

# объединенный ииститут ядерных исследований <br> дубна 

E2-92-176

G. Ganbold, G.V.Efimov

PHASE TRANSITION IN $9 \phi_{2}^{4}$ THEORY

Submitted to "International Journal of Modern Physics $A^{\prime \prime}$

## 1 Introduction

Many problems of modern particle physics rely on spontaneous symmetry breaking as, for instance, the electroweak model with Higgs bosons (see, for example [1]), or the color confinement in QCD which can be explained by vacuum instability [2]. There are many papers [3-12] devoted to investigation of the problem of the vacuum phase structure. using a scalar field model $g \varphi^{4}$ as an example. The theory is simple enough and it is widely used for testing new ideas and methods in quantum field theory.

On the classical level, the theory is stable and has a unique symmetrical trivial ground state. On the other hand, it has been found [3,4] that high order quantum corrections can give rise to the vacuum instability. A useful instrument for investigation of vacuum instability due to quantum effects is the method of the effective potential [5]. A symmetry broken phase of a system is associated with the absolute minimum of the effective potential $V\left(\varphi_{o}\right)$ for which $\varphi_{o}=0$. As the effective potential is described by non-Gaussian functional integrals, one needs to use some approximation schemes. These may be perturbative loop-expansion methods, variational approaches or numerical calculations on lattice.

In two-dimension, the effective potential has been calculated [6] as a partial sum of "cactus-type" diagrams. This approximation method gives the first order phase transition. Nonperturbative Gaussian approaches [7] also lead to similar results. On the other hand, there exist mathematical theorems $[8,9]$ proving that the second order phase transition takes place in this model. There are papers [10-13] where variational methods have been used for investigation of the vacuum stability problem and the correct behavior of the vacuum energy in
the critical region was obtained. The variational methods were applied to the Hamiltonian of the system under consideration but not to the functional integral defining the effective potential.

In this paper we obtain a variational estimation of the effective potential using the methods introduced in [14,15]. We show that there exists the second order phase transition in the $\varphi_{2}^{4}$ model and give the estimation for the critical coupling constant. Our result is in agreement with the Simon-Griffiths theorem.

## 2 Leading Term of Effective Potential

We will consider the scalar field theory $\varphi_{2}^{4}$.The theory is supernormalizable in two-dimension. All ultraviolet divergences in this model can be removed readily by using the quantum Lagrangian density

$$
\begin{equation*}
\mathcal{L}=\frac{1}{2} \varphi(\mathbf{x}) \cdot\left[\partial^{2}-m^{2}\right] \cdot \varphi(\mathbf{x})-\frac{g}{4} \cdot N_{m}\left\{\varphi^{4}(\mathbf{x})\right\} \tag{2.1}
\end{equation*}
$$

where $N_{m}$ denotes the normal product of the fields $\varphi(\mathbf{x})$ with the mass $m$ :

$$
\begin{gathered}
N_{m}\left\{\varphi^{4}(\mathbf{x})\right\}=\varphi^{4}(\mathbf{x})-6 \cdot \varphi^{2}(\mathbf{x}) \cdot D_{m}(0)+3 \cdot D_{m}^{2}(0) \\
D_{m}(\mathbf{x})=\int \frac{d \mathbf{k}}{(2 \pi)^{2}} \frac{\exp \{i \mathbf{k x}\}}{m^{2}+\mathbf{k}^{2}}
\end{gathered}
$$

and $g$ is the self-coupling constant. Here $\mathbf{x} \subset \Omega, \Omega$ is a finite volume in $\mathbf{R}^{\mathbf{2}}$.

