сообщения
объединенного
института
ядерных
исследований
дубна

E2-92-261

A.D.Popov [1], A.G.Sergeev [2]

BOSONIC STRINGS, GHOSTS AND GEOMETRIC QUANTIZATION

[1]E-mail: POPOV@THEOR.JINRC.DUBNA.SU
[2]E-mail: ARMEN@SERGEEV.MIAN.SU

1992

§1. GRADED LOOP SPACE

1.1 Bosonic strings. Let us consider the Lie group $G=R^{d-1,1}\times G_o$, where $R^{d-1,1}$ is the group of translations of d-dimensional Minkowski space and G_o is a simple compact Lie group. It is known (see, e.g. [1-5]), that the phase space of the open bosonic string, moving in the Lie group G, is a manifold $T^*(G)\times\Omega G$. Here $T^*(G)$ is a cotangent bundle over the Lie group G, describing the motion of the string as a whole, and ΩG is a loop space of the Lie group G, describing the oscillator's degrees of freedom of the string. The quantization of the finite dimensional manifold $T^*(G)$ is standard, therefore, we shall consider further on only the manifold ΩG.

1.2. Loop spaces. The space ΩG, which we shall also denote by Ω, is an infinite-dimensional analogue of flag manifolds (see [6,7]). Denote by $\mathfrak{g}$ and $\mathfrak{g}^C$ the Lie algebras of the Lie groups G and G^C respectively, where $G^C=G\otimes C$ is a complexification of G. *The loop space* ΩG of G is defined as

$$\Omega G=LG/G,$$

where LG is the space $Map(S^1,G)$ of C^∞-smooth mappings of the unit circle S^1 into G and G in the denominator is identified with the group of constant maps $S^1\longrightarrow g_o\in G$. The circle S^1 we identify with the set of complex numbers having the modulus 1.

Considering LG as a group with respect to the pointwise multiplication of G-valued functions, we see that ΩG is a homogeneous space with the origin $\circ$, which is the class of the constant mappings $S^1\longrightarrow 1\in G$ in LG. The tangent space of ΩG in the origin $\circ$ is naturally identified with the space $\Omega\mathfrak{g}=L\mathfrak{g}/\mathfrak{g}$.

1.3. Ghosts. Owing to the requirement of invariance of the theory with respect to the changes of the parametrization of the string, the physical phase space is a submanifold in Ω obtained by symplectic dimensional reduction w.r.to $Diff(S^1)$-action (see, e.g. [8]). Here $Diff(S^1)$ is the group of diffeomorphisms of S^1 preserving the orientation of S^1 and the group S^1 is identified with the subgroup of rotations in $Diff(S^1)$.

For the covariant imposing of the condition of $Diff(S^1)$-invariance the phase space Ω must be extended up to the graded manifold (Ω,Gh), containing together with the commuting coordinates $\{\gamma_n^\mu\}$ on Ω the anticommuting coordinates $\{c^n,b^n\}$ called by "ghosts" [1,8]. More precisely, the functions of $0\le\sigma\le2\pi$

$$c(\sigma)=\sum_{n=-\infty}^{\infty} c^n \exp(-in\sigma),\quad b(\sigma)=\sum_{n=-\infty}^{\infty} b^n \exp(-in\sigma),\quad \bar{c}^n=c^{-n},\quad \bar{b}^n=b^{-n}, \qquad (1)$$

with the Fourier-components c^n and b^n, which are the anticommuting variables

$$c^m c^n + c^n c^m = c^m b^n + b^n c^m = b^m b^n + b^n b^m = 0 , \qquad (2)$$

are considered [1]. The line over the letter means a complex conjugation.

We consider the real vector space W with basis $\{c^n, b^n\}$. Let us introduce the Grassmann algebra

$$\Lambda(W) = \mathop{\oplus}_{q=0}^{\infty} \Lambda^q(W),$$

generated by c^n and b^n from (1), (2). Notice, that we identify W and $\Lambda^1(W)$. In the algebra $\Lambda(W)$ one may introduce Z_2-grading. Indeed, $\Lambda(W) = \Lambda_0(W)\oplus\Lambda_1(W)$, where

$$\Lambda_0(W) = \mathop{\oplus}_{p=0}^{\infty} \Lambda^{2p}(W), \qquad \Lambda_1(W) = \mathop{\oplus}_{p=0}^{\infty} \Lambda^{2p+1}(W) .$$

We set $gr\psi=0$ if $\psi\in\Lambda_0(W)$ and $gr\psi=1$ if $\psi\in\Lambda_1(W)$.

On Ω we introduce the constant sheaf Gh of the Grassmann algebras (i.e. $Gh=\Omega\times\Lambda(W)$ with projection $Gh\ni(\gamma,\psi)\longrightarrow\gamma\in\Omega$ and with discrete topology in $\Lambda(W)$), sections of which are the functions on Ω with values in the algebra $\Lambda(W)$. For any open subset U in Ω let us denote by $Gh(U)$ the space of continuous sections of the sheaf Gh over $U\subset\Omega$. In particular $Gh(\Omega)$ is a set of all continuous sections of the sheaf Gh.

A manifold Ω together with a sheaf Gh of graded algebras $\Lambda(W)$ is a simplest example of a *graded manifold* (Ω, Gh) (supermanifold) [9,10]. In our description of graded manifolds and ghosts we follow Kostant [9], Berezin [10], Kostant and Sternberg [8] (see also [11-13]).

1.4. Tangent space. The *observables* on (Ω, Gh) are identified with the sections $f\in Gh(\Omega)$ of the sheaf Gh. The space $Gh(\Omega) = C^\infty(\Omega)\otimes\Lambda(W)$ is the algebra of functions on Ω with values in the Grassmann algebra $\Lambda(W)$.

Let us consider the graded vector space $Gh^*(\Omega)=Hom(Gh(\Omega),\mathbb{R})$. An element $\delta=\delta_0+\delta_1\in Gh^*(\Omega)$ will be said to be a *differentiation* of algebra $Gh(\Omega)$ at $\gamma\in\Omega$ if a graded Leibnitz rule takes place :

$$\delta_i(fg)=\delta_i(f)\,\tilde{g}(\gamma) + (-1)^{i\, grf}\,\tilde{f}(\gamma)\,\delta_i(g),$$

where $i=0,1$, $f = \tilde{f}\otimes\varphi \in Gh(\Omega) = C^\infty(\Omega)\otimes\Lambda(W)$, $g = \tilde{g}\otimes\psi \in C^\infty(\Omega)\otimes\Lambda(W)$, $gr\delta_i=i$, $grf \equiv gr\varphi$, $grg \equiv gr\psi$ (see also [12,13]).

The tangent space $T_\gamma(\Omega, Gh)$ of graded manifold (Ω, Gh) at the point $\gamma\in\Omega$

is defined to be the space of all differentiations of algebra $Gh(\Omega)$ at $\gamma\in\Omega$. In local coordinates $\{\gamma^\mu_n, c^n, b^n\}$ on (Ω, Gh) we have

$$\delta = \sum_{\mu=1}^{\dim G}\sum_{n\neq 0} a^\mu_n \frac{\partial}{\partial\gamma^\mu_n} + \sum_n \left(\varphi^n \frac{\partial}{\partial c^n} + \chi^n \frac{\partial}{\partial b^n}\right),$$

where a^μ_n, φ^n, $\chi^n \in Gh(\Omega)$, $gr\gamma^\mu_n=0$, $grc^n=grb^n=1$.

1.5. For $\delta=\delta_0+\delta_1$, $\delta'=\delta'_0+\delta'_1\in T_\gamma(\Omega, Gh)$ a *graded commutator* is defined by

$$[\delta_i, \delta'_j] = \delta_i\delta'_j + (-1)^{ij}\,\delta'_j\,\delta_i , \quad i,j=0,1.$$

In particular, for the local basis $\left\{ \frac{\partial}{\partial\gamma^\mu_n}, \frac{\partial}{\partial c^n}, \frac{\partial}{\partial b^n} \right\}$ of the space $T_\gamma(\Omega, Gh)$ one has [9,10]:

$$\frac{\partial}{\partial\gamma^\mu_m}\frac{\partial}{\partial\gamma^\nu_n} - \frac{\partial}{\partial\gamma^\nu_n}\frac{\partial}{\partial\gamma^\mu_m} = \frac{\partial}{\partial\gamma^\mu_m}\frac{\partial}{\partial c^n} - \frac{\partial}{\partial c^n}\frac{\partial}{\partial\gamma^\mu_m} = \frac{\partial}{\partial\gamma^\mu_m}\frac{\partial}{\partial b^n} - \frac{\partial}{\partial b^n}\frac{\partial}{\partial\gamma^\mu_m} = 0,$$

$$\frac{\partial}{\partial c^m}\frac{\partial}{\partial c^n} + \frac{\partial}{\partial c^n}\frac{\partial}{\partial c^m} = \frac{\partial}{\partial c^m}\frac{\partial}{\partial b^n} + \frac{\partial}{\partial b^n}\frac{\partial}{\partial c^m} = \frac{\partial}{\partial b^m}\frac{\partial}{\partial b^n} + \frac{\partial}{\partial b^n}\frac{\partial}{\partial b^m} = 0,$$

$$\frac{\partial}{\partial\gamma^\mu_m}(\gamma^\nu_n) = \delta^\nu_\mu\,\delta^m_n, \qquad \frac{\partial}{\partial\gamma^\mu_m}(c^n) = \frac{\partial}{\partial\gamma^\mu_m}(b^n) = \frac{\partial}{\partial c^n}(\gamma^\mu_m) = \frac{\partial}{\partial b^n}(\gamma^\mu_m) = 0,$$

$$\frac{\partial}{\partial c^m}(c^n) = \delta^n_m , \quad \frac{\partial}{\partial c^m}(b^n) = \frac{\partial}{\partial b^m}(c^n) = 0 , \quad \frac{\partial}{\partial b^m}(b^n) = \delta^n_m .$$

1.6. Tangent bundle. The tangent space $T_\gamma(\Omega, Gh)$ may be splitted into the direct sum of the subspaces:

$$T_\gamma(\Omega, Gh) = T_\gamma(\Omega) \oplus T_\gamma(W) ,$$

where $T_\gamma(\Omega)$ coincides with the ordinary tangent space of the manifold Ω at point γ, and the vectors from the subspace $T_\gamma(W)$ do not actually depend on $\gamma\in\Omega$, because the sheaf Gh is a constant sheaf. Finally,

$$T(\Omega, Gh) = \bigcup_{\gamma\in\Omega} T_\gamma(\Omega, Gh)$$

is *a tangent bundle* over (Ω, Gh).

