92-168

объединенный
институт
ядерных
исследований
дубна

E2-92-168

V.A.Bednyakov

ABOUT G-PARITY VIOLATION IN $\tau \to \nu\pi\eta$ DECAY

Submitted to "Ядерная физика"

1992

As is well-known, the proof of the existence of the second class currents [1] which possess 'wrong' G-parity will create a lot of difficulties for modern particle theory [2]. Today there are no any of their manifestations obtained yet. The search for physical processes where manifestations of the second class currents would be found is very important. The weak decay $\tau \to \nu\pi\eta$ is now considered the most possible source of manifestations of second class currents [3]. As estimates show [4], a high production rate of $\tau^+\tau^-$-pairs at planned τ-charm factories [5] will open opportunities for clarification of the question about existence of the second class current in the decay $\tau \to \nu\pi\eta$. It is worth mentioning that some of the decays $\tau \to \nu\pi\eta$ should be registered due to ordinary first class currents. These decays form a first class current background for

second class current contribution to the decay. The G-parity is violated here because of isospin violation. Obviously, the rate of this process is very low. However, keeping in mind the search for manifestations of second class currents one must know the part of the decays due to the isospin violation. One could speak about the observations of the second class current manifestations only if the experiment yields the rate of the decay $\tau \to \nu\pi\eta$ much different from calculations which took into account the isospin symmetry violation.

In the paper we investigate the decay $\tau \to \nu\pi\eta$ in the model which systematically incorporates the isospin violation symmetry effects. In our calculations we used the phenomenological effective meson Lagrangian of the Quark Model of Superconductivity Type [6], which stemmed from the well-known 4-fermion Nambu-Jona-Lasinio theory and uniformly describes interactions of scalar, pseudoscalar, vector and axial-vector meson nonets.

The Lagrangian has the form:

$$
\begin{aligned}
L = & -\frac{g^2}{4}Tr\{[(\bar\sigma - \frac{M}{g})^2 + (\bar\varphi)^2]^2 - [(\bar\sigma - \frac{M}{g}), \bar\varphi]_-^2]\} \\
& - \frac{1}{8}Tr\{G_V^{\mu\nu}G_{V\mu\nu} + G_A^{\mu\nu}G_{A\mu\nu}\} \\
& + \frac{1}{4}Tr\{D_\mu(\bar\sigma - \frac{M}{g}) + \frac{g_\rho}{2}[\bar A_\mu, \bar\varphi]_+\}^2 \\
& + \frac{1}{4}Tr\{D_\mu\bar\varphi - \frac{g_\rho}{2}[\bar A_\mu, (\bar\sigma - \frac{M}{g})]_+\}^2. \qquad (1)
\end{aligned}
$$

Here $\bar A = \lambda_\alpha A^\alpha$, $\bar V = \lambda_\alpha V^\alpha$, $\bar\varphi = \lambda_\alpha\varphi^\alpha$, $\bar\sigma = \lambda_\alpha\sigma^\alpha$; V, A, σ and φ denote vector, axial-vector, scalar and pseodoscalar meson fields respectively, λ_α are Gell-Mann matrices ($0 \le \alpha \le 8$, $\lambda_0 = \sqrt{2/3}I$). $M = diag(m_u, m_d, m_s)$ is the current quark mass matrix, $g_\rho = \sqrt{6}g$ and

$$D_\mu\bar a = \partial_\mu\bar a - i\frac{g_\rho}{2}[\bar V_\mu, \bar a]_-,$$

$$G_V^{\mu\nu} = \partial^\mu\bar V^\nu - \partial^\nu\bar V^\mu - i\frac{g_\rho}{2}([\bar V^\mu, \bar V^\nu]_- + [\bar A^\mu, \bar A^\nu]_-)$$

$$G_A^{\mu\nu} = \partial^\mu\bar A^\nu - \partial^\nu\bar A^\mu - i\frac{g_\rho}{2}([\bar A^\mu, \bar V^\nu]_- + [\bar V^\mu, \bar A^\nu]_-)\,.$$

Further we confine ourselves only to the $U(2)$ sector of the model.

