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MULTIPOLAR-LIKE FORM OF NONABELIAN GAUGE THEORIES

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1. Introduction

The multipolar form (MF) of QED has been presented in papers by Power and Zienau /1/, Atkins and Woolley /2-4/, Healy /5/ et al. It is widely used in different areas of atomic and molecular physics (see cited examples in /6/). In the MF the interaction of charged particles with photons can be written by using the electric and magnetic fields (not their potentials) and relevant multipoles (moments) of atoms or molecules. The most known example is $e\vec{q}\cdot\vec{E}$ interaction. In distinction to many other "potentialless" approaches the MF can be named "minimally nonlocal" /7/.

As a consequence of this "potentialless" feature the free part of the multipolar Hamiltonian is gauge invariant, for details see sect.2. This is taken here as the defining property of MF.

The multipolar Hamiltonian contains an extra potential-like term $P_\perp$ in addition to the initial bounding potential $V(x)$. In the usual presentations of MF $P_\perp$ is divergent. A regularization of MF, which was suggested in /6/, has the consequence that $P_\perp$ becomes finite. It will be shown in sect.2 that $P_\perp(x)$ increases as $x\to\infty$ not slower than const$\cdot x$, irrespective of the choice of the regularization function. The natural question arises: is there a form for nonabelian theories similar to MF for QED which would contain such a confining potential?

This is the main problem which is elaborated here.

Section 2 presents a version of the electrodynamical MF. It is valid for neutral systems interacting with photons, e.g., proton and electron. This new version has some peculiarities but the term $P_\perp$ is the same as in the usual MF. When the proton is infinitely heavy, the version turns into the usual MF. Just this simple case is taken when discussing nonabelian theories in sect.3: one particle is infinitely heavy and is at the centre $\vec{r}$ of a potential $V(\vec{q}-\vec{r})$ for another particle (nonrelativistic quark) that interacts with a nonabelian gauge field. Taking the Hamiltonian of the model in the Coulomb gauge I suggest a unitary transformation of the theory into the form which can be called multipolar (for details see subsect. 3.2). The transformation generalizes the relevant one of QED. The Hamiltonian of this form contains the potential-like term similar to $P_\perp$ of the abelian case. In concluding section 4. I argue that the relevance of such a confining potential to the famous quark confinement problem requires further investigations.

2. Multipolar form for neutral systems of two charges

2.1. Consider two nonrelativistic unlike charges, $+e$ and $-e$ (e.g., proton and electron) which interact with the electromagnetic field in the Coulomb gauge div $\vec{A}=0$. The total Hamiltonian is

$$H=[\vec{p}_--e\vec{A}(\vec{q}_-)]^2/2m_-+[\vec{p}_++e\vec{A}(\vec{q}_+)]^2/2m_++\frac{e^2}{4\pi|\vec{q}_+-\vec{q}_-|}+H_{ph} \quad (1)$$

$$H_{ph}=\frac{1}{2}\int d^3x\,[\vec{E}_\perp^2(\vec{x})+\vec{H}^2(\vec{x})]. \quad (2)$$

It is invariant under the gauge transformation

$$\tilde{\vec{p}}_\mp=\vec{p}_\mp\mp e\vec{\nabla}\chi(\vec{q}_\mp,t)\,;\quad \tilde{\vec{A}}(\vec{x})=\vec{A}(\vec{x})-\vec{\nabla}\chi(\vec{x},t). \quad (3)$$

If $div\,\tilde{\vec{A}}=div\,\vec{A}=0$, then χ must satisfy $\Delta\chi(\vec{x},t)=0$. If this last equation holds everywhere, then it has no nonzero solutions which vanish at infinity. But a nontrivial transformation (3) is possible if $\Delta\chi(\vec{x})=0$ holds only in a restricted region W of $\vec{x}$ values. The bound states of the charges under consideration must be localized in a sense in W.

Let us give an example of realization of a transformation like that. Let an external current $\vec{J}$ be switched on at $t=t_0$ in a bounded region W_J remote from the localized charges. The current produces the Lorentz gauge vector potential

$$\vec{a}(x)=\int_{W_J}\mathcal{D}_{ret}(x-y)\vec{J}(y)\,dy\quad,\ x=(\vec{x},t) \quad (4)$$

which is zero outside the "light ball", i.e. at the points $\vec{x}$ satisfying $|\vec{x}-\vec{y}|\geq c(t-t_0)$, $\vec{y}\in W_J$. However, the transversal part of $\vec{a}(\vec{x})$ turns out to be nonzero outside the "lightball". It equals there the gradient of a function χ which depends on $\vec{J}$ and satisfies the equation $\Delta\chi=0$, see sect.3 in /8/. This gradient is added to the operator $\vec{A}$ of our model, and we have a realization of (3). It was shown in /8/ that the Heisenberg operators $\vec{p}_-$, $\vec{p}_+$ also acquire gradient additions $\mp e\vec{\nabla}\chi$ in the situation. The described transformations were called quasigauge in /8/.