1.7. Forms. *The space of differential q-forms* $\alpha \in \Lambda^q(\Omega, Gh)$ at $\gamma\in\Omega$ is defined as the set of all q-linear maps on $T_\gamma(\Omega, Gh)$ with values in $\Lambda(W)$, characterized by the graded symmetry condition:

$$\alpha(\xi_1,\ldots,\xi_j, \xi_{j+1},\ldots,\xi_q)=(-1)^{(gr\xi_j+1)(gr\xi_{j+1}+1)}\,\alpha(\xi_1,\ldots,\xi_{j+1},\xi_j,\ldots,\xi_q),$$

where $\xi_j\in T_\gamma(\Omega, Gh)$ are homogeneous. In particular, the space $\Lambda^1_\gamma(\Omega, Gh)$ is isomorphic to the space $T^*_\gamma(\Omega, Gh)$, which is dual to the space $T_\gamma(\Omega, Gh)$. For the local basis $\{ d\gamma^\mu_n, dc^n, db^n \}$ of the space $T^*_\gamma(\Omega, Gh)$ the following commutation relations take place [9,10]:

$$d\gamma^\mu_m\, d\gamma^\nu_n + d\gamma^\nu_n d\gamma^\mu_m = d\gamma^\mu_m dc^n - dc^n d\gamma^\mu_m = d\gamma^\mu_m db^n - db^n d\gamma^\mu_m = 0,$$

$$dc^m dc^n - dc^n dc^m = dc^m db^n - db^n dc^m = db^m db^n - db^n db^m = 0,$$

$$(d\gamma^\mu_m)c^n + c^n(d\gamma^\mu_m) = (d\gamma^\mu_m)b^n + b^n(d\gamma^\mu_m) = (dc^n)\gamma^\mu_m - \gamma^\mu_m(dc^n) = 0,$$

$$\gamma^\mu_m\,(db^n) - (db^n)\,\gamma^\mu_m = c^m(dc^n) - (dc^n)c^m = b^m(dc^n) - (dc^n)b^m = 0,$$

$$c^m(db^n) - (db^n)c^m = b^m(db^n) - (db^n)b^m = 0 .$$

Finally, the operator d of external differentiation on the graded manifold (Ω,Gh) in the local coordinates has the form

$$d = \sum_{\mu=1}^{\dim G} \sum_{n\neq 0} d\gamma^\mu_n \frac{\partial}{\partial\gamma^\mu_n} + \sum_n \left(dc^n \frac{\partial}{\partial c^n} + db^n \frac{\partial}{\partial b^n}\right) .$$

§2. SYMPLECTIC STRUCTURE

2.1. Symplectic form. On the graded manifold (Ω,Gh) let us introduce a nondegenerate 2-form $\omega \in \Lambda^2(\Omega,Gh)$ with $d\omega=0$. The manifold (Ω,Gh) provided with such 2-form ω is called the *symplectic graded manifold* and denoted by (Ω,Gh,ω).

To introduce ω we consider the tangent space $T_\circ(\Omega,Gh) = T_\circ(\Omega)\oplus T_\circ(W)$ at the marked point $\gamma=\circ$. Let $\langle\cdot,\cdot\rangle$ be a (nondegenerate) invariant symmetric inner product on the Lie algebra $\mathfrak{g}$. On $L\mathfrak{g}$ let us introduce the following 2-form

$$\omega_\circ(X,Y) = \frac{k}{2\pi}\int_0^{2\pi} d\sigma\, \langle X(\sigma), Y'(\sigma)\rangle \quad , \qquad (3)$$

where $X,Y\in L\mathfrak{g}$, $Y':=dY/d\sigma$, k is an integer number. This form is closed on $L\mathfrak{g}$ and, moreover, $\omega_\circ(X,\ Y)=0$ if at least one of the maps X,Y is constant. Hence (3) defines the closed 2-form $\omega_\circ$ on the tangent space $T_\circ(\Omega)=\Omega\mathfrak{g}=L\mathfrak{g}/\mathfrak{g}$ to Ω in the origin.

Any vector $X\in T_\circ(\Omega,Gh)$ always may be splitted into the sum $X = X^\Omega + X^W$, where $X^\Omega\in T_\circ(\Omega)$ and $X^W\in T_\circ(W)$. Put

$$\omega_\circ(X^\Omega,Y^W) = \omega_\circ(X^W,Y^\Omega) = 0 \qquad (4)$$

for any $X,Y\in T_\circ(\Omega,Gh)$. Notice, that formula (3) defines $\omega_\circ(X^\Omega,Y^\Omega)$. Each vector X^W from $T_\circ(W)$ has the form

$$X^W = \frac{1}{2\pi}\int_0^{2\pi} d\sigma \left(X^c(\sigma)\frac{\partial}{\partial c(\sigma)} + X^b(\sigma)\frac{\partial}{\partial b(\sigma)} \right) .$$

For the vectors X^W, Y^W from the subspace $T_\circ(W)$ of the tangent space $T_\circ(\Omega,Gh)$ we set

$$\omega_\circ(X^W,Y^W) = \frac{1}{4\pi}\int_0^{2\pi} d\sigma \left(X^c(\sigma)\, Y^b(\sigma) + X^b(\sigma)\, Y^c(\sigma) \right) . \qquad (5)$$

2.2. Formulae (3)-(5) define the closed 2-form $\omega_\circ$ on the tangent space $T_\circ(\Omega,Gh)$ at the origin $\circ\in\Omega$. This form is nondegenerate so transferring it to other points of ΩG by left translations of group LG we obtain the (left-)invariant symplectic structure ω on (Ω,Gh). We obtain

$$\omega = \omega_\Omega+\omega_W = \frac{k}{4\pi}\int_0^{2\pi} d\sigma\langle d\gamma(\sigma) \wedge d\gamma'(\sigma)\rangle + \frac{1}{4\pi}\int_0^{2\pi} d\sigma(dc(\sigma)db(\sigma)+db(\sigma)dc(\sigma)), \quad (6)$$

where $\gamma\in\Omega$, $c,b\in W$.

2.3. Reparametrization. Consider the action of the group $\mathrm{Diff}(S^1)$ on the symplectic structure ω. The action of a diffeomorphism $f\in\mathrm{Diff}(S^1)$ on a map $X\in T_\gamma(\Omega,Gh)$ is defined in the usual way $f_*X(\sigma) := X(f(\sigma))$.

The form ω_Ω, which is the restriction on Ω of the symplectic form ω, is invariant with respect to $\mathrm{Diff}(S^1)$ in the sense that

$$\omega_\Omega(\ f_*X^\Omega,\ f_*Y^\Omega) = \omega_\Omega(X^\Omega,Y^\Omega)$$

for any $f\in\mathrm{Diff}(S^1)$.

Under a reparametrization $\sigma \longrightarrow \sigma'=f(\sigma)$ of S^1 the differentials $dc(\sigma)$ and $db(\sigma)$ transform as [1]

$$dc(\sigma) \longrightarrow dc'(\sigma') = \left(\frac{d\sigma}{d\sigma'}\right)^{-1} dc(\sigma),\quad db(\sigma) \longrightarrow db'(\sigma') = \left(\frac{d\sigma}{d\sigma'}\right)^2 db(\sigma). \qquad (7)$$

From (6) and (7) it is obvious that the form ω_W is invariant w.r.to the action of the group $\mathrm{Diff}(S^1)$, therefore the symplectic structure $\omega=\omega_\Omega+\omega_W$ on (Ω,Gh), introduced in (6), is $\mathrm{Diff}(S^1)$-invariant.

2.4. Poisson brackets. Using the fact that the form ω generates an isomorphism between the tangent and cotangent spaces in any point of Ω, we can associate with any observable f from $Gh(\Omega)$ a Hamiltonian graded vector field X_f on (Ω,Gh) according to the rule

$$df(\cdot) = \omega(X_f,\cdot). \qquad (8)$$

Observables on (Ω,Gh) form a Lie superalgebra with a graded Lie bracket given by the Poisson bracket defined by

$$\{f,g\} = \omega(X_f,X_g),$$

where X_f,X_g are the Hamiltonian graded vector fields corresponding to observables $f,g\in Gh(\Omega)$ [9,10]. Notice, that 2-form ω may be locally represented as an exterior differential $\omega=dA$ of 1-form A.

2.5. Bosons and algebra $Vect(S^1)$. The 2-form ω_Ω is invariant under $Diff(S^1)$, so the Lie algebra $Vect(S^1)$ of the group $Diff(S^1)$ may be identified with a subalgebra of the algebra of Hamiltonian vector fields on Ω. A subalgebra of the algebra of the observables corresponding to the algebra $Vect(S^1)$ of Hamiltonian vector fields on Ω is generated by the functions λ_n given by the formula

$$\lambda_n = -\frac{k}{2\pi}\int_0^{2\pi} d\sigma \, \langle\gamma'(\sigma),\gamma'(\sigma)\rangle \, e^{in\sigma} , \qquad (9)$$

where $\gamma\in\Omega G$, $n=0,\pm1,\pm2,\dots$. The functions $\lambda_n\in C^\infty(\Omega)\otimes1$ satisfy the relations

$$\{\lambda_m,\lambda_n\} = (m-n)\,\lambda_{m+n}, \qquad \bar\lambda_n=\lambda_{-n}$$

and give the representation of the algebra $Vect(S^1)$.