The isospin symmetry violation takes place here due to difference of light quark masses[1] m_u and m_d. So, the mixing in the mass sector of neutral axial-vector and pseudoscalar mesons arises. The Lagrangian (1) includes axial-vector–pseudoscalar (φA) and scalar-vector (σV) vertices.

To obtain the effective Lagrangian of physical fields we have to make the diagonalization in the neutral fields mass sector, to eliminate φA- and σV- vertices by shift transformation of axial-vector and vector field and after that to renormalize pseudoscalar and scalar fields [9].

As a result[2] we obtain for the 'vector' hadron vertex:

$$\mathcal{L}_{\rho^-\pi^+\eta} = ig_\rho\rho_\mu^-[c(\pi^+\eta)\eta\partial_\mu\pi^+ + c(\eta\pi^+)\pi^+\partial_\mu\eta], \qquad (2)$$

$$
\begin{aligned}
c(\pi^+\eta) &= -\sqrt{Z_+Z_\eta}\Big\{\frac{\sqrt3}{\sqrt2}\kappa_+[(m_d+m_u)\sin\phi + (m_d-m_u)\cos\phi] + \sin\phi\Big\}, \\
c(\eta\pi^+) &= -\sqrt{Z_+Z_\eta}\Big\{\frac{\sqrt3}{2}[\kappa_{d\eta}(3m_d-m_u) + \kappa_{a\eta}(m_d-3m_u)] - \sin\phi\Big\},
\end{aligned}
$$

and for the 'scalar' hadron vertex:

$$
\begin{aligned}
\mathcal{L}_{a_0^-\pi^+\eta} &= g_\rho\Big\{a_0^-[c_2(\pi^+\eta)\eta\pi^+ + c_3(\pi^+\eta)\partial_\mu\pi^+\partial_\mu\eta] \qquad (3) \\
&+ \partial_\mu a_0^-[c_4(\pi^+\eta)\eta\partial_\mu\pi^+ + c_4(\eta\pi^+)\pi^+\partial_\mu\eta]\Big\},
\end{aligned}
$$

$$
\begin{aligned}
c_2(\pi^+\eta) &= \frac{2}{\sqrt6}\sqrt{Z_+Z_\eta Z_\rho}[(m_d+m_u)\cos\phi + (m_d-m_u)\sin\phi], \\
c_3(\pi^+\eta) &= -\sqrt{Z_+Z_\eta Z_\pi}[\frac{1}{\sqrt2}(\kappa_{a\eta}+\kappa_{d\eta}) + \cos\phi\kappa_+], \\
c_4(\pi^+\eta) &= \sqrt{Z_+Z_\eta Z_\pi}\kappa_+\cos\phi, \\
c_4(\eta\pi^+) &= \sqrt{Z_+Z_\eta Z_\pi}\frac{1}{\sqrt2}(\kappa_{a\eta}+\kappa_{d\eta}).
\end{aligned}
$$

Where $\cos\phi = (\frac12+\frac12\frac{1}{\sqrt{1+\zeta^2}})^{1/2}$ is the π^0-η mixing parameter in the model, $\zeta = \frac{m_d^2-m_u^2}{m_\eta^2-m_\pi^2}$, $Z_+ = (1 - M_{A_1^+}^2|\kappa_+|^2)^{-1}$ is the charged pion renormalization

[1] from here m_u and m_d denote the constituent quark masses
[2] in the leading approximation on light quark mass difference

parameter and $\kappa_+ = -\frac{\sqrt{3}}{\sqrt{2}}\frac{m_d+m_u}{M^2_{A_1^-}}$ is the axial-vector shift parameter. $Z_\eta = (1 - M^2_{A_1^0}|\kappa_{a\eta}|^2 - M^2_{D^0}|\kappa_{d\eta}|^2)^{-1}$ is the η-meson renormalization parameter and the neutral axial-meson shift parameters are $\kappa_{d\eta} = -\frac{\sqrt{3}m_d(\cos\phi - \sin\phi)}{M_D^2}$ and $\kappa_{a\eta} = -\frac{\sqrt{3}m_u(\cos\phi+\sin\phi)}{M^2_{A_1^0}}$.