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In nonabelian theories, nontrivial gauge transformations are possible in the Coulomb gauge in all $\vec{x}$ space [9].

The use of particle variables which are noninvariant under (3) (momentum $\vec{p}$, energy $p^2/2m + V$), can lead to troubles, see [8,7,10]. The troubles are absent if one introduces relevant invariant variables. This can be realized by the unitary transformation S of the canonical variables $\vec{p}_\mp, \vec{q}_\mp$; to the new invariant ones $\vec{p}'_\mp, \vec{q}'_\mp$, : $\vec{p}'=S^+\vec{p}\,S$, etc. Such a transformation preserves commutation relations $[q'_i, p'_j]=i\delta_{ij}$, etc.

I define the MF as a form of the theory which uses gauge-invariant particle energy operators $p'^2_\mp/2m_\mp$.

2.2. A regularized version of the wanted unitary operator S was suggested in [6]:

$$S = exp[+ie\int d^3s\mu(\vec{s})\int_{q_-}^{q_+} d\vec{\ell}\cdot\vec{A}(\vec{\ell})] \tag{5}$$

The line integral $\int d\vec{\ell}\cdot\vec{A}$ is taken over a line connecting the points $\vec{q}_+$ and $\vec{q}_-$

$$\vec{\ell}=\vec{q}_- + \alpha(\vec{q}_+ - \vec{q}_-) + V(\alpha)\,\vec{s}\;;\qquad 0\le\alpha\le 1,\; V(0)=V(1)=0. \tag{6}$$

The vector $\vec{s}$ runs over all three-dimensional space; $\int d^3s\mu(\vec{s})\ldots$ means an averaging over the set (6) of lines; $\int d^3s\,\mu(\vec{s})=1$ [6]. The particular choice $\mu(\vec{s})=\delta^{(3)}(\vec{s})$ means integration over only one line: the straight one. As S depends only on $q_\mp$ and $A(\vec{x})$ we have

$$\vec{q}'_\mp = S^+\vec{q}_\mp S = \vec{q}_\mp\;;\qquad \vec{A}'(\vec{x}) = \vec{A}(\vec{x}). \tag{7}$$

It was shown in [6] that

$$\vec{p}'_\mp = \vec{p}_\mp + e\,\vec{\nabla}_\mp \int d^3s\,\mu(s)\int_{q_-}^{q_+} d\vec{\ell}\cdot\vec{A}(\vec{\ell}) \tag{8}$$

$$E'_{\perp m}(\vec{x}) = E_{\perp m}(\vec{x}) - e\int d^3s\mu(\vec{s})\int_{q_-}^{q_+}\sum_n d\ell_n\,\delta^\perp_{nm}(\vec{\ell}-\vec{x}),\; m=1,2,3 \tag{9}$$

$$\delta^\perp_{nm}(\vec{y}-\vec{x}) = \delta_{nm}\,\delta^{(3)}(\vec{y}-\vec{x}) - \frac{1}{4\pi}\frac{\partial}{\partial y_n}\frac{\partial}{\partial x_m}\,|\vec{y}-\vec{x}|^{-1}. \tag{10}$$

One can prove the $\vec{p}'_\mp$ invariance under (3) using (8) as in Ref. [6], or the defining equation

$$\vec{p}'_- = \tilde{S}^+\tilde{\vec{p}}_-\tilde{S} =$$
$$= S^+ e^{-ie[-\chi(\vec{q}_+)+\chi(\vec{q}_-)]}(\vec{p}_- - e\vec{\nabla}\chi)\,e^{ie[-\chi(\vec{q}_+)+\chi(\vec{q}_-)]}S = \tag{11}$$
$$= \vec{p}'_-\,.$$

Here the equations $\int d^3s\mu(\vec{s})=1$ and $[\vec{p}_-, f(\vec{q}_-)] = -i\,\partial f/\partial\vec{q}_-$ are used.

