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CONSERVED-VECTOR-CURRENT HYPOTHESIS AND THE $\bar{\nu}_e e^- \to \pi^- \pi^0$ PROCESS

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1 Introduction

At the end of fifties, by Gerstein and Zeldovich [1], and independently by Feynman and Gell-Mann [2], the conserved-vector-current (CVC) hypothesis, as a theoretical ground for an explanation of an approximate numerical equality of the muon decay constant G_μ and the neutron decay vector constant G_V, was postulated in the framework of the V-A weak interaction theory. This hypothesis provides a relation between a matrix element of the vector part of the weak charged hadronic current and a corresponding matrix element of the electromagnetic (e.m.) current to be taken between two pion states. As a result of the foregoing a probability of the π^+-meson beta-decay $\pi^+ \to \pi^0 + e^+ + \nu_e$ was predicted [1,2] theoretically. Its agreement with experimental results [3] is now presented as one of brilliant demonstrations of the general validity of the CVC hypothesis in the weak interaction theory. However, there is a release of a negligible amount of energy in the π^+-meson beta-decay and in fact one is authorized to speak only about an experimental proof of the CVC hypothesis in the very narrow interval of momentum transfer squared $m^2_{e^+} \le q^2 \le (m_{\pi^+} - m_{\pi^0})^2$, i.e. practically for $q^2 \approx 0$.

In this paper, based on the CVC hypothesis and a four-ρ-resonance unitary and analytic VMD model of the pion e.m. form factor (ff), $\sigma_{tot}(E^{lab}_\nu)$ and $d\sigma/dE^{lab}_\pi$ of the weak $\bar{\nu}_e e^- \to \pi^- \pi^0$ process are predicted theoretically for the first time. Their experimental verification could validate the CVC hypothesis for investigated energies above the two-pion threshold. Since, contrary to the e.m. $e^+e^- \to \pi^+\pi^-$ process, in which first two excited states of the $\rho(770)$-meson were identified [4,5] and then also the third one [6], there are no isoscalar vector-meson contributions to the weak $\bar{\nu}_e e^- \to \pi^-\pi^0$ reaction and the first excited state of the $\rho(770)$ -meson with a present-day mass somewhere between $m_{\rho'} = 1300$MeV [7] and $m_{\rho'} = 1450$MeV [4] is not shaded with $\rho - \omega$ interference effect tail. Therefore the accurate measurement of the $\sigma_{tot}(E^{lab}_\nu)$ which is, moreover, strengthened with energy E^{lab}_ν linearly, thus could solve now a widely discussed problem of the mass specification of the first excited state of the $\rho(770)$-meson too. By a direct comparison of $\sigma_{tot}(\bar{\nu}_e e^- \to \pi^-\pi^0)$ with $\sigma_{tot}(e^+e^- \to \pi^+\pi^-)$ we predict their equality at $\sqrt{s} \approx 70$GeV.

The paper is organized as follows. The differential and total cross-sections of the $\bar{\nu}_e e^- \to \pi^-\pi^0$ process in the c.m.system are calculated in

Section 2. The CVC hypothesis is formulated in Section 3. Here also a relation between the pure isovector part of the pion e.m. ff $F_\pi^{E,I=1}(s)$ (a transition $\gamma^* \to \pi^+\pi^-$, where γ^* is a virtual photon) and the weak ff $F_\pi^W(s)$ of a charged W^-boson transition $(W^-)^* \to \pi^-\pi^0$ is derived explicitly. Section 4 is devoted to a brief review of the construction of the four ρ-resonance unitary and analytic VMD model of the pion e.m. ff. The behaviour of $\sigma_{tot}(E\nu^{lab})$ and $d\sigma/dE_\pi^{lab}$ of the weak $\bar{\nu}_e e^- \to \pi^-\pi^0$ process is predicted numerically in Section 5. Conclusion and summary are given in Section 6.

2 Calculation of the $\bar{\nu}_e e^- \to \pi^-\pi^0$ cross-section

In general, the differential cross-section of the weak $\bar{\nu}_e e^- \to \pi^-\pi^0$ reaction in the c.m.system is given by the following expression

$$\frac{d\sigma}{d\Omega} = \frac{1}{(2s_{\bar{\nu}}+1)(2s_e+1)} \cdot \frac{1}{64\pi^2 s} \cdot \frac{k}{p} \cdot \sum_{s_{\bar{\nu}},s_e} |M|^2, \qquad (1)$$

where $s \geq 4m_\pi^2$ is the c.m. energy squared, $k = ((s-4m_\pi^2)/4)^{1/2}$ is a length of a 3-dimensional momentum of produced pions, $p = (s/4)^{1/2}$ is a length of a 3-dimensional neutrino-momentum (assuming $m_{\nu_e} = 0$) and $s_{\bar{\nu}}$ and s_e are spins of the antineutrino and electron, respectively. The matrix element M in the lowest order of a perturbation expansion can be calculated from the Feynman diagram presented in Fig.1a (the $F_\pi^W(s)$ is the weak ff of a charged W^--boson transition $(W^-)^* \to \pi^-\pi^0$), which for $s << m_{W^-}^2$ is reduced to a contact diagram shown in Fig.1b.

