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NOTE ON SUPERFIELD FORMULATIONS OF D = 2,3,4,6 AND 10 SUPERPARTICLES

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О суперполевых формулировках суперчастиц в пространстве размерности D = 2,3,4,6 и 10

Показано, что все известные формулировки динамики N = 1 суперчастицы в D = 2,3,4 и 6 как дважды суперсимметричных теорий с n = D - 2 локальной суперконформной симметрией на мировой линии являются частными версиями более общего n = D - 2 суперполевого действия, инвариантного относительно обобщенной (суперполевой) k-симметрии и совпадающего на массовой поверхности с действием Бринка-Шварца для суперчастицы.

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Note on Superfield Formulations of D = 2,3,4,6 and 10 Superparticles

It is shown that all known formulations of N = 1 superparticle dynamics in D = 2,3,4 and 6 space-time dimensions as double supersymmetric theories with n=D-2 local worldline superconformal symmetries [1-3] are particular versions of more general n = D-2 superfield action, which is invariant under generalized (superfield) k-symmetry and coincide (on shell) with the Brink-Schwarz superparticle action. For D = 10 case only n = 1 superfield action is known.

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Recently a double supersymmetric twistor-like version of the $N=1$ Brink-Schwarz superparticle in $D=(2),3,4,6$ and 10 dimensions has been proposed [1], which is invariant under $N=1$ target superspace global transformations, as well as, under the local $n=D-2$ superconformal transformations in worldline superspace (for $D=10$ only $n=1$ action is known[1]). In these particular space-time dimensions [1] such a formulation allowed one to give the well-known κ-symmetry [4,5] a clear geometrical meaning as an on-shell manifestation of the local superconformal symmetry in the proper-time superspace parametrizing the superparticle trajectory. The corresponding $N=1, n=D-2$ superfield actions for $D=3,4$ superparticle were constructed in [1] and the component action for $D=6$ case was discussed in [6].

In a recent elegant paper by Delduc and Sokatchev [2] another superfield version of $D=4$, $n=2$ superparticle in curved target superbackground was proposed and generalized to $D=6$, $n=4$ case. The basic geometrical idea, which allowed the authors to accomplish this program, was to introduce properties of double chirality (in $D=4$,$n=2$) and double harmonic analyticity (in $D=6$,$n=4$) in addition to the double supersymmetry of the particle dynamics considered. Note that this notions have been independently introduced in ref. [3], where, in particular, a superfield action for the massive $N=1, D=2$ superparticle has been discussed.

In series of interesting papers [7,8] it was clarified the connection of the twistor-like approach [1-3;9] to the superparticle dynamics with the Lorentz-harmonic approach developed in parallel with the former one (see [8,10-12] and references therein). One may hope that this will allow to overcome the $D=10$, $n=8$ barrier and close the problem of covariant quantization of the $D=10$ Brink-Schwarz superparticle by constructing its harmonic superfield version.

Let us note also the interesting geometrical interpretation of the superparticle versions of Ref.[1] and some other superparticle twistor formulations [9,12,14] as a supersymmetric Chern-Simons mechanics proposed by Howe and Townsend [15].

Though the connection of superparticle superfield formulations [1-3]

[1]Note that $D=3,4,6,10$ are closely related to the critical space-time dimensions for the classical Green-Schwarz superstring from the one hand side and twistors from the other.

with another approaches is established, the relations between the versions proposed in [1],[2] and [3], as far as we aware, has not been clarified yet.

The main goal of this note is to fill this gap. We will show that all known superfield formulations [1-3] of $N=1$ superparticle dynamics in $D=3,4$ and 6 space-time dimensions are particular versions of more general $n=D-2$ superfield action, which is invariant under generalized superfield κ-symmetry and coincide (on shell) with the $N=1$ Brink-Schwarz superparticle. $D=2, N=1$ massive superparticle action of Ref.[3] arises as a result of dimensional reduction of the $D=3, N=1$ massless superparticle action [1]. When the dimensional reduction of the $D=6, n=4$ action to $D=4$ space-time is carried out one gets an $n=4$ superfield formulation of the $D=4, N=2$ massive superparticle with central charges [4,16]. For $D=10$ superparticle we still possess only the $n=1$ superfield formulation.