2.6. Fermions and $Vect(S^1)$. The 2-form ω_W, which is the restriction on W of the symplectic form ω on (Ω,Gh), is also invariant under $Diff(S^1)$. Therefore in the Lie superalgebra of the observables $Gh(\Omega)$ one may choose a subalgebra $Vect(S^1)$ generated by functions μ_n given by the formula [1]:

$$\mu_n = \frac{i}{2\pi}\int_0^{2\pi} d\sigma \, (c(\sigma)b'(\sigma) + 2c'(\sigma)b(\sigma)) \, e^{in\sigma} , \qquad (10)$$

where $c,b\in W$, $n=0,\pm1,\pm2,\dots$. Functions $\mu_n\in 1\otimes\Lambda(W)$ also satisfy the commutation relations of the algebra $Vect(S^1)$:

$$\{\mu_m,\mu_n\} = (m-n)\,\mu_{m+n} , \qquad \bar\mu_n = \mu_{-n} .$$

With the help of (8) we can associate with any function μ_n from (10) a graded vector field on (Ω,Gh) acting only in odd directions.

2.7. Bosons⊗fermions and $Vect(S^1)$. We introduce the functions

$$\zeta_n = \lambda_n + \mu_n , \qquad (11)$$

which belong to the algebra of the observables $Gh(\Omega) = C^\infty(\Omega)\otimes\Lambda(W)$ on (Ω, Gh). From the definition of the symplectic form ω (see (6)) it follows that $\{\lambda_m,\mu_n\}=0$, and we obtain

$$\{\zeta_m,\zeta_n\} = (m-n)\,\zeta_{m+n} , \qquad \bar\zeta_n = \zeta_{-n} .$$

In other words, ζ also give a representation of the algebra $Vect(S^1)$.

§3. COMPLEX STRUCTURE ON (Ω,Gh)

3.1. The standard definition of (almost) complex structure on manifolds easily generalizes to graded manifolds (see, e.g., [12]). Namely, (almost) complex structure on graded manifold (Ω,Gh) is a graded tensor field J, which is an automorphism of the tangent space $T_\gamma(\Omega,Gh)$ at each point $\gamma\in\Omega$, such that $J_\gamma^2 = -1$.

3.2. Complex structure J^0. Let us define the complex structure J^0 first on the tangent space $T_\circ(\Omega,Gh) = T_\circ(\Omega)\oplus T_\circ(W) = \Omega g\oplus T_\circ(W)$ in the origin $\circ$. We shall consider the complexification $T_\circ^C(\Omega,Gh) = T_\circ^C(\Omega)\oplus T_\circ^C(W)$ of the tangent space $T_\circ(\Omega,Gh)$ and define the action of the operator $J_\circ^0$ on the vectors $X\in T_\circ^C(\Omega,Gh)$.

An arbitrary vector X^Ω belonging to the subspace $T_\circ^C(\Omega)$ in the space $T_\circ^C(\Omega,Gh)$ can be represented by its Fourier series

$$X^\Omega = \sum_{n\neq0} X_n^\Omega z^n ,$$

where $X_n^\Omega\in g^C$. This vector belongs to $T_\circ(\Omega)$ if $\bar X_n^\Omega = X_{-n}^\Omega$. The operator $J_\circ^0$ of the complex structure J^0 in the origin $\circ$ is defined by

$$J_\circ^0 X^\Omega = -i\sum_{n\neq0} sgn(n)\, X_n^\Omega z^n = i\sum_{n<0} X_n^\Omega z^n - i\sum_{n>0} X_n^\Omega z^n . \qquad (12)$$

An arbitrary vector X^W from the subspace $T_\circ^C(W)$ in $T_\circ^C(\Omega,Gh)$ is given by

$$X^W = \sum_n (\varphi^n \frac{\partial}{\partial c^n} + \chi^n \frac{\partial}{\partial b^n}) .$$

This vector belongs to the subspace $T_\circ(W)$ if $\bar\varphi^n = \varphi^{-n}$, $\bar\chi^n = \chi^{-n}$. We define the operator $J_\circ^0$ on $T_\circ^C(W)$ by

$$J_\circ^0 X^W = i\Big(\sum_{n\le-N-1} \varphi^n\frac{\partial}{\partial c^n} + \sum_{n\le N}\chi^n\frac{\partial}{\partial b^n}\Big) - i\Big(\sum_{n\ge-N}\varphi^n\frac{\partial}{\partial c^n} + \sum_{n\ge N+1}\chi^n\frac{\partial}{\partial b^n}\Big), \qquad (13)$$

where the integer-valued N is called the level of the vacuum. The cases N=-1 and N=0 correspond to the U(1)-invariant vacuum. The cases N=-2 and N=1 correspond to the $SL(2,\mathbb{R})$-invariant vacuum (see [1-4]).

3.3. Formulae (12) and (13) define $J_\circ^0$ on $T_\circ(\Omega,Gh)$. We transfer the complex structure $J_\circ^0$ to other points of ΩG using left translations by LG. It should be pointed out that $T_\gamma(W)=T_\circ(W)$ because the vectors from $T_\gamma(W)$ do not depend on $\gamma\in\Omega$. Notice, that expansion (1) actually depends on the complex structure J^0, therefore instead of c^n and b^n in (1), (2), (13) it is necessary to use $c_\circ^n$ and $b_\circ^n$.

3.4. Compatibility of J^0 and ω. The introduced symplectic and complex structures on (Ω, Gh) are compatible in the following sense. If J^0_γ is the operator of complex structure on the tangent space $T_\gamma(\Omega, Gh)$ in a point $\gamma\in\Omega$ and ω_γ is the symplectic structure at this point, then

$$\omega_\gamma(J^0_\gamma X,\ J^0_\gamma Y) = \omega_\gamma(X,\ Y)$$

for any $X, Y\in T_\gamma(\Omega, Gh)$. Moreover, the metric g^0 given by

$$g^0_\gamma\ (X,\ Y) := \omega_\gamma(X,\ J^0_\gamma Y) \tag{14}$$

on $T_\gamma(\Omega, Gh)$ is nondegenerate. In particular, ω_Ω defines a *pseudo-Kähler structure* $g(X,Y) + i\omega(X,Y)$ on Ω. The symplectic manifold (Ω, Gh, ω) with complex structure J^0 we shall call *the graded pseudo-Kähler manifold* with pseudo-Kähler metric (14) and shall denote it by $(\Omega_J o,\ Gh_J o)\equiv(\Omega, Gh, \omega, J^0)$.

3.5. Complex structure J. Let us denote by $T_\gamma(\Omega_o)$, $T_\gamma(W_o)$ the subspaces in $T^{\mathbf{C}}_\gamma(\Omega, Gh)$, corresponding to the eigenvalue $+i$ of the operator J^0, and by $T_\gamma(\bar\Omega_o)$, $T_\gamma(\bar W_o)$ the subspaces in $T^{\mathbf{C}}_\gamma(\Omega, Gh)$, corresponding to the eigenvalue $-i$ of the operator J^0. It is easy to see that the subspaces $T_\gamma(\Omega_o)\oplus T_\gamma(W_o)$ and $T_\gamma(\bar\Omega_o)\oplus T_\gamma(\bar W_o)$ are the isotropic spaces of the symplectic form ω (see (6)).

Consider the change $\sigma \longrightarrow \sigma' = f(\sigma)$ of the parametrization of string. This transformation from the group $Diff(S^1)$ changes the splitting of the space $T^{\mathbf{C}}_\gamma(\Omega, Gh)$ on the isotropic subspaces. The action of $Diff(S^1)$ on the complex structure J^0 is given by the formula

$$J := f^{-1}_* \circ J^0 \circ f_* \tag{15}$$

The new complex structure J defined by (15) coincides with J^0 if and only if the diffeomorphism $f\in Diff(S^1)$ is a rotation, i.e. f belongs to the subgroup $S^1\subset Diff(S^1)$. The complex structure J given by (15) is compatible with ω because ω is invariant under $Diff(S^1)$ and J is also invariant with respect to the left action of LG.

We obtain

$$T^{\mathbf{C}}_\gamma(\Omega, Gh) = \Big(T_\gamma(\Omega_J)\oplus T_\gamma(W_J) \Big) \oplus \Big(T_\gamma(\bar\Omega_J)\oplus T_\gamma(\bar W_J) \Big)$$

The derivatives $\partial/\partial c^m_J$ ($m\le -N-1$) and $\partial/\partial b^m_J$ ($m\le N$) form the basis of the space $T_o(W_J)$. The explicit form of their dependence on $\partial/c^m_o, \partial/\partial b^m_o$ and on the complex structure J, parametrizing by some matrix, is given, for example, in [4,14]. Analogous formulae for $\gamma^\mu_n(J)\in\Omega_J$, $c^m_J, b^m_J\in W_J$ see also in [4,14].

§ 4. THE SPACE OF COMPLEX STRUCTURES

4.1. The space

$$\mathcal{F} = Diff(S^1)/S^1$$

parametrizes the space of compatible complex structures on the graded loop space (Ω, Gh).

The manifold $\mathcal{F}$ and the vector bundles on it have appeared in Bowick-Rajeev [3] and was later studied in the papers [15-17,4,5,14,18-20].