In the standard electro-weak theory an effective Hamiltonian describing the $\bar{\nu}_e e^- \to \pi^-\pi^0$ process is

$$\mathcal{H}_I = \frac{G}{\sqrt{2}}[\bar{\nu}_e \cdot \gamma_\mu \cdot (1+\gamma_5) \cdot e] \cdot j^\mu + h.c., \qquad (2)$$

where $G = 1.1663 \times 10^{-5}\mathrm{GeV}^{-2}$ is the Fermi constant of the weak interactions and j^μ is a pion weak current. Then the matrix element corre-

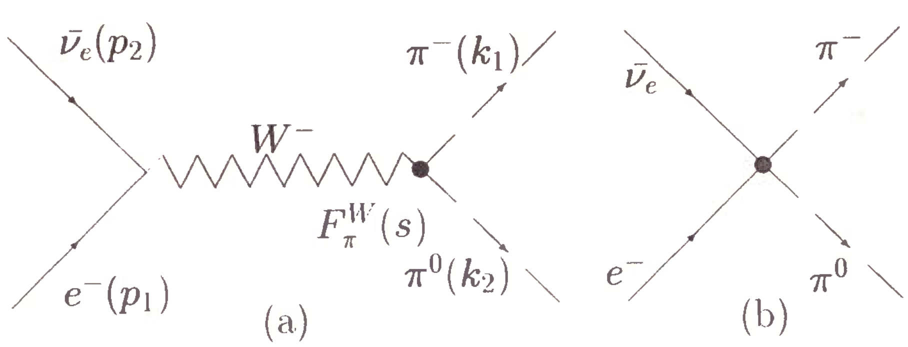

Figure 1: The lowest order perturbation expansion Feynman diagram giving a dominant contribution to the weak $\bar{\nu}_e e^- \to \pi^-\pi^0$ process.

sponding to the diagrams in Fig.1 takes the form

$$M = \frac{G}{\sqrt{2}}\bar{\nu}_e(p_2)\,\gamma_\mu\,(1+\gamma_5)\,e(p_1)(k_1-k_2)^\mu F_\pi^W(s) \qquad (3)$$

and as a result, the differential cross-section (1) can be calculated explicitly

$$\frac{d\sigma}{d\Omega} = \frac{G^2}{128\pi^2} \cdot s \cdot \beta_\pi^3 \cdot |F_\pi^W(s)|^2 \sin^2\vartheta, \qquad (4)$$

where $\beta_\pi = (1-4m_\pi^2/s)^{1/2}$ is the velocity of produced pions and ϑ is the scattering angle in the c.m. system. Considering that $d\Omega = \sin\vartheta\, d\vartheta d\varphi$ and then integrating over angles ϑ and φ in (4) one gets the total cross section

$$\sigma_{tot} = \frac{G^2}{48\pi} \cdot s \cdot \beta_\pi^3 \cdot |F_\pi^W(s)|^2 \qquad (5)$$

of the weak $\bar{\nu}_e e^- \to \pi^-\pi^0$ process. To predict the behaviour of (4) or (5), theoretically one is in need of a knowledge of the weak pion ff $F_\pi^W(s)$ behaviour. Since, there are neither data on the latter, nor accomplished dynamical theory of strong interactions able to predict the $F_\pi^W(s)$ behaviour at the region of $s \geq 4m_\pi^2$, not even a phenomenological approach like in the case of the pion e.m. ff $F_\pi^{el}(s)$ [5,6] is known, to our knowledge no predicted behaviour of $d\sigma(\bar{\nu}e^- \to \pi^-\pi^0)/d\Omega$ and $\sigma_{tot}(\bar{\nu}e^- \to \pi^-\pi^0)$ does exist up to now. However, there is CVC-hyphotesis [1,2] providing a relation between the weak pion ff $F_\pi^W(s)$ and the pure isovector part of the pion e.m. ff $F_\pi^{E,I=1}(s)$, for which we have a well founded a four-ρ-resonance model at our disposal [6]. In this paper the latter is used to predict $\bar\nu_e e^- \to \pi^-\pi^0$ cross-sections for the first time.