Let us start with reminding that the general form of Lagrangians of all known superfield versions of the $N=1$ Brink-Schwarz superparticle except the $D=4, n=2$ version of
Ref. [1], looks as follows:

$$L_0 = P_m \Omega^m, \tag{1}$$

where $m=0,1,\cdots D-1$, P_m is a superfield Lagrange multiplier, which has a physical meaning of superparticle momentum, and Ω^m is a covariant form with respect to global $N=1$ and local $n=D-2$ supersymmetric transformations.

For $D=3$, $n=1$ case superparticle trajectory is parametrized by (τ,η) supertime (with η being real odd superpartner of τ) and $\Omega^m(\tau,\eta)$ has the following form (for the details see refs.[1,6]):

$$\Omega^m = -i(DX^m - i\overline{\Theta}\gamma^m D\Theta). \tag{2}$$

where $D=\frac{\partial}{\partial\eta}+i\eta\frac{\partial}{\partial\tau}$ and $X^m(\tau,\eta), \Theta^\alpha(\tau,\eta)$ are bosonic and fermionic coordinates of the superparticle in $D=3, N=1$ target superspace. Note that Lagrangian (1) with Ω^m represented in the form (2) is valid for all "critical" dimensions $D=3,4,6$ and 10[1,6].

For $D=4, n=2$ η is complex and

$$\Omega^m = \frac{i}{2}(X^m_L - X^m_R) - \Theta^\alpha \sigma^m_{\alpha\dot\alpha}\overline{\Theta}^{\dot\alpha}; \tag{3}$$

with $(X^m_L(\tau,\eta,\overline{\eta}))^* = X^m_R(\tau,\eta,\overline{\eta})$ and $(\Theta^\alpha)^* = \overline{\Theta}^{\dot\alpha}(\tau,\eta,\overline{\eta})$ being (anti)chiral superfields in $(\tau,\eta,\overline{\eta})$ superspace (σ^m are the Pauli matrices). Note that $iX^m_L = V^m$ in the notations of Refs. [1,6]. The chirality conditions can be incorporated into Lagrangian (1) with the help of Lagrange multipliers.

To describe the $D=6, n=4$ superparticle one has to consider harmonic superspace $(\tau,\eta^i,\overline{\eta}_i,u^{\pm i}), (i=1,2)$ and parametrize a particle trajectory by its analytic subspace ($\tau_A = \tau + i\eta^i\overline{\eta}^j u^+_{(i}u^-_{j)}$, $\eta^+ = \eta^i u^+_i$, $\overline{\eta}^+ = \overline{\eta}^i u^+_i, u^{\pm i}$) (see Ref.[2] for the details). In the present case

$$\Omega^{++m} = D^{++}X^m_A - i\Theta^+_A\gamma^m\Theta^+_A, \qquad (4)$$

where D^{++} is a covariant harmonic derivative and $X^m_A(\tau_A,\eta^+,\overline{\eta}^+,u)$, $\Theta^+_A(\tau_A,\eta^+,\overline{\eta}^+,u)$ are analytic quantities in the world-line superspace and in a target $D=6, N=1$ harmonic superspace. This is the way the double analyticity is introduced in the present approach. The necessary harmonic conditions such as $D^{++}\Theta^{\alpha+}_A = 0$ can be incorporated into eq.(1) with the help of corresponding Lagrange multipliers [2].

When $N=n=0$ all theories discussed above are truncated to the massless bosonic particle formulation [13,1] described by the Lagrangian of the form (1) (where $\Omega^m = \frac{d}{d\tau}X^m - \overline{\lambda}\gamma^m\lambda$, and λ^α being a commuting twistor- like spinor).

