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E2-92-272

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IMPULSIVE MOVING MIRROR MODEL IN A SCHROEDINGER PICTURE WITH IMPULSE EFFECT IN A BANACH SPACE

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1992

1 Introduction

In 1948 Casimir [1] showed that when two uncharged plane conductors are placed parallel to each other in vacuum there is an attractive force between them proportional to the inverse forth power of their separation. Introducing boundaries, we change the energy of the electromagnetic vacuum. The dependence of the vacuum state energy on the distance between the capacitor plane is used to calculate the Casimir force. The question considered is whether the motion of one of the mirrors with constant velocity v may change appreciably the structure of the vacuum state and, consequently, the Casimir force.

Another field theoretical approach for studing the properties of the vacuum starts from analysis of the behaviour of local field quantities in Minkowski space with uniformly moving mirrors. The advantage of the local definition is that it permits a different point of view and a deeper anderstanding of the nature of vacuum energy [2,3,4].

The key problem of quantum field theory in arbitrary Riemannian space is that, in general, there is no preferred choice of time-like vector field with respect to which positive and negative frequencies are defined. Different choices lead generally to different Fock spaces and, in particular, to different vacuum states. In the usual discussions one treats the solution of the wave equation as an operator; but there is an implicit dependence in the definition of the local operator algebra upon the timelike vector field. This is reflec ted in the non-trivial Bogoliubov transformation relating the field operators arising from different choices of the "vacuum" [5].

In the Schroedinger picture one must constract a Hamiltonian which propagates the wave-fanctional (or local operators) on the time slices defined by a time like vector field. One selects a family of space-like surfaces, a foliation, $\{\Sigma_t\}$, where the usual tools of differential geometry fail to apply because the space under study is not a "good" manifold (space of leaves of foliation, $\{\Sigma_t\}$), each foil labelled by the "time" parameter, $t_n < t \leq t_{n+1}$, n = 0,1,... and locally orthogonal, $\Psi(\phi, t)$, of "snapshot" (t-independent) field configuration on Σ_t. ϕ is the fundamental variable, analogus to $\vec{x}$ in ordinary single particle quantum mechanics, and the probability of finding configuration ϕ_1 on the slice labelled by t_1 is $|\Psi(\phi_1, t_1)|^2$.

Consider a real massles scalar field $\phi(t,x)$ on two-dimensional Minkowski space with coordinates (t,x) bounded by a mirror (actualy a path) at $x = \dot{x}_n.t, t_n < t \leq t_{n+1}$, $n = 0, 1, \ldots$, $\dot{x}_n = dx/dt \; |t = t_n$. The field is required to fulfiel Dirichlet boundary conditions at the location of the mirror.

$$\phi(t, \dot{x}_n.t) = 0, \; t_n < t \leq t_{n+1}, \; n = 0, 1, \ldots, \tag{1}$$

$\dot{x} = 0$ for $0 \leq t \leq t_0$ assumed that this change of the path takes place by jumps.

The ϕ equation of motion is simply.

$$(\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2})\phi(t,x) = 0, \tag{2}$$

$t_n < t \leq t_{n+1}$, n = 0,1,...

If we quantize the field in the Schroedinger picture with Σ_t chosen as the $t_n = t = const$ lines, for the basis on Σ_t, we can use

$$\phi_\omega(x,t) = \sqrt{2/\pi}\sin(\omega(x - \dot{x}_n.t)), \tag{3}$$

$\omega > 0$, $t_n < t \leq t_{n+1}$, n = 0,1,...,[for t_n and x_n see (2,3,4)], since the set [$\sin \omega x$], $\omega > 0$, is complete for $x > 0$.

Because of the time-dependent conditions the ϕ_ω's are necessarily time-dependent too. Introducing a new spatial coordinate x' = x -$\dot{x}_n.t$, $t_n < t \leq t_{n+1}$, we can get a time independent boundary conditions, but this changes the Minkowski metric to:

$$ds^2 = (1 - \dot{x}_n)dt^2 - 2\dot{x}_n dx'dt - dx'^2 \tag{4}$$

and the field equation to:

$$[\partial^2/\partial t^2 - 2\dot{x}_n \partial/\partial t \partial/\partial x' - \ddot{x}_n \partial/\partial x' - (1 - \dot{x}_n^2)\partial^2/\partial x'^2]\phi(t,x') = 0, \; t_n < t \leq t_{n+1}. \tag{5}$$

For the case $\ddot{x} = 0$, the solutions of eq.(5) are separable as $\phi = \phi_\omega(x')\exp(-i\omega t)$.

Assuming this implies that the foliatian $t_n = const$ is separable and in this case the Schroedinger Hamiltonian decuples into a set of harmonic oscillator Hamiltonian, possibly with time-dependent frequencies. In this case it is fairly easy to study, for instance, cosmological particle creation. Making a Gaussian product ansatz for the ground state wave-functional,

the Schroedinger equation becomes an ordinary differential equation for its covariance. Once this is solved, it is straightforward to obtain particle number expectation values [10].

Identifying t_n with the cosmic time, t, and $\Sigma_s = \Sigma_t$ with the section $t_n = s = t = const$, it is obvious that for every t the surface Σ_t is the euclidean space R. Therefore we can use the same basis on each hypersurface. Foliations of a given space for which this is possible we call separable. In general an arbitrary foliation is non-separable.

The purpose of this paper is two-fold. First, we set up a general formalism for field quantization on non-separable foliations and in this case the impulsive Schrödinger Hamiltonian decouples into a set of oscillators interacting with a "impulsive effect".

Second, we apply this formalism to the impulsive moving mirror model for different foliations, that will be relevant to the discussion of Casimir effect.

With the local field theoretical approach for studing the properties of the vacuum is given a different point of view and a deeper understanding of the nature of vacuum energy. With the application of the equations with impulse effect in the impulsive moving mirror model in Minkowski space has been given another point of view in the local field theoretical approach for studing the properties of the vacuum. The first paper in this direction is the paper by G. Petrov [11].

The equations with impulse effect are models of real processes which are subject to disturbances in its evolution acting in time very short compared with the entire duration of the process.

The first paper in this direction is the paper by Milman and Myshkis [12] (1960). The mathematical theory of ordinary differential equations with impulse effect has been further developed in the papers [13] - [15]. The paper [16] initiates the investigations of the equations with impulse effect in a Banach space. The application of these equations [17,18] in quantum physics is given in the papers [11] and [19-21].

In these problems the fundamental degrees of freedom of the Schroedinger picture necessarly acquire an implicit time dependence. Hence, the RHS of the Schroedinger equation, $H\Psi = id/dt\Psi$, seemingly becomes subject to interpretation by suddenly change of velocity $\dot{x}_n$ for $t = t_n$, (n = 0,1,2,...) of the moving mirror, e.g. $d/dt\Psi \rightarrow \Psi^+(t_n + 0) - \Psi(t_n) = \int_{\dot{x}_n.t}^{\infty} dx[\phi^+(x) - \phi(x)]\delta/\delta\phi(x)\Psi(t_n)$.

We give a rigrous canonical analysis of thes impulse effect in paper [11] which confirms the latter.

2 Covariant impulsive Schroedinger picture for non-separable foliations

A. We briefly repeat the main ingradients of the Schroedinger picture [10].

Given a spacetime manifold, we fix a coordinate system, $x^\mu(\mu = 0,1,...,d)$ and choose a family $\{\Sigma_s\}$ of spacelike hypersurfaces labelled by a parameter s, $t_n < s \le t_{n+1}$. On Σ_s we choose coordinate ξ^i (i = 1,...,d). The embedding of the hypersurfaces into spacetime is defined by a set of equations

$$x^\mu = x^\mu(s,\xi^i).$$

Then the timelike vector-field $\partial x^\mu/\partial s$ is normal to Σ_s.

The volume element of the surface reads:

$$d^d\sigma_\mu = \sqrt{-g}\epsilon_{\mu\nu_1...\nu_d}\frac{\partial x^{\nu_1}}{\partial \xi^1}\cdots\frac{\partial x^{\nu_d}}{\partial \xi^d}d^d\xi \equiv \frac{D\sigma_\mu}{D\xi}d^d\xi. \qquad (6)$$

It is invariant under change of the coordinates ξ^i on $\Sigma_s, t_n < s \le t_{n+1}$, n = 0,1,...