3 CVC-hypothesis and a relation between weak and electromagnetic pion form factors

In the framework of the V-A theory of weak interactions the hadronic charged weak current J_W^μ takes, generally, the form of a sum of vector V^μ and axial-vector A^μ currents

$$J_W^\mu = V^\mu + A^\mu. \qquad (6)$$

On the other hand, the e.m. current of hadrons is composed of the sum

$$J_E^\mu = J_3^\mu + J_S^\mu, \qquad (7)$$

where J_3^μ is a third component of an isotopic vector current $\vec{J}^\mu(J_1^\mu, J_2^\mu, J_3^\mu)$ and J_S^μ is an isoscalar current.

In the second-half of the fifties a very predictable postulation was introduced [1,2]

$$V^\mu = J_1^\mu - iJ_2^\mu, \qquad (8)$$

i.e. that the charged weak vector current V^μ and the isovector part J_3^μ of the e.m. current are components of the same isotopic vector $\vec{J}^\mu$. Since, strong interactions are invariant with respect to the isotopic SU(2) group, the isotopic vector current $\vec{J}^\mu$ obeys the relation

$$\partial_\mu J_i^\mu = 0, \qquad (9)$$

which directly results in the formalism of the conserved-vector-current hypothesis

$$\partial_\mu V^\mu = 0. \qquad (10)$$

Nowadays the relations (8) and (10) find a natural explanation in the framework of the standard theory of electro-weak interactions.

Further, in order to derive a relation between $F_\pi^W(s)$ and $F_\pi^{E,I=1}(s)$, we start with a commutation relation

$$[T_i, J_j^\mu] = i\,\varepsilon_{ijk}\,J_k^\mu \qquad (11)$$

with T_i being an isospin operator. The relation (11) is fulfilled automatically if $\vec{J}^\mu$ is transformed in the isospin space like a vector. Now defining

$$T_- = T_1 - iT_2$$

and

$$J_-^\mu = J_1^\mu - i\,J_2^\mu, \qquad (12)$$

by means of (11) and (8) one can prove the following relation

$$[T_-, J_3^\mu] = J_-^\mu \equiv V^\mu. \qquad (13)$$

If we multiply (13) from the right-hand side by a state vector of π^+ meson $|\pi^+>$ and from the left-hand side by a state vector of π^0 meson $<\pi^0|$, and simultaneously use the relations

$$T_-|\pi^+> = \sqrt{2}|\pi^0>, \qquad <\pi^0|T_- = \sqrt{2}<\pi^+| \qquad (14)$$

following from a general relation

$$T_\pm|T,T_3> = \sqrt{(T\mp T_3)(T\pm T_3+1)}\;|T,T_3\pm 1> \qquad (15)$$

valid for an arbitrary isospin, we get

$$<\pi^0|V^\mu|\pi^+> = <\pi^0|[T_-,J_3^\mu]|\pi^+> = \sqrt{2}<\pi^+|J_3^\mu|\pi^+> - \sqrt{2}<\pi^0|J_3^\mu|\pi^0>. \qquad (16)$$

Because strong interactions are invariant under the charge conjugation, the second term of a difference in (16) is equal to zero. Really, if U_C is a unitary charge conjugation operator, and

$$U_C|\pi^0> = |\pi^0>, \qquad U_C J_3^\mu U_C^{-1} = -J_3^\mu, \qquad (17)$$

then

$$<\pi^0|J_3^\mu|\pi^0> = <\pi^0|U_C^{-1}U_C J_3^\mu U_C^{-1} U_C|\pi^0> = -<\pi^0|J_3^\mu|\pi^0> \equiv 0. \qquad (18)$$

So, finally one gets the relation

$$<\pi^0|V^\mu|\pi^+>=\sqrt{2}<\pi^+|J_3^\mu|\pi^+>. \tag{19}$$

On the other hand, if we multiply (7) from the right-hand side and left-hand side by a state vector of π^+ meson, we get

$$<\pi^+|J_E^\mu|\pi^+>=<\pi^+|J_3^\mu|\pi^+>+<\pi^+|J_S^\mu|\pi^+>. \tag{20}$$

Because strong interactions are invariant under the G-transformation that is a combination of the charge conjugation and an isotopic rotation for an angle π around the second axis in the isospin space, the second term in (20) is equal to zero. Really, considering that

$$U_G|\pi^+>=-|\pi^+>, \qquad U_G J_S^\mu U_G^{-1}=-J_S^\mu \tag{21}$$

one has

$$<\pi^+|J_S^\mu|\pi^+>=<\pi^+|U_G^{-1}U_G J_S^\mu U_G^{-1}U_G|\pi^+>=-<\pi^+|J_S^\mu|\pi^+>\equiv 0. \tag{22}$$