Using (7)-(9) one can rewrite H, see eq.(I), in terms of the new (primed) operators

$$H = [\vec{p}'_- - e\,\vec{a}_-(\vec{q}'_-)]^2/2m_- + [\vec{p}'_+ + e\,\vec{a}_+(\vec{q}'_+)]^2/2m_+ +$$
$$+\frac{e^2}{4\pi|\vec{q}'_- - \vec{q}'_+|} + e\int d^3s\mu(\vec{s})\int_{q_-}^{q_+} d\vec{\ell}\cdot\vec{E}'_\perp(\vec{\ell}) + H'_{ph} + P_\perp \tag{12}$$

$$H'_{ph} = \frac{1}{2}\int d^3x\,[(\vec{E}'_\perp)^2 + (\vec{H}')^2] \tag{13}$$

$$\vec{a}_-(\vec{q}_-) = \vec{A}'(\vec{q}_-) - \vec{\nabla}_-\int d^3s\mu(\vec{s})\int_{q_-}^{q_+} d\vec{\ell}\cdot\vec{A}' \tag{14}$$

$$P_\perp = \frac{e^2}{2}\int d^3s\mu(\vec{s})\int d^3s'\mu(\vec{s}')\sum_{m,n}\int_{q_-}^{q_+} d\ell_m\int_{q_-}^{q_+} d\ell'_n\,\delta^\perp_{mn}(\vec{\ell}-\vec{\ell}'). \tag{15}$$

The operator $\vec{a}(\vec{q}_-)$ can be represented in terms of the magnetic field $\vec{H}'=\vec{H}$ [3,7]. The term $P_\perp$, see eq.(15), has its origin from $\frac{1}{2}\int d^3x\,\vec{E}^2_\perp$ when eq.(9), i.e., $\vec{E}_\perp = \vec{E}'_\perp - e\int\ldots$, is used.

The charged particle Hamiltonian $p'^2_-/2m_- + p'^2_+/2m_+$ is now gauge invariant.

It is implied that one uses the canonical representation $\vec{p}' = -i\,\partial/\partial\vec{q}'$ for the operators $\vec{p}'_\mp$. This means that the old operators $\vec{p}_-, \vec{p}_+$ are represented as $-i\,\partial/\partial\vec{q}_\mp + \vec{\nabla} f$, where f depends on $\vec{q}_-, \vec{q}_+$ and $\vec{A}$, see eq.(8). One chooses also a convenient representation for new electromagnetic fields in terms of new photon creation-destruction operators, see subsect.2.6 in [7].

2.3. Let us note a peculiarity of the discussed version of MF: the interaction term $e\vec{p}'_-\cdot\vec{a}_-(\vec{q}'_-)/2m_-$ now depends not only upon $\vec{q}'_-$ but also upon $\vec{q}'_+$, see eq.(14) (compare with $e\vec{p}_-\cdot\vec{A}(\vec{q}_-)/m_-$ which does not depend upon $\vec{q}_+$)

Let one charge, say the positive one, be infinitely heavy and placed at the point $\vec{r}$, i.e., $\vec{q}_+$ is now a c-number $\vec{q}_+\to\vec{r}$; $\vec{p}_+$ and $\vec{p}'_+$ are absent. The coordinate $\vec{r}$ is the center of the

Coulomb potential $e^2/4\pi|\vec{q}_- - \vec{r}|$ for a negative charge. In this case, our version goes to the usual MF for one charge.

The discussed version of MF cannot be defined for nonneutral systems with unequal numbers of positive and negative charges (e.g., one proton and two electrons). But it can be generalized for the relativistic case of two charged Dirac fields (e.g., electron ψ_e and proton ψ_p fields) interacting with photons provided that only the zero total charge Q sector Φ_0, is considered, $Q\Phi_0 = 0$. To illustrate the possibility I present here the example of the relevant unregularized unitary transformation S (cf. with eq. [3] in [7])

$$S = \exp\left[-ie N^{-1}\int d^3x\, \rho_e(\vec{x}) \int d^3y\, \rho_p(\vec{y}) \int_y^x d\vec{\xi}\cdot\vec{A}(\vec{\xi})\right]$$

$$Q = e\int d^3x\, \rho_p(\vec{x}) - e\int d^3x\, \rho_e(\vec{x}), \quad \rho_e(\vec{x}) \equiv \psi_e^*(\vec{x})\,\psi_e(\vec{x}) \qquad (16)$$

$$N\Phi_0 = \int d^3x\, \rho_p(\vec{x})\,\Phi_0 = \int d^3x\, \rho_e(\vec{x})\,\Phi_0 .$$

2.4. Let us find the asymptotic behaviour of $P_\perp$, eq.(15), as $x \equiv |\vec{q}_- - \vec{q}_+| \to \infty$. It was shown in [6] that (15) can be represented as

$$P_\perp = \frac{e^2}{2(2\pi)^2}\int_0^\infty k^2 dk \int_{-1}^{+1} dt\,[x^2 - x^2t^2]\,\left|\int_0^1 d\alpha\, e^{i\alpha k x t}\,\tilde{\mu}(|v(\alpha)|k)\right|^2 .$$