Now consider a scalar field $\phi(s,\xi) \equiv \phi(x^\mu(s,\xi))$ with action S and energy momentum tensor $T_{\mu\nu}$. The theory is quantized by definig the canonical momentum as:

$$\pi = \mid g \mid^{-1/2}\frac{\partial x^\mu}{\partial s}\frac{\delta S}{\delta(\partial^\mu\phi)}, t_n < s \le t_{n+1}, n = 0,1,... \qquad (7)$$

and requiring the following commutation relations on each hypersurface:

$$[\phi(s,\xi),\pi(s,\xi')] = i\delta_\Sigma(\xi,\xi'), t_n < s \le t_{n+1}, n = 0,1,... \qquad (8)$$

Here the δ-function

$$\delta_\Sigma(\xi,\xi') = \mid \frac{\partial x^\mu}{\partial s}\frac{\partial x_\mu}{\partial s}\mid^{1/2}\mid \frac{D\sigma^\nu}{D\xi}\frac{D\sigma_\nu}{D\xi}\mid^{-1/2}\prod_{i=1}^{d}\delta(\xi^i - \xi^{i'}) \qquad (9)$$

it is invariant under both $s \to s'(s)$ transformations and ξ- reparametrazation on Σ_s. The commutation relation eq.(8) is fulfilled if on each Σ_s the operator ϕ is represented by the multiplication with an s-independent function $\phi(\xi)$ and the momentum by:

$$\pi(\xi) = -i \mid \frac{\partial x^\mu}{\partial s}\frac{\partial x_\mu}{\partial s}\mid^{1/2}\mid \frac{D\sigma^\nu}{D\xi}\frac{D\sigma_\nu}{D\xi}\mid^{-1/2}\frac{\delta}{\delta\phi(\xi)}. \qquad (10)$$

The functional derivative is to be anderstood in a d-dimensional sense.

The evolution of the wave-functional $\Psi(\phi(\xi);s)$ is governed by the Schroedinger equation:

$$\begin{aligned} H\Psi(\phi(\xi);s) &= \int d^d\sigma^\mu\frac{\partial x^\nu}{\partial s}T_{\mu\nu}(\phi(\xi),\pi(\xi))\Psi(\phi(\xi),s) \\ &= id/ds\Psi(\phi(\xi),s). \end{aligned}$$

$$t_n < s \le t_{n+1}, n = 0,1,... \qquad (11)$$

The Hamiltonian H is s-independent only if the spacetime under consideration allows $\partial x^\mu/\partial s, t_n < s \le t_{n+1}$, n = 0,1,... to chosen as a timelike Killing vector field. The Schroedinger equation, eq.(11), is form invariant under change of the spacetime coordinates x^μ, changes of the surface coordinates ξ^i and under reparametrazations $s \to s'(s)$ of the evolution parameter. It is in this sense that the formalism is covariant, however. Different choices of $\{\Sigma_s\}$ lead to different quantum theoris.

We consider now the case of non-separable foliations. By definition this means that it is impossible to use the same system $\{\phi_\omega(\xi)\}$ on all hypersurface Σ_s. One has to introduce a family $\{\phi_\omega(\xi,s)\}$ of complete system, one for each surface. The time evolution of $\Psi[\{a_\omega\},s]$ consists of two pieces then: there is also an explicit time dependence due to the fact that the basis functions are s-dependent. Therefore the Hamiltonian no longer has the simple form of eq. (11). We can study the time evolution of Ψ including the effect of the s-dependence of ϕ_ω. For definiteness we consider a minimally coupled, real scalar field with the action:

$$S = 1/2\int_{t_n}^{t_{n+1}}dt\int d^dx\sqrt{-g}[g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - m^2\phi^2]. \qquad (12)$$

Let $\phi = \phi(x^\mu)$ be an arbitrary field configuration defined on the entire spacetime manifold. Using the relation $x^\mu = x^\mu(s,\xi^i)$ defining the embeding, we write ϕ as function of s and ξ^i; $\phi(s,\xi^i) \equiv \phi(x^\mu(s,\xi^i))$, $t_n < s \leq t_{n+1}$, n = 0,1,...

This function is expanded in terms of the complete sets $\{\phi_\omega(\xi,s)\}$ as:

$$\phi(s,\xi^i) = \sqrt{2/\pi}\int_0^\infty d\omega[a_\omega(s)\phi_\omega(\xi,s) + a^*_\omega(s)\phi^*_\omega(\xi,s)],\ t_n < s \leq t_{n+1}, \tag{13}$$

n=0,1,... Note that the basis vectors $\{\phi_\omega(\xi,s)\}$ are complete on each Σ_s, but this does not imply that they are complete when considered as one function $f_\omega(x^\mu) = \phi_\omega(\xi(x^\mu),s(x^\mu))$ on the spacetime manifold. This obvious from the extreme case of a separable foliation where ϕ_ω has no s-dependence at all. Thus, in the expansion eq. (13) the coefficients are generally required to be s-dependent.

We now express the action in terms of the a_ω's by inserting eq. (13) into eq. (12). One obtains:

$$\begin{aligned} S &= 1/2\int_{t_n}^{t_{n+1}} ds\int_0^\infty d\omega\int_0^\infty d\omega'[(\dot a_\omega,\dot a^*_\omega)\begin{pmatrix} T^{(1)}_{\omega\omega'} & T^{(2)}_{\omega\omega'} \\ T^{(2)}_{\omega\omega'} & T^{(1)*}_{\omega\omega'}\end{pmatrix}(\dot a_{\omega'},\dot a^*_{\omega'})^T - \\ &- (a_\omega,a^*_\omega)\begin{pmatrix} V^{(1)}_{\omega\omega'} & V^{(2)}_{\omega\omega'} \\ V^{(2)}_{\omega\omega'} & V^{(1)*}_{\omega\omega'}\end{pmatrix}(a_{\omega'},a^*_{\omega'})^T + \\ &+ (\dot a_\omega,\dot a^*_\omega)\begin{pmatrix} f^{(1)}_{\omega\omega'} & f^{(2)}_{\omega\omega'} \\ f^{(2)}_{\omega\omega'} & f^{(1)*}_{\omega\omega'}\end{pmatrix}(a_{\omega'},a^*_{\omega'})^T] \end{aligned} \tag{14}$$

where:

$$\begin{aligned} T^{(1)}_{\omega\omega'}(s) &= \int d^d\tau\tilde g^{ss}\phi_\omega\phi_{\omega'} \\ T^{(2)}_{\omega\omega'}(s) &= \int d^d\tau\tilde g^{ss}\phi_\omega\phi^*_{\omega'} \\ V^{(1)}_{\omega\omega'}(s) &= \int d^d\tau\{m^2\phi_\omega\phi_{\omega'} - \tilde g^{ss}\dot\phi_\omega\dot\phi_{\omega'} - 2\tilde g^{sj}\dot\phi_\omega\partial_j\phi_{\omega'} - \tilde g^{ij}\partial_i\phi_\omega\partial_j\phi_{\omega'}\} \\ V^{(2)}_{\omega\omega'}(s) &= \int d^d\tau\{m^2\phi_\omega\phi^*_{\omega'} - \tilde g^{ss}\dot\phi_\omega\dot\phi^*_{\omega'} - 2\tilde g^{sj}\dot\phi_\omega\partial_j\phi^*_{\omega'} - \tilde g^{ij}\partial_i\phi_\omega\partial_j\phi^*_{\omega'}\} \\ f^{(1)}_{\omega\omega'}(s) &= \int d^d\tau\{\tilde g^{ss}\phi_\omega\dot\phi_{\omega'} + \tilde g^{sj}\phi_\omega\partial_j\phi_{\omega'}\} \\ f^{(2)}_{\omega\omega'}(s) &= \int d^d\tau\{\tilde g^{ss}\phi_\omega\dot\phi^*_{\omega'} + \tilde g^{sj}\phi_\omega\partial_j\phi^*_{\omega'}\} \end{aligned} \tag{15}$$

and:

$$\begin{aligned} d^d\tau &= \frac{\partial x^\mu}{\partial s}\frac{D\sigma_\mu}{D\xi}d^d\xi \\ \tilde g^{ss} &= g^{\mu\nu}\frac{\partial s}{\partial x^\mu}\frac{\partial s}{\partial x^\nu} \\ \tilde g^{sj} &= g^{\mu\nu}\frac{\partial s}{\partial x^\mu}\frac{\partial \xi^j}{\partial x^\nu} \\ \tilde g^{ij} &= g^{\mu\nu}\frac{\partial \xi^i}{\partial x^\mu}\frac{\partial \xi^j}{\partial x^\nu} \end{aligned} \tag{16}$$

Here $\dot\phi$ and $\partial_i\phi$ denote the derivative with respect to s and ξ, respectively. In an obvious matrix notation the action can be written as:

$$S = 1/2\int_{t_n}^{t_{n+1}} ds\int_0^\infty d\omega\int_0^\infty d\omega'\{\dot A_\omega T_{\omega\omega'}\dot A_{\omega'} - A_\omega V_{\omega\omega'}A_{\omega'} + 2\dot A_\omega F_{\omega\omega'}A_{\omega'}\} \tag{17}$$

where $A_\omega \equiv (a_\omega, a^*_\omega)$ and the matrices T, V and F can be read off from eq. (14) and eq. (15); in general they depend on s.

The canonical variables are the A_ω's and their conjugate momenta:

$$P_\omega = T_{\omega\omega'}\dot A_{\omega\omega'} + F_{\omega\omega'}A_{\omega'} \tag{18}$$

They are required to satifsfay the canonical commutation relation:

$$[A_\omega, P_{\omega'}] = i\delta(\omega-\omega');\ [A_\omega, A_{\omega'}] = [P_\omega, P_{\omega'}] = 0. \tag{19}$$

Acting on a Schroedinger wave functional $\Psi[\{A_\omega\},s]$ we represent A_ω multiplicatively and P_ω as $-i\delta/\delta A_\omega$.

The Hamiltonian, H, describing the time evolution of Ψ is obtained from eq. (17) via the usual Legandre transformation.

