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MESONS AT FINITE TEMPERATURE IN THE NJL MODEL WITH GLUON CONDENSATE

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1. Introduction

In a recent work two of us (D.Ebert and M.K.Volkov) have studied a simple $U(2)_f$-symmetric variant of the Nambu- Jona- Lasinio (NJL) quark model including a gluon condensate (GC) [1]. In particular, it was shown that conclusion of the gluon condensate does neither change the form of the interaction between mesons nor the expressions for the meson masses. However, it leads to a significant change of the values of the main parameter of the model, as for example, the cut-off parameter Λ which noticeably diminishes and the four- quark coupling κ which increases. Once the gluon condensate is taken into account the quark condensate decreases in magnitude and approaches to standard value.

In the last years there appeared a lot of papers devoted to the investigation of quarks and mesons in hot and dense matter ([2]- [8]). As is now generally believed, with increasing temperature and baryon number density hadronic matter undergoes a phase transition to a quark- gluon plasma which could manifest itself in the ultra-relativistic heavy ion collisions. Of particular interest is here the question how the quark condensate $<\bar{q}q>$ as a chiral order parameter changes in a hot and dense nuclear medium, and at which temperature (and/or chemical potential) the chiral symmetry is restored.

An increasing number of such investigations is based on the NJL model [3]- [8] which turned out to be rather efficient for the study of meson masses and coupling constants as functions of the temperature T and chemical potential μ.

The purpose of this short article is to investigate the influence of the gluon condensate on the temperature behaviour of the constituent quark mass m (or, equivalently, the total quark condensate) and the pion decay constant F_π. Taking the main model parameters be determined from the case $T=0$, we shall further study the temperature dependence of meson masses and coupling constants. The temperature behaviour of the gluon condensate is taken from the Leutwyler work [9]

2. The NJL model

The e ective chiral quark Lagrangian leading to interactions of composite scalar and pseudoscalar mesons in the presence of the condensate of the gluon field G^a_μ is given by [1], [10]- [12]

$$\mathcal{L}(q,G) = \bar{q}\Big[i(\hat{\partial}+ig\frac{\lambda_a}{2}\hat{G}^a)-m^0\Big]q \qquad (1)$$

$$+ \frac{\kappa}{2}\Big[(\bar{q}\tau^a q)^2+(\bar{q}i\gamma_5\tau^a q)^2\Big] .$$

Here g is the QCD coupling constant, λ_a are generators of the color group $SU(N_c)$, τ^a are the Pauli matrices of the flavor group $SU(2)_F$ ($\tau^0\equiv 1$; summation over ν, a and α is understood), and q are fields of current quarks with mass m^0. Upon introducing meson fields, the Lagrangian (1) turns into the equivalent form

$$\mathcal{L}'(q,G,\tilde{\sigma},\phi) = -\frac{(\tilde{\sigma}_a^2+\tilde{\phi}_a^2)}{2\kappa} + \bar{q}\Big[i(\hat{\partial}+ig\frac{\lambda^a}{2}\hat{G}^a)-m^0+\tilde{\sigma}+i\gamma_5\phi\Big]q \qquad (2)$$

with $\tilde{\sigma}=\tilde{\sigma}_\alpha\tau^\alpha, \phi=\phi_\alpha\tau^\alpha$. The vacuum expectation value of the isoscalar- scalar field $\tilde{\sigma}_0$ turns out to be non ero ($<\tilde{\sigma}_0>\neq 0$).

To pass to a physical field σ_0 with $<\sigma_0>=0$, one usually performs a field shift leading to a new quark mass m to be identified with the constituent quark mass

$$-m^0+\tilde{\sigma}_0 = -m+\sigma_0 \;\; ; \;\; \tilde{\sigma}_a=\sigma_a \;\; (a=1,2,3) \; . \qquad (3)$$

Here m is determined from the gap equation (see [1])

$$m = m^0+8\kappa m I_1+\kappa\frac{G^2}{6m} \; , \qquad (4)$$

where

$$G^2 = \frac{\alpha}{\pi}<(G^a_{\mu\nu})^2>_0 \; , \quad \alpha=\frac{g^2}{4\pi} \qquad (5)$$