4.2. Algebra Vect(S^1). The Lie algebra of $Diff(S^1)$ is identified with the *algebra* $Vect(S^1)$ *of smooth vector fields on* S^1 having the bracket

$$\left[\alpha(\sigma)\frac{d}{d\sigma}\ , \beta(\sigma)\frac{d}{d\sigma} \right] = \left(\alpha(\sigma)\beta'(\sigma) - \alpha'(\sigma)\beta(\sigma) \right)\frac{d}{d\sigma} .$$

A basis of the complexified algebra $Vect_c(S^1)$ can be given by the vector fields

$$e_n = ie^{in\sigma}\frac{d}{d\sigma}\ ,\quad n = 0, \pm1, \pm2, \dots\ ,$$

subject to the commutation relations

$$[\, e_m, e_n] = (m - n)e_{m+n}\ .$$

4.3. The group $Diff(S^1)$ acts by left translations on the space $\mathcal{F}$ of complex structures. Denote by L_n the vector fields on $\mathcal{F}$ corresponding under this action to the base vector fields e_n. The fields L_n satisfy to the same commutation relations as e_n

$$[\, L_m,\ L_n] = (m-n)L_{m+n}\ .$$

4.4. Complex structure $\mathcal{J}$ on $\mathcal{F}$. The tangent space $T_o(\mathcal{F})$ in the origin $o \in \mathcal{F}$ consists of vectors

$$v = \sum_{n\neq 0} v_n L_n\ ,$$

subject to the relations $\bar v_n = v_{-n}$. Define a complex structure $\mathcal{J}_o$ on $T_o(\mathcal{F})$ by setting

$$\mathcal{J}_o v = -i \sum_{n\neq 0} sgn(n)\ v_n L_n\ . \tag{16}$$

This structure generates by left translations of $Diff(S^1)$ an invariant (integrable) complex structure $\mathcal{J}$ on $\mathcal{F}$.

4.5. Symplectic form on $\mathcal{F}$. We can also introduce (not uniquely) an invariant Kähler structure on $\mathcal{F}$. In the base $\{ L_n \}$ it can be given by the 2-form

$$\omega'(L_m, L_n) = (\, a\, m^3 + b\, m\,)\,\delta_{m,-n}\ ,$$

which is nondegenerate in two different cases — when $a=0$, $b\neq 0$ or $a\neq 0$ and $-b/a$ is not a square integer. We shall suppose further on that the second possibility is realized.

§ 5. THE GRADED TWISTOR BUNDLE

5.1. The twistor bundle. The bundle $\pi\colon Z \longrightarrow \Omega$ over Ω with the fibre $\mathcal{F}_\gamma$ in a point $\gamma \in \Omega$ consisting of compatible invariant complex structures on $T_\gamma(\Omega)$ will be called the *twistor bundle* over Ω. Points of Z are pairs (γ, J_γ) , where $\gamma \in \Omega$, J_γ is a complex structure on $T_\gamma(\Omega)$. Notice, that the space $\mathcal{F}_\gamma$ also parametrizes the space of the compatible complex structures on $T_\gamma(W)$ and $\mathcal{F}_\gamma \cong \mathcal{F} = \mathrm{Diff}(S^1)/S^1$.

5.2. Double fibration. There is a natural left action of LG on Z induced by the left action of LG on Ω and the orbit space by this action coincides with $\mathcal{F}$ because any complex structure J on Ω is defined by its value J_γ in arbitrary point $\gamma \in \Omega$, e.g. in the origin $\circ$. Hence we have the double fibration

$$\begin{array}{ccc} Z & \xrightarrow{\quad p \quad} & \mathcal{F} \\ \pi \big\downarrow & & \\ \Omega & & \end{array} \qquad (17)$$

where p is the natural projection of Z onto the orbit space $\mathcal{F}$. The fibre Ω_J of p over a point $J \in \mathcal{F}$ is identified with the space (Ω, J), i.e. with the space Ω provided with the complex structure J. It permits to consider the points of Z as the pairs (J, γ_J), where $J\in\mathcal{F}$, $\gamma_J\in\Omega_J$. If one identifies $\mathcal{F}$ with the space $\mathcal{F}_\circ$ of the complex structures on $T_\circ(\Omega)$ then one obtains the complex structure $J=\{J_\gamma\}$ on Ω from $J_\circ\in\mathcal{F}_\circ$ by the left action of LG on Ω. That is way it is convenient to denote $J\in\mathcal{F}$ and the complex structure $J=\{J_\gamma\}$ on Ω by the same letter.

5.3. Ghosts on Z. After the introduction of the complex structure J on the graded manifold (Ω, Gh) we have the constant sheaf $\mathrm{Gh}_J = \Omega_J \times \Lambda(W_J)$. We consider the sheaf $\tilde{\mathrm{Gh}} = \bigcup_{J\in\mathcal{F}} \mathrm{Gh}_J$ on Z with a natural projection $\tilde{\mathrm{Gh}} \longrightarrow Z$. The sections of $\tilde{\mathrm{Gh}}$ will be the functions on Z with values in algebra $\Lambda(W)$. We denote by $\tilde{\mathrm{Gh}}(Z)$ the space of all continuous sections of the sheaf $\tilde{\mathrm{Gh}}$. Pair $(Z, \tilde{\mathrm{Gh}})$ consisting of the manifold Z and the sheaf $\tilde{\mathrm{Gh}}$ over it is a

graded twistor manifold. The coordinates (γ, J_γ) on Z are even coordinates on $(Z,\tilde{\mathrm{Gh}})$ and the coordinates c_J, b_J on W_J are odd coordinates on $(Z,\tilde{\mathrm{Gh}})$.

5.4. Complex structure on $(Z,\tilde{\mathrm{Gh}})$. In order to introduce the almost complex structure $\mathfrak{J}$ on $(Z,\tilde{\mathrm{Gh}})$, we shall define the operator $\mathfrak{J}_z$ on the tangent space $T_z(Z,\tilde{\mathrm{Gh}})=T_z(Z)\oplus T_z(W)$ at each point $z\in Z$. Notice that the vectors from $T_z(W)$ depend only on $J=p(z)$ and it is possible to identify $T_z(W)$ with $T_\gamma(W_J)$, where $\gamma=\pi(z)$. Therefore $\mathfrak{J}_z$ on $T_z(W)$ is actually given by formulae (13) and (15). In order to define $\mathfrak{J}_z$ on $T_z(Z)$ we consider the bundles $\pi^{-1}(T\Omega)$ and $p^{-1}(T\mathcal{F})$ over Z which are the pull-backs of the tangent bundles of Ω and $\mathcal{F}$ respectively.

The projections π and p generate natural bundle homomorphisms

$$d\pi\colon\ TZ \longrightarrow \pi^{-1}(T\Omega), \qquad dp\colon\ TZ \longrightarrow p^{-1}(T\mathcal{F}).$$

We call the kernel of $d\pi$ the *vertical subbundle* V of TZ and kernel of dp — *horizontal subbundle* H of TZ. Then the fibre V_z over a point $z\in Z$ is identified with the tangent space $T_z(\mathcal{F}_\gamma)$ to the fibre $\mathcal{F}_\gamma$ over $\gamma=\pi(z)$ of the bundle $\pi\colon Z \longrightarrow \Omega$ and (by p) with the tangent space $T_J(\mathcal{F})$ where $J=p(z)$. The fibre H_z in a point z is identified with the tangent space $T_z(\Omega_J)$ to the fibre Ω_J of the bundle $p\colon Z \longrightarrow \mathcal{F}$ and (by π) with the tangent space $T_\gamma(\Omega)$. Denote by $\mathfrak{J}^v_z$ the complex structure on V_z induced by the complex structure $\mathcal{J}_J$ on $\mathcal{F}$ in the point J (cf.(16)), and by $\mathfrak{J}^h_z$ — the complex structure on H_z induced by the complex structure J_γ on Ω in the point γ. Now we define the complex structure $\mathfrak{J}_z$ on $T_z(Z)=H_z\oplus V_z$ by setting

$$\mathfrak{J}_z = \mathfrak{J}^h_z \oplus \mathfrak{J}^v_z . \qquad (18)$$

The formula (18) defines an *almost complex structure* $\mathfrak{J}$ on Z. Note that the projection $p\colon Z \longrightarrow \mathcal{F}$ is a holomorphic mapping with respect to $\mathfrak{J}$.

5.5. Real structure on Z. We define now a real structure on Z. Note that the space $\mathcal{F}=\mathrm{Diff}(S^1)/S^1$ has a natural real structure $\theta_\circ$ generated by the mapping $f(e^{i\sigma}) \longrightarrow f(-\,e^{i\sigma})$ for $f\in\mathrm{Diff}(S^1)$, $e^{i\sigma}\in S^1$. The real structure $\theta_\circ$ has no fixed points on $\mathcal{F}$. Introduce a *real structure* θ on Z by setting

$$\theta\colon\ (\gamma, J) \longrightarrow (\gamma, \theta_\circ(J)).$$

It transforms the almost complex structure $\mathfrak{J}$ on Z into the conjugate almost complex structure $-\mathfrak{J}$. Fibers $\mathcal{F}_\gamma$ of $\pi\colon Z \longrightarrow \Omega$ are invariant under

θ so, in this sense, they are real submanifolds of Z. Hence, under the twistor correspondence given by the diagram (17) *points of Ω correspond to real holomorphic sections* of p: $Z \longrightarrow \mathcal{F}$.