and as a result,

$$<\pi^+|J_E^\mu|\pi^+>=<\pi^+|J_3^\mu|\pi^+>. \tag{23}$$

Comparison of(23) with (19) leads to

$$<\pi^0|V^\mu|\pi^+>=\sqrt{2}<\pi^+|J_E^\mu|\pi^+>. \tag{24}$$

Then parametrizing the matrix elements in (24) through corresponding ff's one gets finally the very useful relation between the pure isovector part of the pion e.m. ff $F_\pi^{E,I=1}(s)$ and the weak ff $F_\pi^W(s)$ in the following form

$$F_\pi^W(s)=\sqrt{2}F_\pi^{E,I=1}(s). \tag{25}$$

The relation (25) is valid for squared values of the momentum transfer in the space-like region and time-like region, since it has been derived without any assumption of a dependence of the latter. Substituting (25) into (4) and (5) one gets finally

$$\frac{d\sigma}{d\Omega}=\frac{G^2}{64\pi^2}\cdot s\cdot\beta_\pi^3\cdot|F_\pi^{E,I=1}(s)|^2\sin^2\vartheta \tag{26}$$

and

$$\sigma_{tot}=\frac{G^2}{24\pi}\cdot s\cdot\beta_\pi^3\cdot|F_\pi^{E,I=1}(s)|^2 \tag{27}$$

in the forms that allow us to predict their behaviour theoretically.

4 Four-ρ-resonance unitary and analytic VMD model of the pion electromagnetic structure

The e.m. structure of the charged pions is completely described by one scalar function $F_\pi^E(s)$ depending on the four momentum transfer squared $s=-Q^2$ of the photon and directly measurable in the process $e^+e^-\to\pi^+\pi^-$ through the cross-section

$$\sigma_{tot}(e^+e^-\to\pi^+\pi^-)=\frac{\pi\alpha^2\beta_\pi^3}{3s}|F_\pi^{E,I=1}(s)+Re^{i\Phi}\frac{m_\omega^2}{m_\omega^2-s-im_\omega\Gamma_\omega}|^2, \tag{28}$$

where R and Φ are the $\rho-\omega$ interference amplitude and phase respectively. There are, however, also other processes, like $\pi^-p\to e^+e^-n$ and $J/\Psi\to\pi^+\pi^-$, giving an experimental behaviour of $F_\pi^E(s)$ for $s>0$ and the processes $e^-N\to e^-\pi N, \pi^-e^-\to\pi^-e^-$, from which an experimental information on $F_\pi^E(s)$ for $s<0$ is extracted. The compilation of almost all existing data on $F_\pi^E(s)$ for $-10\mathrm{GeV}^2<s<10\mathrm{GeV}^2$ can be found in ref.[8]. However, all data seemed not to be reliable [9,10].

The most suspicious were 22 space-like experimental points in the range of momenta $-9.77\mathrm{GeV}^2\le s\le 0.18\mathrm{GeV}^2$ which were determined in 1973-1978 at the Cambridge Electron accelerator [11] and at the Cornell University Synchrotron [12,14] from the processes $e^-p\to e^-\pi^+n$ and $e^-n\to e^-\pi^-p$ in a rather model-dependent way. But a careful analysis [15] reveals no global incompatibility of these space-like electroproduction data with the time-like data claimed by other authors [9,10] recently.

Slightly different situation is with three sets of the pion e.m. ff data [16–18] obtained in common Soviet-American measurements of the same $\pi^-e^-\to\pi^-e^-$ process at three different energies. The new measurements [19] carried out at CERN for $-0.253\mathrm{GeV}^2\le s\le -0.015\mathrm{GeV}^2$ unambiguously confirm the conclusion of paper [20] that the first two sets of data [16,17] are charged with a large systematic error and only the third one [18] is reliable. The latter conclusion is definitely confirmed by two independent methods in paper [21] and therefore the first two Soviet-American pion form factor data [16,17] are excluded from any further analysis.

There is no doubt about the reliability of time-like pion e.m. ff data obtained from $e^+e^-\to\pi^+\pi^-$ process, because they are extracted from a measured cross-section without any admixture of a model dependence.

A perfect description of all existing pion e.m. ff data was achieved [6] by a well founded unitary and analytic VMD model with three excited states $\rho'(1450)$, $\rho''(1700)$, $\rho'''(2150)$ of the $\rho(770)$ meson. The latter model which takes into account a two-cut approximation of the correct pion form factor analytic properties is constructed from the standard VMD model

$$F_\pi^{E,I=1}(s) = \sum_{v=\rho,\rho',\rho'',\rho'''} \frac{m_v^2}{m_v^2 - s}\frac{f_{v\pi\pi}}{f_v} \qquad (29)$$

by applying the nonlinear transformation

$$s = s_0 - \frac{4(s_1 - s_0)}{(1/W - W)^2} \qquad (30)$$

and correct incorporation of the nonzero values of vector-meson widths. The $s_0 = 4\,m_\pi^2$ and $s_1 \approx 1\mathrm{GeV}^2$ (in a fit of the data it was left to be a free parameter) are square-root-branch points generating a four-sheeted Riemann surface in s-variable on which the constructed pion e.m. ff model is defined.