and

$$\begin{aligned} I_1 &= -iN_c\int_{reg}\frac{dk}{(2\pi)^4}\frac{1}{(m^2-k^2)} = \frac{N_c}{4\pi^2}\int_0^{\Lambda_3}d\mathrm{p}\frac{\mathrm{p}^2}{\sqrt{\mathrm{p}^2+m^2}} = \\ &= N_c\frac{1}{8\pi^2}\Big[\Lambda_3\sqrt{\Lambda_3^2+m^2}-m^2\ln(\frac{\Lambda_3}{m}+\sqrt{1+\frac{\Lambda_3^2}{m^2}})\Big] \; , \\ I_2 &= -iN_c\int_{reg}\frac{dk}{(2\pi)^4}\frac{1}{(m^2-k^2)^2} = \frac{N_c}{8\pi^2}\int_0^{\Lambda_3}d\mathrm{p}\frac{\mathrm{p}^2}{(\mathrm{p}^2+m^2)^{3/2}} = \\ &= N_c\frac{1}{8\pi^2}\Big[\ln(\frac{\Lambda_3}{m}+\sqrt{1+\frac{\Lambda_3^2}{m^2}})-(1+\frac{m^2}{\Lambda_3^2})^{-1/2}\Big] \; . \end{aligned} \qquad (6)$$

The quark condensates, the constituent quark masses and the meson coupling constants are expressed in the NJL model through diverging integrals I_1 and I_2 (regularized here

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by the cut-off Λ_3) . Indeed, the effective meson Lagrangian corresponding to the σ-model and following from (2) in the one- loop quark approximation has the form

$$\begin{aligned}\mathcal{L}(\sigma,\phi) &= \frac{1}{4}\mathrm{Tr}(\partial_\mu\sigma\partial_\mu\sigma)+\frac{1}{4}\mathrm{Tr}(\partial_\mu\phi\partial_\mu\phi)- \\ &- \frac{g_\sigma^2 m^0}{4\kappa m}\mathrm{Tr}(\sigma^2+\phi^2)-m^2\mathrm{Tr}(\sigma^2)+ \\ &+ g_\sigma m\mathrm{Tr}(\sigma\phi^2)-\frac{g_\sigma^2}{4}\mathrm{Tr}(\sigma^2+\phi^2)^2 ,\end{aligned} \tag{7}$$

where

$$g_\sigma=\frac{1}{2}(I_2+\frac{G^2}{96m^4})^{-\frac{1}{2}} . \tag{8}$$

From (7) one gets the meson masses

$$m_\pi^2=\frac{g_\sigma^2 m^0}{\kappa m} , \; m_\sigma^2=m_\pi^2+4m^2 . \tag{9}$$

That part of the quark condensate which does not explicitly contain the gluon condensate G^2 is determined through the integral I_1

$$<\bar{q}q>=\mathrm{Tr}\Big(\frac{-i}{i\hat{\partial}-m}\Big)=-4mI_1 . \tag{10}$$

Until now we have not considered vector and axial- vector mesons. However, one should bear in mind that since axial- vector mesons do exist, nondiagonal transitions of the type $\pi\to a_1$ play an essential role in the NJL model. If they are taken into account, there arises an additional renormalization of pseudoscalar fields [11, 12] [1]

$$g_\pi=g_\sigma Z^{\frac{1}{2}} , \tag{11}$$

where

$$Z=\Big(1-\frac{6m^2}{m_{a_1}^2}\Big)^{-1}=2\Big[1+\sqrt{1-(\frac{2g_\rho F_\pi}{m_{a_1}})^2}\Big]^{-1} , \tag{12}$$

with m_{a_1} being the mass of the a_1 meson, $F_\pi=93$Mev is the pion decay constant and g_ρ is the ρ meson decay constant ($g_\rho^2/4\pi\approx 3$). In the NJL model the constants g_ρ and g_σ are related by [10]- [12]

$$g_\rho=\sqrt{6}g_\sigma \tag{13}$$

[1] Now in eq.(9) we shall get [11]

$$m_\pi^2=\frac{g_\pi^2 m^0}{\kappa m} , \; m_\sigma^2=Z^{-1}m_\pi^2+4m^2. \tag{9'}$$

Then from the Goldberger- Treiman identity

$$F_\pi^2=\frac{m^2}{g_\pi^2}=\frac{m^2}{Z}\Big(4I_2+\frac{G^2}{24m^4}\Big) \tag{14}$$

and equations (11), (12) and (13) we get

$$m_u^2=\frac{m_{a_1}^2}{12}\Big[1-\sqrt{1-(\frac{2g_\rho F_\pi}{m_{a_1}})^2}\Big]. \tag{15}$$

Using for the mass m_{a_1} the value m_{a_1} = 1.2Gev we get from (12) and (15) the estimates $m_u\approx 300$Mev, $Z\approx 1.6$.