B. From eq. (18) we obtain

$$\dot A_\omega = T^{-1}_{\omega\omega'}P_{\omega'} - T^{-1}_{\omega\omega''}F_{\omega''\omega'}A_{\omega'} \tag{20}$$

and from

$$-\delta H/\delta A_\omega = \dot P_\omega$$

folow

$$d/dt[T_{\omega\omega'}\dot{A}_{\omega'}] = -V_{\omega\omega'}A_{\omega'} - \dot{F}_{\omega\omega'}A_{\omega'} \tag{21}$$

So we obtain the equation:

$$\begin{pmatrix} \dot{A}_{\omega} \\ d/dt[T_{\omega\omega'}\dot{A}_{\omega'}] \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -V_{\omega\omega'} - \dot{F}_{\omega\omega'} & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -T^{-1}_{\omega\omega''}F_{\omega''\omega'} & T^{-1}_{\omega\omega'} \end{pmatrix} \begin{pmatrix} A_{\omega'} \\ P_{\omega'} \end{pmatrix} \tag{22}$$

if $\dot{F}_{\omega\omega'} \neq 0$.

If we take $A_{\omega}(s=0) = A^{+}_{\omega}(0)$ and $P_{\omega}(s=0) = P^{+}_{\omega}(0) = T^{+}_{\omega\omega'}(0)\dot{A}_{\omega'}(0)$ then for $0 < s \leq t_0$ and $P_{\omega}(s) = T_{\omega\omega'}\dot{A}_{\omega'}(s)$ we obtain:

$$\begin{pmatrix} A_{\omega}(s) \\ P_{\omega}(s) \end{pmatrix} = \exp\begin{pmatrix} 0 & \int_0^s ds' T^{-1}_{\omega\omega'}(s') \\ -\int_0^s ds' V_{\omega\omega'}(s') & 0 \end{pmatrix} \begin{pmatrix} A^{+}_{\omega'}(0) \\ P^{+}_{\omega'}(0) \end{pmatrix} \tag{23}$$

and for $s = t_0$: $A^{+}_{\omega}(t_0+0) = A_{\omega}(t_0)$; $P^{+}_{\omega}(t_0+0) = T_{\omega\omega'}(t_0+0)\dot{A}^{+}_{\omega}(t_0+0) = P_{\omega}(t_0) - F_{\omega\omega'}(t_0+0)A_{\omega'}(t_0)$. For $t_0 < s \leq t_1$ the solution is:

$$\begin{pmatrix} A_{\omega}(s) \\ P_{\omega}(s) \end{pmatrix} = \exp\begin{pmatrix} 0 & \int_{t_0}^s ds' T^{-1}_{\omega\omega'}(s') \\ -\int_{t_0}^s ds' V_{\omega\omega'}(s') & 0 \end{pmatrix} \begin{pmatrix} A^{+}_{\omega'}(t_0+0) \\ P^{+}_{\omega'}(t_0+0) \end{pmatrix} \tag{24}$$

The differential evolution equation with impulsive effect for $t_n < s \leq t_{n+1}$(n = 0,1,...) is:

$$\begin{pmatrix} \dot{A}_{\omega} \\ \dot{P}_{\omega} \end{pmatrix} = \begin{pmatrix} 0 & T^{-1}_{\omega\omega'} \\ -V_{\omega\omega'} & 0 \end{pmatrix} \begin{pmatrix} A_{\omega'} \\ P_{\omega'} \end{pmatrix} \tag{25}$$

with $P_{\omega}(s) = T_{\omega\omega'}\dot{A}_{\omega'}(s)$ and where $V(s) \equiv \begin{pmatrix} 0 & T^{-1}_{\omega\omega'}(s) \\ -V_{\omega\omega'}(s) & 0 \end{pmatrix}$ is a continuous operator-function on s, the values of which are linear bounded operators mapping the Banach space B into itself. For $s = t_n$, $(n = 0, 1, \ldots)$ we have

$$\begin{pmatrix} A^{+}_{\omega}(t_n+0) \\ P^{+}_{\omega}(t_n+0) \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -F_{\omega\omega'}(t_n) & 1 \end{pmatrix} \begin{pmatrix} A_{\omega}(t_n) \\ P_{\omega}(t_n) \end{pmatrix} \tag{26}$$

where $F_n \equiv \begin{pmatrix} 1 & 0 \\ -F_{\omega\omega'}(t_n) & 1 \end{pmatrix}$ is continuous with $F_n : B \to B$.

So for $s \neq t_n (n = 0, 1, \ldots)$ and $X(s) = \begin{pmatrix} A_{\omega}(s) \\ P_{\omega}(s) \end{pmatrix}$ we have an impulsive linear equation

$$\dot{X}(s) = V(s)X(s) \tag{27}$$

and with impulsive effect for $s = t_n (n = 0, 1, \ldots)$

$$X^{+}(t_n+0) = F_n X(t_n) \tag{28}$$

3 Statement of the problem

Consider the differential equation with impulse effect of the form

$$\begin{aligned} d/dt X(t) &= V(t)X(t) \,|\, t \neq t_n \, (n = 0, 1, \ldots) & (29) \\ X(t_n+0) &= F_n X(t_n), \, (n = 1, 2, \ldots) & (30) \end{aligned}$$

where

$$V(t) = \begin{pmatrix} 0 & k(t) \\ v(t) & 0 \end{pmatrix}$$

with $v(t) = -V_{\omega\omega'}(t) - \dot{F}_{\omega\omega'}(t)$ and $k(t) = T^{-1}_{\omega\omega'}(t)$. For the impulse operator is

$$F_n = \begin{pmatrix} 1 & 0 \\ -F_{\omega\omega'}(t_n) & 1 \end{pmatrix}$$

and for

$$X(t) = \begin{pmatrix} A_{\omega}(t) \\ P_{\omega}(t) \end{pmatrix}$$

with $P_{\omega}(t) = T_{\omega\omega'}\dot{A}_{\omega'}(t)$.

The operator $V(t)$, for $(t > t_0)$ is a continuous operator-function on t, the values of which are linear bounded operators mapping the complex Banach space B into itself. The operators $F_n : B \to B$, for $(n = 0, 1, \ldots)$ are continuous and the moments t_n, for $(n = 0, 1, \ldots)$ of the impulse effect satisfy the condition $0 < t_0 < t_1 < \ldots$ and $\lim_{n\to\infty} t_n = \infty$.

DEFINITION 1. We say that $X(t)$ is a solution of the equation with impulse effect (29),(30) if X(t) is a piecewise right continuous function with first order discontinuities for $t = t_n$, $(n = 1, \ldots)$, and such that

$$d/dtX(t) = V(t)X(t)$$

for $t \neq t_n$, $(t > t_0)$, $(n = 1, ...)$ and

$$X(t_n + 0) = F_n X(t_n), \text{ for } t = t_n, (n = 1, ...)$$

For $X_0 \in B$ and $t \geq t_0$ the equation with impulse effect (29), (30) has unique solution $X(t)$ which satisfies the condition

$$X(t_0) = X_0 \tag{31}$$

Then we can consider the family of operators $W(t)$ defined by the formula

$$W(t)X_0 = X(t), \ (t_0 < t < \infty).$$

Denote by $U(t, s)$ the evolution operator of the equation

$$d/dtX(t) = V(t)X(t). \tag{32}$$

We note that for $t_n < t \leq t_{n+1}$, $(n = 0, 1, ...)$ the following equality holds

$$W(t) = U(t, t_n)F_nU(t_n, t_{n-1})F_{n-1}...U(t_1, t_0).$$

DEFINITION 2. The equation with impulse effect (29), (30) is called stable if

$$\sup_{t_0 \leq t < \infty} \| W(t) \| < \infty.$$

DEFINITION 3. The equation with impulse effect (29), (30) is called asymptotically stable if

$$\lim_{t \to \infty} \| W(t)X \| = 0. \quad (X \in B).$$

DEFINITION 4. The equation with impulse effect (29), (30) is called uniformly stable if

$$\lim_{t \to \infty} \| W(t) \| = 0.$$

LEMMA 1. Let the following condition hold

$$V(t) \int_{t_0}^{t} dsV(s) = \int_{t_0}^{t} dsV(s) \,.\, V(t) \quad (t \geq t_0). \tag{33}$$

Then the solution X(t) of (32) for $t \geq t_0$ with initial condition has the form

$$X(t) = e^{\int_{t_0}^{t} dsV(s)} X_0.$$

The proof of Lemma 1. is standard.

REMARK 1. For small $t \geq t_0$ the condition is sufficient as well for the solution of (32) to have the form

$$X(t) = e^{\int_{t_0}^{t} dsV(s)} X_0.$$

In fact, let $X(t) = e^{\int_{t_0}^{t} dsV(s)} X_0$. The equality

$$(e^{\int_{t_0}^{t} dsV(s)})' = V(t)e^{\int_{t_0}^{t} dsV(s)}$$

can be represented as

$\sum_{n=0}^{\infty}([(n + 1)!]^{-1}((\int_{t_0}^{t} dsV(s))^{n+1})' - (n!)^{-1}V(s)(\int_{t_0}^{t} dsV(s))^{n}) = 0.$

We set $\Delta(t) = V(t) \int_{t_0}^{t} dsV(s) - \int_{t_0}^{t} dsV(s).\, V(t)$.

