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A NEW DESCRIPTION OF DEEP INELASTIC LEPTON SCATTERING ON BOUND NUCLEONS

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1. The preliminaries

Since the EMC effect was discovered ten years ago, an extensive study of the deep inelastic scattering of leptons on nuclei was carried out. However, a short glance at the experimental programs of the operating and forthcoming accelerators shows that the interest in the lepton-nucleus deep inelastic reactions does not abate.

First of all, we mean experiments at CERN where the renowned EMC and BCDMS collaborations have already obtained a number of interesting results by means of nuclear targets. Among the recent achievements, it is worthwhile to mention the NMC precise data on the F_2^n/F_2^p ratio at small x extracted from the combined proton-deuteron measurements. In view of the so-called "spin crisis", the SMC and NMC results on the μD, $\mu^3 He$ reactions with polarized particles are anticipated impatiently. Next, the research program of the new electron accelerator CEBAF includes the study of the deep inelastic scattering of electrons on nuclei near the boundary of the one-nucleon kinematics ($x \sim 1$). In the present context, the possibility of the charged currents detection on nuclear targets (such as the deuteron or even heavier nuclei) at the HERA set up at DESY, appears to be very important. Finally, new prospects in the investigation of the processes in question may emerge with the creation of the hypothetical machines like UNK or the 10-20 Gev European project.

Since the information about the neutron structure is predominantly obtained by means of the nuclear processes, the importance of studying of the high energy lepton-nucleus reactions is significant not only for the investigation of the "nuclear QCD effects" (short NN-distance phenomena, shadowing etc.) but for the particle physics as well. Both these aspects produce high requirements to the quantitative description of the nuclear structure effects in the deep inelastic scattering. Consequently, there is a need in an accurate method of taking into account such effects.

The discovery of the EMC effect initiated a large variety of theoretical works clarifying the gist both of the detected phenomenon and the role of nuclear effects in deep inelastic scattering in general. Today, after a decade or so this amount of models could be conventionally divided into two large classes being the two faces of the basic idea of the change of nucleon properties in the nuclear medium. These are the well known x-rescaling [1, 2] and Q^2-rescaling models [3].

The confiding parameters determining the EMC-like behavior of the A-dependence of the nuclear structure function (SF) were first identified by Vagradov's group [1] and, a while later, by Birbrair *et al* [2]. This is nothing else but the slight shift ($\sim 5\%$) of the nucleon mass and the Fermi motion, which together lead to the x-rescaling. Here we shall mention that many authors utilized this fruitful idea [4, 5] and sometimes happened the x-rescaling has been "rediscovered". We want to stress that despite the success of this type models in fact they are phenomenological and the problem of their consequent theoretical investigation is still open.

In the present paper, we propose a rigorous theoretical treatment of deep inelastic scattering on nuclei. We consider the simplest nuclear system, the deuteron, within the meson-nucleon model. We analyze the deuteron ground state and the interaction operator in a consistent way [6]. Applying the operator product expansion method we find the explicit form of the deuteron SF moments. The inverse Mellin transformation turns the deuteron structure functions in the convolution form into terms of constituents relevant to determine the NN-potential [7] and the nuclear structure, viz. nucleons and mesons. The nucleon part (with corrections caused by the interaction) could be exactly reduced to the results of the x-rescaling model and the remaining part is the corrections of the meson exchange currents.

The calculations are made in the following approximations:

1. up to the second order in the meson-nucleon coupling constant g, which corresponds to the usual approximations in nuclear physics in deriving the potential and Schrödinger equation, g^2-approximation;
2. in the leading twist approximation, i.e. when corrections $\sim m^2/Q^2$ are negligible, "*twist two*"-approximation.

