92-195

сообщения
объединенного
института
ядерных
исследований
дубна

E2-92-195

Kh.M.Beshtoev

ON A COMPLEX VACUUM STRUCTURE IN NON-ABELIAN GAUGE THEORIES AND ON THE PLACE OF MAJORANA PARTICLES IN THESE THEORIES

1992

I. Introduction

In non-Abelian gauge theories there are special solutions that are called instantons, solitons, etc. (they have topological characteristics)/I/.

By analogy with quantum mechanics, one relates these quasiparticles to a complex structure of the vacuum (or, to be more specific, to transitions between different vacuum states)/2/.

At present there is no alternative point of view.

We think, however, that there can be an alternative approach to interpretation of topological solutions appearing in non-Abelian gauge theories. Our opinion is based on the fact that in non-Abelian gauge theories the existence of vector fields is postulated. The particles corresponding to these fields are point-like and structureless. In non-Abelian gauge theories these vector particles interact non-linearly. These non-linear interactions between particles result in non-local objects (quasiparticles) with topological characteristics. These quasiparticles are not at all identical to initial point-like structureless vector particles. Roughly, we obtain a non-local quasiparticle,which is a non-linear superposition of initial point like vector particles. Now we see that quasiparticles are derivatives of initial vector fields and thus they cannot change the vacuum structure of initial vector (fields) particles.

A more adeguate interpretation of these non-local quasiparticles is formation of non-local superpositional objects in plasma (of zero temperature) of vector point-like particles (this point of view corresponds to the traditional point of view on this problem in physics). Then some questions arise, two of them being basic ones:

I. If the whole plasma of vector particles changes into these non-local particles, the complete restructuring of the initial vacuum state and transition to a new vacuum state is possible.

2. It the plasma of vector particles is populated with non-local quasiparticles, interaction between initial vector fields and these non-local quasiparticles is possible, and this effect must be taken into account in the final Lagrangian.

Now let's return to the problem of the complex vacuum structure in non-Abelian gauge theories and the place of Majorana particles in these theories.

2. On the vacuum structure in non-Abelian gauge theories and Majorana particles

Let's consider the grounds for the assumption of the complex vacuum structure in non-Abelian gauge theories. First, we consider Bloch functions in condensed matter physics (it is this example that is usually given as an analogue of the complex vacuum structure), and then we proceed to non-Abelian gauge theories.

a) The Bloch function for solid state physics [3] has the form

$$\varphi_K(x) = e^{iKx} u_K(x), \tag{1}$$

where $u_K(x)$ is the periodical function of the straight lattice period. The property of this function is its transitional invariance

$$T_{mnp}\, x = x + t_{mnp} = x + ma + nb + pc$$

$$T_{mnp}\, \varphi_K(x) = \varphi_K(x + ma + nb + pc) = C_{mnp}\varphi_K(x), \tag{2}$$

where m, n, p are integer numbers. If periodical boundary conditions are satisfied,

$$\varphi_K(x + Na) = \varphi_K(x)$$
$$C_\xi = \exp(2\pi i \xi / N), \quad \xi = 1, 2, \ldots, N. \tag{3}$$

These properties of the Bloch function are equivalent to gauge transformations.

Note one significant point: the electron function $\varphi_K(x)$ is a complex one, and confinement of the electron to periodical fields does not put the electron function beyond the given class of functions and only changes its periodicity properties.

b) Now let's proceed to non-Abelian SU(N) gauge theories.

In non-Abelian SU(N) gauge theories gauge fields A^α_μ $\alpha = 1 \div N^2 - 1$ are real (or reducible to real fields), which is clear from the definition $M = 2N^2 - N^2 - 1 = N^2 - 1$. The vector fields A^α_μ being real, vectors of the state $\Psi[A_\mu]$, built from these fields, cannot have any phase transformations. So it is necessary to make these fields complex ones for the state vectors to have phase transformations. For this purpose one should either build a gauge theory based on the SL(N) group or make the SU(N) group a complex one.

