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NJL MODEL WITH GLUON CONDENSATE AT FINITE TEMPERATURE

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In a recent work' by two authors (D.E. and M.V.) of this article the simplest $U(2)$- symmertic variant of the NJL model with gluon condensate (GC) [1] has been studied. It is shown that the GC does not change the form of the interaction between mesons and the expressions for meson masses (in ref. [1] only scalar and pseudoscalar mesons were be considered). The influence of the GC is revealed only through the change of values for main parameters of the model, for instance, the cut-off parameter Λ which noticeably diminishes and four- quark coupling constant which on the contrary decreases twice. Once the GC is taken into account, quark condensate increases in magnitude and approaches a standard value $<\bar{q}q>_G \approx (-253\mathrm{Mev})^3$.

In the last years there appear many articles devoted to the investigation of mesons in hot and dense matter [2]- [8]. In part of these works the NJL model [3]- [8] is used. This model was found to be very convenient for the description of meson masses and coupling constants at a finite temperature and a finite chemical potential. In this short article we show that the influence of GC on the temperature behaviour of the constituent quark mass m and pion decay constant F_π may be easily studied. Using these main parameters of the model one can extend the influence of GC to meson masses and their coupling contants.

The effective chiral Lagrangian describing interactions of composite scalar and pseudoscalar mesons in the presence of the condensate of the gluon field G^a can be written as [1], [9]- [11]

$$\begin{aligned} \mathcal{L}(q,G) &= \bar{q}\Big[i\gamma^\nu(\partial_\nu + ig\frac{\lambda_a}{2}G^a_\nu) - m^0\Big]q \\ &+ \frac{\kappa}{2}\Big[(\bar{q}\tau^\alpha q)^2 + (\bar{q}i\gamma_5\tau^\alpha q)^2\Big] , \end{aligned} \tag{1}$$

where g is the QCD coupling constant, λ_a are generators of the color group $SU(N_c)$, τ^a are the Pauli matrices of the flavor group $SU(2)_F$ ($\tau^0 \equiv 1$; summation over ν, a and α is understood), and q are fields of current quarks with mass m^0. Upon introducing meson fields, the Lagrangian (1) turns into the form

$$\begin{aligned} \mathcal{L}'(q,G,\tilde{\sigma},\phi) &= -\frac{(\tilde{\sigma}_a^2 + \tilde{\phi}_a^2)}{2\kappa} + \\ &+ \bar{q}(i(\hat{\partial} + ig\frac{\lambda^a}{2}\hat{G}^a) - m^0 + \tilde{\sigma} + i\gamma_5\phi)q \end{aligned} \tag{2}$$

with $\tilde{\sigma} = \tilde{\sigma}_\alpha\tau^\alpha, \phi = \phi_\alpha\tau^\alpha$. The vacuum expectation value of the isoscalar- scalar field $\tilde{\sigma}_0$ turns out to be nonzero ($<\tilde{\sigma}_0> \neq 0$). To pass to a physical field σ_0 with $<\sigma_0> = 0$, one usually performs a field shift leading to a new quark mass m to be identified with the constituent quark mass

$$-m^0 + \tilde{\sigma}_0 = -m + \sigma_0 \; ; \; \tilde{\sigma}_a = \sigma_a, \; (a = 1,2,3) \; . \tag{3}$$

Here m is determined from the gap equation (see [1])

$$m = m^0 + 8\kappa m I_1 + \kappa\frac{G^2}{6m} , \tag{4}$$

where

$$G^2 = \frac{\alpha}{\pi} < (G^a_{\mu\nu})^2 >_0 \; , \; \alpha = \frac{g^2}{4\pi} \tag{5}$$

and

$$\begin{aligned} I_1 &= -iN_c\int^\Lambda \frac{dk}{(2\pi)^4}\frac{1}{(k^2-m^2)} = \frac{N_c}{4\pi^2}\int_0^{\Lambda_3} d\mathbf{p}\frac{\mathbf{p}^2}{\sqrt{\mathbf{p}^2+m^2}} = \\ &= N_c\frac{1}{8\pi^2}[\Lambda_3\sqrt{\Lambda_3^2+m^2} - m^2\ln(\frac{\Lambda_3}{m} + \sqrt{1+\frac{\Lambda_3^2}{m^2}})] , \\ I_2 &= -iN_c\int^\Lambda \frac{dk}{(2\pi)^4}\frac{1}{(k^2-m^2)^2} = \frac{N_c}{8\pi^2}\int_0^{\Lambda_3} d\mathbf{p}\frac{\mathbf{p}^2}{(\mathbf{p}^2+m^2)^{3/2}} = \\ &= N_c\frac{1}{8\pi^2}\Big[\ln(\frac{\Lambda_3}{m} + \sqrt{1+\frac{\Lambda_3^2}{m^2}}) - (1+\frac{m^2}{\Lambda_3^2})^{-1/2}\Big] . \end{aligned} \tag{6}$$