We can deform $V^{tr}(t).C. \int_{t_0}^{t} dsV(s) - \int_{t_0}^{t} dsV^{tr}(s).C.V(t) = \Delta^C(t)$ where $C = \begin{pmatrix} 0 & -q^{-1/2} \\ q^{1/2} & 0 \end{pmatrix}$ and $\Delta^C(t) = 0$ if the following conditions hold

$$k(t) \int_{t_0}^{t} dsv(s) = \int_{t_0}^{t} dsv(s).\, k(t) \tag{34}$$

$$v(t) \int_{t_0}^{t} dsk(s) = \int_{t_0}^{t} dsk(s).\, v(t) \tag{35}$$

Then the following equalities hold:

$$\sum_{n=0}^{\infty}([(n + 1)!]^{-1}(\sum_{k=0}^{n}(\int_{t_0}^{t} dsV(s))^{k}V(t)(\int_{t_0}^{t} dsV(s))^{n-k} -$$
$$(n + 1)V(t)(\int_{t_0}^{t} dsV(s))^{n})) =$$
$$= \sum_{n=0}^{\infty}[(n + 1)!]^{-1} \sum_{k=1}^{n}((\int_{t_0}^{t} dsV(s))^{k}V(t)(\int_{t_0}^{t} dsV(s))^{n-k} -$$
$$- V(t)(\int_{t_0}^{t} dsV(s))^{n}) = \sum_{n=0}^{\infty}[(n + 1)!]^{-1} \sum_{k=1}^{n} k(\int_{t_0}^{t} dsV(s))^{n-k}\Delta(t).$$
$$(\int_{t_0}^{t} dsV(s))^{k-1} = 0, \text{i.e.}$$

$1/2\Delta^C(t) = - \sum_{n=2}^{\infty}[(n+1)!]^{-1} \sum_{k=1}^{n} k(\int_{t_0}^{t} dsV(s))^{n-k}\Delta^C(t)(\int_{t_0}^{t} dsV(s))^{k-1}.$

Finally we get $1/2 \| \Delta^C(t)\| \leq \sum_{n=2}^{\infty}[(n + 1)!]^{-1} \| \Delta^C(t)\| k(k + 1)/2$ $\| \int_{t_0}^{t} dsV(s)\|^{n-1} = \| \Delta^C(t)\|/2(e^{\|\int_{t_0}^{t} dsV(s)\|} - 1).$

Then for sufficiently small t

$$|e^{\|\int_\tau^t dsV(s)\|} - 1| < 1$$

and there exists an interval $[t_0, t^*]$ such that $\Delta^C(t) = 0$. $(t \in [t_0, t^*])$.

Applying the step method we come to the conclusion that the equality $\Delta^C(t) = 0$ holds as well if t belongs to an interval containing $[t_0, t^*]$. So the equations (34) and (35) holds.

4 Main result

We say that the condition (A) holds if the following condition is fulfilled:

(A) $\int_\tau^t dsV(s).C.V^{tr}(t) = V^{tr}(t).C.\int_\tau^t dsV(s)$, $(t_0 < \tau < t < \infty)$. with

$$V^{tr}(t) = \begin{pmatrix} 0 & -v(t) \\ k(t) & 0 \end{pmatrix}$$

and $\int_\tau^t dsk(s).v(t) = v(t).\int_\tau^t dsk(s)$; $\int_\tau^t dsv(s).k(t) = k(t)\int_\tau^t dsv(s)$

REMARK 2. Let the condition (A) hold. Then for $t_0 < \tau < t < \infty$ the following equality holds

$$V^{tr}(t).C.V(\tau) = V(\tau).C.V^{tr}(t) \text{ with}$$
$$k(t)v(\tau) = v(\tau)k(t); \; v(t)k(\tau) = k(\tau)v(t)$$

We say that conditions (B) are satisfied if:

B1: Condition (A) holds.

B2: There exists a constant $T > 0$ for which $t_{n+1} - t_n \leq T$ for n = 1,2,...

B3: $e^{\|\int_\tau^t dsV(s)\|} \leq e^{\gamma(t-\tau)}$ for $0 \leq t - \tau \leq T$ where γ is a constant.

B4. $\| F_n X\| \leq q_n \| X\| + h_n$ for n = 1,... and $X \in B$ so that $q_1 q_2 ... q_{n(t)} \leq Le^{\delta.t}$ where $n(t) = n$ for $t_n < t \leq t_{n+1}$, while L and δ are constants.

If the conditions (B) hold, for the solution X(t) of the equation with impulse effect (1),(2) with the initial condition (3) we obtain the following estimation

$$\begin{aligned} \| X(t)\| &\leq e^{\gamma(t-t_n)}\{q_n e^{\int_{t_{n-1}}^{t_n} dsV(s)} F_{n-1} e^{\int_{t_{n-2}}^{t_{n-1}} dsV(s)} F_{n-2}...F_1 e^{\int_{t_0}^{t_1} dsV(s)} X_0\| + h_n\} \\ &\leq e^{\gamma(t-t_{n-1})} q_n \{q_{n-1} \| e^{\int_{t_{n-2}}^{t_{n-1}} dsV(s)} F_{n-2}...F_1 e^{\int_{t_0}^{t_1} dsV(s)} X_0\| + h_{n-1}\} + \end{aligned}$$

$$\begin{aligned} & + e^{\gamma(t-t_n)} h_n \leq \\ \leq \; & e^{(t-t_{n-2})} q_n q_{n-1} \| F_{n-2}...F_1 e^{\int_{t_0}^{t_1} dsV(s)} X_0\| + e^{\gamma(t-t_{n-1})} q_n h_{n-1} + \\ & + e^{\gamma(t-t_n)} h_n \leq \\ \leq \; & ... \leq \qquad (36) \\ \leq \; & e^{\gamma(t-t_0)} q_n q_{n-1}...q_1 \| X_0\| + \{e^{\gamma(t-t_n)} h_n + e^{\gamma(t-t_{n-1})} q_n \\ & h_{n-1} + e^{\gamma(t-t_{n-2})} q_n q_{n-1} h_{n-2} + ... + e^{\gamma(t-t_1)} q_n q_{n-1}...q_2 h_1\}. \end{aligned}$$

Using this estimation we will find sufficient conditions under which the solution of the equation with the impulse effect are bounded.

THEOREM 1. Let the conditions (B) hold. Let $h_1 = h_2 = ... = 0$.

Then for $\delta + \gamma < 0$, $\lim_{t\to\infty} X(t) = 0$ and for $\delta + \gamma = 0$ the solution X(t) is bounded for $t \geq t_0$.

The proof of the theorem 1 follows immediately from the estimation (37).

THEOREM 2. Let the condition (B) be satisfied. Suppose additionally that

$$q \models q_1 = q_2 ...; h = h_1 = h_2 = ...$$

and let

$$t_n = t_0 + nk \, (n = 1, 2, ...), \; t = t_0 + \tau k,$$

where $n \leq \tau \leq n + 1$, $k \leq T$ is a constant and the inequality

$$qe^{\gamma k} < 1$$

holds. Then for $\delta + \gamma \leq 0$ the solution X(t) is bounded on the half axis $t \geq t_0$.

Proof. We can denote by M the expression in braces in the right hand side in (8) and, having made certain transformations for nonrelativistic case, we obtain

$$M = e^{\gamma(\tau-n)k}(qe^{\gamma k} - 1)^{-1}.(q^n e^{\gamma k n} - 1)h.$$

Under the assumption made, M is bounded. This fact and the condition A_2 imply the assertion of the Theorem 2.

THEOREM 3. Suppose that the following conditions hold:

1. The condition (A) is fulfilled.

2.There exist constants $p \geq 0$ such that $0 \leq i(t_0, t) - p(t - t_0) \leq \sigma$ for $0 \leq t_0 \leq t < \infty$ where i(a, b) is the number of points t_n which lie in the interval $(a, b)\,(t_0 < a < b < \infty)$.

3. $F = F_1 = F_2 = ...$ where the operator F is linear and commutes with $\int_\tau^t dsV(s)$ for $t_0 \leq \tau \leq t < \infty$.

4. The operator F has a logarithm, i.e. there exists a linear and bounded operator $\ln F : B \to B$ such that $F = e^{\ln F}$.

Then the equation with impulse effect is bounded, asymptotycally bounded, uniformly asymptotically bounded if and only if respectively

$$a)\ \sup_{t \geq t_0} \| e^{\int_{t_0}^t ds(V(s) + p \ln F)} \| < \infty.$$

$$b)\ \lim_{t \to \infty} \| e^{\int_{t_0}^t ds(V(s) + \ln F)} X \| = 0 \ (X \in B).$$

$$c)\ \lim_{t \to \infty} \| e^{\int_{t_0}^t ds(V(s) + \ln F)} \| = 0.$$

Proof. The operator-function $W(t)$ has the form

$$W(t) = e^{\int_{t_0}^t dsV(s)} F^{i(t_0,t)} = e^{\int_{t_0}^t dsV(s)} F^{[p(t-t_0)]} F^{i(t_0-t)-[p(t-t_0)]} =$$

$$= e^{\int_{t_0}^t dsV(s)} e^{\ln F[p(t-t_0)]} F^{i(t_0,t)-[p(t-t_0)]} = \qquad (37)$$

$$= e^{\int_{t_0}^t dsV(s)} e^{\ln Fp(t-t_0)} e^{\ln F[p(t-t_0)] - \ln Fp(t-t_0)} F^{i(t_0,t)-[p(t-t_0)]} =$$

$$= e^{\int_{t_0}^t ds(V(s) + \ln Fp)} e^{\ln F\{[p(t-t_0)] - p(t-t_0)\}} F^{i(t_0,t)-[p(t-t_0)]}.$$

Taking into account the represontation (37) the assertions a). b) and c) follows immediately.

