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STRING-LIKE EXCITATIONS IN QED

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1 Introduction

It is shown in refs.[1]–[4] that the only object of gauge theories (Abelian and non-Abelian), transforming as a bilocal tensor [1], is the exponential line integral $P\exp(i\int A_\mu dx^\mu)$. All gauge invariant structures must be built of it. The configuration pointed out corresponds to the coherent excitation of the field A_μ on an integration contour. It is easy to be convinced [2]–[6] that the energy corresponding to the excitation is proportional to the contour length. It is natural to identify the state created by the exponential line integral with the string that is usually done. The fact of the proportionality of the excitation energy to the contour length is not yet sufficient for this identification. It is necessary for this configuration to keep the string properties during its time evolution. The present letter is devoted to the investigation of this problem.

By itself the problem proposed is rather difficult. Fortunately, there are explicitly solvable gauge theories where there exist "strings" like that and where this question can be precisely investigated. There are free quantum electrodynamics (QED) and QED with static sources in the framework of which the problem formulated above can be solved.

In Sec.2, the evolution of an arbitrary coherent state for a massless scalar field taken as an example is studied. All calculations can be easily extended to electrodynamics and allow us to elucidate the future of the "closed string" $\exp(i\oint A_\mu dx^\mu)$ in free electrodynamics (Sec.3). In Sec.4, the problem of the "string" evolution in the model of static sources (the "string with charges at its ends") is solved, which is important for understanding the nature of these excitations. It is shown that such "strings" break down, converting into the field of two Coulomb sources after emission of a surplus energy. The questions of the nature of strings in elementary particle physics and of the correspondence between lattice and pure field calculations are discussed in Sec.5.

2 A free scalar massless field

For clarifying the main point of the problem under investigation, consider the simplest model of a free scalar field φ with zero mass. The Lagrangian density and the Hamiltonian operator, respectively, read

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\varphi)^2 \tag{1}$$

$$\hat{H} = \frac{1}{2}\int d^3x[\hat{\pi}^2 + (\partial\hat{\varphi})^2] \tag{2}$$

where the operators $\hat{\varphi}$ and $\hat{\pi}$ obey the canonical equal-time commutation relation

$$[\hat{\varphi}(x), \hat{\pi}(y)]_{x_0=y_0} = i\delta(\mathbf{x}-\mathbf{y}). \tag{3}$$

We are interested in the evolution of the coherent excitation

$$|J\rangle = \hat{E}_J|0\rangle, \qquad \hat{H}|0\rangle = E_0|0\rangle \tag{4}$$

with $|0\rangle$ being the ground state and

$$\hat{E}_J = \exp\left[i\int d^3x\hat{\varphi}(\mathbf{x},0)J(\mathbf{x})\right] \tag{5}$$

where $J(\mathbf{x})$ is a classical real function.

Let us find out what happens with the state $|J\rangle$ in due course. At the initial time moment $t=0$ we have

$$\varphi_J(\mathbf{x},0) = \langle J|\hat{\varphi}(\mathbf{x},0)|J\rangle = 0\,; \tag{6}$$

$$\pi_J(\mathbf{x},0) = \langle J|\hat{\pi}(\mathbf{x},0)|J\rangle = J(\mathbf{x})\,, \tag{7}$$

i.e., the field oscillators are excited in the space region where $J(\mathbf{x}) \neq 0$; and $J(\mathbf{x})$ is the average value of the canonical momentum oscillations. In other words, $J(\mathbf{x})$ is the displacement of the oscillation center on the phase plane $(\varphi(\mathbf{x},0), \pi(\mathbf{x},0))$. At some time t, the state $|J\rangle$ becomes $|J,t\rangle = \exp(-it\hat{H})|J\rangle$ so that the averages (6),(7) of the canonical variables change

$$\varphi_J(\mathbf{x},t) = \langle J,t|\hat{\varphi}(\mathbf{x},0)|J,t\rangle = \langle J|\hat{\varphi}(\mathbf{x},t)|J\rangle = \int d^3y D(\mathbf{x}-\mathbf{y},t)J(\mathbf{y}) \tag{8}$$