We will investigate the effective potential defined as

$$
\begin{align*}
V\left(\varphi_{o}\right) & =-\lim _{\Omega \rightarrow \infty} \frac{1}{\Omega} \ln I_{\Omega}\left(\varphi_{o}\right) \\
I_{\Omega}\left(\varphi_{o}\right)=C_{m} \int \delta \varphi \cdot \delta\left\{\varphi_{o}\right. & \left.-\frac{1}{\Omega} \int_{\Omega} d^{2} \mathbf{x} \varphi(\mathrm{x})\right\} \exp \int_{\Omega} d^{2} \mathbf{x} \cdot \mathcal{L}[\varphi(\mathrm{x})]  \tag{2.3}\\
C_{m} & =\sqrt{\operatorname{det}\left\{-\partial^{2}+m^{2}\right\}}
\end{align*}
$$

It has the meaning of the vacuum energy density $[16,17]$ in the vacuum state of which the expectation value of the field is $\varphi_{o}$. The functional
integral in (2.3) is normalized in the following way

$$
I_{\Omega}\left(\varphi_{o}, g=0\right)=1
$$

All integrations are performed in the Euclidean space.
Let us do some transformations of the functional integral $I_{\Omega}\left(\varphi_{o}\right)$ in (2.3). First, we introduce [15] a transformation of the field variable:

$$
\begin{equation*}
\varphi(\mathbf{x})=\phi_{o}+\phi(\mathbf{x}) \tag{2.4}
\end{equation*}
$$

where $\phi_{o}$ is a constant field and $\phi(\mathbf{x})$ satisfies the condition

$$
\begin{equation*}
\cdot \int_{\Omega} d^{2} \mathbf{x} \phi(\mathbf{x})=0 \tag{2.5}
\end{equation*}
$$

We can substitute (2.4) into (2.3) and perform integration over $d \phi_{o}$ taking into account the functional differential $\delta \varphi=d \phi_{o} \delta \phi$. Then, we obtain

$$
\begin{equation*}
I_{\Omega}\left(\varphi_{o}\right)=C_{m} \int \delta \phi \cdot \exp \left\{\int_{\Omega} d^{2} \mathbf{x} \cdot \mathcal{L}\left[\varphi_{o}+\phi(\mathbf{x})\right]\right\} \tag{2.6}
\end{equation*}
$$

Second, we go over to the normal ordering in the new fields $\phi(\mathbf{x})$ with a new mass $\mu$ using the well-known [3] formula as

$$
\begin{gather*}
N_{m}\{\exp \{\beta \varphi(\mathbf{x})\}\}=N_{\mu}\left\{\exp \left\{\beta\left(\varphi_{o}+\phi(\mathbf{x})\right)+\frac{\beta^{2}}{2} \Delta(m, \mu)\right\}\right\} \\
\Delta(m, \mu)=D_{m}(0)-D_{\mu}(0)  \tag{2.7}\\
D_{\mu}(\mathbf{x})=\int \frac{d \mathbf{k}}{(2 \pi)^{2}} \frac{\exp \{i \mathbf{k} \mathbf{x}\}}{\mu^{2}+\mathbf{k}^{2}}-\frac{1}{\mu^{2} \Omega}
\end{gather*}
$$

Then, substituting (2.7) into (2.6) we obtain

$$
\begin{gather*}
I_{\Omega}\left(\varphi_{0}\right)=e^{-\Omega V_{o}\left(\varphi_{0}\right)} \int d \sigma_{\mu} \cdot \exp \left\{\int_{\Omega} d^{2} \mathbf{x}\right. \\
\left.N_{\mu}\left\{\frac{1}{2} \cdot\left[\mu^{2}-m^{2}+3 g\left(\Delta-\varphi_{o}^{2}\right)\right] \cdot \phi^{2}(\mathbf{x})-\frac{g}{4}\left[\phi^{4}(\mathbf{x})+4 \varphi_{o} \phi^{3}(\mathbf{x})\right]\right\}\right\}, \tag{2.8}
\end{gather*}
$$

$$
\begin{aligned}
& \text { FREven } 2
\end{aligned}
$$

$$
\int d \sigma_{\mu}=C_{\mu} \int \delta \phi \cdot \exp \left\{-\frac{1}{2} \int_{\Omega} d^{2} \mathbf{x} \phi(\mathbf{x})\left(-\partial^{2}+\mu^{2}\right) \phi(\mathbf{x})\right\}=1
$$