In the next section we write the differential equation with impulse effect down for a special exampl. In general it is rather complicated, since it not only contains s-dependent kinetic and potential energy matrices T and V, but also a term coupling A_ω to $\dot{A}_\omega$ or P_ω, respectively. As is seen from eq. (15), the origin of this term is the s-dependence of the basis functions. It couples A_ω to a "impulsive functional gauge potential" $\mathcal{A}_\omega = \int d\omega' F_{\omega\omega'} A_{\omega'}$ with a constant, i.e. A-independent impulsive field

strength tensor $\mathcal{F}_{[\omega,\omega']}$. This is reminiscent of a term $L_{int} = e\, x^i F^{ij} \dot{x}_j$ in particle mechanics, where $A^i = 1/2 F^{ij} x_j$ is the vector potential of a constant magnetic field F^{ij}. A consequence of this "functional magnetic field" is that the velocity operators do not commute:

$$[\dot{A}_\omega, \dot{A}_{\omega'}] = 2i\, T^{-1}_{\omega\omega''} T^{-1}_{\omega'\omega'''} \mathcal{F}_{[\omega''',\omega'']}. \qquad (38)$$

Hence there are correlations between the modes A_ω similar to the corre lation between the x- and y-motion of a charged particle in a magnetic field directed along the z-axis. By the standard argument a "pure gauge"

$$\mathcal{A}_\omega = \frac{\delta\Lambda[A]}{\delta A_\omega},$$

does not affect the dynamics. By incpection of $f^{(1)}_{\omega\omega'}$, and $f^{(2)}_{\omega\omega'}$ in eq.(15) one can readly be convinced that a pure gauge would correspond to a situation where are using an s-dependent basis system without being forced to do so, i.e. when on each Σ_s there exists an s-dependent basis $\{\phi'_\omega(\xi)\}$ to a new s-independent basis $\{\phi_\omega(\xi)\}$. By definition this means that $\{\Sigma_s\}$ is separable. Hence the statement $\mathcal{F}_{\omega\omega'} \neq 0$ is a coordinate system independent characterization of non-separable foliations.

5 Impulsive Moving Mirrors: Timelike Surfaces

A. To illustrate the method of section (2) we consider two dimensional Minkowski space with coordinates (t,x) bounded by a impulsive moving mirror at $x(t) = \dot{x}_n(t_n).t,\ t_n < t \leq t_{n+1}, n = 0, 1, ...$ For $x > x(t)$ we define a scalar field $\phi(t, x)$ with the action:

$$S = 1/2 \int_{t_n}^{t_{n+1}} dt \int_{\dot{x}_n . t}^{\infty} dx\{(\partial_t\phi)^2 - (\partial_x\phi)^2 - m^2\phi^2\},\ \dot{x}_n = const. \qquad (39)$$

ϕ is assumed to obey Dirichlet boundary conditiodns of the location of the mirror:

$$\phi(t, \dot{x}.t) = 0,\ t_n < t \leq t_{n+1}, n = 0, 1, ... \qquad (40)$$

The hypersurface $\Sigma_s = \Sigma_t$ are taken to be lines t = const. As a basis on Σ_t we use the function of eq. (3). As general configuration $\phi = \phi(t,x)$, $x > \dot{x}_n.t$ is expended as:

$$\phi(t,x) = \sqrt{2/\pi} \int_0^\infty d\omega [a_\omega(t) \sin(\omega(x - \dot{x}_n.t))]. \tag{41}$$

The action reads now:

$$S = 1/2 \int_{t_n}^{t_{n+1}} dt \int_0^\infty d\omega \{\dot{a}_\omega^2 - \omega^2(1 - \dot{x}_n^2)a_\omega^2 - m^2 a_\omega^2\} \tag{42}$$
$$- \int_{t_n}^{t_n+1} dt \dot{x}_n \int_0^\infty d\omega \int_0^\infty d\omega' F(\omega,\omega')\dot{a}_\omega a_{\omega'}$$

with the antisymmetric "impulsive strength tensor":

$$F(\omega,\omega') = 2/\pi \int_0^\infty dx \sin\omega x \omega' \cos\omega' x. \tag{43}$$

The momenta conjugate to a_ω are:

$$p_\omega = \dot{a}_\omega - \dot{x}_n \int_0^\infty d\omega' F(\omega,\omega') a_\omega. \tag{44}$$

B. The impulsive linear differential equation becomes:

$$\begin{aligned} \dot{X}(t) &= VX(t),\ t \neq t_n,\ n = 0,1,\dots \\ X_n^+ &= F_n X_n,\ t = t_n\ n = 1,2,\dots \end{aligned} \tag{45}$$

with $X = \begin{pmatrix} a_\omega(t) \\ p_\omega(t) \end{pmatrix}$ and $X^+ = \begin{pmatrix} a_\omega^+(t_n+0) \\ p_\omega^+(t_n+0) \end{pmatrix}$, by

$$V = \begin{pmatrix} 0 & 1 \\ -(m^2 + \omega^2(1 - \dot{x}_n^2)) & 0 \end{pmatrix}$$

and

$$F_n = \begin{pmatrix} 1 & 0 \\ -\dot{x}_n \int_0^\infty d\omega' F(\omega,\omega') & 1 \end{pmatrix}$$

with the evolutions operator:

$$U(t) = \begin{pmatrix} \cos(t - t_n)\sqrt{v} & \frac{\sin(t-t_n)\sqrt{v}}{\sqrt{v}} \\ -\sqrt{v}\sin(t - t_n)\sqrt{v} & \cos(t - t_n)\sqrt{v} \end{pmatrix} \tag{46}$$

with $v = -(m^2 + \omega^2(1 - \dot{x}^2))$ and

$$X(t) = U(t - t_n)X_n^+(t_n + 0) = U(t - t_n)F_n X(t_n),\ n = 0,1,\dots \tag{47}$$

We again note the similarity with the Hamiltonian of a particle subject to the Lorentz force and an additional harmonic restoring impulsive force. In the Schroedinger picture the system is quantized by maintaing a_ω as a c-number and replacing p_ω by the derivative operator $-i\delta/\delta a_\omega$. Then the velocity commutators are:

$$[\dot{a}_\omega, \dot{a}_{\omega'}] = 2i\dot{x}_n F(\omega,\omega'),\ t_n < t \leq t_{n+1},\ n = 0,1,\dots. \tag{48}$$

The Schroedinger equation becomes:

$$H_{n+1}\Psi[\{a_\omega(t_n)\}, t] = d/dt\Psi[\{a_\omega(t_n)\}, t],\ t_n < t \leq t_{n+1},\ n = 0,1,\dots \tag{49}$$

with the free Hamiltonian:

$$H_{n+1} = -i1/2 \int_0^\infty d\omega \{-\delta^2/\delta a_\omega^2(t_n) + (m^2 + \omega^2(1 - \dot{x}_n^2))a_\omega^2(t_n)\},\ t_n < t \leq t_{n+1}, \tag{50}$$

$n = 0,1,\dots$ and the impulsive operator:

$$F_n = 1 - \dot{x}_n t_n \int_0^\infty d\omega \int_0^\infty d\omega' F(\omega,\omega') a_{\omega'}(t_n)\delta/\delta a_\omega(t_n),\ t = t_n, \tag{51}$$

$n = 1,2,\dots$ with

$$\Psi_n^+[\{a_\omega(t_n)\}, t_n + 0] = F_n\Psi[\{a_\omega(t_n)\}, t_n] = F_n\Psi_n[\{a_\omega(t_n)\}],\ t = t_n,\ \tag{52}$$

$n = 0,1,\dots$

We have omitted a divergent normal ordering constant arising from $\delta a_\omega/\delta a_\omega = \delta(0)$.

The aim of the present paper is to study the solution of a linear homogeneous Schroedinger equation with impulse effect at fixed moments and to establish a dependence between the asymptotic behaviour of its solutions and the spectrum of the linear Hamiltonian operator with impulse effect.

6 Problem statement

Consider the linear homogeneous Schroedinger equation with impulse effect

$$
\begin{aligned}
d/dt\Psi[\{a_\omega(t_n)\},t] &= H_{n+1}\Psi[\{a_\omega(t_n)\},t],\ t_n < t \le t_{n+1}. &(53)\\
\Psi_n^+[\{a_\omega^+(t_n+0)\},t_n+0] &= F_n\Psi[\{a_\omega(t_n)\},t_n],\ t = t_n,\ n = 0,1,.. &(54)
\end{aligned}
$$

where the Hamiltonian H is a linear continuous operator with spectrum Sp(H), mapping the complex Banach space B into itself, the continuous mappings F_n, (n = 1,2,...) map B into B, while

$0 < t_1 < t_2 < ...$,

$\lim t_n = \infty, n \to \infty$.