Here $\tilde{\mu}(k)$ is the Fourier transform of $\mu(\vec{s}) \equiv \mu(|\vec{s}|)$. [6].
Introducing the variable $\tau = kxt$ instead of t one obtains

$$P_\perp = \frac{e^2}{2(2\pi)^2}\, x \int_0^\infty k\,dk \int_{-kx}^{+kx} d\tau\,\left[1 - \tau^2/(kx)^2\right]\Pi(\tau,k)$$

$$\Pi(\tau,k) \equiv \int_0^1 d\alpha\, e^{i\alpha\tau}\,\tilde{\mu}(vk)\int_0^1 d\alpha\, e^{-i\alpha\tau}\,\tilde{\mu}^*(vk) .$$

Here $\tilde{\mu}$ is real if $\mu(s)$ is (and depends only on $|\vec{s}|$). Therefore $\Pi(\tau,k) = \Pi(-\tau,k)$ and as $x \to \infty$

$$P_\perp \cong \frac{e^2}{2(2\pi)^2}\, x \int_0^\infty k\,dk\; 2\int_0^{kx} d\tau\; \Pi(\tau,k) . \qquad (17)$$

Here $\Phi(kx,k) = \int_0^{kx} d\tau\; \Pi(\tau,k)$ is a nondecreasing function of

x at any fixed k value because Π is positive (note that Φ must be a decreasing function of k in order that integral over k in eq. (17) would converge [6]). So $P_\perp$ as $x \to \infty$ increases not slower than x at any choice of $\mu(s)$, e.g.

$$P_\perp = const\cdot x^{1+\beta} + \dots \quad ; \quad const > 0 \;, \quad \beta \geq 0 .$$

This means that $P_\perp$ as a function of $x \equiv |\vec{q}_- - \vec{q}_+|$ is an attractive confining potential. For a discussion of this fact see Conclusion.

3. Multipolar-like form for a nonabelian gauge theory

My purpose is to search for a multipolar-like form of nonabelian theories which contains a confining potential between a pair of particles in a similar way as was done in sect.2.

3.1. I consider the simple model: one particle is heavy and located at a point $\vec{r}$, the other is nonrelativistic and spinless but possesses isotopic or colour degrees of freedom. Its free Hamiltonian is $p^2/2m + V(\vec{q}-\vec{r})$ and its interaction with the nonabelian fields $\vec{A}^a$ is introduced by replacing $\vec{p}$ by $\vec{p} - g\,\Sigma_a \vec{A}^a T^a$ by analogy with QED. I need not specify the gauge group. For the simplest SU(2) case one has three group generators T^a, a=1,2,3. The Hamiltonian of the gauge field is taken in the Coulomb gauge div $\vec{A}^a = 0$, $\forall a$:

$$H_g = \frac{1}{2}\int d^3x\; \Sigma_a\left[(\vec{E}_\perp^a(x))^2 + (\vec{H}^a(x))^2 + (\vec{E}_L^a(x))^2\right], \qquad (18)$$

e.g., see sect. 13 in [11] and [12]. Here the term containing $\vec{E}_L^a$ replaces the Coulomb interaction of QED but now this term depends on $\vec{A}^a$ and is known only as a series over g [11]. The canonical gauge operators are $\vec{A}^a(x)$ and $\vec{E}_\perp^a(x)$, their commutation relation being

$$[A_n^a(\vec{y}),\, E_{\perp m}^b(\vec{x})] = -i\,\delta_{ab}\,\delta_{nm}^\perp(\vec{y}-\vec{x}) . \qquad (19)$$

The total Hamiltonian is

$$H = [\vec{p} - g\,\Sigma_a \vec{A}^a(\vec{q})\,T^a]^2/2m + V(\vec{q}-\vec{r}) + H_g . \qquad (20)$$

3.2. Consider the gauge transformation of the model operators. The quark wave function $\phi(\vec q)$ transforms as the relevant quark field

$$\phi(\vec q) \to \tilde\phi(\vec q) = \mathcal U(\vec q)\phi(\vec q) \equiv \exp[-ig\Sigma_a\Theta_a(\vec q)T^a]\phi(\vec q) \quad (21)$$