The value of the gluon condensate is taken from the Shifman, Vainshtein, Zakharov paper [13]

$$G^2=\frac{\alpha}{\pi}<G_{\mu\nu}^a G_a^{\mu\nu}>=[330\mathrm{Mev}]^4 . \tag{16}$$

Then, from (14) we obtain

$$\Lambda_3=680\mathrm{Mev} . \tag{17}$$

3. NJL model at finite temperature and chemical potential

In [1] the behaviour of di erent physical quantities was described after introducing the gluon condensate. Let us now show the behaviour of these quantities in hot and dense matter.

For this purpose we need the free quark propagator at finite temperature and baryon number density which in the "real time" formali m takes the form [6, 8]

$$\begin{aligned}S_F(p,T,\mu) &= (\hat{p}+m)\Big[\frac{1}{p^2-m^2+i\epsilon}+ \\ &+ 2\pi i\delta(p^2-m^2)\big[\theta(p_0)n(p,\mu)+\theta(-p_0)\bar{n}(p,\mu)\big]\Big] ,\end{aligned} \tag{18}$$

Here $n,\bar{n}$ are Fermi functions for quark and antiquarks,

$$\begin{aligned}n(p,\mu) &= \Big[1+\exp(\beta(E-\mu))\Big]^{-1} , \\ \bar{n}(p,\mu) &= \Big[1+\exp(\beta(E+\mu))\Big]^{-1} ,\end{aligned} \tag{19}$$

$\beta = T^{-1}$, $E = \sqrt{\mathbf{p}^2 + m^2}$, and μ is the chemical potential. Then, instead of the integrals I_1 and I_2 of eq.(6) we get the following T- and μ- dependent quantities

$$\begin{aligned} I_1(m,T,\mu) &= \frac{N_c}{4\pi^2}\int_0^{\Lambda_3} dp \frac{p^2}{E}(1-n-\bar{n}) \ , \\ I_2(m,T,\mu) &= \frac{N_c}{8\pi^2}\int_0^{\Lambda_3} dp \frac{p^2}{E^3}(1-n-\bar{n}) \ . \end{aligned} \tag{20}$$

Finally, let us use the following temperature behavior of the gluon condensate [9] [2]

$$G^2(T) = G^2(0) - \frac{\pi^2}{135}\frac{N_f^2(N_f^2-1)}{(11N_c - 2N_f)}\frac{T^8}{F_\pi^4}\left\{\ln\frac{\Lambda_p}{T} - \frac{1}{4}\right\} + \cdots \ , \tag{21}$$

where $\Lambda_p = 275$Mev is the logarithmic scale occuring in the temperature expansion of the pressure P [9].

Now from eqs (4) and (14) and using I_1, I_2 and G^2 from (20) and (21) we can obtain the temperature dependence of the constituent quark mass m and the decay constant F_π. The behavior of the total quark condensate which includes also gluon condensate corrections is defined by the equation [1]

$$< \bar{q}q >^{tot} = < \bar{q}q > - \frac{G^2(T)}{12m} = -4mI_1(m,T,\mu) - \frac{G^2(T)}{12m} \ . \tag{22}$$

The temperature behaviour of the meson masses and the coupling constant g_σ and g_π are defined by eqs (8), (9′) and (11) with $m = m(T,\mu)$ and $I_2 = I_2(m,T,\mu)$. In our approach we will neglect a possible temperature dependence of κ and Λ_3.

By using (10) and (22) we get the following estimates of quark condensates at $T = 0$

$$\begin{aligned} < \bar{q}q >_0 &= (-255\text{Mev})^3 \ , \\ < \bar{q}q >_0^{tot} &= (-271\text{Mev})^3 \ . \end{aligned}$$

After this, from equations (4) and (9) we obtain the estimates for κ and m^0

$$\kappa^{-1} = (\frac{m_\pi F_\pi}{m})^2 - \frac{2 < \bar{q}q >_0^{tot}}{m} \approx 7.4\text{Gev}^{-2} \tag{23}$$

$$m^0 = \frac{m_\pi^2 F_\pi^2 \kappa}{m} = m + 2\kappa < \bar{q}q >_0^{tot} \approx 4.2\text{Mev} \tag{24}$$

The temperature behavior of $m, < \bar{q}q >, F_\pi, g_\sigma, g_\pi, m_\sigma$ and m_π is shown in Figures 1-5.

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[2] We neglect a possible dependence on μ. Equation (21) is legitimate in the region $0 < T < 150$Mev. The weak dependence of the gluon condensate on the temperature was found also in $SU(2)$ lattice gauge theory in the papers [14, 15].