where the leading term $V_{o}\left(\varphi_{o}\right)$ of the effective potential is:

$$
\begin{align*}
V_{o}\left(\varphi_{o}\right)= & -\frac{1}{2} \int \frac{d \mathbf{k}}{(2 \pi)^{2}}\left[\ln \left(1+\frac{m^{2}-\mu^{2}}{\mu^{2}+\mathbf{k}^{2}}\right)-\frac{m^{2}-\mu^{2}}{\mu^{2}+\mathbf{k}^{2}}\right] \\
& +\frac{m^{2}}{2} \varphi_{o}^{2}+\frac{g}{4}\left(\varphi_{o}^{4}-6 \Delta \varphi_{o}^{2}+3 \Delta^{2}\right) \tag{2.9}
\end{align*}
$$

There are no linear field configurations $\sim \phi$ in (2.8) due to the condition (2.5).

According to our method [15], we require that all the quadratic field configurations be concentrated in the Gaussian measure $d \sigma_{\mu}$. The requirement leads to the following constraint equation for the parameter $\mu$ :

$$
\begin{equation*}
\mu^{2}-m^{2}+3 g\left(\Delta-\varphi_{o}^{2}\right)=0 \tag{2:10}
\end{equation*}
$$

Thus, we finally obtain the formula for the effective potential

$$
\begin{align*}
V\left(\varphi_{o}\right) & =V_{o}\left(\varphi_{o}\right)+V_{s c}\left(\varphi_{o}\right) \\
V_{s c}\left(\varphi_{o}\right) & =-\lim _{\Omega \rightarrow \infty} \frac{1}{\Omega} \ln J_{\Omega}\left(\varphi_{o}\right) \tag{2.11}
\end{align*}
$$

where

$$
\begin{equation*}
J_{\Omega}\left(\varphi_{o}\right)=\int d \sigma_{\mu} \cdot \exp \left\{-\frac{g}{4} \int_{\Omega} d^{2} \mathbf{x} N_{\mu}\left[\phi^{4}(\mathbf{x})+4 \phi^{3}(\mathbf{x}) \varphi_{o}\right]\right\} \tag{2.12}
\end{equation*}
$$

Eqs (2.9)-(2.12) define completely the effective potential at arbitrary coupling $g$.

We note that our leading term $V_{o}\left(\varphi_{o}\right)$ and constraint equation (2.10) are identical with the Hartree-type potential and corresponding condition of its minimum on the parameter $\mu$, obtained in [10]. This coincidence may be explained by our particular choices of linear field transformation (2.4) and Gaussian type of measure $d \sigma_{\mu}$ in (2.8). It is well known [6,7] that the potential (2.9) within the limitation for $\mu$ by (2.10) corresponds to the sum of 'cactus-type' diagrams and indicates in favor of a first-order phase transition at :

$$
\begin{equation*}
\left(\frac{g_{i}}{m^{2}}\right)_{c a c t u s}=10.211 \quad \text { or } \quad G_{c a c t u s}=\left(\frac{g}{2 \pi m^{2}}\right)_{c a c t u s}=1.6251 \tag{2:13}
\end{equation*}
$$

The 'cactus-type' potential has the following asymptotic behavior

$$
\begin{equation*}
V\left(\varphi_{o}\right)=\frac{m^{2}}{2}\left\{\varphi_{o}^{2}+O\left(\varphi_{o}^{4}\right)\right\} \tag{2.14}
\end{equation*}
$$

as $\varphi_{o} \rightarrow 0$ at any $g$.