Using the matrix $\mathcal U$ the corresponding transformation of the potentials can be represented as

$$\tilde{\vec A}^a_{(\vec x)}T^a = \mathcal U(\vec x)\vec A^a(\vec x)T^a\mathcal U^{-1}(\vec x) - \frac{i}{g}(\vec\nabla\mathcal U(\vec x))\mathcal U^{-1}(\vec x) \quad (22)$$

see Sect.1 in [11]. As in the electrodynamical case, one can consider instead of (21) the corresponding transformation of the quark operators, defined by the equation

$$\langle\mathcal U(q)\phi_1(q), \vec p\,\mathcal U(q)\phi_2(q)\rangle = \langle\phi_1, \mathcal U^+\vec p\,\mathcal U\phi_2\rangle, \ \forall\phi_1,\phi_2$$

in the case of the momentum $\vec p$. So $\tilde{\vec p} = \mathcal U^+\vec p\,\mathcal U$. As the consequence of this definition, one obtains that the operators T^a also suffer a transformation $\tilde T^a = \mathcal U^+T^a\mathcal U \neq T^a$. So one can consider along with (22) the transformations

$$\tilde{\vec p} = \mathcal U^+(\vec q)\,\vec p\,\mathcal U(\vec q), \qquad \tilde T^a = \mathcal U^+(\vec q)T^a\mathcal U(\vec q). \quad (23)$$

The Hamiltonian H, eq. (20), is invariant under (22),(23). In particular

$$\tilde{\vec p} - g\tilde{\vec A}^a_{(\vec q)}\tilde T^a = \mathcal U^+\vec p\,\mathcal U - g\,\mathcal U^+\{\tilde{\vec A}^aT^a\}\mathcal U = p - gA^a(\vec q)T^a.$$

Here, I use eq.(22) and

$$\mathcal U^+\vec p\,\mathcal U \equiv \mathcal U^+\{\mathcal U\vec p - [\vec p,\mathcal U]\} = \vec p + \mathcal U^+(-i\vec\nabla_q)\mathcal U.$$

But the quark part of the Hamiltonian, i.e., $\vec p^2/2m + V$ is not gauge invariant along with the momentum $\vec p$.

3.3. The introduction of gauge-invariant momentum will be realized by analogy with sect.2 by means of a canonical transformation using a unitary operator S. To give an example of the wanted S, I make an Ansatz and verify that it turns out suitable

$$S = P\exp[ig\int_q^r d\vec l\cdot\vec A^a(\vec l)T^a] = P\exp[ig\int_0^1 d\alpha\,\frac{\partial\vec l}{\partial\alpha}\cdot\vec A^a_{(\vec l)}T^a]. \quad (24)$$

The symbol P will be explained later, see eq. (29); T^a are the same quark operators as in eq.(20); other notations are as in eqs. (5),(6) where $\vec q_+ \to \vec r$. One must prove now that the new momentum $\vec p' = S^+\vec pS$ is gauge invariant, i.e.,

$$\tilde{\vec p}' \equiv \tilde S^+\tilde{\vec p}\,\tilde S = \tilde S^+\mathcal U^+\vec p\,\mathcal U\tilde S = \vec p'. \quad (25)$$

For this purpose one must determine the S behaviour under the gauge transformation (22),(23). One has

$$\tilde S = P\exp[ig\int_q^r d\vec l\,\tilde{\vec A}^a\tilde T^a] = \mathcal U^+(\vec q)\,P\exp[ig\int_q^r d\vec l\,\tilde{\vec A}^aT^a]\,\mathcal U(\vec q) \quad (26)$$

because of $\tilde T^a = \mathcal U^+T^a\mathcal U$ and of the fact that $P\exp$ can be expanded in a series over powers of T^a. The $P\exp$, which is sandwiched between $\mathcal U^+$ and $\mathcal U$ in eq.(26), has the following property:

$$P\exp[ig\int_q^r d\vec l\cdot\tilde{\vec A}^aT^a] = \mathcal U(\vec r)\,P\exp[ig\int_q^r d\vec l\cdot\vec A^aT^a]\,\mathcal U^{-1}(\vec q). \quad (27)$$

To derive it one may devide the line of integration upon N small segments $(\vec l_n,\vec l_{n+1}) = \vec\Delta_n$ so that $\vec l_0 = \vec q$, $\vec l_N = \vec r$. One can verify that

$$\begin{aligned}\tilde S(\vec l_{n+1},\vec l_n) &\cong \exp ig\vec\Delta_n\tilde{\vec A}^a(\vec l_n)T^a \cong \\ &\cong 1 + ig\vec\Delta_n\tilde{\vec A}^aT^a = 1 + ig\vec\Delta_n[\mathcal U(\vec l_n)\vec A^a(\vec l_n)T^a\mathcal U^{-1}(\vec l_n) - \\ &- \frac{i}{g}\vec\nabla\mathcal U(\vec l_n)\mathcal U^{-1}(\vec l_n)] = \mathcal U(\vec l_{n+1})[1 + ig\vec\Delta_n\cdot\vec A^a(\vec l_n)T^a]\mathcal U^{-1}(\vec l_n) = \\ &= \mathcal U(\vec l_{n+1})S(\vec l_{n+1},\vec l_n)\mathcal U^{-1}(\vec l_n).\end{aligned} \quad (28)$$