Practically, besides (30) we use in (29) also the relations

$$m_v^2 = s_0 - \frac{4(s_1 - s_0)}{(1/W_{v0} - W_{v0})^2} \quad \text{and} \quad 0 = s_0 - \frac{4(s_1 - s_0)}{(1/W_N - W_N)^2}, \qquad (31)$$

where W_{v0} is the zero width (therefore a subindex 0) of VMD poles and W_N is the normalization point (corresponding to s = 0) in the W-plane.

The relations (30) and (31) transform the standard VMD model (29) in the zero-width approximation into the following factorized form

$$F_\pi^{E,I=1}(s) = \left(\frac{1-W^2}{1-W_N^2}\right)^2 \cdot$$
$$\cdot \left[\sum_{v=\rho,\rho',\rho'',\rho'''} \frac{(W_N - W_{v0})(W_N + W_{v0})(W_N - 1/W_{v0})(W_N + 1/W_{v0})}{(W - W_{v0})(W + W_{v0})(W - 1/W_{v0})(W + 1/W_{v0})}(f_{v\pi\pi}/f_v)\right] \qquad (32)$$

where the asymptotic behaviour of (29) is now completely determined by the term in front of the square brackets, because the sum inside the brackets for $s \to \pm\infty$ turns out to be a real constant. Now, using the relations between complex and complex conjugate values of corresponding

pole positions

$$W_{\rho_0} = -W_{\rho_0}^*;\quad W_{\rho_0'} = 1/W_{\rho_0'}^*;\quad W_{\rho_0''} = 1/W_{\rho_0''}^*;\quad W_{\rho_0'''} = 1/W_{\rho_0'''}^* \qquad (33)$$

following from the fact that in a fitting procedure we find

$$(m_\rho^2 - \Gamma_\rho^2/4) < s_1; \qquad (m_{\rho'}^2 - \Gamma_{\rho'}^2/4) > s_1$$
$$(m_{\rho''}^2 - \Gamma_{\rho''}^2/4) > s_1; \qquad (m_{\rho'''}^2 - \Gamma_{\rho'''}^2/4) > s_1 \qquad (34)$$

and incorporating the nonzero values of vector meson widths $\Gamma_v \neq 0$ in a correct way we get the unitary and analytic VMD model of the pion electromagnetic structure in the form

$$F_\pi^{E,I=1}(s) = \left(\frac{1-W^2}{1-W_N^2}\right)^2 \cdot \qquad (35)$$
$$\left[\frac{(W_N - W_\rho)(W_N - W_\rho^*)(W_N - 1/W_\rho)(W_N - 1/W_\rho^*)}{(W - W_\rho)(W - W_\rho)(W - W_\rho^*)(W - 1/W_\rho)(W - 1/W_\rho^*)}(f_{\rho\pi\pi}/f_\rho) + \right.$$
$$\left. + \sum_{v=\rho',\rho'',\rho'''} \frac{(W_N - W_v)(W_N - W_v^*)(W_N + W_v)(W_N + W_v^*)}{(W - W_v)(W - W_v^*)(W + W_v)(W + W_v^*)}(f_{v\pi\pi}/f_v)\right]$$

which governs the asymptotic behaviour as predicted for the pion by QCD up to the logarithmic correction and depends on m_v, Γ_v, $(f_{v\pi\pi}/f_v)$ $v = \rho, \rho', \rho'', \rho'''$ and s_1 as free parameters.

The number of four coupling constant ratios is reduced to three independent by the relation

$$\sum_{v=\rho,\rho',\rho'',\rho'''} (f_{v\pi\pi}/f_v) = 1 \qquad (36)$$

following from the pion form factor normalization condition $F_\pi^{E,I=1}(0) = 1$. The reproduction of all existing data on the pion e.m. ff [8,19,22] by means of (35), where also the isospin violating $\omega \to \pi^+\pi^-$ decay contribution (the so-called ρ-ω interference effect) was taken into account by means of (28) with R as an additional free parameter and the phase to be [23]

$$\phi = \arctan\frac{m_\rho \Gamma_\rho}{(m_\rho^2 - m_\omega^2)} \qquad (37)$$

is presented in Fig.2. The latter results on the pion e.m. ff allow us by using (35) to predict the behaviour of the cross-sections (26) and (27) from the previous section.