One can see that eq.(1) has the form of the geometrical relations (Ω^m-quantity) defining the structure of the corresponding target superspaces times the Lagrange multipliers P_m [1,2].

We argue here that this form of the Lagrangian is, in fact, not necessary for the superfield description of the $N=1$ Brink-Schwarz superparticles and propose its generalization which looks as a conventional relativistic (super)particle first-order Lagrangian:

$$L_1 = P_m\Omega^m - \frac{1}{2}EP_mP^m, \qquad (5)$$

where for the $n=1$ case E is an odd superfield, for $D=4, n=2$ it is a real scalar superfield, which, in particular, can be a constant due to the invariance properties of the integration measure in $(\tau,\eta,\overline{\eta})$ superspace. In the case of the $D=6, n=4$ superparticle the analytic superfield E^{++} bears two harmonic $U(1)$ charges to compensate two minus charges of the analytic superspace measure in the action integral. One can identify superfields E with the "reverse" supereinbeins of the corresponding one-dimensional supergravity theories in the sense that their *last* components coincide with the einbein of the underlying bosonic massless particle dynamics.

The equation of motion for unconstrained superfield P_m, which follows from eq.(5), is:

$$\Omega^m = EP^m, \qquad (6)$$

whereas

$$\Omega^m = 0 \qquad (7)$$

when Lagrangian (1) is varied over P_m. Actually, the both equations describe theories with the same physical content, i.e. the $N=1$ Brink-Schwarz superparticle, which can be straightforwardly checked by solving all the equations of motions following from Lagrangians (1) and (5). The nonzero r.h.s. of eq.(6) results only in a shift of auxiliary components of the superfields integral to the Ω^m. The basic reason of this equivalence lies in the fact that Lagrangian (5) is invariant (up to a full derivative) under additional gauge transformations which allow one to get rid of superfield E. For example, in target $D=3$, $N=1$ superspace these transformations look as follows (generalization to $D=4,6$ is straightforward):

$$\delta X^m = \Lambda(\tau,\eta)P^m \quad, \delta E = -iD\Lambda, \quad \delta\Theta_\alpha = \delta P^m = 0; \quad (8)$$

$$\delta\Theta_\alpha = P_m\gamma^{m\beta}_\alpha K_\beta(\tau,\eta) \quad, \delta X^m = -i\overline{\Theta}\gamma^m\delta\Theta, \quad \delta E = 4\overline{K}D\Theta, \qquad (9)$$

where $\Lambda(\tau,\eta)$ is an even superfield gauge parameter and $K_\alpha(\tau,\eta)$ is an odd one. In transformations (9) one can easily recognize a superfield generalization of the conventional κ-symmetry [4,5]. Both symmetries (8),(9), which form an algebra, occur to be enough for choosing the gauge in which superfield $E(\tau,\eta)$ turns to zero. Thus, Lagrangian (1) is nothing but eq.(5) written in the particular gauge ($E=0$).

For $D=4,6$ and $n=D-2$ one can substitute P^m in eq.(5) for its expression (6) in terms of Ω^m and get the second-order form of the superparticle formulations considered:

$$L_2 = \frac{\Omega_m\Omega^m}{2E}. \qquad (10)$$

For example, in the case of $D = 4, n = 2$ superparticle one can put $E = 1$ and obtain the action proposed in [1]:

$$S^{n=2}_{D=4} = \frac{1}{2}\int d\tau\; d\eta\; d\overline{\eta}\; (\frac{i}{2}(X^m_L - X^m_R) - i\Theta\sigma^m\overline{\Theta})^2. \qquad (11)$$

The generalization of Lagrangians (5),(10) to describe superparticle propagating in supersymmetric Maxwell and gravitational background is straightforward and implies the replacement of $\overline{\Theta}\gamma^m\Theta$ terms in Ω^m,eqs.(2)-(4) by the corresponding Maxwell and supergravity prepotentials (see [2] for the details).