DEFINITION 1. The piecewise continuous function with first order discontinuities for $t = t_n$ and such that

$$d/dt\Psi(t) = H_{n+1}\Psi(t)$$

for $t \ne t_n$, while for $t = t_n$

$$\Psi_n^+(t_n+0) = F_n\Psi(t_n) = F_n\Psi_n$$

will be called a solution of the equation with impulse effect (53),(54).

$\Psi(t)$ will be assumed to be right continuous at the discontinuity points t_n.

Consider the Cauchy problem with initial condition

$$\Psi(t_0) = \Psi_0,\ 0 < t_0 < t_1. \tag{55}$$

where in the free Hamiltonian we have for $t = t_0$ the $a_\omega = a_\omega(t_0)$ and the solution

$$
\begin{aligned}
&\Psi(\{a_\omega(t_0)\},t_0) = \Psi_0(\{a_\omega\}) = \\
&\exp(-1/2\int_0^\infty d\omega\sqrt{(}m^2+\omega^2)(a_\omega(t_0))^2 - iE_0t_0).
\end{aligned}
$$

with E_0 the ground state energy, for the equation with impulse effect (53),(54). Its solution is given by the formula:

$$\Psi(t) = e^{H_{n+1}(t-t_n)}F_ne^{H_n(t_n-t_{n-1})}\dots F_1e^{H_1(t_1-t_0)}\Psi_0. \tag{56}$$

for $t_n < t \le t_{n+1}$, n = 1,2,...

7 Main results

We will find sufficient conditions for boundedness of the Cauchy problem with initial condition (55) for the Schroedinger equation with impulse effect (53),(54) for $t > t_0$.

We will say that conditions (A) hold provided the following conditions are fulfilled:

A_1.

$$\| e^{Ht}\| \le e^{\gamma t}$$

for $t > t_0$

where γ is a constant.

A_2.

$$\| F_n\Psi\| \le q_n \| \Psi\| + h_n$$

for $n = 1,2,...$, and $\Psi \in B$, and

$$q_1q_2...q_{n(t)} \le Le^{\delta t}$$

where $n(t) = n$ for $t_n < t \le t_{n+1}$, while L and δ are constants. Let conditions (A) hold. We will estimate the solution of the Cauchy problem with the initial condition (55) for the Schroedinger equation with impulse effect (53),(54):

$$
\begin{aligned}
\| \Psi(t)\| &\le e^{\gamma(t-t_n)}\{q_n \| e^{H_n(t-t_{n-1})}F_{n-1}e^{H_{n-1}(t_{n-1}-t_{n-2})}F_{n-2}\dots\\
&\quad F_1e^{H_1(t_1-t_2)}F_0\Psi_0\| + h_n\}\\
&\le e^{\gamma(t-t_{n-1})}q_n\{q_{n-1} \| e^{H_{n-1}(t_{n-1}-t_{n-2})}F_{n-2}\dots\\
&\quad F_1e^{H_1(t_1-t_0)}F_0\Psi_0\| + h_{n-1}\} + e^{\gamma(t-t_n)}h_n\\
&\le e^{\gamma(t-t_{n-2})}q_nq_{n-1} \| F_{n-2}\dots F_1e^{H_1(t_1-t_0)}F_0\Psi_0\| +\\
&\quad e^{\gamma(t-t_{n-1})}q_nh_{n-1} + e^{\gamma(t-t_n)}h_n\\
&\le \dots \qquad\qquad (57)\\
&\le e^{\gamma(t-t_0)}q_nq_{n-1}\dots q_2q_1 \| \Psi_0\| + \{e^{\gamma(t-t_n)}h_n + e^{\gamma(t-t_{n-1})}q_nh_{n-1} +\\
&\quad e^{\gamma(t-t_{n-2})}q_nq_{n-1}h_{n-2} + \dots + e^{\gamma(t-t_0)}q_nq_{n-1}\dots q_2h_1\}
\end{aligned}
$$

The estimate made allows to consider partial cases where the solutions of the Cauchy problem with initial conditions (55) for the Schroedinger equation with impulse effect (53),(54), are bounded. Here we will mention two of them.

THEOREM 1. Let conditions (A) hold. Moreover, let $h_1 = h_2 = \ldots = 0$. Then for $\delta + \gamma = 0$ the solution $\Psi[\{a^+_\omega(t_0)\}, t], (t \geq t_0)$ is bounded, and for $\delta + \gamma < 0$, $\lim_{t\to\infty} \Psi(t) = 0$.

The proof of the theorem is implied immediately by (57).

THEOREM 2. Let condition (A) be fulfilled. Let

$$q = q_1 = q_2 = \ldots ; h = h_1 = h_2 = \ldots$$

and let

$$t_{2n-1} = t_0 + f(n, x_0)\kappa (n = 1, 2, \ldots).$$

From the paper [2,3,4] we have

$$t_{2n-1} = \cosh ns.t_0 - \sinh ns.x_0$$

$$t_{2n} = \cosh ns.t_0 + \sinh ns.x_0$$

and then $t_{2n-1} < t \leq t_{2n}$, $t = t_0 + u(\tau, x_0)\kappa$,

where $n \leq \tau \leq n+1$, while κ is a constant. Besides, let $qe^{\gamma\kappa} < 1$. Then for $\delta + \gamma \leq 0$ the solution $\Psi(t)$, $(t \geq t_0)$ is bounded.

Proof. By M denote the expression in braces in the right hand side in (57) and, having made certain transformations by $f(n, x_0) \approx n$ and $u(\tau, x_0) \approx \tau$,(from $\cosh(ns/2) = 1 + n^2/2(v^2/c^2) + \ldots$, $\sinh(ns/2) = nv/2 + (n/3 + n^3/6).v^3/c^3 + \ldots$), we obtain

$$M = e^{\gamma(\tau-n)\kappa}\{[q^n e^{n\gamma\kappa} - 1]/[qe^{\gamma\kappa} - 1]\}h.$$

Under the assumption made, M, is bounded. This fact and condition A_2 imply the assertion of Theorem 2.

THEOREM 3. Let the following conditions be fulfilled:

1.

Positive constants $q_1, q_2, \ldots$ exist such that

$$\| F_n\Psi\| \geq q_n \| \Psi\| \text{ for } n = 1, 2, \ldots, \Psi \in B.$$

2. $e^{H_{n+1}\tau}F_n = F_n e^{H_{n+1}\tau}$, for $n = 1, 2, \ldots, \tau > 0$ and $\| e^{(H_{n-1}-H_n)t_{n-1}}\| \geq e^{\gamma_{n-1}t_{n-1}}$ where γ_n, $(n = 1, 2, \ldots)$ are a constants

3. For all $\Psi_0 \in B$ and $t \geq t_0 = 0$ the solution of the Cauchy problem with initial condition (55) for the Schroedinger equation with impulse effect (53),(54), are bounded, i.e. for $t \geq 0$, $\| \Psi(t)\| \leq C(\Psi_0)$.

Then the spectrum Sp(H) of the operator H lies in the halfplane $\mathcal{R}\lambda \leq \mathcal{Q}$ where

$$\mathcal{Q} = -\lim_{t\to\infty} \inf 1/t \sum_{n=1}^{n(t)} \gamma_n t_n \ln q_n.$$

Proof. For k = 1,2,... introduce the notation $\Psi^+_k = F_k\Psi(t_k)$. Let $t \in [t_n, t_{n+1}]$. For $\Psi^+_0 \in B$,

$$\begin{aligned} C(\Psi^+_0) &\geq \| e^{H_n(t-t_{n-1})}\Psi^+_{n-1}\| \\ &= \| e^{H_n(t-t_{n-1})}F_{n-1}e^{H_{n-1}(t_{n-1}-t_{n-2})}F_{n-2}\Psi_{n-2}\| \\ &= \| e^{H_n(t-t_{n-1})}e^{H_{n-1}(t_{n-1}-t_{n-2})}F_{n-1}\Psi^+_{n-2}\| \\ &= \| e^{H_n t+(H_{n-1}-H_n)t_{n-1}-H_{n-1}t_{n-2}}F_{n-1}\Psi^+_{n-2}\| \\ &= \ldots \\ &= \| e^{H_n t+(H_{n-1}-H_n)t_{n-1}+(H_{n-2}-H_{n-1})t_{n-2}}\ldots \\ &\quad e^{(H_1'-H_2)t_1+(H_0-H_1)t_0-H_0t_0}F_{n-1}F_{n-2}\ldots F_1\Psi_0\| \\ &\geq e^{(\gamma_{n-1}t_{n-1}+\ldots+\gamma_0t_0)}\| F_{n-1}F_{n-2}\ldots F_1e^{H_nt}\Psi_0\| \\ &\geq e^{(\gamma_{n-1}t_{n-1}+\ldots+\gamma_0t_0)}q_{n-1}q_{n-2}\ldots q_1\| e^{H_nt}\Psi_0\| \end{aligned} \tag{58}$$

(57)yelds

$$\| e^{H_nt}\Psi\| \leq C(\Psi_0)/[e^{(\gamma_{n-1}t_{n-1}+\ldots+\gamma_0t_0)}q_1\ldots q_{n-1}]$$

whence it follows that for $\epsilon > 0$ and for sufficiently large values of t,

$$\| e^{H_nt}\Psi_0\| \leq C(\Psi_0)e^{(Q+\epsilon)t}$$

Hence, for $\lambda \in Sp(H)$, the inequality $\mathcal{R} \leq \mathcal{Q} + \epsilon$ holds, i.e. $\mathcal{R}\lambda \leq \mathcal{Q}$. Thus, Theorem 3 is proved.