For calculations we need to determine three parameters g_ρ, m_u and m_d. We use $g_\rho = 5.6$, which gives good width $\Gamma(\tau \to \nu\rho)$. There are two possible choices of the light quark masses in the model. First, where $m_u = 280$ MeV and m_d differs from that value by 5 MeV, so $m_d = 285$ MeV [9], and second, where m_u is about 320 MeV [10] and here we hold the same mass difference 5 MeV. For other particle masses we use the experimentally obtained values[3].

The formulae above allow us to calculate widths of the decays $\rho \to \pi\eta$ and $a_0 \to \pi\eta$. The amplitudes are:

$$T(\rho \to \pi\eta) = -g_\rho \epsilon^\lambda_\mu(p_\rho)\frac{c(\pi^+\eta) - c(\eta\pi^+)}{2}(p_\pi - p_\eta),$$

$$\begin{aligned} T(a_0 \to \pi\eta) &= g_\rho[c_2(\pi^+\eta) - c_3(\pi^+\eta)(p_\pi p_\eta) \\ &+ c_4(\pi^+\eta)(p_\pi p_{a_0}) + c_4(\eta\pi^+)(p_{a_0}p_\eta)]. \end{aligned} \qquad (4)$$

Some of the obtained results are given in Table 1. The decay $a_0 \to \pi\eta$ is calculated in the SU(2) limit. The experimental value for the decay $a_0 \to \pi\eta$ is (0.057 ± 0.011) GeV [7].

There are two weak vertices (Fig.1) contributing to the decay $\tau \to \nu\pi\eta$ in the model after the fulfilled transformations. The first one is the well-known vertex [8] including vector meson — $\tau^-\nu_\tau\rho^+$ (Fig.1a). The second one (Fig.1b) is the result of σV-mixing — $\tau^-\nu_\tau\partial a_0^+$ (full dot denotes σV-transition).

The full weak Lagrangian could be written in the form:

$$\mathcal{L}_{weak} = G_F \cos\theta_c \frac{m_\rho^2}{g_\rho}\bar{\nu}_\tau\gamma_\mu(1-\gamma_5)\tau\left[\rho^+_\mu + \kappa_\rho Z_\rho^{1/2}\partial_\mu a_0^+\right] + h.c., \qquad (5)$$

where $Z_\rho = (1 - m_\rho^2|\kappa_\rho|^2)^{-1}$ is the scalar field renormalization parameter, $\kappa_\rho = i\frac{g_\rho}{g}\frac{m_d - m_u}{2m_\rho^2}$ is the vector shift parameter.

[3] $m_{A_1} = 1.260$ GeV and others from [7]

Table 1: Calculated widths (all in GeV)

m_u	m_d	$\Gamma(\rho \to \pi\eta)$	$\Gamma(a_0 \to \pi\eta)$
0.280	0.283	$0.22\cdot10^{-7}$	
	0.285	$2.28\cdot10^{-7}$	0.48
	0.287	$6.53\cdot10^{-7}$	
0.320	0.322	$0.07\cdot10^{-7}$	
	0.325	$2.32\cdot10^{-7}$	0.82
	0.327	$7.78\cdot10^{-7}$	

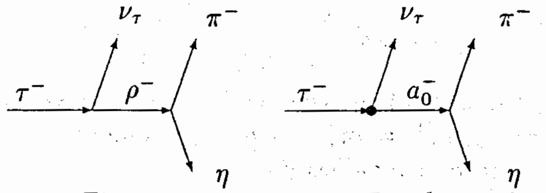

Fig.1a *Fig.1b*

Figure 1: Diagrams contributing to $\tau \to \nu\pi\eta$

Using the first part of (5), we obtained the width of ρ production in the weak τ decay: $\Gamma(\tau \to \nu\rho) = 0.50 \cdot 10^{-12} GeV$, or the branching ratio $Br(\tau \to \nu\rho) = 0.23$, while the experimental data is equal to $(.49 \pm .01)\cdot 10^{-12}GeV$ [7].

The second term in (5) gives the width of a_0 production in the weak τ decay. For the light quark mass difference equal to 5 MeV we obtain:

$$\Gamma(\tau \to \nu a_0) = \frac{G^2\cos^2_\theta}{8\pi g_\rho^2}m_\tau^3\frac{3}{2}(m_d - m_u)^2(1 + 2\frac{m_\rho^2}{m_\tau^2}) = 1.7\cdot 10^{-16}GeV,$$

or the branching ratio equal to $Br(\tau \to \nu a_0) = 8\cdot 10^{-6}$.