5 The $\bar{\nu}_e e^- \to \pi^-\pi^0$ cross-section behaviour

It is now straightforward to calculate the $\sigma_{tot}(\bar{\nu}_e e^- \to \pi^-\pi^0)$ in the c.m. system given by the relation (27) using the pion e.m. ff model [6] a brief construction of which was outlined in the previous section. However, before that it is interesting to compare the relations (27) and (28) as $\sigma_{tot}(\bar{\nu}_e e^- \to \pi^-\pi^0) \sim s\,|F_\pi^{E,I=1}(s)|^2$ and $\sigma_{tot}(e^+e^- \to \pi^+\pi^-) \sim 1/s\,|F_\pi^{E,I=1}(s)|^2$.

We note that the $\omega \to \pi^+\pi^-$ decay term contribution in (28) can be disregarded for $s \gg m_\omega^2$. Then we get the relation

$$\frac{G^2}{24\pi}s \approx \frac{\pi\alpha^2}{3s} \tag{38}$$

from which we find an equality of the total cross-section of the weak process $\bar{\nu}_e e^- \to \pi^-\pi^0$ with the total cross-section of the e.m. process $e^+e^- \to \pi^-\pi^0$ at

$$\sqrt{s} \approx 70\text{GeV}. \tag{39}$$

Graphical presentation of this result with a simultaneous comparison of $\sigma_{tot}(\bar{\nu}_e e^- \to \pi^-\pi^0)$ and $\sigma_{tot}(e + e^- \to \pi^-\pi^0)$ is presented in Fig.3.

However, the results in Fig.3 are not interesting for experimentalists because the weak $\bar{\nu}_e e^- \to \pi^-\pi^0$ process can be (if at all) practically measured only in an interaction of antineutrino beams with atomic electrons in the laboratory system. The threshold energy of the antineutrino beam is then

$$E_\nu^{(0)} = \frac{4m_\pi^2 - m_e^2}{2m_e} = 76.7\text{GeV}. \tag{40}$$

The behaviour of $\sigma_{tot}(\bar{\nu}_e e^- \to \pi^-\pi^0)$ in the laboratory system can be found by the substitution of

$$s = m_e^2 + 2m_e\, E_\nu^{lab} \tag{41}$$

into (27) which leads to the following relation

$$\sigma_{tot}(E_\nu^{lab}) = \frac{G^2}{24\pi}\cdot(m_e^2 + 2m_e E_\nu^{lab})\cdot\beta_\pi^3\cdot|F_\pi^{E,I=1}(E_\nu^{lab})|^2. \tag{42}$$

The behaviour of $\sigma_{tot}(E_\nu^{lab})$ is predicted in Fig.4, where a manifestation of all four ρ-resonances is clearly seen.

It is also interesting to predict the energy distribution of the pions created in the final state of the weak $\bar{\nu}_e e^- \to \pi^-\pi^0$ process given by the differential cross-section $d\sigma/dE_\pi^{lab}$ in the laboratory system. The latter is obtained by a direct transformation of (26) into the laboratory system. With this aim we first use the relation $d\Omega = -d\cos\vartheta\, d\varphi$ and integrate the corresponding expression over the angle φ. As a results we have

$$\frac{d\sigma}{d\cos\vartheta} = -\frac{G^2}{32\pi}\cdot\beta_\pi^3\cdot s\cdot|F_\pi^{E,I=1}(s)|^2\sin^2\vartheta. \tag{43}$$

Then we insert into (43) the following expressions

$$\begin{aligned} d\cos\vartheta &= -\frac{m_e}{k^{c.m.}\cdot p^{c.m.}}dE_\pi^{lab}; \qquad s \approx 2m_e E_\nu^{lab}; \\ \sin^2\vartheta &= 1 - \frac{(E_\pi^{c.m.}E_e^{c.m.} - m_e E_\pi^{lab})^2}{(k^{c.m.})^2\cdot(p^{c.m.})^2}, \end{aligned} \tag{44}$$

where

$$E_\pi^{c.m} \approx E_e^{c.m.} \approx p^{c.m} \approx \sqrt{\frac{m_e}{2}E_\nu^{lab}} \quad \text{and} \quad k^{c.m.} \approx \sqrt{\frac{m_e}{2}(E_\nu^{lab} - E_\nu^{(0)})}. \tag{45}$$