REMARK 1. If the operators F_n are linears surjections for then condition 1 of Theorem 3 is equivalent to the condition for the operators F_n to have inverse ones. In this case for $n = 0, 1, 2, \ldots$, $\| F_n^{-1}\| \leq q_n^{-1}$.

THEOREM 4. Let the following conditions to be fulfilled:

$$1.\| F_n\Psi\| \geq q_n \| \Psi\|, \text{ for } n = 1,2,...,$$

where the numbers q_n satisfy the inequalities

$$q_{1,2}...q_m \geq \alpha,$$

for $m = 0, 1, 2, ...,$ and $\alpha > 0$ is a constant.

$$2.\ F_n e^{H\tau} \neq e^{H\tau} F_n, \text{ for } n = 1,2,...\tau \in (-\infty, \infty).$$

3. For all $\Psi_0 \in B$ and $t \in (0,\infty)$, the soulution of the Cauchy problem with initial condition (55) for the Schroedinger equation with impulse effect (53),(54), are bounded,i.e.$\| \Psi(t)\| \leq C(\Psi_0)$.

Then Sp(H) lies on the imaginery axis.

Proof. With arguments analogous to those employed in the proof of Theorem 3, we find the estemate

$$\| e^{H_n t}\Psi_0\| \leq C(\Psi)/\alpha \text{ for } t \in (0,\infty).$$

Hence a constant M exists for which the inequality

$$\| e^{Ht}\| \leq M \, for \; t \in (0,\infty) \tag{59}$$

holds. (59) yields that Sp(H) lies on the imaginary axis.

This completes the proof of Theorem 4.

REMARK 2. The assertion of Theorem 4 still remains true if in condition 3 the requirement $\Psi_0 \in B$ is replaced by the condition $\Psi_0 \in \delta\Omega$, where $\delta\Omega$ is the boundary of a bounded subset Ω of B with nonempty interior.

For the case when the Hamiltonian is time-dependent the stability of the solution of the Schroedinger equation with impulse effect must be prooved.

8 Statement of the Problem

Consider the linear Schroedinger equation with impulse effect

$$d/dt\Psi(t) = H(t)\Psi(t)| t \neq t_n,\ (n = 0, 1, ...) \tag{60}$$

$$\Psi^+(t_n + 0) = F_n\Psi(t_n),\ t = t_n,\ (n = 1, 2, ...) \tag{61}$$

where $H(t) = -i/2 \int_0^\infty d\omega \int_0^\infty d\omega' \{-T^{-1}_{\omega\omega'}\delta/\delta A_\omega.\delta/\delta A_{\omega'} + A_\omega(t)V_{\omega\omega'}(t)A_{\omega'}(t)\}$ and the impulsive operator $F_n = 1 - t_n/2 \int_0^\infty d\omega \int_0^\infty d\omega' \int_0^\infty d\omega'' \{T^{-1}_{\omega\omega'} F_{\omega\omega''} A_{\omega''}\delta/\delta A_{\omega'} + T^{-1}_{\omega\omega'} F_{\omega'\omega''} A_{\omega''}\delta/\delta A_\omega\}$.

The Hamiltonian H(t), for ($t > t_0$)) is a continuous operator-function on t, the values of which are linear bounded operators mapping the complex Banach space into itself. The operators $F_n : B \rightarrow B$, for (n=1,2,...) are continuous and the moments t_n, for (n=0,1,...) of the impulse effect satisfy the condition $0 < t_0 < t_1 < ...$ and $\lim_{n\rightarrow\infty}(t_n) = \infty$.

DEFINITION 1. We say that $\Psi(t)$ is a solution of the Schroedinger equation with impulse effect (60),(61) if $\Psi(t)$ is a piecewise left continuous function with first order disconinuities for $t = t_n$, (n=0,1,...) and such that

$$d/dt\Psi(t) = H(t)\Psi(t) \text{ for } t \neq t_n,\ (t \geq t_0),\ (n = 1,2,...)$$

and

$$\Psi^+(t_n + 0) = F_n\Psi(t_n),\ \text{ for } t = t_n,\ (n = 1,2,..).$$

For $\Psi \in B$ and $t \geq t_0$ the Schroedinger equation with impulse effect (60),(61) has unique soulution $\Psi(t)$ which satisfies the condition

$$\Psi(t_0) = \Psi_0. \tag{62}$$

Then we can consider the family of operators W(t) defined by the formula

$$W(t)\Psi_0 = \Psi(t),\ (t_0 < t < \infty).$$

Denote by U(t,s) the evolution operator of the Schroedinger equation

$$d/dt\Psi(t) = H(t)\Psi(t),\ t \neq t_n. \tag{63}$$

We note that for $t_n < t \leq t_{n+1}$, (n=1,2,...) the following equality holds

$$W(t) = U(t,t_n)F_nU(t_n,t_{n-1})F_{n-1}\dots F_1U(t_1,t_0).$$

DEFINITION 2. The Schroedinger equation with impulse effect (60),(61) is called stable if

$$\sup_{t_0\le t<\infty} \| W(t)\| < \infty.$$

DEFINITION 3. The Schroedinger equation with impulse effect (60),(61) is called asymptotically stable if

$$\lim_{t\to\infty} \| W(t)\Psi\| = 0,\ (\Psi \in B).$$

DEFINITION 4. The Schroedinger equation with impulse effect (60),(61) is called uniformly stable if

$$\lim_{t\to\infty} \| W(t)\| = 0.$$

LEMMA 1. Let the following condition hold

$$H(t)\int_{t_0}^{t} dsH(s) = \int_{t_0}^{t} dsH(s).H(t),\ (t \ge t_0). \qquad (64)$$

Then the solution $\Psi(t)$ of (63) for $t \ge t_0$ with initial condition has the form

$$\Psi(t) = \exp(\textstyle\int_{t_0}^{t} dsH(s))\Psi_0.$$

The proof of Lemma 1. is standard.

REMARK 1. For small $t \ge t_0$ the condition (64) is sufficient a well for the solution of (4) to have the form

$$\Psi(t) = \exp(\textstyle\int_{t_0}^{t} dsH(s))\Psi_0.$$

In fact, let $\Psi(t) = \exp(\int_{t_0}^{t} dsH(s))\Psi_0$. The equality

$$[exp \textstyle\int_{t_0}^{t} dsH(s)]' = H(t)\exp\int_{t_0}^{t} dsH(s)$$

can be presented as

$$\textstyle\sum_{n=0}^{\infty} 1/(n+1)!((\int_{t_0}^{t} dsH(s))^{n+1})' - 1/n!H(t)(\int_{t_0}^{t} dsH(s))^n) = 0.$$

We set $\Delta(t) = H(t)\int_{t_0}^{t} dsH(s) - \int_{t_0}^{t} dsH(s)H(t)$. Then the following equalities hold:

$$\sum_{n=0}^{\infty} 1/(n+1)!(\sum_{k=0}^{n}(\int_{t_0}^{t} dsH(s))^k H(t)(\int_{t_0}^{t} dsH(s))^{n-k} - (n+1)H(t)(\int_{t_0}^{t} dsH(s))^n) =$$

$$= \sum_{n=0}^{\infty} 1/(n+1)!\sum_{k=1}^{n}((\int_{t_0}^{t} dsH(s))^k H(t)(\int_{t_0}^{t} dsH(s))^{n-k} - H(t)(\int_{t_0}^{t} dsH(s))^n) =$$

$$= \sum_{n=0}^{\infty} 1/(n+1)!\sum_{k=1}^{n} k(\int_{t_0}^{t} dsH(s))^{n-k}\Delta(t)(\int_{t_0}^{t} dsH(s))^{k-1} = 0.$$

i.e.

$1/2\Delta(t) = -\sum_{n=2}^{\infty} 1/(n+1)!\sum_{k=1}^{n} k(\int_{t_0}^{t} dsH(s))^{n-k}\Delta(t)(\int_{t_0}^{t} dsH(s))^{k-1}$.

Finally we get

$1/2\| \Delta(t)\| \le \sum_{n=2}^{\infty} \| \Delta(t)\|/(n+1)!.k(k+1)/2 \| \int_{t_0}^{t} dsH(s)\|^{n-1} =$

$= 1/2 \| \Delta(t)\|(exp \| \int_{t_0}^{t} dsH(s)\| - 1)$.