## 3 Variational Upper Bound of Effective Potential

In the previous section we have defined an expression for the effective potential consisting of two parts. Considering of only the 'cactus-type' part $V_{o}\left(\varphi_{o}\right)$ leads to a conclusion in favor of a first order phase transition $[6,7]$ in the scalar $\varphi_{2}^{4}$ theory. This is in contradiction with statements of mathematical theorems $[8,9]$. To answer the question about the nature of phase transition in this theory, one should consider also the other part $V_{s c}\left(\varphi_{o}\right)$ of the effective potential, clefined in (2.12). At weak coupling limit one can estimate it expanding the exponential in (2.12) in perturbative series. But explicit calculation of the non-Gaussian functional integral $J_{\Omega}\left(\varphi_{0}\right)$ in (2.12) at arbitrary values of coupling constant $g$ and $\varphi_{0}$ is a very complicated problem. However, we are able to estimate it for small values of $\varphi_{o}$ at arbitrary $g$.

Let us rewrite (2.12) in the form which is correct for small $\varphi_{0} \sim 0$ :

$$
\begin{equation*}
J_{\Omega}\left(\varphi_{o}\right)=\int d \sigma_{\mu} \cdot \exp \left\{-\frac{g}{4} \int_{\Omega} d^{2} \mathbf{x} N_{\mu} \cdot \phi^{4}(\mathbf{x})+\frac{g^{2} \varphi_{o}^{2}}{2}\left[\int_{\Omega} d^{2} \mathbf{x} N_{\mu} \cdot \phi^{3}(\mathbf{x})\right]^{2}\right\} \tag{3.1}
\end{equation*}
$$ lowing transformation in (2.12):

$$
\exp \left(-\varphi_{o} W\right)=\cosh \left(\varphi_{o} W\right) \simeq \exp \left\{\frac{1}{2} \varphi_{o}^{2} W^{2}+O\left(\varphi_{o}^{4}\right)\right\}
$$

for infinitesimal $\varphi_{o}$ and finite functional $W$. Then, applying to the integral (3.1) the variational techniques $[14,18]$ one can get

$$
\begin{gathered}
V_{s c}\left(\varphi_{o}\right)=-\lim _{\Omega \rightarrow \infty} \frac{1}{\Omega} \ln J_{\Omega}\left(\varphi_{o}\right) \leq V_{s c}^{+}\left(\varphi_{o}\right), \\
V_{s c}^{+}\left(\varphi_{o}\right)=\min _{q, A}\left\{\frac{1}{2} \int \frac{d \mathbf{k}}{(2 \pi)^{2}}\left[\ln \left(1+q\left(\mathbf{k}^{2}\right)\right)-\frac{q\left(\mathbf{k}^{2}\right)}{1+q\left(\mathbf{k}^{2}\right)}\right]\right.
\end{gathered}
$$

$$
\begin{gather*}
+\frac{\mu^{2} A^{2}}{2}+\frac{g}{4}\left[A^{4}-6 A^{2} \Delta_{q}+3 \Delta_{q}^{2}\right]  \tag{3.2}\\
\left.\frac{-3 g^{2} \varphi_{\rho}^{2}}{\Omega} \int_{\Omega} d^{2} \mathrm{x} \int_{\Omega} d^{2} \mathrm{y}\left[D_{q}^{3}(\mathrm{x}-\mathrm{y})+3 A^{2} D_{q}^{2}(\mathrm{x}-\mathrm{y})\right]\right\}
\end{gather*}
$$

where

$$
\begin{align*}
\Delta_{q}= & \int \frac{d \mathbf{k}}{(2 \pi)^{2}} \frac{q\left(\mathbf{k}^{2}\right)}{1+q\left(\mathbf{k}^{2}\right)} \tilde{D}\left(k^{2}\right) \\
D_{q}(\mathbf{x}-\mathbf{y})= & \int \frac{d \mathbf{k}}{(2 \pi)^{2}} \frac{\exp \{i \mathbf{k}(\mathbf{x}-\mathbf{y})\}}{1+q\left(\mathbf{k}^{2}\right)} \tilde{D}\left(k^{2}\right) \\
& -\frac{1}{\mu^{2}(1+q(0)) \Omega},  \tag{3.3}\\
& \tilde{D}\left(k^{2}\right)=\frac{1}{\mu^{2}+k^{2}}
\end{align*}
$$

Here $\mu$ is defined by equation (2.10). The constant $A$ and the function $q\left(\mathrm{k}^{2}\right)$ are variational parameters ( see Appendix ). The optimal form of the function $q\left(\mathbf{k}^{2}\right)$ is

$$
\begin{equation*}
q\left(\mathbf{k}^{2}\right)=f \mu^{2} \tilde{D}\left(k^{2}\right) \tag{3.4}
\end{equation*}
$$

as it follows from the variational equation. Here $f$ is a variational parameter.