The quark condensates, the constituent quark masses and the meson coupling constants are expressed in the NJL model through diverging integrals (regularized by the parameter Λ) I_1 and I_2. Indeed the effective meson Lagrangian corresponding to the σ- model and following from (2) in the one- loop quark approximation has the form

$$\mathcal{L}(\sigma,\phi) = -\frac{g_\sigma^2 m^0}{4\kappa m}\mathrm{Tr}(\sigma^2+\phi^2) - m^2\mathrm{Tr}(\sigma^2) + + g_\sigma m\mathrm{Tr}(\sigma\phi^2) - \frac{g_\sigma^2}{4}\mathrm{Tr}(\sigma^2+\phi^2)^2 , \quad (7)$$

where

$$g_\sigma = \frac{1}{2}(I_2 + \frac{G^2}{96m^4})^{-\frac{1}{2}} . \quad (8)$$

From these formulae for the meson masses we obtain

$$m_\pi^2 = \frac{g_\sigma^2 m^0}{\kappa m} , \quad m_\sigma^2 = m_\pi^2 + 4m^2 . \quad (9)$$

The quark condensate is determined through the integral I_1

$$< \bar{q}q >= \mathrm{Tr}\Big(\frac{1}{i\hat{\partial} - m}\Big) = -4mI_1 . \quad (10)$$

Until now we have not considered vector and axial- vector mesons. However, one should bear in mind that since axial- vector mesons do exist, nondiagonal transitions of the type $\pi \longrightarrow a_1$ play an essential role in the NJL model. If they are taken into account, there arises an additional renormalization of pseudoscalar fields [10, 11]

$$g_\phi = g_\sigma Z^{\frac{1}{2}} , \quad (11)$$

where

$$Z = \Big(1 - \frac{6m^2}{m_{a_1}}\Big)^{-1} = \Big(1 + \frac{6m^2}{m_{a_1}}\Big) = 2\Big[1 + \sqrt{1 - (\frac{2g_\rho F_\pi}{m_{a_1}})^2}\Big]^{-1} \quad (12)$$

with m_{a_1} being the mass of the a_1 meson, $F_\pi = 93$Mev is the pion decay constant and g_ρ is the ρ meson decay constant ($g_\rho^2/4\pi \approx 3$). In the NJL model the constants g_ρ and g_σ are related by [9]- [11]

$$g_\rho = \sqrt{6}g_\sigma \quad (13)$$

Then from the Goldberger- Treiman identity

$$F_\pi^2 = \frac{m^2}{g_\phi^2} = \frac{m^2}{Z}\Big(4I_2 + \frac{G^2}{24m^4}\Big) \quad (14)$$

and equations (11), (12) and (13) we get

$$m_u^2 = \frac{m_{a_1}^2}{12}\Big[1 - \sqrt{1 - (\frac{2g_\rho F_\pi}{m_{a_1}})^2}\Big] \approx \frac{g_\rho^2 F_\pi^2}{3} \approx \frac{m_\rho^2}{6} \quad (15)$$

From (15) and (12) the estimates $m_u = 315\mathrm{Mev}, Z = 2$ follow if $m_{a_1} = 2m_\rho^2$ (Weinberg relation) and $m_\rho^2 = 2g_\rho^2 F_\pi^2$ (KSFR relation).

The value of the gluon condensate is taken from the data on the hadron process $e^+e^- \longrightarrow$ hadrons [13]

$$G^2 = \frac{\alpha}{\pi} < G^a_{\mu\nu}G^{\mu\nu}_a >= [(410 \pm 80)\mathrm{Mev}]^4 \approx [400\mathrm{Mev}]^4 . \quad (16)$$

Then, from (14) we obtain

$$\Lambda_3 = 530\mathrm{Mev} . \quad (17)$$

In [1] the behaviour of different physical quantities was described after introducing GC. Now we show the behaviour of these quantities in hot and dense matter.