using $\mathcal U(\vec l_{n+1}) \cong \mathcal U(\vec l_n) + \vec\Delta_n\cdot\vec\nabla\mathcal U(\vec l_n)$ and neglecting terms of the order Δ_n^2 (cf. [13,14]). Eq. (28) leads to eq.(27) if one defines

$$\lim_{N\to\infty} S(\vec r,\vec l_{N-1})\dots S(\vec l_{n+1},\vec l_n)\dots S(\vec l_1,\vec q) \equiv P\exp[ig\int_q^r d\vec l\,\vec A^aT^a]. \quad (29)$$

Here $P\exp$ cannot be written simply as exp because in the non-abelian case $\vec A^a(\vec l)T^a$ does not commute with $\vec A^a(\vec l')T^a$, $l'\neq l$. Combining (26) and (27) one obtains

$$\tilde S \equiv \tilde S(\vec r,\vec q) = \mathcal U^+(\vec q)\,\mathcal U(r)\,S(\vec r,\vec q) =$$
$$= \mathcal U^+(\vec q)\,S S^+\,\mathcal U(\vec r)\,S(\vec r,\vec q) = \mathcal U^+(\vec q)\,S\,\mathcal U'(\vec r). \tag{30}$$

Here $\mathcal U' \equiv S^+\mathcal U(\vec r)S = \exp[-ig\theta_a(r)T'^a]$ does commute with $\vec p'$, and therefore

$$\tilde{\vec p}' = \tilde S^+\,\tilde{\vec p}\,\tilde S = \mathcal U'^+(\vec r)\,S^+\,\mathcal U(\vec q)\,\mathcal U^+(\vec q)\,\vec p\,\mathcal U\,\mathcal U^+(\vec q)\,S\,\mathcal U'(\vec r) =$$
$$= \mathcal U'^+(\vec r)\,\vec p'\,\mathcal U'(\vec r) = \vec p'. \tag{31}$$

Note that $T'^a = S^+T^aS$ turns out to be noninvariant under gauge transformation (but only globally noninvariant, not locally). But T^a does not enter into the quark Hamiltonian $p'^2/2m + V$ and the latter is guage invariant along with $\vec p'$.

3.4. The next task is to establish the existence of a potential-like term which arises when $\frac12\int d^3x\sum_a(\vec E^a_\perp)^2$ is expressed in terms of new operators. To this end, I need to calculate

$$E'^a_{\perp m}(\vec x) = S^+\,E^a_{\perp m}(\vec x)\,S \quad,\quad m = 1,2,3. \tag{32}$$

I shall give a brief derivation of a formula for $\mathcal O' = S^+\mathcal O S$, where $S = P\exp$, see eq.(24). Introduce the auxiliary operator $S(\beta)$, $S = S(\beta=1)$:

$$S(\beta) = P\exp\Big[ig\int_0^\beta d\alpha\;\partial\vec\ell(\alpha)/\partial\alpha\cdot\vec A^a(\vec\ell(\alpha))T^a\Big] \equiv P\exp\Big[ig\int_0^\beta d\alpha\,K(\alpha)\Big]. \tag{33}$$

Differentiation gives for $S(\beta)$ and $S^+(\beta)$ the identities (equations)

$$dS(\beta)/d\beta = ig\,K(\beta)\,S(\beta)\;,\qquad dS^+(\beta)/d\beta = -ig\,S^+(\beta)\,K(\beta). \tag{34}$$

Following Schwinger's approach (see /15/, eqs. (1.10)-(1.15)), I use the identity

$$S^+(\beta)\,\mathcal O\,S(\beta) = \mathcal O + \int_0^\beta d\alpha_1\,\frac{d}{d\alpha_1}\big[S^+(\alpha_1)\,\mathcal O\,S(\alpha_1)\big]. \tag{35}$$

Using (34) gives

$$\frac{d}{d\alpha_1}\big[S^+(\alpha_1)\,\mathcal O\,S(\alpha_1)\big] = -ig\,S^+(\alpha_1)\big[K(\alpha_1),\mathcal O\big]S(\alpha_1).$$

Further one deals with $S^+(\alpha_1)[K(\alpha_1),\mathcal O]S(\alpha_1)$ in the same manner as with $S^+(\beta)\,\mathcal O\,S(\beta)$ in eq.(35) and so on. One obtains

$$S^+(\beta)\,\mathcal O\,S(\beta) = \mathcal O +$$
$$+\sum_{n=1}^{\infty}(-ig)^n\int_0^\beta d\alpha_1\int_0^{\alpha_1}d\alpha_2\dots\int_0^{\alpha_{n-1}}d\alpha_n\,[K(\alpha_n),[\dots[K(\alpha_1),\mathcal O]\dots]. \tag{36}$$

The wanted formula $\mathcal O' = S^+\mathcal O S$ follows when $\beta = 1$.