Then for sufficiently small t

$$|\exp \| \textstyle\int_{t_0}^{t} dsH(s)\| - 1| < 1$$

and there exists an interval $[t_0, t*]$ such that $\Delta(t) = O(t \in [t_0, t*])$.

Applying the step method we come to the condition that the equality $\Delta(t) = 0$ holds as well if t belongs to an interval côntaining $[t_0, t*]$.

9 Main Result

We say that the condition (A) holds if the following condition is fulfilled:

$$(A)\ \textstyle\int_{\tau}^{t} dsH(s).H(t) = H(t)\int_{\tau}^{t} dsH(s),\ (t_0 < \tau < t < \infty).$$

REMARK 2. Let the condition (A) hold. Then for $t_0 < \tau < t < \infty$ the following equality holds

$$H(t)H(\tau) = H(\tau)H(t).$$

We say that the conditions (B) are satisfied if:

B1: Condition (A) holds.

B2: There exists a constant $T > 0$ for which $t_{n+1} - t_n \leq T$ for n=1,2,..

B3: $\exp \| \int_\tau^t dsH(s)\| \leq \exp\gamma(t-\tau)$ for $0 \leq t-\tau \leq T$ where γ is a constant.

B4. $\| F_n\Psi\| \leq q_n \| \Psi\| + h_n$ for n = 1,2,... and $\Psi \in B$ so that $q_1q_2...q_{n(t)} \leq L\exp\delta t$ where $n(t) = n$ for $t_n \leq t < t_{n+1}$ and L and δ are constants.

If the conditions (B) hold, for the solution $\Psi(t)$ of the Schroedinger equation with impulse effect (60),(61) we obtain the following estimation

$$
\begin{aligned}
\| \Psi(t)\| \leq\ & \exp\gamma(t-t_n)\{q_n \| \exp(\int_{t_{n-1}}^{t_n} dsH(s))F_{n-1}[\exp(\int_{t_{n-2}}^{t_{n-1}} dsH(s))]. \\
& F_{n-2}...F_1[\exp(\int_{t_0}^{t_1} dsH(s))]\Psi_0\| + h_n\} \leq \qquad (65) \\
\leq\ & [\exp\gamma(t-t_{n-1})]q_n\{q_{n-1} \| [\exp(\int_{t_{n-2}}^{t_{n-1}} dsH(s))]F_{n-2}...F_1 \\
& [\exp(\int_{t_0}^{t_1} dsH(s))]\Psi_0\| + h_{n-1}\} + [\exp\gamma(t-t_n)]h_n \leq \\
\leq\ & [\exp\gamma(t-t_{n-2})]q_nq_{n-1} \| F_{n-2}...F_1[\exp(\int_{t_0}^{t_1} dsH(s))]\Psi_0\| + \\
& +[\exp\gamma(t-t_{n-1})]q_nh_{n-1} + [\exp\gamma(t-t_n)]h_n \leq \\
\leq\ & ... \leq [\exp\gamma(t-t_0)]q_nq_{n-1}...q_1 \| \Psi_0\| + \{[\exp\gamma(t-t_n)]h_n + \\
& +[\exp\gamma(t-t_n)]q_nh_{n-1} + [\exp\gamma(t-t_n)]q_nq_{n-1}h_{n-2} + ... \\
& +[\exp\gamma(t-t_1)]q_nq_{n-1}...q_2h_1\}.
\end{aligned}
$$

Using this estimation we will find sufficient conditions under which the solutions of the Schroedinger equation with impulse effect (60),(61) are bounded.

THEOREM 1. Let the conditions (B) hold. Let $h_1 = h_2 = ... = 0$. Then for $\delta + \gamma < 0, \lim_{t\to\infty} \Psi(t) = 0$ and for $\delta + \gamma = 0$ the solution $\Psi(t)$ is bounded for $t \geq t_0$.

The proof of the theorem follows immediately from the estimation (65).

THEOREM 2. Let the conditions (B) be satisfied. Suppose additionally that

$$q = q_1 = q_2 = ...;\ h = h_1 = h_2 = ...$$

and let

$$t_n = t_0 + nk(n = 1,2,...),\ t = t_0 + \tau k.$$

[that is for nonrelativistic case] where $n \leq \tau \leq n+1, k \leq T$ is a constant and the inequality

$$qe^{\gamma k} < 1$$

holds. Then for $\delta + \gamma \leq 0$ the solution $\Psi(t)$ is bounded on the half axis $t \geq t_0$.

Proof. We denote by K the expression in the big brackets in (65) and after certain transformations we get

$$K = e^{\gamma(\tau-n)}\frac{q^ne^{\gamma kn}-1}{qe^{\gamma k}-1}h.$$

From the assumtions it follows that K is bounded. This conclusion combined with the condition B4 complets the proof.

THEOREM 3. Suppose that the following conditions hold:

1. The condition (A) is fulfilled.

2. There exist constants $p \geq 0, r \geq 0$ and $\sigma > 0, \rho > 0$ such that $0 \leq i(t_0,t) - p(t-t_0) \leq \sigma, 0 \leq i(t_0,t) - r(t-t_0) \leq \rho$ for $0 \leq t_0 \leq t < \infty$ where $i(a,b)$ is the number of the points t_n which lie in the interval $(a,b)(t_0 < a < b < \infty)$.

3. $F = F_1 = F_2 = ...$ where the operator F is linear and

$$[\int_\tau^t dsH(s)]F = C.F. \int_\tau^t dsH(s) \text{ for } t_0 \leq \tau \leq t < \infty \text{ and } C^{n(t)} \leq Re^{\rho t}$$
where $n(t) = n$ for the interval (a,b).

4. The operator F has a logaritm, i.e. there exists a linear and bounded operator $\ln F : B \to B$ such that $F = \exp\ln F$.

Then the Schroedinger equation with impulse effect is bounded, asymptotically bounded, uniformly asimptotically bounded if and only if respectively

$$\text{a) } \sup_{t\geq t_0} \| \exp\int_{t_0}^t ds(H(s) + p\ln F + r\ln C)\| < \infty.$$

$$\text{b) } \lim_{t\to\infty} \| [\exp\int_{t_0}^t ds(H(s) + p\ln F + r\ln C)]\Psi\| = 0 (\Psi \in B).$$

c) $\lim_{t\to\infty} \| \exp \int_{t_0}^{t} ds(H(s) + p\ln F + r\ln C)\| = 0.$

Proof. The operator-function W(t) has the form

$$
\begin{aligned}
W(t) = & [\exp \int_{t_0}^{t} dsH(s)][F.C]^{i(t_0,t)} = [\exp \int_{t_0}^{t} dsH(s)]F^{[p(t-t_0)]}F^{i(t_0,t)-[p(t-t_0)]} \\
& C^{[r(t-t_0)]}C^{i(t_0,t)-[r(t-t_0)]} = [\exp \int_{t_0}^{t} dsH(s)][\exp \ln F[p(t-t_0)]] \\
& F^{i(t_0,t)-[p(t-t_0)]}[\exp \ln C[r(t-t_0)]]C^{i(t_0,t)-[r(t-t_0)]} = [\exp \int_{t_0}^{t} dsH(s)]. \\
& [\exp \ln Fp(t-t_0)][\exp(\ln F[p(t-t_0)] - \ln Fp(t-t_0))]F^{i(t_0,t)-[p(t-t_0)]}. \\
& [\exp \ln Cr(t-t_0)][\exp(\ln C[r(t-t_0)] - \ln Cr(t-t_0))C^{i(t_0,t)-[r(t-t_0)]} = \\
= & [\exp \int_{t_0}^{t} ds(H(s) + p\ln F + r\ln C)][\exp(\ln F\{[p(t-t_0)] - p(t-t_0)\})]. \\
& F^{i(t_0,t)-[p(t-t_0)]}[\exp(\ln C\{[r(t-t_0)] - r(t-t_0)\})]C^{i(t_0,t)-[r(t-t_0)]}. \qquad (66)
\end{aligned}
$$

Taking ito account the representation (66) the assertions a), b) and c) follows immediately.

10 Conclusions

In this paper the covariant functional Schroedinger formalism with the impulse effect in a Banach space is extended of non-separable spacetime foliations.

In principle, at least, one can solve the Schroedinger equation resulting from the situation where we are using an time-dependent basis system and one then could compute arbitrary expectation values. However, the association of "particles" with the excitation modes A_ω is even more dubious than it is for separable foliations alredy. There we can choose the basis functions on Σ_s to be the spatial part of a complete set of solutions of the classical field equations. Because these modes diagonalize the Hamiltonian, they do not mix during the time evolution. In this sens they preserve their identity and could be considered as "particles" in a restricted sense. In the non-separable cases, however, it is not possible to find modes with this property. The closest analogue one could imagine is that it is possible (in general it is not) for the impulsive moving

mirror model to diagonalize kinetic and potential energy matrices T and V, and to simultaneously skew- diagonalize A-independent field strength matrice F.

In a typical separable situation we have a spatial momentum conservation, which, in our case, is spoiled by the presence of the impulsive moving mirror. For the case $\dot{x}_n = constant$ for $t \neq t_n$ and assumed that this change in $t = t_n$ by jumps is possible to obtain closed form solutions to the Schroedinger equation with impulse effect in Banach space.

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Received by Publishing Department
on June 29, 1992.