This situation reminds the one for hyper-Kähler manifolds [21]. The twistor space Z is a holomorphic bundle p: $Z \longrightarrow \mathcal{F}$ over the Kähler manifold $\mathcal{F}$. This bundle has a family of real (holomorphic) sections parametrized by points of Ω. The normal bundle of any of these sections is identified (as in [21]) with the tensor product of the trivial bundle with the fibre Ωg and complex line bundle $K^{-1} \longrightarrow \mathcal{F}$ which is the anticanonical bundle of $\mathcal{F}$. It would be interesting to know if this construction can be reversed as in the hyper-Kähler case. Or, in other words, how to describe manifolds Ω which arise as the spaces of parameters of families of real holomorphic sections of holomorphic bundles p: $Z \longrightarrow \mathcal{F}$ where Z is an infinite-dimensional complex manifold with a real structure ? Such manifolds should have a collection of covariantly constant complex structures parametrized by the space $\mathcal{F}$ so they may be considered as infinite-dimensional analogues of hyper-Kähler manifolds.

5.6. We shall denote by π the projection π: $(\gamma, J_\gamma, c_J, b_J) \longrightarrow (\gamma, c, b)$, where c,b are odd coordinates on the graded manifold (Ω,Gh). In § 3 we have introduced the graded pseudo-Kähler manifold $(\Omega_J, Gh_J) \equiv (\Omega, Gh, \omega, J)$ with coordinates (γ_J, c_J, b_J). The action of the group LG on $(Z, \tilde{G}h)$ permits to define a projection p: $(J, \gamma_J, c_J, b_J) \longrightarrow J$ on the space $\mathcal{F}$. Thus, we have a double bundle

$$\begin{array}{ccc} (Z,\tilde{G}h) & \xrightarrow{\quad p \quad} & \mathcal{F} \\ \pi \downarrow & & \\ (\Omega, Gh) & & \end{array} \qquad (19)$$

The fibres of the bundle $(Z, \tilde{G}h) \longrightarrow (\Omega, Gh)$ are identified with the space $\mathcal{F}_\gamma$ of the (compatible) complex structures on the space $T_\gamma(\Omega, Gh)$. The fibre (Ω_J, Gh_J) of p over a point $J \in \mathcal{F}$ is identified with the graded symplectic space (Ω, Gh, ω) provided with the complex structure J.

§ 6. PREQUANTIZATION

6.1. The prequantization bundle. Let L be a complex line bundle over Ω with an Hermitian structure $(\cdot,\cdot)$. Suppose that L is provided with a connection ∇^Ω compatible with the Hermitian structure (i.e. $(\nabla^\Omega_X s, t) + (s, \nabla^\Omega_X t) = X(s,t)$ for any sections s,t of L and any vector field X on Ω). It is necessary to choose a connection ∇^Ω so that its curvature F_{∇^Ω} coincides with the symplectic form ω_Ω on Ω. The form ω_Ω is the curvature form of the complex line bundle L over Ω with a connection ∇^Ω if and only if the cohomology class of ω_Ω is integral (see [22,23]). The bundle $L \longrightarrow \Omega$ is called in this case the *prequantization bundle*. By definition $F_{\nabla^\Omega}(X,Y) = = i([\nabla^\Omega_X, \nabla^\Omega_Y] - \nabla^\Omega_{[X,Y]})$ and locally we have $\nabla^\Omega_X = X - iA(X)$, where A is a connection 1-form.

6.2. The prequantization sheaves. Let us consider a complexified Grassmann algebra $\Lambda^c(W)$ $(=\Lambda(W^c))$ and direct products

$$\Omega_{gh} = \Omega \times \Lambda^c(W), \quad \tilde{\Omega}_{gh} = Z \times \Lambda^c(W)$$

of the manifolds Ω,Z and of the vector space $\Lambda^c(W) = \Lambda(W) \otimes \mathbb{C}$. We introduce trivial vector bundle

$$\Omega_{gh} \longrightarrow \Omega\,, \qquad (20a)$$

$$\tilde{\Omega}_{gh} \longrightarrow Z\,, \qquad (20b)$$

with a natural projection on the first factor.

Now let us introduce a tensor product of the bundle $L \longrightarrow \Omega$ and of bundle (20a):

$$L \otimes \Omega_{gh} \longrightarrow \Omega \qquad (21)$$

A sheaf of sections of this bundle will be denoted by Gh^L. This sheaf is called a *line bundle sheaf* over the graded manifold (Ω,Gh) (see [9]). Analogously, let us introduce a bundle

$$\tilde{L} \otimes \tilde{\Omega}_{gh} \longrightarrow Z\,, \qquad (22)$$

where $\tilde{L}$ is the pull-back of the bundle L over Ω to the twistor space Z. It is obvious that $\tilde{L} \otimes \tilde{\Omega}_{gh}$ is the pull-back of the vector bundle $L \otimes \Omega_{gh}$ over Ω to Z. A sheaf of sections of the bundle $\tilde{L} \otimes \tilde{\Omega}_{gh}$ we shall denote by $\tilde{G}h^L$; it is a line bundle sheaf over the graded twistor manifold $(Z, \tilde{G}h)$.

6.3. The sheaf Gh^L is called also the *prequantization sheaf*, and the space $Gh^L(\Omega)$ is called the *prequantization space*. If we denote by $H = \Gamma(\Omega, L)$ the space of sections of the bundle L, then $Gh^L(\Omega) = H \otimes \Lambda^c(W)$.

6.4. Let us introduce a connection ∇ in the sheaf Gh^L, i.e. for any open set $U \subset \Omega$ and for any graded vector field X we will define a linear map ∇_X: $Gh^L(U) \longrightarrow Gh^L(U)$ locally given by formula

$$\nabla_X f = Xf - iA(X)f,$$

where 1-form $A \in \Lambda^1(\Omega, Gh)$ has degree $grA=1$, $f \in Gh^L(U)$. The curvature of this connection is calculated with the help of the graded commutator [9,10].

6.5. Connection ∇. We shall choose a connection ∇ so that its curvature F_∇ coincides with the symplectic form ω on the graded manifold (Ω, Gh). We can do this if and only if the cohomology class, defined by the restriction ω_Ω of the form ω to the manifold Ω, is integral (see [9]). Let X be a vector field on Ω. As the components ∇_X of the connection ∇ we take the components of the connection having the curvature equal to ω_Ω (see Sect.6.1). If X coincides with one of the basic vector fields $X_{c_j^m} = \partial/\partial b_j^{-m}$ or $X_{b_j^m} = \partial/\partial c_j^{-m}$ from $T_\circ^c(W)$ then we put

$$\nabla^{(1,0)}_{X_{c_j^n}} = \frac{\partial}{\partial b_j^{-n}} - ic_j^n, \ n \geq -N, \quad \nabla^{(1,0)}_{X_{b_j^n}} = \frac{\partial}{\partial c_j^{-n}} - ib_j^n, \ n \geq N+1,$$

$$\nabla^{(0,1)}_{X_{c_j^n}} = \frac{\partial}{\partial b_j^{-n}}, \ n \leq -N-1, \quad \nabla^{(0,1)}_{X_{b_j^n}} = \frac{\partial}{\partial c_j^{-n}}, \ n \leq N \qquad (23)$$

It is not difficult to verify that for the curvature tensor F_∇ of the connection ∇ we have

$$F_\nabla(X_{c_j^m}, X_{b_j^n}) \equiv i(\nabla_{X_{c_j^m}} \nabla_{X_{b_j^n}} + \nabla_{X_{b_j^n}} \nabla_{X_{c_j^m}}) = \delta_{m,-n},$$

$$F_\nabla(X_{c_j^m}, X_{c_j^n}) = F_\nabla(X_{b_j^m}, X_{b_j^n}) = 0, \qquad (24)$$

and $F_\nabla(X,Y) = 0$ if $X \in T_\circ^c(\Omega)$ and $Y \in T_\circ^c(W)$. This connection ∇ can be transferred to other points of Ω using left translations by LG. Its curvature form F_∇ coincides with the symplectic 2-form ω on (Ω, Gh).

6.6. As already noted, the space $Gh^L(\Omega)$ of sections of the sheaf Gh^L may be written as a tensor product

$$Gh^L(\Omega) = H \otimes \Lambda^c(W)$$

of the space H of sections of the bundle L (the Fock space of the prequantization of the string) and of the algebra $\Lambda^c(W)$ (the Fock space

of the prequantization of ghosts). That is why the connection ∇ is written as a tensor product

$$\nabla = \nabla^\Omega \otimes 1 + 1 \otimes \nabla^W$$

of the connections ∇^Ω and ∇^W, where the connection ∇^W is given by (23).

6.7. Quantum ghosts. Following the prequantization construction of Kostant and Souriau, to each observable f from the Poincare superalgebra $Gh(\Omega)$ one may correspond the Hamiltonian graded vector field X_f by formula (8) and the operator

$$\hat{f} \equiv O(f) = f - i\nabla_{X_f}, \qquad (25)$$

acting in the space $Gh^L(\Omega)$, $O(1) = id$. In particular, to the "classical" ghosts variables c_j^n and b_j^n there correspond the "quantum" ghosts

$$\hat{c}_j^n = c_j^n - i\nabla_{X_{c_j^n}}, \quad \hat{b}_j^n = b_j^n - i\nabla_{X_{b_j^n}}. \qquad (26)$$

It is relatively easy to check that

$$\hat{c}_j^m \hat{b}_j^n + \hat{b}_j^n \hat{c}_j^m = -i\omega\left(\frac{\partial}{\partial c_j^m}, \frac{\partial}{\partial b_j^n}\right) = -i\delta_{m,-n},$$

$$\hat{c}_j^m \hat{c}_j^n + \hat{c}_j^n \hat{c}_j^m = \hat{b}_j^m \hat{b}_j^n + \hat{b}_j^n \hat{b}_j^m = 0.$$

§ 7. GRADED ANALOG OF THE WARD'S CONSTRUCTION

7.1. Anti-self-duality. Let E be a homogeneous (with respect to LG-action) vector bundle over Ω provided with a left-invariant connection ∇^Ω. A connection ∇^Ω is called *anti-self-dual* (ASD) if its curvature F_{∇^Ω} has the type (1,1) with respect to any invariant compatible complex structure J on Ω. This definition is analogous to the one in the conventional twistor theory, cf. [24,21,25].