Using it and eq.(19) one obtains

$$E'^a_{\perp m}(\vec x) = E^a_{\perp m}(\vec x) - ig\,T^a\int_q^r\sum_n d\ell_n\,\delta^\perp_{nm}(\vec\ell-\vec x)+\dots \tag{37}$$

The unwritten terms of this infinite series contain $\vec A^a = \vec A'^a$. Therefore, eq. (37) allows one to express $\vec E^a_\perp$ in terms of $\vec A'^a$ and $\vec E'^a_\perp$ and to get

$$\frac12\int d^3x\sum_a[\vec E^a_\perp(\vec x)]^2 = \frac12\int d^3x\sum_a[\vec E'^a_\perp(\vec x)]^2 + g\sum_a T^a\int_q^r d\vec\ell\cdot\vec E'^a_\perp(\vec\ell) + P_\perp+\dots \tag{38}$$

$$P_\perp = \frac{g^2}{2}\sum_a T^aT^a\sum_{m,n}\int_q^r d\ell_m\int_q^r d\ell'_n\;\delta^\perp_{mn}(\vec\ell-\vec\ell'). \tag{39}$$

Here $P_\perp$ is the only potential-like term (depending only on q), other unwritten terms in eq.(38) contain $\vec A'^a$ and $\vec E'^a_\perp$. It is essentially the same as in the unregularized electrodynamical MF because $\sum_a T^aT^a$ is the Casimir operator, i.e. a number for each specific nonabelian gauge group.

I shall not rewrite the Hamiltonian H, eq. (20) in terms of the new operators. This cumbersome job may be the subject of another paper. My goal here is to obtain the term $P_\perp$, eq.(39).

3.5. The r.h.s. of eq.(39) is divergent and therefore mathematically meaningless. Let us discuss the problem of its regularization. By analogy with the abelian case one may seek for a regularized transformation operator S_R obtained by averaging eq.(24) over a set of lines. This is a more difficult task than in the abelian case: eq.(24) is $P\exp$ instead of exp and the operators (24) corresponding to different lines do not commute.

For a finite set of lines

$$\vec\ell_i(\alpha) = \vec q + (\vec r-\vec q)\alpha + v(\alpha)\,\vec s_i \quad,\quad i = 0,1,2,\dots N \tag{40}$$

the problem is to construct $S_R(S_0, S_1, \ldots S_N)$ starting from the operators S_i defined by eq.(24) with $l(\alpha) = l_i(\alpha)$. Besides being unitary S_R must also have the following behaviour under the gauge transformation

$$\tilde{S}_R \equiv S_R(\tilde{S}_0, \tilde{S}_1, \ldots \tilde{S}_N) = S_R(WS_0, WS_1, \ldots WS_N) =$$
$$= W S_R(S_0, S_1, \ldots S_N), \qquad (41)$$

where $W \equiv \mathcal{U}^+(\vec{q})\,\mathcal{U}(\vec{r})$ is the operator which is present in equations $\tilde{S}_i = \mathcal{U}^+(\vec{q})\,\mathcal{U}(\vec{r})\,S_i$, see eq.(30), which must hold for each S_i. This behaviour is needed in order to get gauge-invariant new momentum operator $\vec{p}' = S_R^+ \vec{p}\, S_R$, cf. eq.(31). Eq.(41) defines a homogeneous (operator) function (of the degree one) of N+1 (operator) variables. The general solution of eq.(41) is of the form

$$S_R(S_0, S_1, \ldots S_N) = S_0\, \varphi(S_0^{-1}S_1, S_0^{-1}S_2, \ldots S_0^{-1}S_N),$$

where φ is an arbitrary operator-valued function. It is natural to assume that the distinguished operator S_0 is that corresponding to the straight line of integration: let $\vec{s}_0 = 0$ in eqs.(40). In order that φ be unitary one can try to take φ as the product $\prod_i (S_0^{-1}S_i)^{\mu_i}$, where μ_i are arbitrary real numbers. The product is unitary along with $S_0^{-1}S_i$ and $(S_0^{-1}S_i)^{\mu_i}$. But $S_0^{-1}S_i$, i=1,2,... N do not commute and one must define an order of multipliers in the product. The expression

$$\varphi = \exp \Sigma_i \mu_i \left[-\ln S_0 + \ln S_i\right] \qquad (42)$$

allows one to avoid the trouble. The r.h.s. of eq. (42) is not equal to the product above: remember the Baker – Campbell – Hausdorff formula, e.g. see /16,17/.