The free quark propagator at a finite temperature and a finite baryon number density takes the form

$$S_F(p,T,\mu) = (\hat{p} + m)\Big[\frac{1}{p^2 - m^2 + i\epsilon} + \quad (18)$$

$$+ 2\pi i\delta(p^2 - m^2)\big[\theta(p_0)n(p,\mu) + \theta(-p_0)\bar{n}(p,\mu)\big]\Big] \quad (19)$$

where Fermi functions for quark and antiquarks

$$n(p,\mu) = \Big[1 + \exp(\beta(E - \mu))\Big]^{-1} ,$$
$$\bar{n}(p,\mu) = \Big[1 + \exp(\beta(E + \mu))\Big]^{-1} , \quad (20)$$

have been introduced and $\beta = T^{-1}$, $E = \sqrt{\mathbf{p}^2 + m^2}$, μ is the chemical potential. Then for the $I_1(m, T, \mu)$ and $I_2(m, T, \mu)$ we get

$$
\begin{aligned}
I_1(m, T, \mu) &= \frac{N_c}{4\pi^2} \int_0^{\Lambda_3} dp \frac{p^2}{E} (1 - n - \bar{n}) , \\
I_2(m, T, \mu) &= \frac{N_c}{8\pi^2} \int_0^{\Lambda_3} dp \frac{p^2}{E^3} (1 - n - \bar{n}) .
\end{aligned}
\tag{21}
$$

The temperature behavior of the gluon condensate has the form [14]-[15]

$$G^2(T) = G^2(0) \left[1 - \left(\frac{T}{T_c}\right)^4 \right] , \tag{22}$$

where T_c is the critical temperature of gluon condensate ($150\text{Mev} < T_c < 250\text{Mev}$).

Now from eqs (4) and (14) we can obtain the temperature dependence of the constituent quark mass m and the decay constant F_π using I_1, I_2 and G^2 from (21) and (22). The behavior of the total quark condensate which includes also gluon condensate corrections is defined by the equation

$$< \bar{q}q >^{tot} = < \bar{q}q > - \frac{G^2(T)}{12m} = -4mI_1(m, T, \mu) - \frac{G^2(T)}{12m} . \tag{23}$$

The temperature behaviour of the meson masses and the coupling constant g_ρ and g_ϕ are defined by eqs (8), (9) and (11) with $m = m(T, \mu)$ and $I_2 = I_2(m, T, \mu)$. In our approach we will neglect the temperature dependence of m^0 and κ.

By using (10) and (23) we get the estimations of quark condensates at $T = 0$

$$
\begin{aligned}
< \bar{q}q >_0 &= (-212\text{Mev})^3 , \\
< \bar{q}q >_0^{tot} &= (-253.5\text{Mev})^3 .
\end{aligned}
$$

After this, from equations (4) and (9) we can obtain the estimations for κ and m^0

$$\kappa^{-1} = \left(\frac{m_\pi F_\pi}{m}\right)^2 - \frac{2 < \bar{q}q >_0^{tot}}{m} \approx 9.5\text{Gev}^{-2} \tag{24}$$

$$m^0 = \frac{m_\pi^2 F_\pi^2 \kappa}{m} = m + 2\kappa < \bar{q}q >_0^{tot} \approx 5\text{Mev} \tag{25}$$

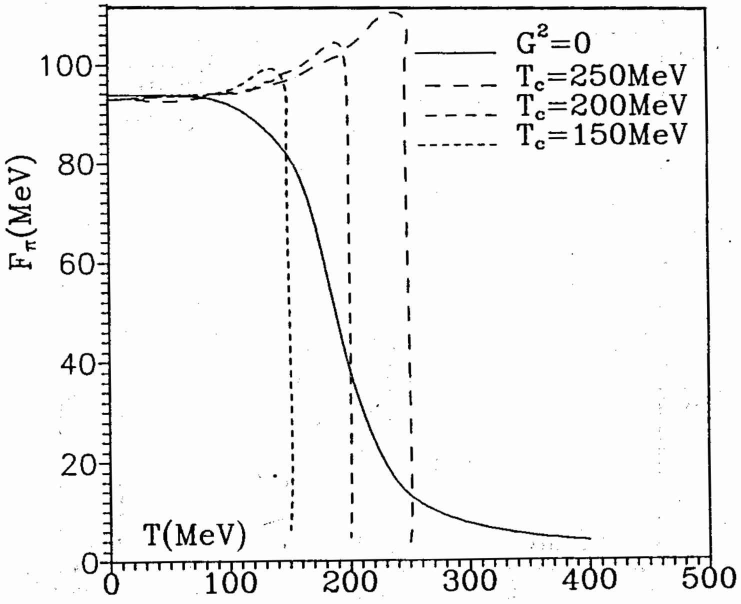

Figure 3. The T- dependence of the pion decay constant F_π.