7.2. Ward's construction. Let the connection ∇^Ω on E be ASD. Consider the pull-back $\tilde{E} = \pi^*E$ of the bundle E to the twistor space Z and denote by $\tilde{\nabla}^\Omega$ the pull-back of ∇^Ω to $\tilde{E}$. The curvature $\tilde{F}_{\tilde{\nabla}^\Omega}$ of $\tilde{\nabla}^\Omega$ is horizontal and has, obviously, type (1,1) with respect to the almost complex structure $\mathfrak{J}$ on Z. We show that $\tilde{E} \longrightarrow Z$ is holomorphic. Indeed, we can define the $\bar{\partial}$-operator on sections $\tilde{s}$ of $\tilde{E}$ by setting

$$\bar{\partial}\tilde{s} := \tilde{\nabla}^{(0,1)}\tilde{s}, \qquad (27)$$

where $\tilde{\nabla}^{(0,1)}$ is the (0,1)-component of the connection $\tilde{\nabla}^{\Omega}$. Then $\bar{\partial}^2$ coincides with the (0,2)-component of the curvature $\tilde{F}_{\tilde{\nabla}^{\Omega}}$ which vanishes because of the anti-self-duality of ∇^{Ω}. So $\bar{\partial}^2=0$ and it means that the corresponding almost complex structure on $\tilde{E}$ is integrable. By the construction, the bundle $\tilde{E}$ is trivial when restricted to fibres $\mathcal{F}_\gamma$ of π: $Z \longrightarrow \Omega$. Hence, under the pull-back to the twistor space *anti-self-dual vector bundle* E *over* Ω *corresponds to holomorphic vector bundles* $\tilde{E}$ *over* Z *trivial on fibres* $\mathcal{F}_\gamma$.

7.3. Anti-self-duality of ω_Ω. Let us consider now the Ward's construction for a prequantization bundle L over Ω. This bundle has a connection ∇^{Ω} with the curvature F_{∇^Ω} equal to the symplectic form ω_Ω. The curvature F_{∇^Ω} has the type (1,1) with respect to any invariant compatible complex structure J on Ω because ω_Ω is compatible with any of such structures. So the connection ∇^{Ω} on L is ASD and we can apply the Ward's construction. Denoting by $\tilde{L}$ the pull-back of L to the twistor space Z we shall obtain that $\tilde{L} \longrightarrow Z$ is a holomorphic line bundle. In particular, if $G=R^{d-1,1}$ then $\tilde{L}$ is trivial and the *almost complex structure* $\mathfrak{J}$ *on* Z *is integrable in this case.*

7.4. Graded Ward's construction. Now let $\mathcal{E}$ be a sheaf on (Ω,Gh) of Gh^{C}-modules provided with a connection ∇. A connection ∇ is called *anti-self-dual* if its curvature F_∇ has the type (1,1) with respect to any invariant compatible complex structure on the graded manifold (Ω,Gh). As shown in Sec.7.3, the connection ∇^{Ω} in the prequantization bundle L satisfies this definition. Notice that the components F_{∇^W} of the connection ∇^W given by (24) also have the type (1,1) w.r.to any complex structure J on (Ω,Gh). Therefore the connection $\nabla=\nabla^{\Omega}+\nabla^W$ in the prequantization sheaf Gh^L is ASD. Repeating the discussion of sect. 7.2, we obtain that *anti-self-dual sheaf* Gh^L *on* (Ω,Gh) *corresponds to the holomorphic sheaf* $\tilde{Gh}^{\tilde{L}}$ *on* $(Z,\tilde{Gh})$ *trivial on fibres* $\mathcal{F}_\gamma$ *of the bundle* π: $(Z,\tilde{Gh}) \longrightarrow (\Omega,Gh)$.

§8. QUANTIZATION

8.1 Kähler polarization. The introducing of the complex structure J on the graded symplectic manifold (Ω,Gh,ω) defines a *Kähler polarization*, i.e. a subbundle of vectors of the type (0,1) in the complexified tangent

bundle $T^{C}(\Omega,Gh)$. Then the connection ∇ in Gh^L splits into the direct sum $\nabla={}^J\nabla^{(1,0)}\oplus{}^J\nabla^{(0,1)}$ of the components ${}^J\nabla^{(1,0)}$ of the type (1,0) and of the components ${}^J\nabla^{(0,1)}$ of the type (0,1) w.r.to J. Let us introduce a graded quantization space

$$Gh_J(\Omega) = \left\{ s \in Gh^L(\Omega):\ {}^J\nabla^{(0,1)}s = 0 \right\}, \qquad (28)$$

corresponding to the polarization $J\in\mathcal{F}=\mathrm{Diff}(S^1)/S^1$. It is the space of the holomorphic (w.r.to J) sections of the sheaf Gh^L. Remind that $Gh^L(\Omega) = H\otimes\Lambda^C(W)$, $\nabla=\nabla_\Omega\otimes 1 + 1\otimes\nabla_W$ and ${}^J\nabla^{(0,1)} = {}^J\nabla_\Omega^{(0,1)}\otimes 1 + 1\otimes{}^J\nabla_W^{(0,1)}$. Let us introduce

$$H_J = \left\{ f\in H:\ {}^J\nabla_\Omega^{(0,1)}f=0 \right\}, \qquad \Lambda(W_J) = \left\{ \varphi\in\Lambda^C(W):\ {}^J\nabla_W^{(0,1)}\varphi=0 \right\}.$$

It is clear that $H=H_J\oplus\bar{H}_J$, $\Lambda^C(W)=\Lambda(W_J)\otimes\Lambda(\bar{W}_J)$ and $Gh_J(\Omega)=H_J\otimes\Lambda(W_J)$, where H_J is the bosonic quantum space and $B_J\equiv\Lambda(W_J)$ is the fermionic quantum space, corresponding to the polarization J. Notice that instead of $\Lambda(W_J)$ the space $\Lambda(W_J)\otimes\Lambda^{max}(\bar{W}_J)$ is often considered (see, e.g. [11,18,26]). Here $\Lambda^{max}(\bar{W}_J)$ (vacuum) is a one-dimensional vector space obtained by taking the wedge product of all the basis elements of $\bar{W}_J$. The spaces $\Lambda(W_J)$ and $\Lambda(W_J)\otimes\Lambda^{max}(\bar{W}_J)$ are isomorphic. By definition these spaces are the spaces of the *spin representation* of the Clifford algebra $Cl(W^C)$ of the space W^C (see [8,9,11]).

8.2. Inner product. Remind that $G=R^{d-1,1}\times G_o$ and $\Omega G = \Omega R^{d-1,1}\times\Omega G_o$. Therefore the prequantization bundle $L \longrightarrow \Omega G$ is a tensor product $L=L'\otimes L''$ of the bundle $L' \longrightarrow \Omega R^{d-1,1}$ and the bundle $L'' \longrightarrow \Omega G_o$. The bundles L' and L'' are provided with connections, the curvatures of which are equal to ω' and ω'', where ω' is a restriction of the symplectic form ω on (Ω,Gh) to $\Omega R^{d-1,1}$ and ω'' is a restriction of ω to manifold ΩG_o.

The bundle L is provided with a Hermitian structure $(\cdot,\cdot)$. Let us introduce the inner product

$$((s,t)) = \int_\Omega (s,t)\,\det\omega_\Omega \qquad (29)$$

on the space $H=\Gamma(\Omega,L)$ of smooth sections of the bundle L. We introduce the notations $(w^n_A)=(w^n_1,w^n_2)$ where $A,B,\ldots=1,2$ and $w^n_1=c^n$, $w^n_2=b^n$. In §3 the metric g on (Ω,Gh) was defined (see (14)). We introduce the inner product $((\cdot,\cdot))$ of elements w^m_A, $w^n_B\in W$ by formula

$$((w_A^m, w_B^n)) = g\left(\frac{\partial}{\partial w_A^m}, \frac{\partial}{\partial w_B^n} \right) .$$

Then the inner product on $\Lambda(W)$ is defined by (see [6,8]):

$$((w_{A_1}^{m_1} w_{A_2}^{m_2} \dots w_{A_p}^{m_p} , w_{B_1}^{n_1} w_{B_2}^{n_2} \dots w_{B_q}^{n_q})) = \begin{cases} \det g(w_{A_i}^{m_i}, w_{B_j}^{n_j}), \ i,j=1,\dots,p, \text{ if } q=p \\ 0, \text{ if } q\neq p . \end{cases} \qquad (30)$$

Of course, this inner product may be also defined in term of integral (see [10]). The inner product (30) is $\mathbb{C}$-linear extended on $\Lambda^{\mathbb{C}}(W)$. Finally, we put

$$((s,\varphi)) = 0, \qquad (31)$$

if $s\in H\otimes 1$ and $\varphi \in 1\otimes\Lambda^{\mathbb{C}}(W)$. Thus, formulae (29)-(31) define the inner product on $H\otimes\Lambda^{\mathbb{C}}(W)$.

8.3. Metaplectic representation. We consider the representation of the group $\mathrm{Diff}(S^1)$ in the space $H\subset H\otimes\Lambda^{\mathbb{C}}(W)$, the generators $O(\lambda_n)$ of which are defined by formulae (8), (9) and (25). Remind that $H=H_J\oplus\bar{H}_J$ and denote $H_o:=H_{J^o}$. As discussed in §3, the complex structure J^0 is not invariant under the action of the group $\mathrm{Diff}(S^1)$, therefore the action of the operators $O(\lambda_n)$ do not preserve the space H_o. More precisely, the operators $O(\lambda_n)$ with $n\geq 0$ still preserve H_o, but the operators $O(\lambda_n)$ with $n<0$ – not.