$$\pi_J(\mathbf{x},t) = \langle J,t|\hat{\pi}(\mathbf{x},0)|J,t\rangle = \langle J|\hat{\pi}(\mathbf{x},t)|J\rangle = \partial_t\varphi_J(\mathbf{x},t) \tag{9}$$

where $\hat{\pi}(\mathbf{x},t) = \partial_t\hat{\varphi}(\mathbf{x},t)$ and

$$\hat{\varphi}(\mathbf{x},t) = \exp(it\hat{H})\hat{\varphi}(\mathbf{x},0)\exp(-it\hat{H}) = \partial_t\int d^3y D(\mathbf{x}-\mathbf{y},t)\hat{\varphi}(\mathbf{y},0) + \int d^3y D(\mathbf{x}-\mathbf{y},t)\hat{\pi}(\mathbf{y},0) \tag{10}$$

are the Heisenberg field operators being solutions of the Heisenberg equations of motion. The kernel $D(\mathbf{x}-\mathbf{y},t)$ is the commutator of the field operator $\hat{\varphi}(x)$

$$D(x-y) = i[\hat{\varphi}(x),\hat{\varphi}(y)] = \frac{\epsilon(x_0-y_0)}{2\pi}\,\delta((x-y)^2), \tag{11}$$

where $t = x_0 - y_0$. Therefore, the averages (8) and (9) obey the D'Alembert equation

$$\Box\varphi_J(\mathbf{x},t) = 0, \qquad \Box \equiv -\partial_\mu^2 = -\partial_t^2 + \Delta \tag{12}$$

under the initial conditions

$$\partial_t\varphi_J(\mathbf{x},t)|_{t=0} = J(\mathbf{x}), \qquad \varphi_J(\mathbf{x},t)|_{t=0} = 0. \tag{13}$$

Now the meaning of the time evolution for the state $|J\rangle$ becomes clear. Equalities (8) and (9) show that, even if the field is excited inside a compact region at the initial time moment $t=0$, in due course the excitation can reach infinitely distant points, i.e. the energy confined into the region where $J(\mathbf{x}) \neq 0$ is outgoing to the spatial infinity. Thus, we conclude that a local field excitation of the field breaks down.

3 "Closed strings" in free QED

Now it is easy to predict what happens with the state generated by the gauge-invariant operator $\hat{E}_C = \exp(i\oint_C \hat{A}_\mu dx^\mu)$ (the "closed string", C is the integration contour) in free QED determined by the Lagrangian density

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2, \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \tag{14}$$

Defining the quantity

$$\mathbf{J}(\mathbf{x}) = \int_0^1 \delta(\mathbf{x}-\mathbf{x}(s))\dot{\mathbf{x}}(s)ds = \oint_C \delta(\mathbf{x}-\mathbf{y})d\mathbf{y}, \qquad \mathbf{x}(0) = \mathbf{x}(1) \tag{15}$$

we see that the evolution problem for the state $\hat{E}_C|0\rangle$ is reduced to the one considered in Sec.2. Indeed, by taking into account (15) $\hat{E}_C$ can be written in the form

$$\hat{E}_C = \exp\left[i\int d^3x\, \hat{\mathbf{A}}(\mathbf{x},0)\mathbf{J}(\mathbf{x})\right] \tag{16}$$

(for the integration contour being on the plane $t=0$). Introducing the transverse field $\mathbf{A}_\perp$ ($\mathbf{A} = \mathbf{A}_\perp + \mathbf{A}_\parallel$, $\partial\mathbf{A}_\perp = 0$) we obtain the Hamiltonian

$$\hat{H}_0 = \frac{1}{2}\int d^3x\left[\hat{\mathbf{E}}_\perp^2 + \hat{\mathbf{B}}^2\right] \tag{17}$$

and

$$\hat{E}_C = \exp\left[i\oint_C \hat{\mathbf{A}}_\perp d\mathbf{x}\right] = \exp\left[i\int d^3x\, \hat{\mathbf{A}}_\perp(\mathbf{x},0)\mathbf{J}_\perp(\mathbf{x})\right] \tag{18}$$