It will be convenient to work in units of $m$ dealing with numerical results. We define

$$
\begin{equation*}
\xi=(\mu / m)^{2}, \quad \Phi_{o}^{2}=4 \pi \varphi_{o}^{2} \quad \text { and } \quad B^{2}=4 \pi A^{2} \tag{3.5}
\end{equation*}
$$

For $\Phi_{o}^{2} \ll 1$ eq. (2.10) has the solution :

$$
\begin{equation*}
\xi=1+\frac{3 G}{2+3 G} \Phi_{o}{ }^{2}+O\left(\Phi_{o}{ }^{4}\right) \tag{3.6}
\end{equation*}
$$

All integrals in (3.2) and (3.3) for the function (3.4) are calculated explicitly. An upper bound of the "strong-connected" potential can be written for $\Phi_{o}^{2} \ll 1$ in the notations (2.11) as follows:

$$
\begin{equation*}
V_{s c}^{+}\left(\Phi_{o}\right)=\frac{m^{2}}{8 \pi}\left\{E_{s c}^{+}(G)+\alpha_{s c}^{+}(G) \Phi_{o}^{2}+O\left(\Phi_{o}^{4}\right)\right\} \tag{3.7}
\end{equation*}
$$

where

$$
\begin{equation*}
E_{s c}^{+}(G)=\min _{q, A}\left\{f-\ln (1+f)+B^{2}+\frac{G}{4}\left[B^{4}-6 B^{2} \ln (1+f)+3 \ln ^{2}(1+f)\right]\right\} \tag{3.8}
\end{equation*}
$$

and

$$
\begin{gather*}
\alpha_{s c}^{+}(G)=\frac{3 G}{2+3 G}\left[f-\ln (1+f)+B^{2}\right]-\frac{3 G^{2}}{2(1+f)}\left[Q+3 B^{2}\right]  \tag{3.9}\\
Q=\iiint_{0}^{1} d \alpha d \beta d \gamma \frac{\delta(1-\alpha-\beta-\gamma)}{\alpha \beta+\alpha \gamma+\beta \gamma}=2.3439 .
\end{gather*}
$$

The functions $f(G)$ and $B(G)$ define the minimum $E_{s c}^{+}(G)$ in (3.8). They satisfy the following equations

$$
\begin{gather*}
2 f-3 G\left[B^{2}-\ln (1+f)\right]=0 \\
B\left\{2+G\left[B^{2}-3 \ln (1+f)\right]\right\}=0 \tag{3.10}
\end{gather*}
$$

Equations (3.10) have nontrivial real solutions at

$$
\begin{equation*}
G>G_{o}=\min _{\rho} \frac{f+3}{3 \ln (1+f)}=1.4397 \tag{3.11}
\end{equation*}
$$

When $G<G_{o}$ the solutions are trivial: $f(G)=B(G)=0$.
Substituting (2.14), (3.6) and (3.7) into (2.11) one gets an upper bound of the effective potential :

$$
\begin{align*}
& V\left(\Phi_{o}\right) \leq V^{+}\left(\Phi_{o}\right)= \frac{m^{2}}{8 \pi}\left\{E^{+}(G)+\alpha^{+}(G) \cdot \Phi_{o}{ }^{2}+O\left(\Phi_{o}{ }^{4}\right)\right\} \\
& E^{+}(G)=E_{s c}^{+}(G)  \tag{3.12}\\
& \alpha^{+}(G)=1+\alpha_{s c}^{+}(G)
\end{align*}
$$

for small $\Phi_{o}$. The coefficient $\alpha^{+}(G)$ plays an important role [13,14]: it depends on $\alpha^{+}(G)$ whether we obtain a minimum at the origin $\Phi_{o}=0$ or a maximum. At weak coupling limit $\alpha^{+}(G)=1$. For increasing $G$ this coefficient becomes smaller and vanishes at $G=G_{c}^{+}$. It indicates that there is a second order phase transition provided that $V^{+}\left(\Phi_{o}\right)$ is positive at finite $\Phi_{o}$.