Define the operators $r_o(\lambda_n)$ by setting

$$r_o(\lambda_n) = O(\lambda_n) \text{ for } n\geq 0, \quad r_o(\lambda_n) = -r_o^*(\lambda_{-n}) \text{ for } n<0,$$

where r_o^* denotes the Hermitian conjugate of r_o with respect to the inner product $((\cdot,\cdot))$. The operators $r_o(\lambda_n)$ act now in H_o for any n and are the generators of *metaplectic representation* (the Shale-Weyl representation) of the group $\mathrm{Diff}(S^1)$ in the space $H_o\subset H\otimes\Lambda^{\mathbb{C}}(W)$. This is a projective representation because (see [1-5])

$$[r_o(\lambda_m), r_o(\lambda_n)] - r_o(\{\lambda_m,\lambda_n\}) = \left(\frac{d+c}{12} m^3 - (\beta - \frac{c}{12}) \right) \delta_{m,-n} , \qquad (32)$$

where β is the normal ordering constant responsible for the ambiguity in the definition of $r_o(\lambda_0)$ (which will be chosen later) and c is the number given by

$$c = \frac{k \dim \mathfrak{g}_o}{k + \varkappa(\mathfrak{g}_o)} .$$

Here $\varkappa(\mathfrak{g}_o)$ is the so-called dual Coxeter number which can be computed for simple Lie algebras from the following table

$$\varkappa(sl_n) = n, \quad \varkappa(so_n) = n-2, \quad \varkappa(sp_n) = n+1,$$
$$\varkappa(e_6)=12, \quad \varkappa(e_7)=18, \quad \varkappa(e_8)=30, \quad \varkappa(f_4)=9, \quad \varkappa(\mathfrak{g}_2)=4.$$

The right-hand side of (32) is called an *anomaly*. Using left translations by $\mathrm{Diff}(S^1)$ we can define operators $r_J(\lambda_n)$ which generate a projective representation of $\mathrm{Vect}(S^1)$ in the space H_J with the polarization given by a complex structure J on Ω. The explicit form of operators $r_J(\lambda_n)$ for $G=R^{d-1,1}$ and $\Omega=\Omega R^{d-1,1}$ is given for example in [19].

8.4. Spin representation. Let us consider the representation of the group $\mathrm{Diff}(S^1)$ in the space $\Lambda^{\mathbb{C}}(W)\subset H\otimes\Lambda^{\mathbb{C}}(W)$, which generators $O(\mu_n)$ are given by (8), (10) and (25). Remind that $\Lambda^{\mathbb{C}}(W)=\Lambda(W_J)\otimes\Lambda(\bar{W}_J)$ and denote $\Lambda(W_o):=\Lambda(W_{J^o})$. The operators $O(\mu_n)$ do not preserve the space $\Lambda(W_o)$ and repeating the discussion of Sect. 8.3 we define the operators

$$s_o(\mu_n) = O(\mu_n) \text{ for } n\geq 0, \quad s_o(\mu_n) = -s_o^*(\mu_{-n}) \text{ for } n<0,$$

where s_o^* denotes the Hermitian conjugate of s_o w.r.to the inner product $((\cdot,\cdot))$ (see, e.g. [8,9]). The operators $s_o(\mu_n)$ act now in $\Lambda(W_o)$ for any n and are the generators of the *spin representation* of the group $\mathrm{Diff}(S^1)$ in the space $B_o=\Lambda(W_o)$. The explicit form of $s_o(\mu_n)$ in terms of $\hat{c}_o^m$ and $\hat{b}_o^n$ is given, for example, in [1,11,12,27]. The operators $s_J(\mu_n)$ may be obtained by the left action of the group $\mathrm{Diff}(S^1)$. This representation is projective and the commutation relations have the form [1,4,14]:

$$\left[s_J(\mu_m), s_J(\mu_n)\right] - s_J(\{\mu_m,\mu_n\}) = \left(-\frac{26}{12} m^3 + (N^2+N+\frac{1}{6}) m \right) \delta_{m,-n} . \qquad (33)$$

8.5. Physical Fock space. Now we introduce the generators

$$\rho_J(\zeta_n) = r_J(\lambda_n) \otimes 1 + 1 \otimes s_J(\mu_n) \qquad (34)$$

of the *tensor product* of the metaplectic and spin representations of the group $\mathrm{Diff}(S^1)$. The operators $\rho_J(\zeta_n)$ act in the space $H_J\otimes\Lambda(W_J)$. From (32) - (34) it follows that

$$\left[\rho_J(\zeta_m),\rho_J(\zeta_n)\right]-\rho_J(\{\zeta_m,\zeta_n\}) = \left(\frac{d+c-26}{12} m^3+(N^2+N+\frac{1}{6}+\frac{c}{12}-\beta)m\right) \delta_{m,-n} . \qquad (35)$$

The parentheses in the right-hand side of Eq. (35) are equal to zero if

$$d = 26-c, \quad \beta = N^2 + N + \frac{1}{6} + \frac{c}{12} . \qquad (36)$$

If the parameters d, c, N and β satisfy the relations (36) one may impose the conditions of the reparametrization invariance $\rho_J(\zeta_n)f=0$ on the elements f from the quantum Fock space $H_J\otimes\Lambda(W_J)$ of the string with ghosts. If we consider the space $H\otimes\Lambda^c(W)$ of sections of the prequantization sheaf Gh^L on the graded phase manifold (Ω, Gh), then the physical Fock space $\hat{H}^{ph}$ is given as the union of the spaces of solutions of the system of differential equations

$$\hat{H}^{ph} = \bigcup_{J\in\mathcal{F}} \hat{H}^{ph}_J, \quad \hat{H}^{ph}_J = \left\{ \Psi \in H\otimes\Lambda^c(W) : {}^J\nabla^{(0,1)}\Psi = 0, \ \rho_J(\zeta_n)\Psi = 0 \right\}, \qquad (37)$$

depending on auxiliary parameter $J\in\mathcal{F}$. Notice that ${}^J\nabla^{(0,1)}$ are the first-order differential operators on (Ω, Gh) and $\rho_J(\zeta_n)$ are the second-order differential operators on the graded manifold (Ω, Gh). The introduction of the $Diff(S^1)$-invariant Fock space $\hat{H}^{ph}$ completes the procedure of geometric quantization. In the space $\hat{H}^{ph}$ one may set the subspace $\hat{H}^{ph}_+$ of the elements with positive-definite norm. This subspace consists of the eigenfunctions of the ghost number operator (see [1]) with the eigenvalue -1/2.

8.6. The Fock bundle. As discussed above, the connection $\tilde{\nabla}$ in the prequantization sheaf $\tilde{Gh}^{\tilde{L}}$ on $(Z, \tilde{Gh})$ is the pull-back of the connection ∇ in the prequantization sheaf Gh^L on (Ω, Gh). The connection $\tilde{\nabla}$ may be splitted into (1,0)- and (0,1)-components w.r.to the (almost) complex structure $\mathfrak{J}$ on $(Z, \tilde{Gh})$. Let us denote by $\tilde{Gh}^{\tilde{L}}_{\mathcal{O}}(Z)$ the space of all holomorphic sections of the sheaf $\tilde{Gh}^{\tilde{L}}$ on $(Z, \tilde{Gh})$. By definition the space $\mathcal{H}\otimes\mathcal{B} \equiv \tilde{Gh}^{\tilde{L}}_{\mathcal{O}}(Z)$ coincides with the union of the spaces $H_J\otimes B_J$:

$$\mathcal{H}\otimes\mathcal{B} = \bigcup_{J\in\mathcal{F}} H_J\otimes B_J = \bigcup_{J\in\mathcal{F}} \left\{ \Psi \in H\otimes\Lambda^c(W) : {}^J\nabla^{(0,1)}\Psi = 0 \right\},$$

where $B_J \equiv \Lambda(W_J)$. In other words, the space $\mathcal{H}\otimes\mathcal{B}$ coincides with the total space of the bundle over the manifold $\mathcal{F}$:

$$\mathcal{H}\otimes\mathcal{B} \longrightarrow \mathcal{F}, \qquad (38)$$

where the fibres over $J\in\mathcal{F}$ are the vector spaces $H_J\otimes B_J$. The restriction of the connection $\tilde{\nabla}$ on the fibre (Ω_J, Gh_J) of the bundle $(Z, \tilde{Gh}) \longrightarrow \mathcal{F}$ coincides with the connection ${}^J\nabla$ in the sheaf Gh^L and

$$\mathcal{H}\otimes\mathcal{B} = \left\{ \Phi \in \tilde{Gh}^{\tilde{L}}(Z) : \tilde{\nabla}^{(0,1)}\Phi = 0 \right\}.$$

8.7. "Twistor" representation of $Diff(S^1)$. Now we consider the pull-back of the family $\{\rho_J(\zeta_n)\}$ of the second-order differential operators on (Ω, Gh), parametrized by $J\in\mathcal{F}$, to the graded manifold $(Z, \tilde{Gh})$. Following Bowick and Rajeev [3,18], we consider the Lie derivatives $\mathcal{L}_{L_n}$ on $\mathcal{F}$ along the vector fields L_n and the operators $\rho_J(\zeta_n)$ acting in $H_J\otimes B_J$. Let us introduce the connection D in the bundle (38), which in the base $\{L_n\}$ is given by the formula

$$D_{L_n} = \mathcal{L}_{L_n} + \rho_J(\zeta_n). \qquad (39)$$

The curvature F_D of D in the base $\{L_n\}$ was computed at origin in [3-5] and equals (cf.(35)):

$$F_D(L_m, L_n) = \left(\frac{d+c-26}{12} m^3 + \left(N^2 + N + \frac{1}{6} + \frac{c}{12} - \beta \right) m \right) \delta_{m,-n}. \qquad (40)$$

Obviously, that $F_D=0$ if the parameters d, c, N, and β satisfy the relations (36).