As a result, we arrive at the expression

$$\frac{d\sigma}{dE_\pi^{lab}} = \frac{m_e G^2}{8\pi}\left(\frac{E_\nu^{lab} - E_\nu^{(0)}}{E_\nu^{lab}}\right)\left\{1 - \frac{(E_\nu^{lab} - 2E_\pi^{lab})^2}{E_\nu^{lab}(E_\nu^{lab} - E_\nu^{(0)})}\right\}|F_\pi^{E,I=1}(E_\nu^{lab})|^2 \tag{46}$$

from which the energy distribution of the final state pions of the weak $\bar{\nu}_e e^- \to \pi^-\pi^0$ reaction at four different energies

$$\begin{aligned} E_\nu^{lab}(\rho) &= 568\text{ GeV} \\ E_\nu^{lab}(\rho') &= 2057\text{ GeV} \\ E_\nu^{lab}(\rho'') &= 2828\text{ GeV} \\ E_\nu^{lab}(\rho''') &= 4523\text{ GeV} \end{aligned} \tag{47}$$

corresponding always to one of the four ρ-resonances is calculated. The results are graphically presented in Fig.5, from which one can see immediately the restricted energy distributions of pions created in the final state depending on the incident antineutrino energy (47). The experimental verification of the behaviour of $\sigma_{tot}(E_\nu^{lab})$ in Fig.4 and the energy distribution of pions in Fig.5 will mean verification of the CVC hypothesis for all investigated energies above the two-pion threshold.

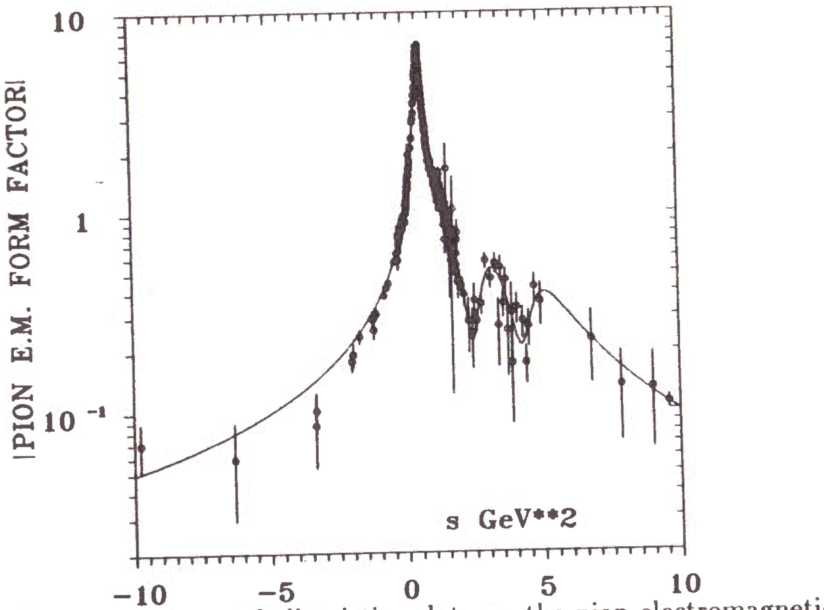

Fig.2. The reproduction of all existing data on the pion electromagnetic form factor.

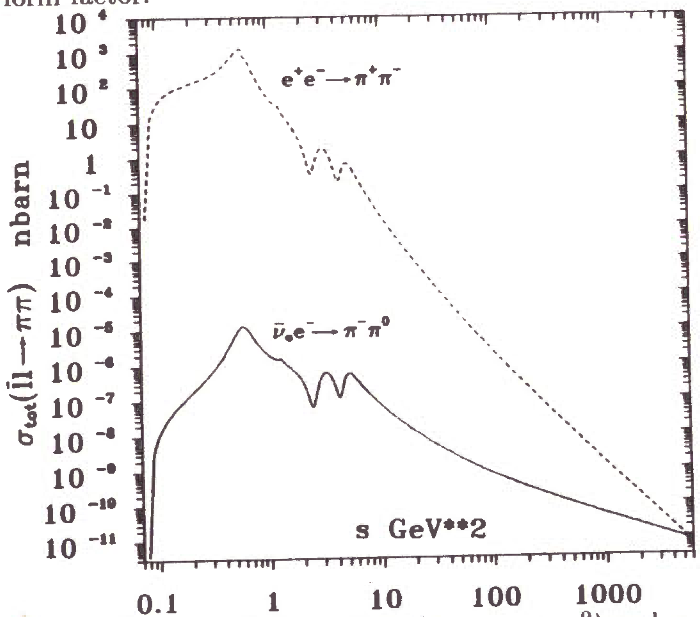

Fig.3. Simultaneous comparison of $\sigma_{tot}(\bar{\nu}_e e^- \rightarrow \pi^-\pi^0)$ and $\sigma_{tot}(e + e^- \rightarrow \pi^-\pi^0)$.