2. The x-rescaling model

The main idea of the x-rescaling model is based on the well known fact that the properties of quasiparticles-nucleons differ from those of free nucleons. In particular, the bound nucleons have an effective mass depending on the shell energy. This leads to the renormalization of the scaling variable $x \to m/m^* x$. The original formula of the model is [1]

$$F_2^{N/A}(x) = \int_x^{M_A/m} f^{N/A}(y) \cdot F_2^N(x/y)dy, \tag{1}$$

$$f^{N/A}(y) = \int \frac{d\mathbf{k}}{(2\pi)^3}\, d\varepsilon\, S(\mathbf{k},\varepsilon) \cdot \left(1+\frac{k_3}{m}\right) \delta\left(y - \left[1 + \frac{\varepsilon}{m} + \frac{k_3}{m}\right]\right), \qquad A > 2; \tag{2}$$

[1] We omit here the detailed discussions. For details see e.g. [4, 5]

$$f^{N/D}(y) = \int \frac{d\mathbf{k}}{(2\pi)^3}\, |\Psi_D(\mathbf{k})|^2 \cdot \left(1+\frac{k_3}{m}\right) \delta\left(y - \left[1 + \frac{\varepsilon_D}{m} - \frac{\mathbf{k}^2}{2m^2} + \frac{k_3}{m}\right]\right), \qquad A = 2; \tag{3}$$

where M_A and m are the nucleus and nucleon mass; $f^{N/A}(y)$ is the "nucleon distribution function" versus the "longitudinal fraction of the nuclear momentum"; $S(\mathbf{k},\varepsilon)$ and $\Psi_D(\mathbf{k})$ are the nuclear spectral function and the deuteron wave function respectively; $\mathbf{k}$ and ε are the momentum and the energy of the nucleon inside the nucleus. All the nuclear structure effects in (1,2,3) are encoded in the definition of y via δ-function. The main distinction of the model is that the nucleons carry out only a part of the total momentum of the nuclear target, i.e. $\langle y \rangle = (1 + \langle\varepsilon\rangle/m + \langle k_3^2\rangle/m^2) < 1$ (*vide supra* m^*/m !).

In conclusion of the section note once again that the x-rescaling approach presented by the relations like (1), (2) is consistent with the experimental data (see e.g. [4, 5]), but to the point is the phenomenlogical. However it faithfully catches the essence of the effect.

3. The method

We start with the hadronic tensor $W_{\mu\nu}$, that is the imaginary part of the forward Compton amplitude $W^D_{\mu\nu} \propto \mathrm{Im}T^D_{\mu\nu}$. The amplitude $T_{\mu\nu}$ is given by the time-ordered product of two hadron currents:

$$T^D_{\mu\nu}(p_D, q) = i \int d^4x\, e^{iqx} \langle p_D \mid T(J_\mu(x)J_\nu(0)) \mid p_D\rangle, \tag{4}$$

where q is the virtual photon momentum and M_D and p_D are the mass and momentum of the target (*the deuteron*). To calculate the r.h.s (4), one needs a self-consistent model describing both (i) the current operator and (ii) the nuclear ground state.

The most rigorous analysis of the product of two currents at high momentum transfers is accomplished by the Wilson's operator product expansion (OPE) method. In order to define the brackets in (4), we use the tempting theoretical field approach suggested in refs. [8, 6]. The approach operates with the effective meson-nucleon theory and describes the deuteron ground state and exchange effects to advantage. Besides, it gives a fit set of operators required in the OPE, hence the consistency of computations is maintained.

The applying approach is based on the procedure of nonrelativistic reduction of the nucleon fields with the elimination of the irrelevant antinucleon degrees of freedom. Then the Tamm-Dankoff method to derive the physical states is used. For the deuteron one gets:

$$| p_D\rangle = \sqrt{1-Z_D}\varphi_0^D \mid \bar{N}\bar{N}\rangle + \varphi_1^D \mid \bar{N}\bar{N}\sigma\rangle + \varphi_2^D \mid \bar{N}\bar{N}\sigma\sigma\rangle + O(g^3), \qquad (5)$$

where Z_D is a normalization constant; $\mid \bar{N}\bar{N}\ldots\rangle$ denotes the states of two nonrelativistic bare nucleons and different number of relativistic mesons σ; φ_i^D are the corresponding wave functions. An analogous expression could be written for the state of the physical nucleon. As an example, we present below the calculations with the scalar mesons. The generalization to the other mesons (determining the realistic NN-potential and the deuteron wave function [7]) is straightforward.