where $\hat{\mathbf{A}}_\perp = \hat{\mathbf{A}} - \partial\Delta^{-1}(\partial\hat{\mathbf{A}})$, analogously for $\mathbf{J}_\perp(\mathbf{x})$; Δ^{-1} is the inverse Laplace operator. The electric field and vector potential operators have the following commutation relations

$$\left[\hat{A}_k(\mathbf{x},0), \hat{E}_j(\mathbf{y},0)\right] = i\delta_{kj}\delta(\mathbf{x}-\mathbf{y})\,; \tag{19}$$

the transverse operator $\hat{\mathbf{E}}_\perp$ is defined as $\hat{\mathbf{A}}_\perp$.

Using calculations (6)–(13) of Sec.2, we immediately discover that the evolution of the electromagnetic field excitation generated by the "closed string" ($|C\rangle = \hat{E}_C|0\rangle$ with $|0\rangle$ being the ground state of the Hamiltonian (17), $\langle\hat{\mathbf{A}}\rangle_0 = \langle\hat{\mathbf{E}}\rangle_0 = 0$), is described by the field averages

$$\mathbf{E}_C(\mathbf{x},t) = \langle C,t|\hat{\mathbf{E}}_\perp(\mathbf{x},0)|C,t\rangle = \partial_t\int d^3y D(\mathbf{x}-\mathbf{y},t)\mathbf{J}_\perp(\mathbf{y}) \tag{20}$$

$$\mathbf{B}_C(\mathbf{x},t) = \langle C,t|\hat{\mathbf{B}}(\mathbf{x},0)|C,t\rangle = \mathbf{curl}\int d^3y D(\mathbf{x}-\mathbf{y},t)\mathbf{J}_\perp(\mathbf{y}) \tag{21}$$

that obey the Maxwell wave equations with the initial conditions $\mathbf{B}_C(\mathbf{x},t)|_{t=0} = 0$, $\mathbf{E}_C(\mathbf{x},t)|_{t=0} = \mathbf{J}_\perp(\mathbf{x})$ (i.e. the magnetic field is absent but the electric field is excited on the closed contour C). Therefore, for a while t, the state $|C\rangle$ with the field oscillators excited just on the contour C turns into a state with the oscillators excited inside a 3-dimensional region that widens with the light velocity [7].

Thus, the "closed string" $\hat{E}_C$ in free QED breaks down, i.e. there are no stable one-dimensional extended excitations in the theory described by the Lagrangian (14).

4 "Strings" in QED with static sources

A quantum field theory with a static source is also explicitly solvable. Let us investigate what happens with the electromagnetic field excited on a line connecting two opposite charge sources. The theory is determined by the action

$$S = \int d^4y\left[-\frac{1}{4}F_{\mu\nu}^2 - A_0J_0\right];\quad J_0(\mathbf{y}) = -g\left[\delta(\mathbf{y}-\mathbf{x}) - \delta(\mathbf{y}-\mathbf{x}')\right] \tag{22}$$

with g being an electric charge. Action (22) is invariant under gauge transformations $A_\mu \to A_\mu + \partial_\mu\omega$ with an arbitrary function ω, $[\omega(\mathbf{x},t) - \omega(\mathbf{x},-t)] \to 0$ when $t\to\infty$. The constraints and the Hamiltonian, respectively, read

$$\pi_0 = \frac{\partial\mathcal{L}}{\partial\dot{A}_0} = 0, \qquad \partial\mathbf{E} - J_0 = 0 \tag{23}$$

$$H = \frac{1}{2}\int d^3y\,[\mathbf{E}^2 + \mathbf{B}^2]. \tag{24}$$

In quantum theory the first-class constraints (23) must annihilate vectors Φ from the physical Hilbert subspace $\mathcal{H}_{ph}$

$$\hat{\pi}_0\Phi = 0, \qquad (\partial\hat{\mathbf{E}} - J_0)\Phi = 0. \tag{25}$$