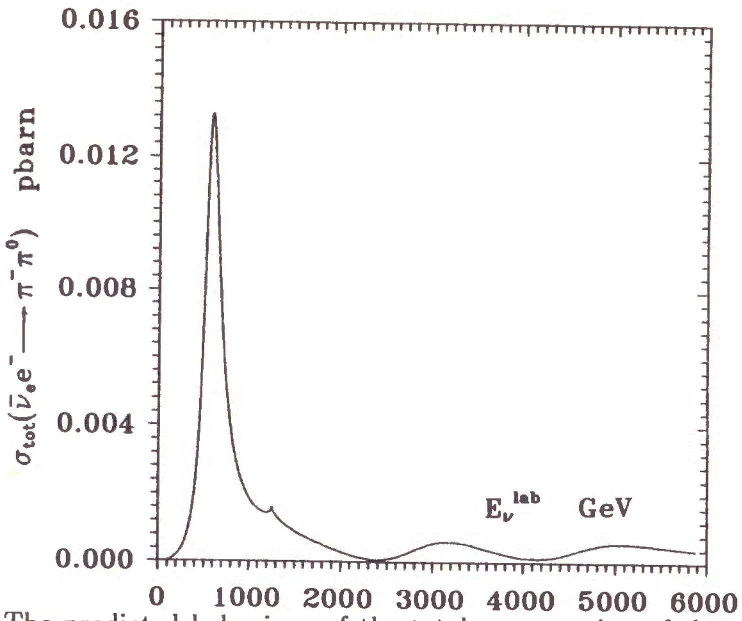

Fig.4. The predicted behaviour of the total cross-section of the $\bar{\nu}_e e^- \rightarrow \pi^-\pi^0$ process at the laboratory system.

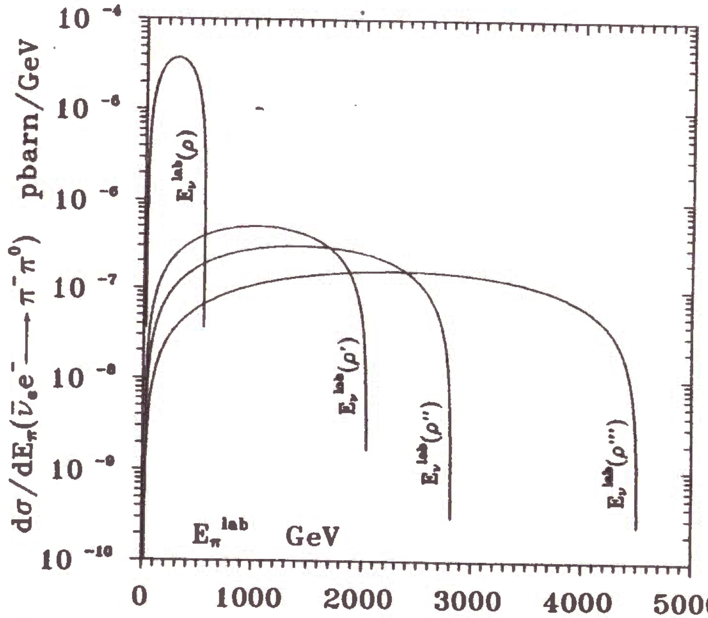

Fig.5. The predicted behaviour of the energy distribution of pions.

6 Conclusion and summary

The CVC hypothesis [1,2], though now naturally following from the standard electro-weak theory, was to our knowledge not adequately verified experimentally. Based on the latter only the pion beta-decay probability was predicted theoretically [1,2] up to now and its agreement with experimental results [3] is presented as a glorious demonstration of the general validity of the CVC hypothesis in the weak interaction theory. Therefore in this paper we have proposed to investigate experimentally a related reaction to the pion beta-decay, the weak $\bar{\nu}_e e^- \rightarrow \pi^-\pi^0$ process, which allows, unlike the pion beta-decay, to verify CVC hypothesis in a broad interval of energies for all s above the two-pion threshold. To realize this program, we first calculate the lowest order of a perturbation expansion contribution to the amplitude of the $\bar{\nu}_e e^- \rightarrow \pi^-\pi^0$ process and found expressions for corresponding differential and total cross-sections expressed through the weak pion ff $F_\pi^W(s)$. Then the relation between $F_\pi^W(s)$ and the e.m. pion ff $F_\pi^{E,I=1}(s)$ was explicitly derived and the construction of the four-ρ-resonance unitary and analytic VMD model of the pion e.m. ff was briefly sketched. The latter finally allowed us to predict the behaviour of $\sigma_{tot}(E_\nu^{lab})$ and $d\sigma/dE_\pi^{lab}$ in the laboratory system. Their experimental verification could just validate the CVC hypothesis for all investigated energies above the two-pion threshold. Moreover, the accurate measurement of $\sigma_{tot}(E_\nu^{lab})$, that moreover is strengthened with E_ν^{lab} linearly, could give a precise information about the mass of the first excited state of the $\rho(770)$ meson, the value of which is now after new experimental results [7] slightly confused.

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