Let us now consider massive superparticle dynamics in $D = 2$ and $D = 4$ space-time dimensions which possess the κ-symmetry. Note that just in the dynamics of the
$D = 4, N = 2$ massive superparticle with central charge the κ-symmetry was first discovered [4].

To obtain the description of massive $N = 2$ superparticle in $D = 2$ one can make use of the dimensional reduction procedure applied to the $D = 3$ Lagrangian (5)

$$L_{D=3} = P_m(DX^m - i\overline{\Theta}\gamma^m D\Theta) - \frac{1}{2}EP_mP^m, \;\; (m = 0,1,2) \qquad (12)$$

. In eq.(12) nothing is supposed to depend on the X^2 coordinate and P_2 is identified with the $D = 2$ superparticle mass m. Then eq.(12) takes the form:

$$\begin{aligned} L_{D=2} \;&=\; P_m(DX^m - i\overline{\Theta}\gamma^m D\Theta) - im\overline{\Theta}\gamma^2\Theta - \frac{E}{2}(P_mP^m - m^2), (13) \\ &(m = 0,1; \gamma^2 = \gamma^0\gamma^1). \end{aligned}$$

When $E = 0$ Lagrangian (13) coincides with the one considered in [3].

The analogous procedure applied to the $D = 6, n = 4$ action

$$L^{N=1}_{D=6} = \int d\tau du d\eta^+ d\overline{\eta}^+ \left[P_m(D^{++}X^m_A - i\Theta^+_A\gamma^m\Theta^+_A) - \frac{E}{2}P_mP^m\right], \; (14)$$

(where $P_4 = P_5 = \frac{m}{2}$ are identified with the equal central charges of the $D = 4, N = 2$ supersymmetry, which value equals the superparticle mass) results in the action

$$\begin{aligned} S^{N=2}_{D=4} \;&=\; \int d\tau\; du\; d\eta^+ d\overline{\eta}^+ \Big[P_m(D^{++}X^m_A - i\Theta^+_A\sigma^m\overline{\Theta}^+_A) - \\ &-\frac{im}{2}(\Theta^+_{A\alpha}\Theta^{+\alpha}_A + \overline{\Theta}^+_{A\dot\alpha}\overline{\Theta}^{+\dot\alpha})_A - \frac{E^{++}}{2}(P_mP^m - m^2)\Big] \;(15) \end{aligned}$$

describing the $N = 2, D = 4$ massive superparticle of ref.[4] and generalizing its harmonic superspace formulation proposed in [16].

To resume, the superfield Lagrangian (5) generalizes the known superfield versions of the Brink-Schwarz superparticle and allows one to clarify relations between them. Note, also that Lagrangian (5) belongs to the Lagrangian classes discussed in [15], which admit the interpretation in terms of gauged supersymmetric Chern-Simons mechanics.

In conclusion we would like to give some motivation in favor of using Lagrangians of the type (5) for a generalization of the discussed superfield approach to describe superstring theories. The well-known superfield versions of the $N = 1$ and $N = 2$ Green-Schwarz superstrings [17,18,3] suffer from the difficulty to incorporate constraints of the type (7), which are imposed by hand, into superstring actions. To our point of view this problem is connected with the fact that the superfield superstring actions [17,18,3] are constructed in some world-line supersymmetry gauge (for instance, superconformal one) which puts too rigid limits on the possibility to manipulate with the quantities available. Thus, in contrast to the (super)particle case where the term with supereinbein E can be gauged away without losing the local supersymmetry, one has to manifestly incorporate the geometrical objects of the superstring world-sheet superspace into its action. Then one can hope to obtain the necessary constraints as some superstring equations of motion. For the heterotic superstrings the step in this direction was performed in a recent papers by Tonin [19] (see also ref.[20] by Berkovits). Let us explain the above arguments with the example of twistor-like formulation of the classical bosonic string discussed by the Kharkov group (see, for example, [21]). The proposed action for $D = 3,4,6$ and 10 classical bosonic string looks as:

$$S = \int d\tau\; d\sigma\; det(e^a_\nu)\overline{\lambda}\gamma_m\rho^a e^\mu_a\lambda(\partial_\mu X^m - \frac{T^2}{2}\overline{\lambda}\gamma^m\rho_b e^b_\mu\lambda), \qquad (16)$$

where T is a string tension parameter, $e^a_\nu(\tau,\sigma)$ is a world-sheet zweinbein, $\rho^a (a = 0, 1)$ are world-sheet Dirac matrices and $\lambda^i_\alpha (i = +, -)$ are commuting spinors in both, world-sheet and target, spaces.

Due to the equations of motion following from (16) one obtains a solution of the Virasoro constraints $(\partial_\pm X)^2 = 0$ in the twistor-like form:

$$\partial_\pm X^m = T^2 \overline{\lambda}_\pm \gamma^m \lambda_\pm . \tag{17}$$

One could naively think that eq.(17) also arises from the conformal-gauge form of actions (16), (17), where the dependence of $e^a_\nu(\tau, \sigma)$ vanishes similar to the twistor particle case as well, but , in fact, that is not the case unless some additional constraints on the solutions of corresponding equations of motion are imposed.

We argue that action (16), or some its modification, has to be considered as the basis for superfield formulations of Green-Schwarz superstrings with world-sheet supergeometry being explicitly incorporated into their actions. After the elimination of auxiliary variables the twistor-like $N = 1, 2$ superstring actions should take the form:

$$S_{N=1} = \int d\tau\, d\sigma\, det(e^a_\nu) \left[\overline{\lambda} \gamma_m \rho^a e^\mu_a \lambda (\partial_\mu X^m + i \overline{\Theta} \gamma^m \partial_\mu \Theta - \frac{T^2}{2} \overline{\lambda} \gamma^m \rho_\mu \lambda) - \right.$$
$$\left. - \frac{i}{det(e^a_\nu)} \varepsilon^{\mu\kappa} \overline{\Theta} \gamma_m \partial_\mu \Theta \partial_\kappa X^m \right], \tag{18}$$

$$S_{N=2} = \int d\tau\, d\sigma\, det(e^a_\nu) \left[\overline{\lambda} \gamma_m \rho^a e^\mu_a \lambda (\partial_\mu X^m + i \overline{\Theta}_1 \gamma^m \partial_\mu \Theta_1 + i \overline{\Theta}_2 \gamma_m \partial_\mu \Theta_2 - \right.$$
$$- \frac{T^2}{2} \overline{\lambda} \gamma^m \rho_\mu \lambda) - \frac{1}{det(e^a_\nu)} \varepsilon^{\mu\kappa} \left(i (\overline{\Theta}_1 \gamma_m \partial_\mu \Theta_1 - \overline{\Theta}_2 \gamma_m \partial_\mu \Theta_2) \partial_\kappa X^m + \right.$$
$$\left. \left. + (\overline{\Theta}_1 \gamma_m \partial_\mu \Theta_1)(\overline{\Theta}_2 \gamma^m \partial_\kappa \Theta_2) \right) \right], \tag{19}$$

where $\Theta_{1,2}$ are $d = 2$ scalars. The main goal one may hope to achieve by considering superstring formulations of eqs.(18) type is the possibility to trade the κ-symmetry for the local world-sheet supersymmetry (in compliance with the superparticle case)[17] and to accomplish the superstring covariant quantization.

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Note added. When the work was completed the authors became aware of the paper [22], where the dimensional reduction procedure has also been discussed in application to superfield superparticle actions. We thank E.Ivanov for bringing this paper to our attention.

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