Let's consider the so-called large gauge transformations [4] in SU(N)

$$\vec{A}^{\upsilon}_n = U_n \vec{A} U_n^{-1} + \frac{i}{g} U_n \vec{\nabla} U_n^{-1}, \tag{4}$$

where $U_n(x) = [-\exp[-i\upsilon]]^n, \; n = 0, \pm 1, \pm 2, \ldots$

where $U_n(x) \to 1$; $\vec{A} = \vec{A}^a T^a$, $\upsilon(x) = \upsilon^a(x) T^a$.
$|x| \to \infty$

The explicit form of $\upsilon(x)$ for the SU(2) group is

$$\upsilon(x) = \frac{\pi x_a T^a}{(x^2 + \rho^2)^{1/2}}; \quad x^2 = |x|^2, \; x \to x - x_0. \tag{5}$$

Now we give the standard proof of the complex vacuum structure in non-Abelian gauge theories.

In view of the fact that $\Psi[\vec{A}_n]$ is not obligatory invariant under gauge transformations from the class $n \neq 0$ and the Hamiltonian is locally gauge-invariant, all eigenfunctions corresponding to the given eigenvalue of the energy can be determined with an accuracy to the constant phase shift (the phase shift must be the same for all eigenfunctions)

$$\Psi[\vec{A}_n] = e^{in\theta}\Psi[\vec{A}]. \tag{6}$$

Now the Hilbert space of the theory is divided into sectors numbered by a continuous parameter θ . Each sector contains states constructed over the corresponding θ vacuum. In this case the wave function of the vacuum state $\Psi_\theta[\vec{A}_\mu]$ can be intuitively represented as

$$\Psi_\theta[\vec{A}] = \sum_{n=-\infty}^{\infty} e^{in\theta}\Psi_n[\vec{A}] \tag{7}$$

There exists tunnelling between different $\Psi_n[\vec{A}]$ provided by instantons. We have given a standard way of construction of the complex vacuum structure in non-Abelian gauge theories.

Remember that fields A_μ in SU(N) theories are real (or reducible to real fields), so phase transformations (6), (7) do not exist for them. The picture appearing in these theories is described in the Introduction. A detailed description of the above-mentioned picture will be given in our future papers. Here we note that this problem is touched upon in ref. [5].

Phase transformations (6),(7) can arise in non-Abelian theories based on comlexified SU(N) groups and SL(N) groups.

c) Now let's consider a neutral fermion (the so-called Majorana particle).

The kinetic term of the Lagrangian of a Majorana particle has the form [6]:

$$\mathcal{L} = -\frac{1}{2}\bar{\varphi}\gamma_\mu \frac{\partial}{\partial x_\mu}\varphi$$

$$\varphi = \varphi_L + (\varphi_L)^c, \quad \varphi = \varphi^c \equiv C\bar{\varphi}^T \tag{8}$$

C-charge conjugation operator;
T -transpose.
From (8) we have

$$\varphi(x) = \int \frac{1}{(2\pi)^{3/2}}\frac{1}{\sqrt{2p_0}}\left(u^\tau(p)a_\tau(p)e^{ipx} + u^\tau(-p)a^+_\tau(p)e^{-px}\right)d^3p, \tag{9}$$

$u^\tau(p)$ spinor with momentum p and helicity τ,

$$u^\tau(-p) = C(\bar{u}^\tau(-p))^\tau.$$

a_τ and $a^+_\tau(p)$ are operators of destruction and production of particles with momentum p and helicity τ.

It follows from (9) the $\varphi(x)$ is a field of a really neutral particle with spin I/2.

It means that the gauge transformation

$$\Psi'(x) = e^{i\alpha}\Psi(x), \quad \bar{\Psi}'(x) = \bar{\Psi}(x)e^{-i\alpha}$$

which existed for Dirac spinors Ψ, $\bar{\Psi}$ does not exist for fields $\varphi(x)$, i.e. a field $\varphi(x)$ has no gauge charge transformation because it is real, unlike a field $\Psi(x)$.

Now, since the field $\varphi(x)$ is a really neutral field, we must comprehend the possibility of this field appearing in the theory of (electro) weak (N = 2) interaction.