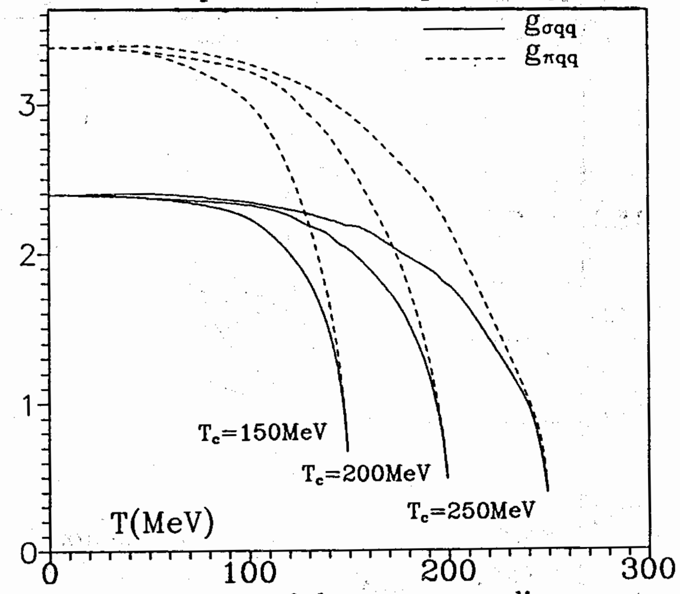

Figure 4. The behaviour of the meson coupling constants $g_{\sigma\bar{q}q}, g_{\pi\bar{q}q}$ as functions of temperature T. The values of the coupling constants at critical temperatures increase rapidly: $g_{\sigma\bar{q}q}(T = T_c = 150) = g_{\pi\bar{q}q}(T = T_c = 150) \approx 2.2, g_{\sigma\bar{q}q}(T = T_c = 200) = g_{\pi\bar{q}q}(T = T_c = 200) \approx 2.4, g_{\sigma\bar{q}q}(T = T_c = 250) = g_{\pi\bar{q}q}(T = T_c = 250) \approx 2.7$.

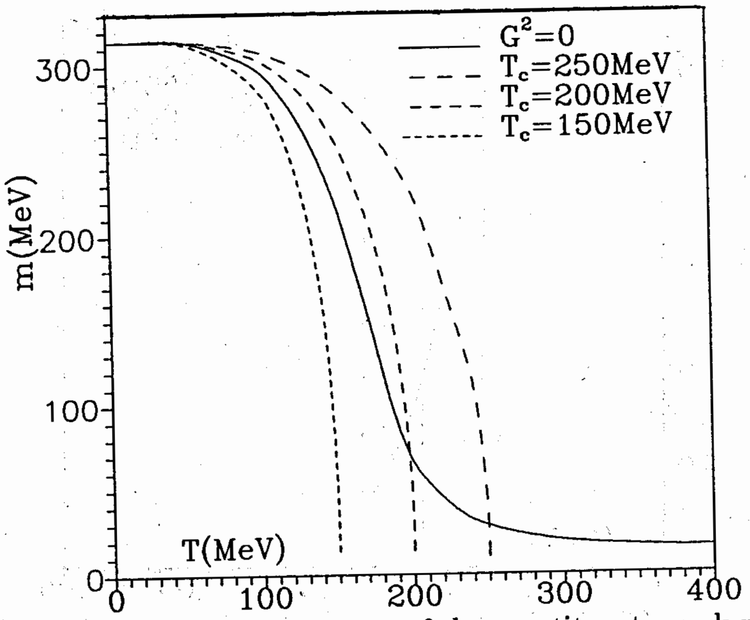

Figure 1. The T- dependence of the constituent quark mass m.