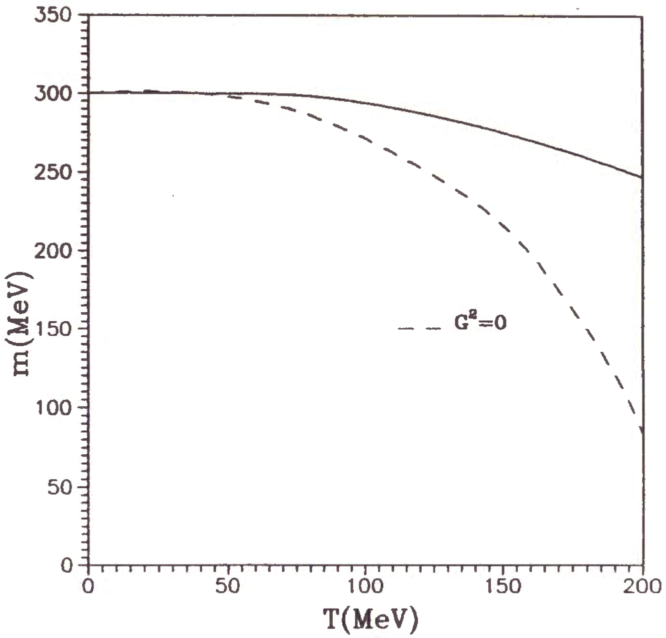

Figure 1. The T- dependence of the constituent quark mass m.

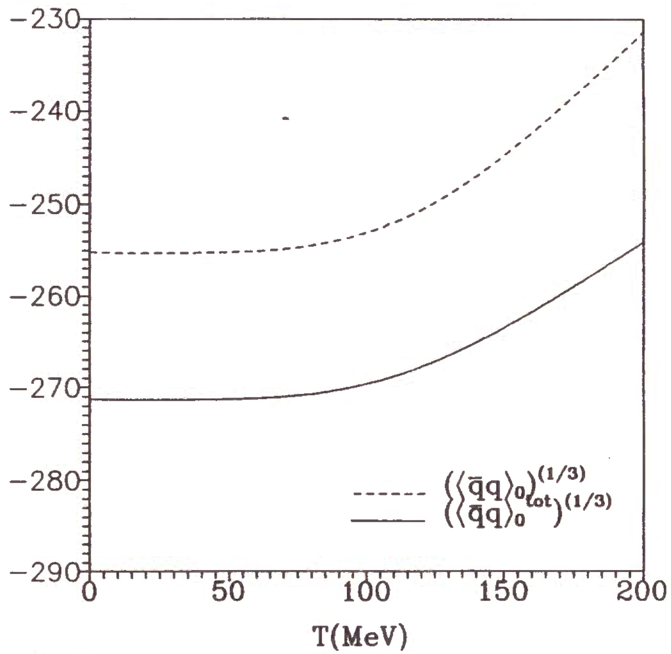

Figure 2. The T- dependence of the quark condensates; $< \bar{q}q >_0$ is the quark condensate without gluon corrections; $< \bar{q}q >_0^{tot}$ is the quark condensate with gluon corrections.

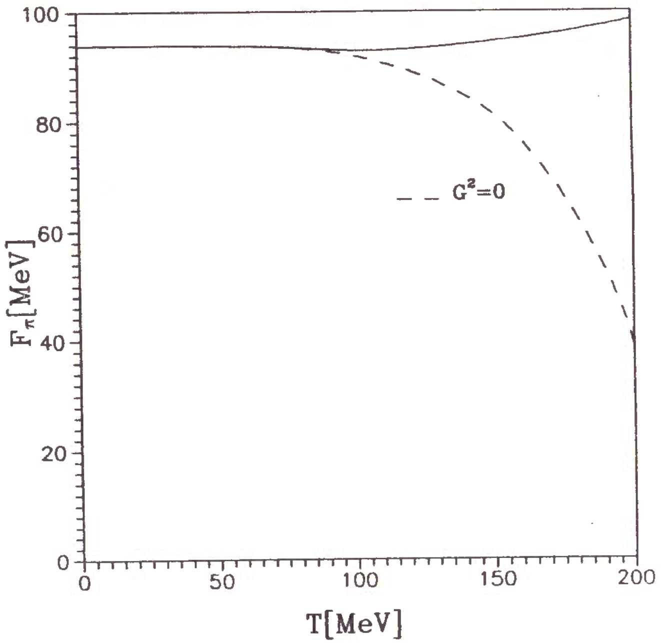

Figure 3. The T- dependence of the pion decay constant F_π.