The numerical value of the critical coupling constant $G_{c}^{+}$can be found from our formulae. Let us consider the region $G<G_{o}$. The only solutions of (3.10) are

$$
\begin{equation*}
f(G)=0, \quad B(G)=0 \tag{3.13}
\end{equation*}
$$

and the coefficient

$$
\begin{equation*}
\alpha^{+}(G)=1-\frac{3}{2} Q G^{2} \tag{3.14}
\end{equation*}
$$

becomes negative for

$$
\begin{equation*}
G>G_{c}^{+}=\left[\frac{2}{3 Q}\right]^{1 / 2}=0.5333 \tag{3.15}
\end{equation*}
$$

or

$$
\left(g / m^{2}\right)>\left(g / m^{2}\right)_{c}^{+}=3.3508
$$

Note that the critical value (3.15) of the coupling constant is calculated only for the upper estimation $V^{+}\left(\Phi_{o}\right)$ but not for the true effective potential $V\left(\Phi_{o}\right)$. Nevertheless, we believe that the true critical coupling constant $G_{c}$ lies not far from $G_{c}^{+}$. Then, we pay one's attention to the hierarchy (see (2.14) and (3.11) ):

$$
\begin{equation*}
G_{c}^{+}<G_{o}<G_{\text {cactus }} \tag{3.16}
\end{equation*}
$$

This means that a second order phase transition comes earlier than a first order one.

## 4 Conclusion

In this paper we have investigated the problem of phase transition in two-dimensional quantum field theory $\varphi_{2}^{4}$.The functional integral describing the effective potential is estimated by a variational approximation. We have obtained the expression for an upper bound of the effective potential at small values of its argument. We have shown that it describes only a second order phase transition in contrast with the "cactus-type" approximation of the effective potential giving a first order phase transition at a larger coupling constant. Thus, in the theory under consideration the symmetry $\varphi \longleftrightarrow-\varphi$ turns out to be spontaneously broken through the second order phase transition.

## Acknowledgement

The authors are indebted to the Referee for critical reading and valuable notes. They would like to thank Prof. V.N. Pervushin, Drs. A.A. Vladimirov and D.O'Connor for discussions.

## Appendix

Here we formulate our variational techniques (for details see [14,18]), i.e. show how to obtain (3.2) and (3.3) from (3.1). We work in a Euclidean space volume $\Omega \rightarrow \mathbf{R}^{\mathbf{2}}$. Let the integral (3.1) be given

$$
\begin{equation*}
\quad J_{\Omega}(g)=\int d \sigma_{\Phi} \cdot \exp \left\{-g \int_{\Omega} d^{2} \mathbf{x} U(\Phi)\right\} \tag{A.1}
\end{equation*}
$$

with the measure

$$
\begin{equation*}
d \sigma_{\Phi}=C_{\mu} \delta \Phi \cdot \exp \left\{-\frac{1}{2} \int_{\Omega} d^{2} \mathbf{x} \Phi(\mathbf{x})\left(-\square+\mu^{2}\right) \Phi(\mathbf{x})\right\} \tag{A.2}
\end{equation*}
$$

where $U(\Phi)$ is a real functional, $C_{\mu}=\operatorname{det}^{1 / 2}\left(-\square+\mu^{2}\right)$ and $\Phi(\mathbf{x})$ satisfies (2.4). Let us diagonalize the quadratic form in (A.2) by introducing the functional variables $\Phi(\mathbf{x})$ :

