end{aligned} \tag{15}
$$

and:

$$
\begin{aligned}
d^d\tau &= \frac{\partial x^\mu}{\partial s}\frac{D\sigma_\mu}{D\xi}d^d\xi \\
\dot g^{ss} &= g^{\mu\nu}\frac{\partial s}{\partial x^\mu}\frac{\partial s}{\partial x^\nu}
\end{aligned} \tag{16}
$$

$$\dot{g}^{sj} = g^{\mu\nu}\frac{\partial s}{\partial x^\mu}\frac{\partial \xi^j}{\partial x^\nu}$$

$$\dot{g}^{ij} = g^{\mu\nu}\frac{\partial \xi^i}{\partial x^\mu}\frac{\partial \xi^j}{\partial x^\nu}$$

Here $\dot{\phi}$ and $\partial_i\phi$ denote the derivative with respect to s and ξ, respectively. In an obvious matrix notation the action can be written as:

$$S = 1/2\int_{t_n}^{t_{n+1}} ds \int_0^\infty d\omega \int_0^\infty d\omega' \{\dot{A}_\omega T_{\omega\omega'}\dot{A}_{\omega'} - A_\omega V_{\omega\omega'}A_{\omega'} + 2\dot{A}_\omega F_{\omega\omega'}A_{\omega'}\} \tag{17}$$

where $A_\omega \equiv (a_\omega, a^*_\omega)$ and the matrices T, V and F can be read off from eq. (14) and eq. (15); in general they depend on s.

The canonical variables are the A_ω's and their conjugate momenta:

$$P_\omega = T_{\omega\omega'}\dot{A}_{\omega\omega'} + F_{\omega\omega'}A_{\omega'} \tag{18}$$

They are required to satifsfay the canonical commutation relation:

$$[A_\omega, P_{\omega'}] = i\delta(\omega - \omega');\ [A_\omega, A_{\omega'}] = [P_\omega, P_{\omega'}] = 0. \tag{19}$$

Acting on a Schrödinger wave functional $\Psi[\{A_\omega\}, s]$ we represent A_ω multiplicatively and P_ω as $-i\delta/\delta A_\omega$.

The Hamiltonian, H, describing the time evolution of Ψ is obtained from eq. (17) via the usual Legandre transformation.

B. From eq. (18) we obtain

$$\dot{A}_\omega = T^{-1}_{\omega\omega'}P_{\omega'} - T^{-1}_{\omega\omega''}F_{\omega''\omega'}A_{\omega'} \tag{20}$$

and from

$$-\delta H/\delta A_\omega = \dot{P}_\omega$$

folow

$$d/dt[T_{\omega\omega'}\dot{A}_{\omega'}] = -V_{\omega\omega'}A_{\omega'} - \dot{F}_{\omega\omega'}A_{\omega'} \tag{21}$$

So we obtain the equation:

$$\begin{pmatrix} \dot{A}_\omega \\ d/dt[T_{\omega\omega'}\dot{A}_{\omega'}] \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -V_{\omega\omega'} - \dot{F}_{\omega\omega'} & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -T^{-1}_{\omega\omega''}F_{\omega''\omega'} & T^{-1}_{\omega\omega'} \end{pmatrix}\begin{pmatrix} A_{\omega'} \\ P_{\omega'} \end{pmatrix} \tag{22}$$

if $\dot{F}_{\omega\omega'} \neq 0$.

If we take $A_\omega(s=0) = A^+_\omega(0)$ and $P_\omega(s=0) = P^+_\omega(0) = T^+_{\omega\omega'}(0)\dot{A}_{\omega'}(0)$ then for $0 < s \leq t_0$ and $P_\omega(s) = T_{\omega\omega'}\dot{A}_{\omega'}(s)$ we obtain:

$$\begin{pmatrix} A_\omega(s) \\ P_\omega(s) \end{pmatrix} = \exp\begin{pmatrix} 0 & \int_0^s ds' T^{-1}_{\omega\omega'}(s') \\ -\int_0^s ds' V_{\omega\omega'}(s') & 0 \end{pmatrix}\begin{pmatrix} A^+_{\omega'}(0) \\ P^+_{\omega'}(0) \end{pmatrix} \tag{23}$$

and for $s = t_0$; $A^+_\omega(t_0+0) = A_\omega(t_0)$; $P^+_\omega(t_0+0) = T_{\omega\omega'}(t_0+0)\dot{A}^+_\omega(t_0+0) = P_\omega(t_0) - F_{\omega\omega'}(t_0+0)A_{\omega'}(t_0)$. For $t_0 < s \leq t_1$ the solution is:

$$\begin{pmatrix} A_\omega(s) \\ P_\omega(s) \end{pmatrix} = \exp\begin{pmatrix} 0 & \int_{t_0}^s ds' T^{-1}_{\omega\omega'}(s') \\ -\int_{t_0}^s ds' V_{\omega\omega'}(s') & 0 \end{pmatrix}\begin{pmatrix} A^+_{\omega'}(t_0+0) \\ P^+_{\omega'}(t_0+0) \end{pmatrix} \tag{24}$$