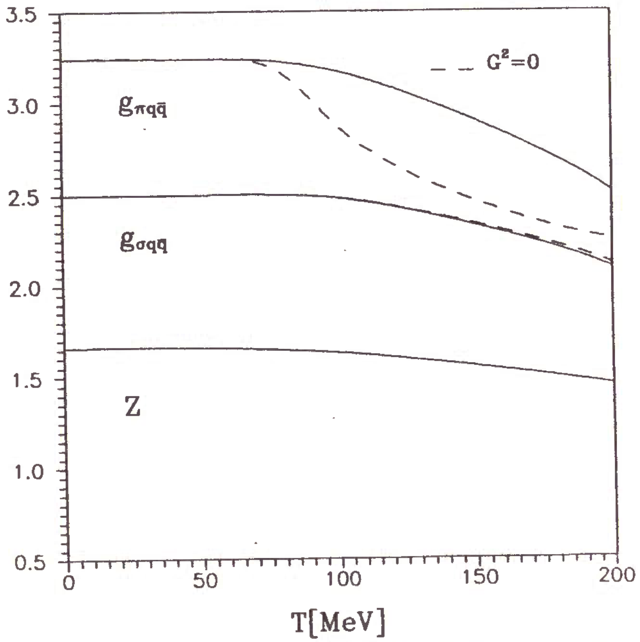

Figure 4. The behaviour of the meson coupling constants $g_{\sigma q\bar{q}}, g_{\pi q\bar{q}}$ and Z as functions of temperature T.

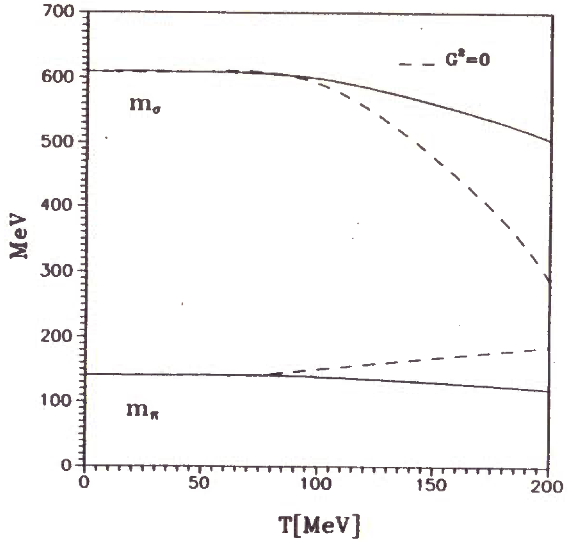

Figure 5. The behaviour of the meson masses m_σ, m_π as functions of temperature T.

4. Discussions

The dependence of the masses of constituent quarks and mesons, as well as of the decay constant F_π and coupling constants $g_{\sigma q\bar{q}}, g_{\pi q\bar{q}}$ on temperature is determined in our model by the two characteristic integrals I_1 and I_2 (see (20)) and the gluon condensate (21). The first interesting question is the behaviour of the constituent quark mass $m(T)$ as a function of T. The answer is found by a self- consistent solution of the thermal gap equation leading to the expression (4) with I_1, G^2 given by (20), (21). The behaviour of $m(T)$ is shown in Figure 1. Figure 2 shows the T- dependence of the quark condensates (with gluon corrections and without gluon corrections). Figure 3 shows the T- dependence of the pion constant F_π. Finally, Figs 4,5 show the T- dependence of the coupling constants $g_{\pi q\bar{q}}, g_{\sigma q\bar{q}}$ and of the meson masses m_π, m_σ.

For comparison we give the behavior of these values also for the case where the gluon condensate is equal to ero. We can see that the gluon condensate plays a stabili ing role for the behavior of di erent physical quantities at changing temperature in the region $0 < T < 150$Mev.

It takes plays because the term with gluon condensate plays more and more important role when the temperature increases. Indeed, this term has in the numerator the weakly changing with temperature function $G^2(T)$ and more rapidly decreasing function $m^4(T)$ in the denominator. (See, for instance, equation (14)).

At a temperature larger than 150Mev the Leutwyler's presentation (21) for the gluon condensate is not correct and one needs to use another approach (maybe, going beyond the scope the G^2- approximation).

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Эберт Д., Калиновский Ю.Л., Волков М.К. E2-92-268
Мезоны в модели Намбу-Иона-Лазинио
с глюонным конденсатом при
конечной температуре

В работе рассматривается модель Намбу-Иона-Лазинио с глюонным конденсатом при конечной температуре и конечной барионной плотности. Изучено поведение конституэнтной массы кварка, кваркового конденсата, F_π, констант связи $g_{\pi q\bar{q}}$, $g_{\sigma q\bar{q}}$ и Z как функций температуры. Показано, что при изменении температуры глюонный конденсат играет стабилизирующую роль в поведении различных физических величин.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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