The width of the decay $\tau \to \nu a_0$ strongly depends on the light quark mass difference and can be used for its experimental determination.

Now let us consider the decay $\tau \to \nu\pi\eta$. The amplitude of the decay can be expressed in the form

$$T(\tau \to \nu\pi\eta) = -\frac{G_F}{2}\cos\theta_c\bar{\nu}_\tau(1+\gamma_5)\gamma_\mu\tau\{f_+(q^2)p^\mu + f_-(q^2)q^\mu\}, \quad (6)$$

where $p = p_- - p_0$, $q = p_- + p_0$ and p_-, p_0 are the momenta of the charged pion and neutral η-meson ($m_0^2 = m_\eta^2 = p_0^2$ and $m_-^2 = m_\pi^2 = p_-^2$). Two formfactors $f_+(q^2)$, $f_-(q^2)$ are determined by common contribution of weak and strong vertices.

$$f_+ = \frac{m_V^2}{q^2 - m_V^2}(c(\pi^+\eta) - c(\eta\pi^+)), \quad (7)$$

$$f_- = -\frac{m_\pi^2 - m_\eta^2}{m_V^2} f_+ - (c(\pi^+\eta) + c(\eta\pi^+)) \quad (8)$$

$$-2\kappa_\rho Z_\rho^{1/2}\frac{m_\rho^2}{q^2 - m_\sigma^2}\frac{T(a_0 \to \pi\eta)}{ig_\rho}.$$

Here squared masses of intermediate mesons are complex quantities $m_V^2 = m_\rho^2 - im_\rho\Gamma_\rho$, $m_\sigma^2 = m_{a_0}^2 - im_{a_0}\Gamma_{a_0}$, and m_ρ^2, Γ_ρ, $m_{a_0}^2$, Γ_{a_0} are experimentally determined values [7].

The total widths, Γ, branching ratios, Br, contributions from f_+-proportional term, $\Gamma(f_+)$, and f_--proportional term, $\Gamma(f_-)$, contributions from ρ-meson, $\Gamma(\rho)$, and a_0-meson, $\Gamma(a_0)$, intermediate states and the expected number of useful events per year, N, are collected in Table 2.

We used the input value of planned $2 \cdot 10^7$ τ-pairs per year and the total τ-decay width equal to $0.2 \cdot 10^{-11}$ GeV [7].

So, the width of the decay $\tau \to \nu\pi\eta$ due to isospin violation in the first class currents is about $2 \cdot 10^{-17}$ GeV.

The f_--formfactor contribution dominates due to existence of the scalar intermediate meson a_0. The contribution of the intermediate vector ρ-meson and f_+-formfactor is small.

So, without second class current manifestations one could have about 200 $\tau \to \nu\pi\eta$ decays per year at a $c\tau$-factory.

If the registered number of the decays $\tau \to \nu\pi\eta$ will appear strongly different from the values calculated above, then one could speak about registration of the second class currents.

The author is sincerely grateful to Dr A.A.Osipov and Dr Yu.P.Ivanov for their support and fruitful discussions.

Table 2: Calculated values (in GeV)

$(m_u,\ m_d)$	(0.280, 0.283)	(0.280, 0.285)	(0.280, 0.287)	(0.320, 0.325)
$\Gamma(f_+)$	$0.025 \cdot 10^{-17}$	$0.263 \cdot 10^{-17}$	$0.738 \cdot 10^{-17}$	$0.258 \cdot 10^{-17}$
$\Gamma(f_-)$	$0.575 \cdot 10^{-17}$	$1.639 \cdot 10^{-17}$	$3.421 \cdot 10^{-17}$	$2.718 \cdot 10^{-17}$
$\Gamma(\rho)$	$0.022 \cdot 10^{-17}$	$0.307 \cdot 10^{-17}$	$0.930 \cdot 10^{-17}$	$0.354 \cdot 10^{-17}$
$\Gamma(a_0)$	$0.626 \cdot 10^{-17}$	$1.763 \cdot 10^{-17}$	$3.500 \cdot 10^{-17}$	$3.031 \cdot 10^{-17}$
Γ	$0.581 \cdot 10^{-17}$	$1.600 \cdot 10^{-17}$	$3.255 \cdot 10^{-17}$	$2.697 \cdot 10^{-17}$
Br	$0.29 \cdot 10^{-5}$	$0.83 \cdot 10^{-5}$	$1.63 \cdot 10^{-5}$	$1.35 \cdot 10^{-5}$
N	58	166	326	270