For deep inelastic scattering the leading operators in the OPE are twist two. For unpolarized scattering within the model with the nucleons and scalar meson fields these are

$$O_N^{\mu_1\ldots\mu_n} = \left(\frac{i}{2}\right)^{n-1} \mathcal{S}\left\{\bar{N}(0)\gamma^{\mu_1}\overleftrightarrow{\partial}^{\mu_2}\ldots\overleftrightarrow{\partial}^{\mu_n}N(0)\right\},$$

$$O_\Phi^{\mu_1\ldots\mu_n} = \left(\frac{i}{2}\right)^{n} \mathcal{S}\left\{\Phi(0)\overleftrightarrow{\partial}^{\mu_1}\ldots\overleftrightarrow{\partial}^{\mu_n}\Phi(0)\right\}, \qquad (6)$$

where $\mathcal{S}$ symmetrizes the subsequent operator and removes all traces in $\mu_1\ldots\mu_n$.

Drawing the Lorentz structure in a convenient manner [9] we present the amplitude (4) via the OPE in the effective meson-nucleon model:

$$T_{\mu\nu}^D(p_D,q) = \sum_{a;n=2,4,\ldots}^{\infty} C_{a,n}^{(1)}\left(-g_{\mu\nu}+\frac{q_\mu q_\nu}{q^2}\right)\frac{2m_a 2^n q_{\mu_1}\cdots q_{\mu_n}}{(-q^2)^n}\langle p_D \mid O_a^{\mu_1\ldots\mu_n}(0)\mid p_D\rangle + \qquad (7)$$

$$+\sum_{a;n=2,4,\ldots}^{\infty} C_{a,n}^{(2)}\left(g_{\mu\mu_1}-\frac{q_\mu q_{\mu_1}}{q^2}\right)\left(g_{\nu\mu_2}-\frac{q_\nu q_{\mu_2}}{q^2}\right)\frac{2m_a 2^n q_{\mu_3}\cdots q_{\mu_n}}{(-q^2)^{n-1}}\langle p_D \mid O_a^{\mu_1\ldots\mu_n}(0)\mid p_D\rangle,$$

where $a = N,\ \Phi$; $C_{a,n}^{(1,2)}$ are the Wilson coefficient functions.

For the large momentum transfers q OPE factorizes the amplitude into pieces depending on short and long distance physics. The concrete scales are controlled by the properties of the chosen model. For instance, in asymtotically free theories those are the perturbative and nonberturbative regions or the quark and hadron scales in QCD. In the effective meson-nucleon model the short and long distances correspond to the hadron and nuclear scales. In (7) these two pieces are $C_{a,n}^{(1,2)}$ and $\langle p_D \mid O_a^{\mu_1\ldots\mu_n}(0)\mid p_D\rangle$ respectively.

In contrast to the QCD the matrix elements of the operators (6) sandwiched between the deuteron states could be computed explicitly. By direct computation of the matrix

elements over the bare states $\mid \bar{N}\rangle$ or $\mid \sigma\rangle$ one can easily show that the coefficients $C_{a,n}^{(1,2)}$ are identical with the moments of bare nucleons ($a = N$) or mesons ($a = \Phi$). Note that $C_{a,n}^{(1,2)}$ are target-independent and the same quantities define the moments of the physical nucleon and deuteron structure functions. Since below the deuteron moments will be expressed in terms of the moments of the *physical* nucleons and the nuclear structure characteristics (such as the potential and kinetic energies, wave function etc.), the dependence upon the unphysical bare moments falls out throughout.

To present the results in a compact form, we define the reduced matrix elements $\bar{a}_{a,n}^D$:

$$\langle p_D \mid O_a^{\mu_1\ldots\mu_n}\mid p_D\rangle = p_D^{\mu_1}\cdots p_D^{\mu_n}\cdot\bar{a}_{a,n}^D, \qquad (8)$$

which are related with the structure function (e.g. F_2^D) moments and coefficients $C_{a,n}^{(i)}$, $i = 1,2$ by:

$$\mathsf{M}_{n-1}(F_2^D) = \sum_a C_{a,n}^{(2)}\cdot\bar{a}_{a,n}^D, \quad \text{with} \quad \mathsf{M}_n(F) = \int_0^1 F(x)x^{n-1}dx. \qquad (9)$$