Representing the electric field operator in the form $\hat{\mathbf{E}} = \hat{\mathbf{E}}_\perp + \hat{\mathbf{E}}_\parallel$, $\partial\hat{\mathbf{E}}_\perp = 0$, we derive the Hamiltonian operator in the physical subspace

$$\hat{H} = \frac{1}{2}\int d^3y\left[\hat{\mathbf{E}}_\perp^2 + \hat{\mathbf{B}}^2 - J_0\Delta^{-1}J_0\right] \equiv \hat{H}_0 + C \tag{26}$$

where $\hat{H}_0$ coincides with (17) and C is a constant equal to the Coulomb energy of the sources (including the self-interaction). The ground state energy is equal to E_0+C with E_0 being the vacuum energy for (17). The corresponding vacuum functional has the form

$$\Phi_0[\mathbf{A}] = \exp\left[-i\int d^3y d^3y' J_0(\mathbf{y})\Delta^{-1}_{\mathbf{yy}'}\partial\mathbf{A}(\mathbf{y}')\right]\phi_0[\mathbf{A}], \tag{27}$$

where $\phi_0[\mathbf{A}]$ is the vacuum functional for free QED. It is easy to be convinced that $\Phi_0[\mathbf{A}]$ satisfies the constraint equations (25), i.e. $\Phi_0 \in \mathcal{H}_{ph}$ if $(\partial\hat{\mathbf{E}})\phi_0[\mathbf{A}] = \hat{\pi}_0\phi_0[\mathbf{A}] = 0$ $(\hat{\pi}_0 = -i\delta/\delta A_0,\ \hat{\mathbf{E}} = -i\delta/\delta\mathbf{A})$.

We are interested in the state generated by the operator

$$\hat{E}_{\mathbf{xx}'} = \exp\left[ig\int_{\mathbf{x}'}^{\mathbf{x}} \hat{A}_j dy^j\right], \qquad j = 1,2,3\,, \tag{28}$$

being applied to the vacuum ϕ_0 of the free theory,

$$\Phi_{\mathbf{xx}'}[\mathbf{A}] = \hat{E}_{\mathbf{xx}'}\phi_0[\mathbf{A}]. \tag{29}$$

Let us demonstrate that $\Phi_{\mathbf{xx}'} \in \mathcal{H}_{ph}$. Substituting the following obvious identities

$$\mathbf{A} = \mathbf{A} - \partial\Delta^{-1}(\partial\mathbf{A}) + \partial\Delta^{-1}(\partial\mathbf{A}) = \mathbf{A}_\perp + \partial\Delta^{-1}(\partial\mathbf{A}), \tag{30}$$

$$g\int_{\mathbf{x}'}^{\mathbf{x}} d\mathbf{y}\,\partial\int \Delta^{-1}_{\mathbf{yy}'}(\partial\mathbf{A}(\mathbf{y}'))d^3y' = -\int d^3y d^3y' J_0(\mathbf{y})\Delta^{-1}_{\mathbf{yy}'}\partial\mathbf{A}(\mathbf{y}') \tag{31}$$

into (29) we obtain

$$\Phi_{\mathbf{xx}'}[\mathbf{A}] = \exp\left[ig\int_{\mathbf{x}'}^{\mathbf{x}} A_{\perp j}dy^j\right]\Phi_0[\mathbf{A}]. \tag{32}$$

The exponent in (32) is invariant under the gauge transformations, i.e. it is a bundle of closed strings with the integration contours consisting of the segment of the straight line $\mathbf{x}-\mathbf{x}'$ and the field lines of two Coulomb charges (the latter occurs through the term $\partial\Delta^{-1}(\partial\mathbf{A})$ in $\mathbf{A}_\perp$ (see (30)). One can easily check this using the representation of the Coulomb field by the exponential line integrals [1]–[2].

The state $\Phi_0[\mathbf{A}]$ is the ground state of the system, and the evolution of the exponential factor in (32) is described, in fact, by the free Hamiltonian (17), i.e. the problem comes to the one solved in Sec.3. The field configuration corresponding to the state (32) is the Coulomb field of the sources (the ground state of the system with the Hamiltonian (26)) plus the field of the above mentioned bundle of the "closed strings". The energy surplus of the latter is radiated to the spatial infinity. Therefore, in this model there are no stable string-like excitations of the field too. The field, being concentrated on a line, breaks down into the Coulomb field of two opposite charges and radiation.