References

[1] S .Weinberg, Phys.Rev. 112 (1958) p.1375-1379,

[2] P. Langacker, Phys.Rev. D15 (1977) p.2386-2400, H. Stremnitzer, Phys.Rev. D10 (1974) p.1327-1339, S.L. Adler et al., Phys.Rev. D12 (1975) p.3522-3532, B.R. Holstein and S.B. Treiman, Phys.Rev. D13 (1976) p.3059-3066, A. Haprlin, B.W. Lee and P. Sorba, Phys.Rev. D14 (1976) p.2343-2348, V.P. Barannik, A.P. Korzh and M.P. Rekalo, A.Phys.Pol. B13 (1982) p.835.

[3] A. Bramon, S. Narison and A. Pich, Phys.Lett. B196 (1987) p.543-549, Bing-An Li and Keh-Fei Lui, Z.Phys.C 40 (1988) p.559-563, S. Fajfer and R.J.Oakes, Phys.Lett. B213 (1988) p.376-378.

B.C. Barish, Proc. of the Tau Charm Factory Workshop, SLAC-Report-343, June 1989, p.113.

[4] Proc. of the Tau-Charm Factory Workshop, SLAC-report-3436, June 1989; J Kirkby, Preprint CERN-PPE/91-13 17 January, 1991; M.L. Perl Preprint SLAC-PUB-5428, February 1991; M.V. Danilov et al., ITEP-Preprint 67-90, Moscow, 1990.

[5] J.M. Jowett, Initial Design of a Tau-Charm Factory at CERN, Preprint CERN LEP-TH/87-56. Y. Baconnier et al., A Tau-Charm Factory Laboratory is Spain Combined with a Synchrotron Light Source. Preprint CERN/AC/90-07.

[6] D. Ebert and M.K. Volkov, Z.Phys.C v.16 (1983) p.205-210, M.K. Volkov, Annals of Physics v.157 (1984) p.282

[7] Particle Data Group, Phys.Lett.B239 (1990) p.1

[8] D. Ebert and H. Reinhardt, Nucl.Phys. B271 (1986) p.188

[9] Yu.P. Ivanov, A.A. Osipov, M.K. Volkov, Z.Phys.C49,563-567 (1990) and references therein.

[10] V. Bernard, A.A. Osipov, Ulf-G. Meissner, Preprint University of Louis Paster CRN 92-11, Strasbourg, 1992.

Received by Publishing Department

on April 14, 1992.

Бедняков В.А. E2-92-168

О нарушении G-четности в распаде $\tau \to \nu\pi\eta$

На основе последовательного учета эффектов нарушения изотопической симметрии в феноменологической кварковой модели вычислены характеристики запрещенного законом сохранения G-четности распада $\tau \to \nu\pi\eta$. Изучена роль векторного и скалярного промежуточных состояний. Исследованный механизм является "фоновым" при поиске возможных проявлений токов второго рода в распаде $\tau \to \nu\pi\eta$. При интенсивностях $c\tau$-фабрик за счет нарушения изотопической симметрии следует ожидать около 200 распадов $\tau \to \nu\pi\eta$ в год.

Работа выполнена в Лаборатории ядерных проблем ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1992

Bednyakov V.A. E2-92-168

About G-Parity Violation in $\tau \to \nu\pi\eta$ Decay

The characteristics of the G-parity forbidden $\tau \to \nu\pi\eta$ decay were calculated in the phenomenological quark model with allowance for isospin symmetry violation. Contributions of vector and scalar intermediate states to the decay width were studied. The investigated mechanism is a background for searching for the second class current in the decay $\tau \to \nu\pi\eta$. The background should be about 200 events per year at the $c\tau$-factory.

The investigation has been performed at the Laboratory of Nuclear Problems, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1992