We are now in a position to evaluate the deuteron structure functions moments given by (9). The main problem here is to compute $\bar{a}_{a,n}^D$ explicitly. We do this by the nonrelativistic reduction of the operators (6) and averaging over the deuteron physical states (5). To separate different contributions to the deuteron moments it is convenient to classify the resulting matrix elements as it is depicted in fig. 1. The diagrams a) denote the moments of the physical nucleons, moving inside the deuteron. As it is seen, they consist of the moments of bare nucleons plus the self-energy corrections. The diagrams b) are the so-called renormalization and recoil terms that cancel each other in the g^2-approximation. Next diagrams c) are the interaction corrections to the nucleons contribution. The remaining diagrams d) are the pure meson exchange current corrections. The corresponding explicit form of the moments of the deuteron structure function is:

$$\frac{1}{2}\left(\frac{M_D}{m}\right)^n\cdot\mathsf{M}_n(F_2^D) = \qquad (10)$$

$$= \mathsf{M}_n(F_2^N)\int\frac{d^3\mathbf{p}}{(2\pi)^3}\mid\Psi_D(\mathbf{p})\mid^2\left(1+\frac{p_z}{m}\right)\cdot\left(1+\frac{p_z}{m}+\frac{\mathbf{p}^2}{2m^2}\right)^n + \qquad (11)$$

$$+ \mathsf{M}_n(F_2^N)\int\frac{d^3\mathbf{p}d^3\mathbf{k}}{(2\pi)^6}\Psi_D^+(\mathbf{p})V(\mathbf{k})\Psi_D(\mathbf{p}+\mathbf{k})\frac{1}{k_z}\left[\left(1+\frac{k_z}{2m}\right)^n-\left(1-\frac{k_z}{2m}\right)^n\right] + \qquad (12)$$

$$+ \mathsf{M}_n(F_2^M)\int\frac{d^3\mathbf{p}d^3\mathbf{k}}{(2\pi)^6}\Psi_D^+(\mathbf{p})V(\mathbf{k})\Psi_D(\mathbf{p}+\mathbf{k})\frac{m}{\omega^2(\mathbf{k})}\cdot\frac{(1+(-1)^{n+1})}{2}\cdot\left(\frac{k_z}{m}\right)^{n+1}, \qquad (13)$$

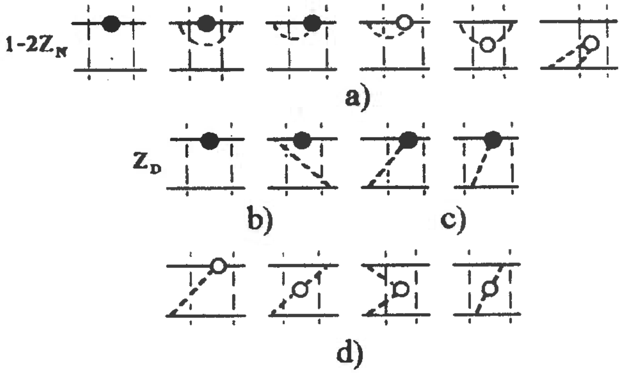

Fig. 1: *The deuteron moments: a) - impulse approximation, b) - renormalization and recoil diagrams c) and d) - meson exchange currents. Closed and open circles denote the moments of bare nucleon and meson respectively; the vertical dash-lines separate the operator and wave function in the corresponding matrix element.*

where $\Psi_D \equiv \varphi_0^D$ is the conventional deuteron wave function; $\omega^2(\mathbf{k}) = (\mathbf{k}^2+\mu^2)$; $V(\mathbf{k})$ is the one-boson-exchange potential generated by the interaction term in the Lagrangian of the model.