$$
\begin{equation*}
\Phi(\mathbf{x})=\left(-\square+\mu^{2}\right)^{-1 / 2} \phi(\mathbf{x})=\int_{\Omega} d^{2} \mathbf{y} \Delta(\mathbf{x}, \mathbf{y}) \phi(\mathbf{y})=(\Delta, \phi)(\mathbf{x}) \tag{A.3}
\end{equation*}
$$

where

$$
\begin{gather*}
\Delta(\mathbf{x}, \mathbf{y})=\int \frac{d \mathbf{k}}{(2 \pi)^{2}}\left(\mathbf{k}^{2}+\mu^{2}\right)^{-1 / 2} \exp [-i \mathbf{k}(\mathbf{x}-\mathbf{y})]  \tag{A.4}\\
\int_{\Omega} d^{2} \mathbf{x} \phi(\mathbf{x})=0
\end{gather*}
$$

Then, (A.1) can be rewritten

$$
\begin{equation*}
J_{\Omega}(g)=\int d \sigma_{\Phi} \cdot \exp \left\{-g \int_{\Omega} d^{2} \mathbf{x} U[(\Delta, \phi)(\mathrm{x})]\right\} \tag{A.5}
\end{equation*}
$$

$$
d \sigma_{\phi}=C \delta \phi \cdot \exp \left\{-\frac{1}{2} \int_{\Omega} d^{2} \mathbf{x} \phi^{2}(\mathbf{x})\right\}
$$

$C$ obeys the condition $\int d \sigma_{\phi}=1$.
Now we will proceed to the variational estimation of the integral (A.5). Let us introduce the new variables $v(x)$ and $A(x)$

$$
\begin{equation*}
\phi(\mathbf{x})=(1+q(\square))^{-1 / 2} v(\mathbf{x})+\left(-\square+\mu^{2}\right)^{1 / 2} A(\mathbf{x}) \tag{A.6}
\end{equation*}
$$

where the variational function $q\left(\mathbf{k}^{2}\right)$ satisfies the condition

$$
\begin{equation*}
\int \frac{d \mathbf{k}}{(2 \pi)^{2}} q^{2}\left(\mathbf{k}^{2}\right)<\infty \tag{A.7}
\end{equation*}
$$

Substituting (A.6) into (A.5) we have the equivalent form of Eq.(A.1):

$$
\begin{gathered}
J_{\Omega}(g)=\prod_{q}(1+q(\square))^{-1 / 2} \int d \sigma_{v} \\
\cdot \exp \left\{\frac{1}{2} \int_{\Omega} d^{2} \mathrm{x} v(\mathbf{x}) q(\square)[1+q(\square)]^{-1} v(\mathrm{x})\right. \\
-\frac{1}{2} \int_{\Omega} d^{2} \mathrm{x}\left[2 A(\mathbf{x})\left(-\square+\mu^{2}\right)\left(\Delta_{q}, v\right)(\mathrm{x})+A(\mathbf{x})\left(-\square+\mu^{2}\right) A(\mathrm{x})\right] \\
\left.-g \int_{\Omega} d^{2} \mathbf{x} U\left[\left(\Delta_{q}, v\right)(\mathbf{x})+A(\mathbf{x})\right]\right\}
\end{gathered}
$$

where

$$
\begin{equation*}
\Delta_{q}(\mathbf{x})=\int \frac{d \mathbf{k}}{(2 \pi)^{2}}\left[\left(\mathbf{k}^{2}+\mu^{2}\right)\left(1+q^{2}\left(\mathbf{k}^{2}\right)\right)\right]^{-1 / 2} \exp (-i \mathbf{k x}) \tag{A.9}
\end{equation*}
$$