The differential evolution equation with impulsive effect for $t_n < s \leq t_{n+1}(n = 0,1,...)$ is:

$$\begin{pmatrix} \dot{A}_\omega \\ \dot{P}_\omega \end{pmatrix} = \begin{pmatrix} 0 & T^{-1}_{\omega\omega'} \\ -V_{\omega\omega'} & 0 \end{pmatrix}\begin{pmatrix} A_{\omega'} \\ P_{\omega'} \end{pmatrix} \tag{25}$$

with $P_\omega(s) = T_{\omega\omega'}\dot{A}_{\omega'}(s)$ and where $V(s) \equiv \begin{pmatrix} 0 & T^{-1}_{\omega\omega'}(s) \\ -V_{\omega\omega'}(s) & 0 \end{pmatrix}$ is a continuous operator-function on s, the values of which are linear bounded operators mapping the Banach space B into itself. For $s = t_n$, $(n = 1, 2, ...)$ we have

$$\begin{pmatrix} A^+_\omega(t_n+0) \\ P^+_\omega(t_n+0) \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -F_{\omega\omega'}(t_n) & 1 \end{pmatrix}\begin{pmatrix} A_\omega(t_n) \\ P_\omega(t_n) \end{pmatrix} \tag{26}$$

where $F_n \equiv \begin{pmatrix} 1 & 0 \\ -F_{\omega\omega'}(t_n) & 1 \end{pmatrix}$ is continuous with $F_n : B \to B$.

So for $s \neq t_n(n = 0, 1, ...)$ and $X(s) = \begin{pmatrix} A_\omega(s) \\ P_\omega(s) \end{pmatrix}$ we have an impulsive linear equation

$$\dot{X}(s) = V(s)X(s) \tag{27}$$

and with impulsive effect for $s = t_n(n = 1, 2, ...)$

$$X^+(t_n+0) = F_n X(t_n) \tag{28}$$

In the next section we write it down for a special exampl. In general it is rather complicated, since it not only contains s-dependent kinetic and potential energy matrices T and V, but also a term coupling A_ω to $\dot{A}_\omega$ or P_ω, respectively. As is seen from eq. (15), the origin of this term is the s-dependence of the basis functions. It couples A_ω to a "impul sive functional gauge potential" $\mathcal{A}_\omega = \int d\omega' F_{\omega\omega'} A_{\omega'}$ with a constant, i.e. A-independent impulsive field strength tensor $\mathcal{F}_{[\omega,\omega']}$. This is reminiscent of a term $L_{int} = \epsilon\, x^i F^{ij} \dot{x}_j$ in particle mechanics, where $A^i = 1/2 F^{ij} x_j$ is the vector potential of a constant magnetic field F^{ij}. A consequence of this "functional magnetic field" is that the velocity operators do not commute:

$$[A_\omega, A_{\omega'}] = 2i\, T^{-1}_{\omega\omega''} T^{-1}_{\omega'\omega'''} \mathcal{F}_{[\omega''',\omega'']}. \tag{29}$$

Hence there are correlations between the modes A_ω similar to the corre lation between the x- and y-motion of a charged particle in a magnetic field directed along the z-axis. By the standard argument a "pure gauge"

$$\mathcal{A}_\omega = \delta\Lambda[A]/\delta A_\omega.$$

does not affect the dynamics. By incpection of $f^{(1)}_{\omega\omega'}$, and $f^{(2)}_{\omega\omega'}$ in eq.(15) one can readly be convinced that a pure gauge would correspond to a situation where are using an s-dependent basis system without being forced to do so, i.e. when on each Σ_s there exists an s-dependent basis $\{\phi'_\omega(\xi)\}$ to a new s-independent basis $\{\phi'_\omega(\xi)\}$. By definition this means that $\{\Sigma_s\}$ is separable. Hence the statement $\mathcal{F}_{\omega\omega'} \neq 0$ is a coordinate system independent characterization of non-separable foliations.