3. The nucleon contribution to the nuclear structure functions

The sum of terms (11) and (12) (diagrams a) and c) in fig. 1) is the contribution of the Fermi motion of interacting nucleons. Applying the inverse Mellin transformation to (11,12) we reconstruct the nucleon contribution to the deuteron structure functions in the convolution form (1) with $f^{N/D} = f_{IA}^{N/D} + f_{int}^{N/D}$ given by:

$$f_{IA}^{N/D}(y) = \int \frac{d^3\mathbf{p}}{(2\pi)^3} |\Psi_D(\mathbf{p})|^2 \left(1+\frac{p_z}{m}\right) \delta\left(y - \left[1 + \frac{\mathbf{p}^2}{2m^2} + \frac{p_z}{m}\right]\right) \qquad (14)$$

$$f_{int}^{N/D}(y) = \int \frac{d^3\mathbf{p} d^3\mathbf{k}}{(2\pi)^6} \Psi_D^+(\mathbf{p}) V(\mathbf{k}) \Psi_D(\mathbf{p}+\mathbf{k}) \frac{1}{k_z} \times$$

$$\times \left\{ \delta\left(y - \left|1+\frac{k_z}{2m}\right|\right) - \delta\left(y - \left|1-\frac{k_z}{2m}\right|\right)\right\} \qquad (15)$$

The distribution function $f_{IA}^{N/D}$ describes the Fermi motion of the on-mass-shell nucleons and it is quite similar to the conventional formula of nuclear physics usually referred to as the "impulse approximation". Taking into account only this type of "Fermi smearing" results in wrong nuclear structure functions. In particular, it breaks

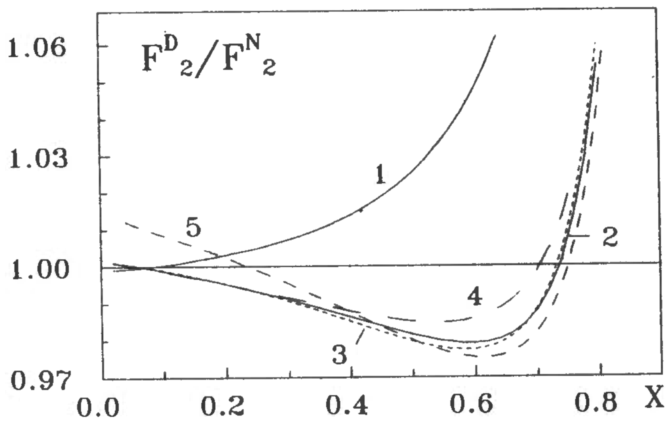

Fig. 2. The ratio of the deuteron and isoscalar nucleon structure functions. Curves: 1 – the Fermi motion of the "on shell mass" nucleons; 2 – the Fermi motion with taking into account the boundness effects (full line); 3 – calculations on the basis of approximate formula (16) *(short dash-line); 4 –impulse approximation* (3) *(long dash-line); 5 – the summary contribution of the impulse approximation and meson exchange currents obtained within present approach.*

the sum rule for the four-momentum ($\langle y \rangle > 1$) and does not give the EMC like A-dependence (see fig. 2, curve 1).

Instead of the modification of the impulse approximation by the reasonable redefinition of variable y like (2), (3) we get the pure interaction term (15) of the exchange origin, $f_{int}^{N/D}$. The sum of $f_{IA}^{N/D}$ and $f_{int}^{N/D}$ gives the final result for the nucleon contribution to the deuteron compared with the result of the earlier x-rescaling calculation [6] utilizing eq. (3) in fig. 2 (curves 2 and 3 resp.). The small difference at $x \sim 0.7$ is caused by the g^4-terms which are mixed in eqs. (3) and (14)-(15). Indeed, expanding these expressions around the "on-mass-shell" $y = 1 + \mathbf{p}^2/2m^2 + p_z/m$ and formally keeping only the g^2-terms we derive the same approximate expressions in both cases:

$$\frac{1}{2}F_2^{N/D} = F_2^{N/D}(IA) + \frac{\langle V \rangle}{m} x \cdot \frac{dF_2^N}{dx} = F_2^{N/D}(IA) + \frac{\varepsilon_D - \langle T \rangle}{m} x \cdot \frac{dF_2^N}{dx}, \qquad (16)$$

where $F_2^{N/D}(IA)$ is the impulse approximation contribution computed by (14). The structure function $F_2^{N/D}$ obtained by means of (16) is given in fig. 2 (curve 3). As is