Now we choose the function $A(\mathbf{x})$ in the form

$$
\begin{equation*}
\int_{\Omega} d^{2} \mathrm{x} A(\mathrm{x})=0, \quad A^{2}(\mathrm{x})=A^{2} \tag{A.10}
\end{equation*}
$$

where $A$ is an arbitrary number. Let us use the inequality:

$$
\begin{equation*}
\int d \sigma \exp \{-W\} \geq \exp \left\{-\int d \sigma W\right\} \tag{A.11}
\end{equation*}
$$

which is valid for any positively defined measures $d \sigma$ and any real functionals $W$. Then, taking into account (3.1) one can obtain as $\Omega \rightarrow \infty$

$$
\begin{gather*}
V_{s c}\left(\varphi_{o}\right) \leq \min _{q, A} \frac{1}{\Omega}\left\{L(q)+\frac{\mu^{2} A^{2}}{2} \Omega\right. \\
\left.+g \int d \sigma_{\mathbf{v}} \int_{\Omega} d^{2} \mathbf{x} U\left[\left(\Delta_{q}, v\right)(\mathbf{x})+A(\mathbf{x})\right]\right\},  \tag{A.12}\\
L(q)= \\
\frac{\Omega}{2} \int \frac{d \mathbf{k}}{(2 \pi)^{2}}\left[\ln \left(1+q\left(\mathbf{k}^{2}\right)\right)-\frac{q\left(\mathbf{k}^{2}\right)}{1+q\left(\mathbf{k}^{2}\right)}\right] .
\end{gather*}
$$

After integration over $d \sigma_{v}$ we obtain (3.2).

## References

[1] E. Abers and B.W. Lee, Phys.Rep. 9C(1973)1;
[2] G.K. Savvidy, Phys.Lett. B71(1977)133;
[3] S. Coleman and E. Weinberg, Phys.Rev. D7(1973)1888;
[4] R. Jackiw, Phys.Rev. D9(1984)1686;
[5] G. Iona-Lasino, Nuovo Cim. 34(1964)1790;
[6] S.-J. Chang, Phys.Rev. D12(1975)1071;
[7] P.M. Stevenson, Phys.Rev. D32(1985)1389; D30(1984)1712;
[8] B. Simon and R.B. Griffiths, Comm.Math.Phys. 33(1973)145;
[9] O.A. McBryan and J. Rosen, Comm.Math.Phys. 51(1979)97;
[10] S.-J. Chang, Phys.Rev., D13(1976)2778; D16(1977)1979;
[11] S.D. Drell,M. Weinstein and S. Yankielowicz, Phys.Rev. D14(1976)487;
[12] M. Funke,U. Kaulfuss and H. Kummel, Plys.Rev. D35(1987)631;
[13] L. Polley and U. Ritschel, Phys.Lett. B221(1989)44;
[14] G.V. Efimov and G. Ganbold, Int.J.Mod.Phys. A5(1990)531;
[15] G.V. Efimov and G. Ganbold, phys.stat.sol.(b) 165(1991)168;
[16] S. Coleman, in Proc.Int.Summ.Sc.Phys.'Ettore Majorana' (Erice,Bologna,1973);
[17] R. Fukuda and E. Kyriakopoulos, Nucl.Plyss. B85(1975)354;
[18] G.V. Efimov, Problems of QFT of Nonlocal Interactions (Nauka,Moscow,1985).

[^0]Ганбопд Г., Ефимов Г.В.
Фазовый переход в теории $9 \phi_{2}^{4}$
Устойчивость вакуума в скалярной $\phi^{4}$ теории исследуется в двумере. Найдена вариационная оценка зффективного потенциала, которая указывает на существование фазового перехода второго рода. Полученный результат находится в согласии с теоремой Саймона-Гриффитса.

Работа выполнена в Лаборатории теоретической физйки ОИЯИ.

Препринт Оо́ьединенного института ядерньх исследований. Дубна 1992

## Ganbold G., Efimov G.V.

E2-92-176
Phase Transition in $g \phi^{4}$ Theory
The vacuum stability of a scalar g $\phi^{4}$ theory in twodimension is studied. A variational approach is applied to estimation of the effective potential in this model. We find that the second-order phase transition takes place. It is in complete agreement with the Simon-Griffiths theorem.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.


[^0]:    on April 17, 1992.