The above considerations suggest the following Ansatz in the case of the continuous set of lines, numbered by the values of the vectors $\vec{s}$ in eq.(6)

$$S_R = S_0 \exp \int d^3s\, \mu(s)\, \left[-\ln S_0 + \ln S\right]. \qquad (43)$$

Here S is defined by eq.(24) and corresponds to a particular choice of the vector $\vec{s}$ in eq.(6). The operator S_R is unitary, possesses the property $\tilde{S}_R = W S_R$ and turns into the operator (5) in the abelian case if $\int d^3s\, \mu(s) = 1$.

It can be shown that the S_R expansion in powers of g is of the form

$$S_R = 1 + ig \int d^3s\, \mu(s) \int_q^r d\vec{l}\cdot \vec{A}^a T^a + \ldots$$

Therefore

$$S_R^+ E^a_{\perp m}(\vec{x})\, S_R = (1 - ig\int \;+\ldots)\, E^a_{\perp m}(\vec{x})\,(1 + ig\int \;+\ldots) =$$
$$= E_{\perp m}(\vec{x}) - g\int d^3s\, \mu(s)\, T^a \int_q^r \Sigma_n\, dl_{sn}\, \delta^{\perp}_{nm}(\vec{l}_s - \vec{x}) + \ldots$$

and eq. (39) for $P_\perp$ turns into the r.h.s. of eq.(15) multiplied by $\Sigma_a T^a T^a$, i.e., the regularized $P_\perp$ for the nonabelian case is essentially the same as in the electrodynamical MF.

4. Conclusion

In this paper a multipolar-like form of nonabelian theories was shown to exist for a simple nonabelian model. Generalizations seem to be possible by analogy with /7/. This form contains a specific potential $P_\perp$ which is essentially the same as in the abelian case. The regularized $P_\perp$ is shown to be a confining potential.

The first puzzle with $P_\perp$ is the following: as we know, the confinement is absent in electrodynamics. In particular, existence of free electrons is the experimental fact. One can suggest the following theoretical resolution of this trouble: electron-photon interactions may give in the MF (radiative) corrections to potential terms which would cancel $P_\perp$ in a sense. The evidence for this possibility is provided by the following note by Power and Zienau /1/. Consider the correction to the atomic level $|m\rangle$ energy E_m which follows from the interaction $e\int d\vec{l}\cdot\vec{E}$ in the second order of perturbation theory. The correction ΔE_m turns out to have a part which is equal to $(-1)\langle m | P_\perp | m\rangle$. Here $\langle m | P_\perp | m\rangle$ is the first order correction to the level energy resulting from $P_\perp \sim e^2$ considered as the perturbation term. For an improved version see /4/, sect. VIB. Our regularization of $P_\perp$ /5/ allows one to attach strict sense to this compensation.

But only diagonal $P_\perp$ elements are considered in this note. Other interaction terms of the multipolar Hamiltonian (12) also must contribute to the radiative correction to potential terms. So the problem still waits its full consideration in electrodynamics. In the nonabelian case, nothing is known about the $P_\perp$ compensation.

The interrelation between the existence of the confining potential $P_{\perp}$ and the problem of quark confinement has to be still investigated.

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Широков М.И. E2-92-269
Мультипольная форма неабелевых калибровочных теорий

В квантовой оптике широко используется мультипольная форма квантовой электродинамики. Построен ее вариант, пригодный для нейтральных систем (например, две противоположно заряженные частицы). В мультипольном гамильтониане возникает новый потенциалоподобный член $P_{\perp}$. Показано, что он является потенциалом конфайнмента в регуляризованной версии мультипольной формы. Это придает интерес исследованию возможности существования мультипольной формы для неабелевых теорий. Такая форма построена для случая нерелятивистских кварков, взаимодействующих с глюонами. Показано, что в ней появляется потенциал конфайнмента, аналогичный $P_{\perp}$. Однако связь этого факта с проблемой конфайнмента кварков требует дальнейшего исследования.

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Shirokov M.I. E2-92-269
Multipolar-Like Form of Nonabelian Gauge Theories

A notion of the multipolar form of QED is exemplified by using its version valid for neutral systems such as two unlike charges. A new potential-like term $P_{\perp}$ arises in the multipolar Hamiltonian. It is shown to be a confining potential in a regularized variant of the multipolar form. This attracts interest to the discussion of multipolar-like form of nonabelian theories. The form is constructed for the case of nonquantized quarks interacting with gluons. A confining potential is shown to arise which is similar to $P_{\perp}$. But its relevance to the quark confinement problem requires further investigations.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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