3 Impulsive Moving Mirrors: Timelike Surfaces

A. To illustrate the method of section (2) we consider two dimensional Minkowski space with coordinates (t,x) bounded by a impulsive moving mirror at $x(t) = \dot{x}_n(t_n).t$, $t_n < t \leq t_{n+1}, n = 0,1,\ldots$ For $x > x(t)$ we define a scalar field $\phi(t,x)$ with the action:

$$S = 1/2 \int_{t_n}^{t_{n+1}} dt \int_{\dot{x}_n.t}^{\infty} dx\{(\partial_t\phi)^2 - (\partial_x\phi)^2 - m^2\phi^2\}.\ \dot{x}_n = const. \tag{30}$$

ϕ is assumed to obey Dirichlet boundary conditiodns of the location of the mirror:

$$\phi(t, \dot{x}_n.t) = 0,\ t_n < t \leq t_{n+1},\ n = 0, 1, \ldots \tag{31}$$

$\dot{x} = 0$, for $0 \leq t \leq t_0$

The hypersurface $\Sigma_s = \Sigma_t$ are taken to be lines t = const. As a basis on Σ_t we use the function of eq. (3). As general configuration $\phi = \phi(t,x)$, $x > \dot{x}_n.t$ is expended as:

$$\phi(t,x) = \sqrt{2/\pi} \int_0^\infty d\omega[a_\omega(t)\sin(\omega(x - \dot{x}_n.t))]. \tag{32}$$

The action reads now:

$$S = 1/2 \int_{t_n}^{t_{n+1}} dt \int_0^\infty d\omega\{\dot{a}_\omega^2 - \omega^2(1 - \dot{x}_n^2)a_\omega^2 - m^2 a_\omega^2\} \tag{33}$$
$$- \int_{t_n}^{t_n+1} dt\,\dot{x}_n \int_0^\infty d\omega \int_0^\infty d\omega'\, F(\omega,\omega')\dot{a}_\omega a_{\omega'}$$

with the antisymmetric "impulsive strength tensor":

$$F(\omega, \omega') = 2/\pi \int_0^\infty dx \sin\omega x \omega' \cos\omega' x. \tag{34}$$

The momenta conjugate to a_ω are:

$$p_\omega = \dot{a}_\omega - \dot{x}_n \int_0^\infty d\omega' F(\omega,\omega')a_\omega. \tag{35}$$

B. The impulsive linear differential equation becomes:

$$\begin{aligned} \dot{X}(t) &= VX(t),\ t \neq t_n,\ n = 0, 1, \ldots \\ X^+ &= FX,\ t = t_n\ n = 0, 1, \ldots \end{aligned} \tag{36}$$

with $X = \begin{pmatrix} a_\omega(t) \\ p_\omega(t) \end{pmatrix}$ and $X^+ = \begin{pmatrix} a_\omega^+(t_n + 0) \\ p_\omega^+(t_n + 0) \end{pmatrix}$, by

$$V = \begin{pmatrix} 0 & 1 \\ -(m^2 + \omega^2(1 - \dot{x}_n^2)) & 0 \end{pmatrix}$$

and

$$F_n = \begin{pmatrix} 1 & 0 \\ -\dot{x}_n \int_0^\infty d\omega' F(\omega,\omega') & 1 \end{pmatrix}$$

with the evolutions operator:

$$U(t) = \begin{pmatrix} \cos(t-t_n)\sqrt{v} & \frac{\sin(t-t_n)\sqrt{v}}{\sqrt{v}} \\ -\sqrt{v}\sin(t-t_n)\sqrt{v} & \cos(t-t_n)\sqrt{v} \end{pmatrix} \tag{37}$$

with $v = -(m^2 + \omega^2(1 - \dot{x}^2))$ and

$$X(t) = U(t-t_n)X_n^+(t_n+0) = U(t-t_n)F_n X(t),\ n = 0, 1, \ldots \tag{38}$$

We again note the similarity with the Hamiltonian of a particle subject to the Lorentz force and an additional harmonic restoring impulsive force. In the Schrödinger picture the system is quantized by maintaing a_ω as a c-number and replacing p_ω by the derivative operator $-i\delta/\delta a_\omega$. Then the velocity commutators are:

$$[\dot{a_\omega}, \dot{a}_{\omega'}] = 2i\dot{x}_n F(\omega,\omega'),\ t_n < t \leq t_{n+1},\ n = 0, 1, \ldots. \tag{39}$$

The Schrödinger equation with impulse effect becomes:

$$H_{n+1}\Psi[\{a_\omega(t_n)\}, t] = d/dt\Psi[\{a_\omega(t_n)\}, t],\ t_n < t \leq t_{n+1},\ n = 0, 1, \ldots \tag{40}$$

with the free Hamiltonian:

$$H_{n+1} = -i1/2\int_0^\infty d\omega\{-\delta^2/\delta a_\omega^2(t_n) + (m^2+\omega^2(1-\dot{x}_n^2))a_\omega^2(t_n)\},\ t_n < t \leq t_{n+1}, \tag{41}$$