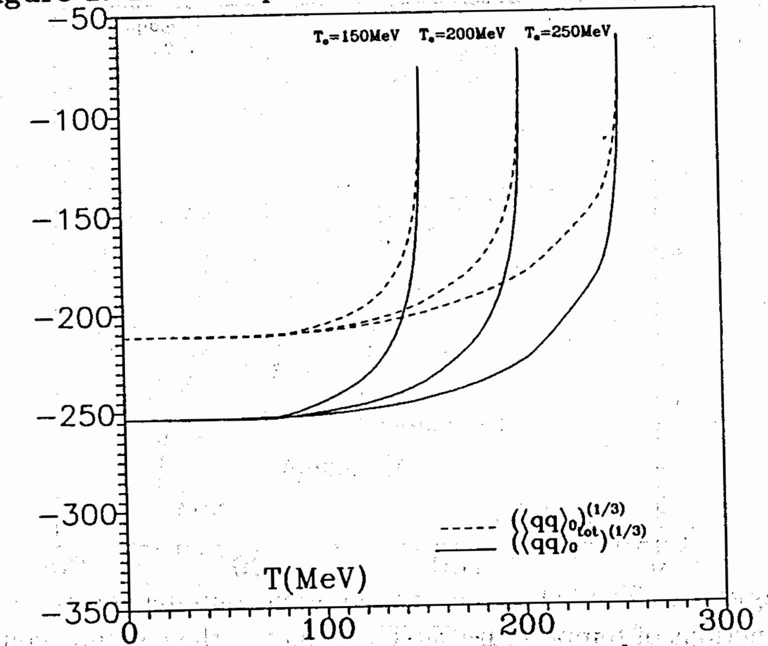

Figure 2. The T- dependence of the quark condensates; $<\bar{q}q>_0$ is the quark condensate without gluon corrections; $<\bar{q}q>_0^{tot}$ is the quark condensate with gluon corrections.

The temperature behavior of $m, <\bar{q}q>, F_\pi, g_\sigma, g_\phi, m_\pi$ and m_σ is shown in Figures 1-5.

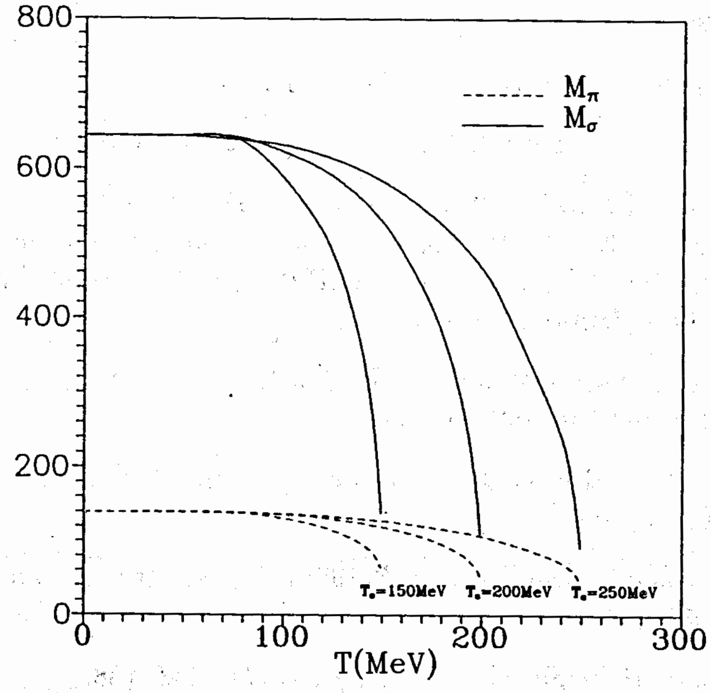

Figure 5. The behaviour of the meson masses M_σ, M_π as functions of temperature T. The values of the meson masses at critical temperatures increase rapidly: $M_\sigma(T = T_c = 150) \approx M_\pi(T = T_c = 150) \approx 440, M_\sigma(T = T_c = 200) \approx M_\pi(T = T_c = 200) \approx 530, M_\sigma(T = T_c = 250) \approx M_\pi(T = T_c = 250) \approx 630$.

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Эберт Д., Калиновский Ю.Л., Волков М.К. E2-92-173
Модель Намбу - Иона - Лазинио с глюонным конденсатом при конечной температуре

Рассмотрена модель Намбу - Иона - Лазинио с глюонным конденсатом при конечной температуре и конечной барионной плотности. В горячей и плотной среде изучено поведение массы кварка, глюонного конденсата, масс мезонов и констант связи мезонов.

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NJL Model with Gluon Condensate at Finite Temperature

The QCD-motivated NJL-model with gluon condensate at finite temperature and baryon number density are considered. We studied the behaviour of the constituent quark mass, quark condensates, meson masses and coupling constants of mesons embedded into a hot and dense medium.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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