5 Conclusion

We see that the stable string-like excitations are absent in free QED as well as in QED with static sources. The exponential line integral $\exp(i\oint A_\mu dx^\mu)$ naturally appearing in the fiber bundle theory represents the unstable field configuration completely converting into radiation (Sec. 3). Does it mean that the notion of the electric charge is not compatible with the sting-like structures? Presumably, not. There are two possible points of view on electrodynamics. Firstly, it can be considered as the field theory determined by the Lagrangian (14) (or the more complicated one). In this case, the field is a fundamental object and the "strings" are unstable. The other viable point of view is to consider the string as a fundamental object, but the field as a notion arising when investigating a large number of the strings (like the description of matter consisting of atoms by continuous functions). Unfortunately, this approach has not yet been worked out.

Another question is connected with the Wilson result [8] about the confinement phase in the lattice electrodynamics of massive fermions in the strong coupling limit. A charged particle in the limit $m \to \infty$ turns into the static source. As all calculations in Sec.4 do not depend on the magnitude of the charge g, the model (22) contains the case of the Wilson limit [8]. However, the stable string-like excitations are impossible in QED with static sources and, as a consequence, there is no a linearly rising potential. This circumstance rises the question of the correspondence between lattice and field calculations. The difference is obvious. So, it is important to clarify the situation. In the models considered above it is easy to do.

The potential energy (that corresponds to $1/2 \int d^3x \mathbf{B}^2$ in QED) in the Wilson model is bounded from above by a constant proportional to g^{-2} [9]; therefore, in the strong coupling limit, $g \to \infty$, it becomes inessential as compared with the kinetic energy $\hat{H}_{kin} = 1/2 \int d^3x \hat{\mathbf{E}}^2$. As a result, the state (32) turns into the eigenstate of the Hamiltonian $\hat{H} = \hat{H}_{kin}$, i.e. the string-like excitations become stable (the field oscillators are not coupled to each other; hence, excitation of some of them does not excite the others).

The difference of the field and lattice calculations just pointed out may occur in non-Abelian gauge theories too. In our opinion, this feature forces to be more cautious in comparing the results of lattice calculations with those of the field theory.

References

[1] L.V.Prokhorov, *Vestnik LGU* , N 18 (1990) 3 (in Russian).

[2] L.V.Prokhorov and S.V.Shabanov, *Invariant structures in gauge theories and confinement*, JINR preprint E2-91-195, JINR, Dubna, 1991.

[3] L.V.Prokhorov and S.V.Shabanov, *Invariant structures and static forces in gauge theories*, JINR preprints E2-91-266, JINR, Dubna, 1991.

[4] L.V.Prokhorov, *Vestnik LGU*, N 4 (1992) 3 (in Russian).

[5] L.V.Prokhorov, in: *The Proceedings of the X Seminar on High Energy Physics and Field Theory* , Nauka, Moscow, 1988, p.131.

[6] L.V.Prokhorov, *The string model of electric charge*, Preprint 89-04, Carleton University, Ottawa, 1989.

[7] V.I.Smirnov, *The Course of Higher Mathematics*, v.2, GITTL, Moscow, 1953 (in Russian).

[8] K.Wilson, *Phys.Rev.* 10 (1974) 2445.

[9] A.M.Polyakov, *Gauge Fields and Strings*, Hardwood Academic Publishers, Switzerland, Chur, 1987, p.42.

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Струноподобные возбуждения в КЭД

Изучена эволюция струноподобных возбуждений полей (упорядоченных вдоль пути экспонент) в свободной КЭД и КЭД со статическими источниками. Показано, что эти возбуждения нестабильны. Они распадаются в электромагнитное излучение и кулоново поле. Результаты сравниваются с вычислениями в решеточной КЭД.

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String-Like Excitations in QED

The evolution of string-like excitations of fields (exponential line integrals) in free QED and in QED with static sources is investigated. It is shown that these excitations are not stable. They break down into electromagnetic radiation and the Coulomb field. The results are compared with calculations in the lattice QED.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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