$n = 0, 1, \ldots$

and the impulsive operator:

$$F_n = 1 - \dot{x}_n t_n \int_0^\infty d\omega \int_0^\infty d\omega' F(\omega,\omega') a_{\omega'}(t_n)\delta/\delta a_\omega(t_n),\ t = t_n, \tag{42}$$

$n = 0, 1, \ldots$

with

$$\Psi_n^+[\{a_\omega^+(t_n+0)\}, t_n+0] = F_n\Psi[\{a_\omega(t_n)\}, t_n] = F_n\Psi_n[\{a_\omega(t_n)\}],\ t = t_n, \tag{43}$$

$n = 0, 1, \ldots$

We have omitted a divergent normal ordering constant arising from

$$\delta a_\omega/\delta a_\omega = \delta(0). \tag{44}$$

4 Conclusions

In this paper the covariant functional Schroedinger formalism with the impulse effect in a Banach space is extended of non-separable spacetime foliations.

In principle, at least, one can solve the Schroedinger equation resulting from the situation where we are using an time-dependent basis system and one then could compute arbitrary expectation values. However, the association of "particles" with the excitation modes A_ω is even more dubious than it is for separable foliations alredy. There we can choose the basis functions on Σ_s to be the spatial part of a complete set of solutions of the classical field equations. Because these modes diagonalize the Hamiltonian, they do not mix during the time evolution. In this sens they preserve their identity and could be considered as "particles" in a restricted sense. In the non-separable cases, however, it is not possible to find modes with this property. The closest analogue one could imagine is that it is possible (in general it is not) for the impulsive moving mirror model to diagonalize kinetic and potential energy matrices T and V, and to simultaneously skew- diagonalize A-independent field strength matrice F.

In a typical separable situation we have a spatial momentum conservation, which, in our case, is spoiled by the presence of the impulsive moving mirror. For the case $\dot{x}_n = constant$ for $t \neq t_n$ and assumed that this change in $t = t_n$ by jumps is possible to obtain closed form solutions to the Schroedinger equation with impulse effect in Banach space.

Acknowledgements. The author thanks J.A. Smorodinski and D. Kasakov for important remarks and comments.

References

[1] Casimir, H.B.G., Proc. Kon. Nederl. Akad. Wet., 51, 793 (1948).

[2] Bordag, M., Petrov, G., Robaschik, D., Yad. Fiz., 39, 1315 (1984).

[3] Petrov, G., Bulg. J. Phys., 12, 4, 355 (1985).

[4] Petrov, G., Rev. Roum. Phys., 34, 5, 453 (1989).

[5] Birrel, N.D., Davies, P.G.W., Quantum Fields in Curved Space, Cambridge University Press, 1982.

[6] Reuter, M. Hill, C.T., Moving Mirrors in Schrödinger Picture, CERN-TH.5126/88.

[7] Bainov, D.D., Kostadinov, S.I., and Myshkis, A.D. (1988b). Bounded and periodic solutions of differential equations with impulse effect in a Banach space, Differential and Integral Equations, 1, 223-230.

[8] Bainov, D.D., Kostadinov, S.I., Nguyen Hong Thai, and Zabreiko, P.P.,(1989). Existence of Integral Manifolds for Impulsive Differential Equations in a Banach Space, International Journal of Theoretical Physics, 28,7, 815-833.

[9] Bainov, D.D., Kostadinov, S.I. and Zabreiko, P.P., (1988). International Journal of Theoretical Physics, 27, 731-743.

[10] Freese, K., Hill, C.T., Mueller, M., Nucl. Phys. B255 (1985), 693.

Received by Publishing Department

on June 29, 1992.

Петров Г. E2-92-276

Модель импульсно движущегося зеркала и импульсные дифференциальные уравнения в пространстве Банаха

Получены импульсные дифференциальные уравнения в пространстве Банаха как следствие зависимости вакуумной энергии от расстояния между равномерно движущимися зеркалами. Сформулирована проблема движущегося зеркала с внезапным изменением скорости v для $t = t_n$, $n = 0,1,2,\ldots$, на котором квантовое поле выполняет условие Дирихлета, связанное с эффектом Казимира в функциональной картине Шредингера.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Сообщение Объединенного инсти

Petrov G.

Impulsive Moving...

E2-92-276.

Petrov G.

Impulsive Moving

and Impulsive Di

in Banach Space

The dependence

ce between the u

used to calculate

Banach space. We

rors, possibly wi

$t = t_n$, n = 0,1,2

fy Dirichlet boun

Casimir effect, i

The investigat

tory of Theoretic

Communication of th