The second-order differential operators D_{L_n} on the graded manifold $(Z, \tilde{Gh})$ are the pull-back of the family $\{\rho_J(\zeta_n)\}$ of the second-order differential operator on (Ω, Gh) to the graded twistor space $(Z, \tilde{Gh})$. If the relations (36) take place, then the operators D_{L_n} are the generators of faithful representation of the group $Diff(S^1)$ in the space $\mathcal{H}\otimes\mathcal{B}$. Accordingly, the definition (37) of the space $\hat{H}^{ph}$ may be rewritten in terms of the space of sections of the prequantization sheaf $\tilde{Gh}^{\tilde{L}}$ on $(Z, \tilde{Gh})$ in the following way:

$$\hat{H}^{ph} = \left\{ \Phi \in \tilde{Gh}^{\tilde{L}}(Z) : \tilde{\nabla}^{(0,1)}\Phi = 0, \quad D_{L_n}\Phi = 0, \ n=0,\pm1,\dots \right\}. \qquad (41)$$

Thus, we obtain the formulation of the conditions of the reparametrization invariance and the definition of $\hat{H}^{ph}$ in the twistor terms.

REFERENCES

1. Green M.B., Schwarz J.H., Witten E. *Superstring theory.* Cambridge University Press, 1987.
2. Gepner D., Witten E. *String theory on group manifolds* // Nucl. Phys. **B278** (1986), 493-549.
3. Bowick M.J., Rajeev S.G. *The holomorphic geometry of closed bosonic string theory and Diff(S¹)/S¹* // Nucl.Phys. **B293** (1987), 348-384.

4. Pilch K.,Warner N.P. *Holomorphic structure of superstring vacua //* Class.Quant.Grav. **4** (1987),1188-1192.

5. Mickelsson J. *String quantization on group manifolds and the holomorphic geometry of* $Diff(S^1)/S^1$ // Commun.Math.Phys. **112** (1987), 653-661.

6. Pressley A.,Segal G. *Loop group.* Clarendon Press, Oxford, 1986.

7. Freed D. S. *Flag manifolds and infinite dimensional Kähler geometry.* In: Infinite Dimens.Groups and Appl. Berlin, Springer,1985; 83-124.

8. Kostant B.,Sternberg S. *Symplectic reduction, BRS cohomology, and infinite-dimensional Clifford algebras //* Ann.Phys. **176** (1987),49-113.

9. Kostant B. *Graded manifolds, graded Lie theory and prequantization//* Lect.Notes Math. **570** (1977), 177-306.

10. Berezin F.A. *Introduction to superanalysis.* Reidel, Dordrecht, 1987.

11. Frenkel I.B.,Garland H.,Zuckerman G.J. *Semi-infinite cohomology and string theory//* Proc.Nat.Acad.Sci.USA **83** (1986), 8442-8446.

12. Schaller P.,Schwarz G. *Anomalies from geometric quantization of fermionic field theories//* J.Math.Phys. **31** (1990), 2366-2377.

13. Loll R. *The extended phase space of the BRS algebras//* Commun.Math. Phys. **119** (1988), 509-527; Schaller P.,Schwarz G. *Geometric quantization and field theories - the chiral anomaly example//* Mod.Phys.Lett. **A4** (1989), 483-490; Ewen H.,Schaller P.,Schwarz G. *Schwinger terms from geometric quantization of field theories//* J.Math.Phys. **32** (1991), 1360-1367.

14. Yu Y. *Symplectic geometry and geometric quantization for the open bosonic string in the BRST formalism//* Phys.Lett. **B216** (1989),75-80.

15. Kirillov A.A.,Juriev D.V. *Kähler geometry of the infinite-dimensional homogeneous space* $M = Diff(S^1)/Rot(S^1)$// Funct.Anal.Appl. **21** (1987) N 4, 35-46 (in Russian)

16. Nag S.,Verjovsky A. $Diff(S^1)$ *and the Teichmüller spaces //* Commun. Math.Phys. **130** (1990), 123-138.

17. Hong D.K.,Rajeev S.G. *Universal Teichmüller space and* $Diff(S^1)/S^1$// Commun.Math.Phys. **135** (1991), 401-411.

18. Bowick M.J.,Rajeev S.G. *Anomalies as curvature in complex geometry //* Nucl.Phys. **B296** (1988), 1007-1033.

19. Juriev D. *Gauge invariance of Banks-Peskin differential forms (flat background)//* Lett.Math.Phys. **19** (1990), 59-64.

20. Popov A.D. *BRST-quantization of bosonic strings and hypertwistors //* Mod.Phys.Lett. **A25** (1990), 2057-2062; Popov A.D. *Generalized twistors and geometric quantization//* Theor.Math.Phys. **89** (1991), 331-344.

21. Hitchin N.J.,Karlhede A.,Lindström U.,Roček M. *HyperKähler metrics and supersymmetry//* Commun.Math.Phys. **109** (1987), 535-589.

22. Woodhouse N.J.M. *Geometric quantization.* Oxford,Clarendon Press,1980.

23. Sniatycki J. *Geometric quantization and quantum mechanics.* N.Y., Springer, 1980.

24. Atiyah M.F. *Geometry of Yang-Mills fields.* Pisa, Scuola normale superiore, 1979.

25. Salamon S. *Quaternionic Kähler manifolds//* Invent.Math. **67** (1982), 148-171; Capria M.M.,Salamon S. *Yang-Mills fields on quaternionic spaces//* Nonlinearity **1** (1988), 517-530.

26. Witten E. *Quantum field theory, grassmanians and algebraic curves//* Commun.Math.Phys. **113** (1988), 529-600.

27. Neretin Yu.A. *Representations of Virasoro and affine algebras.* In: Itogi nauki i tehn. Sovr.probl.matem. Fund.naprav. T.22. M.,VINITI, 1983, 163-224 (in Russian).

Received by Publishing Department
on June 22, 1992.

SUBJECT CATEGORIES OF THE JINR PUBLICATIONS

Index	Subject
1.	High energy experimental physics
2.	High energy theoretical physics
3.	Low energy experimental physics
4.	Low energy theoretical physics
5.	Mathematics
6.	Nuclear spectroscopy and radiochemistry
7.	Heavy ion physics
8.	Cryogenics
9.	Accelerators
10.	Automatization of data processing
11.	Computing mathematics and technique
12.	Chemistry
13.	Experimental techniques and methods
14.	Solid state physics. Liquids
15.	Experimental physics of nuclear reactions at low energies
16.	Health physics. Shieldings
17.	Theory of condenced matter
18.	Applied researches
19.	Biophysics

Попов А.Д., Сергеев А.Г.
E2-92-261

Бозонные струны, духи и геометрическое квантование

Мы рассматриваем открытую бозонную струну, движущуюся в группе Ли $G = R^{d-1,1} \times G_0$, где G_0 - простая компактная группа Ли. Пространство $T^*(G) \times \Omega G$ является фазовым многообразием струны. Здесь $\Omega := \Omega G$ - *пространство петель* группы Ли G. Требование Diff(S^1)-инвариантности приводит к необходимости введения духов и расширения многообразия Ω до градуированного многообразия (Ω, Gh) струны с духами. Мы описываем геометрическое квантование супермногообразия (Ω, Gh). Чтобы наложить условие Diff(S^1)-инвариантности *ковариантным образом*, мы расширяем супермногообразие (Ω, Gh) до супермногообразия *твисторов* (Z, $\tilde{G}h$), где $Z = \Omega \times \mathfrak{J}$, а $\mathfrak{J} = \mathrm{Diff}(S^1)/S^1$ - пространство комплексных структур J на (Ω, Gh). Условие Diff(S^1)-инвариантности на твисторном языке эквивалентно условию независимости от выбора комплексной структуры $J \in \mathfrak{J}$ (*антиавтодуальность*). Это, в свою очередь, эквивалентно равенству нулю кривизны связности в расслоении фоковских пространств над пространством комплексных структур $\mathfrak{J}$.

Работа выполнена в Лаборатории теоретической физики ОИЯИ и Математическом институте им. В.А.Стеклова.

Сообщение Объединенного института ядерных исследований. Дубна 1992

Popov A.D., Sergeev A.G.
E2-92-261

Bosonic Strings, Ghosts and Geometric Quantization

We consider an open bosonic string moving in the Lie group $G = R^{d-1,1} \times G_0$, where G_0 is a simple compact Lie group. The space $T^*(G) \times \Omega G$ is a phase manifold of the string. Here $\Omega := \Omega G$ is a *loop space* of the Lie group G. The requirement of Diff(S^1)-invariance leads to the necessity of introduction of ghosts and of enlargement of the manifold Ω to the graded manifold (Ω, Gh) of string with ghosts. We describe the geometric quantization of the supermanifold (Ω, Gh). To put the condition of Diff(S^1)-invariance in the *covariant way*, we enlarge the supermanifold (Ω, Gh) to the *twistor* supermanifold (Z, $\tilde{G}h$), where $Z = \Omega \times \mathfrak{J}$ and $\mathfrak{J} = \mathrm{Diff}(S^1)/S^1$ is the space of complex structures J on (Ω, Gh). In twistor terms the condition of Diff(S^1)-invariance is equivalent to the condition of independence on the choice of the complex structure $J \in \mathfrak{J}$ (*anti-self-duality*). This is equivalent to the equality to zero of the curvature of connection in the bundle of the Fock spaces over the space of complex structures $\mathfrak{J}$.

The investigation has been performed at the Laboratory of Theoretical Physics JINR and V.A.Steklov Mathematical Institute.

Communication of the Joint Institute for Nuclear Research. Dubna 1992