The weak interaction current has the form

$$j^\mu_i = \bar{\Psi}\gamma^\mu\tau_i\Psi, \quad i = 1 \div 3 \tag{IO}$$

where Ψ is the Dirac spinor field. This field has a charge, and it justifies existence of current (IO) in the theory of weak (electro-weak) interaction.

If now we formally re-write expression (IO) as

$$j^\mu = \bar{\varphi}(x)\gamma^\mu\varphi(x), \tag{II}$$

where $\varphi(x)$ is the spinor field of a really neutral Majorana particle, it will be completely meaningless, because the $\varphi(x)$ field of a really neutral particle and expression (II) cannot be connected with any charge and thus with any interaction like $\mathcal{L}_{Int} = g j^\mu A_\mu$ (see ref [7] about relations between particle masses and charges).

Besides, we know from experiments on decays of leptons and quasi-elastic interactions of neutrinos that the corresponding lepton numbers (in the theory of (electro)weak interaction fermions appear as duplets $\binom{e}{\nu}$) are well conserved.

So, if we want to use the purely neutral Majorana spinor field $\varphi(x)$ instead of the Dirac charged spinor field $\Psi(x)$, in this theory we cannot have an interaction, i.e. to construct an analogue of the weak interaction theory.

Now let's proceed the problems related to masses of Majorana particles.

Since the $\varphi(x)$ is a field of a pure neutral particle, we cannot get the mass of this particle through interactions with gauge fields, i.e. by a standard method.

The field $\varphi(x)$ being really neutral, we cannot get the mass by means of the Yukawa mechanism.

$$\mathcal{L} = -\frac{1}{2}g\bar{\varphi}h\varphi = -\frac{1}{2}g\bar{\varphi}(h_0 + h'(x))\varphi(x) \equiv 0, \tag{I2}$$

where h_0 is constant.

Besides, the Yukawa mechanism demands that the field $\varphi(x)$ should be a complex one.

Thus, we arrive at a conclusion that the Majorana spinor fields $\varphi(x)$ must be massless

$$m_\varphi \equiv 0. \tag{I3}$$

Since the total probability of neutrinoless double beta decay includes masses of Majorana particles

$$\Gamma_{0\nu} = \frac{1}{2}\cdot\frac{G^2 m_e^2}{(2\pi)^3}\,|\langle m\rangle|^2 F(Z,..), \quad \langle m\rangle = \sum_{k=e,\mu,\tau} U_k^2 m_k,$$

then

$$\Gamma_{0\nu} \equiv 0. \qquad (14)$$

Thus, it is difficult to include a Majorana particle in the theory within the standard approach.

3.Conclusion

In the Introduction we gave a possible interpretation of topological non-local solutions of classical SU(N) gauge theories, namely treating these solutions as non-local (classical) objects arising in plasma from vector particles (at zero temperature). Owing to absence of poles these solutions can be analytically continued from the Euclidean space to the Minkowski space. In the authors opinion, the necessity of interpreting the solution in this way arises from the fact that reality of vector field in SU(N) non- Abelian gauge theories prevents them from having gauge transformations which could be associated with new conserved numbers or with a new transformation parameter of the θ type. A possibility of this appears in extended theories (e.g. in complexified SU(N) theories) . Besides, the solutions obtained in these SU(N) theories correspond to a new type of particles (fields), and the direct relation between the initial vector fields and this solution is not observed.

In subsection b) of section 2 in this paper it was illustrated with the Bloch function for solid state physics that because of reality of vector fields in non-Abelian SU(N) theories the complex vacuum structure does not appear in these theories.

In section 2 (subsection c)) it was show that because of rigid requirements to obtaining Majorana particles (fields) $\varphi(x)$ from Dirac particles (fields) $\Psi(x)$, $\bar{\Psi}(x)$ the fields $\varphi(x)$ become real and lose charge gauge invariance, turning into really neutral spinor particles. As a result, these particles cannot be included in SU(N) gauge theories. Besides, these Majorana particles cannot get a mass in a standard way, i.e. they remain massless.

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Received by Publishing Department
on May 5, 1992.