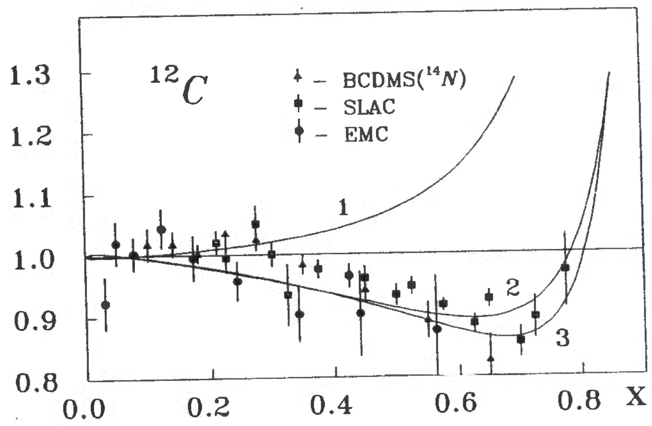

Fig. 3. *The ratio of the carbon and isoscalar nucleon structure functions. Curves: 1 – the Fermi motion of the "on shell mass" nucleons; 2–impulse approximation within CDFM; 3 – the Fermi motion with taking into account the boundness effects within the present approach. Experimental data are taken from* [11].

seen, the present result excellently fits the calculation by exact formulae (14) - (15).

Heavy nuclei. Notice that according to (16) the deuteron structure function is determined with high accuracy by the momentum distribution of nucleons and mean value of the potential $\langle V \rangle$. The Schrödienger equation allows one to express $\langle V \rangle$ through the binding energy and well defined mean value of the kinetic energy of nucleons. Due to this fact eq. (16) is straightforwardly extended to the scattering on heavy nuclei and formidable computations of the nuclear spectral function are avoided. With the two-body origin of $NN-$interaction in mind we have:

$$\langle V \rangle_A = 2(\varepsilon_A - \langle T \rangle_A), \qquad (17)$$

where $\varepsilon_A \approx 8\ MeV$ is the binding energy of a nucleus per nucleon, $\langle T \rangle_A$ is the mean of kinetic energy of a nucleon inside the nucleus. The structure function of ^{12}C calculated on the basis of (16), (17) is shown in fig. 3 (curve 1). The function $F_2^{N/A}(IA)$ and $\langle T \rangle_A$ have been calculated with the realistic momentum distribution obtained in the coherent-fluctuation density model ($CFDM$) [5]. This figure displays also the comparison of present calculations with the result of the $x-$rescaling model where the spectral

function has been taken from $CDFM$ as well (curve 2) [5]. Both curves in fig. 3 are in a reasonable agreement with the data. The slight discrepancy between two curves has the same origin as in the deuteron case.

4. Concluding remarks

We have proposed a quite rigorous theoretical method that describes the nuclear structure effects in the deep inelastic lepton-nucleus scattering and substantiates the x-rescaling idea. The internal self-consistency peculiar to the method assures the energy-momentum conservation, the role of the meson exchange currents being found to be important. The explicit expression for the mesonic correction derived from (13) in g^2-approximation coincides with the one previously presented in ref. [6]. An almost model independent representation of the nuclear SF has been succeeded.

Some possible applications and improvements of the method are:

1.The most topical application of the method is the possibility of accurate extraction of the neutron structure function from the combined proton-deuteron data. The effects of the Fermi motion and meson exchange currents result in significant corrections to the extracted neutron structure function [10]. Note, the partial cancellation of the meson correction and the nuclear shadowing effect at small x is expected.

2. The performed analysis of the structure function moments persuades us into the existence of a tight relation between Q^2- and $x-$rescaling models. Equating the nuclear moments M_n found in two approaches makes it possible to obtain the QCD motivated parameters of the Q^2-rescaling in terms of the nuclear structure.

3. The study of the nuclear structure functions at $x \sim 1$ and beyond is an interesting theoretical problem. Obviously, the nuclear structure effects here are predominant and the application of our approach is rather appropriate. In this region other degrees of freedom become relevant ($\Delta-$isobars, multiquarks, ...) and the OPE and Tamm-Dankoff methods should merely be applied with taking into account such fields.

4. The enlargement of the basis (6) of twist two operators by attaching the axial operators (γ_5-terms) [9] allows us to extend our method to the consideration of polarization processes and the spin-dependent structure functions of nuclei.

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