

СОобщения ОбЬEДИНЕННОГО

ИНСТИTYTa
ядерных
исследований

дубна

V.A.Bednyakov, A.A.Osipov

STRANGENESS-CHANGING VECTOR CURRENTS
IN $\tau$-LEPTON DECAYS

Бедняков В.А., Осипов А.А.

Рассмотрены запрещенные слабые Кабиббо-распады т-лептонов на каон и нестранный псевдоскалярный мезон в U(3)версии кварковой модели сверхпроводящего типа с учетом фА-смешивания. Вычислены полные и дифференциальные ширины четырех распадов т-пептонов указанного типа. Получено ожидаемое число таких распадов в год на с-т-фабрике. Отмечена роль фА-смешивания.

Работа выполнена в Лаборатории ядерных проблем ОИяИ.

Сообщение Оо́ьединенного институга ядерных исслежований. Дуо́на 1992

Bednyakov V.A., Osipov A.A.
E2-92-16
Strangeness-Changing Vector Currents
in $\tau$-Lepton Decays
The Cabibbo suppressed $\tau$-lepton decays into a kaon and a non-strange pseudoscalar meson have been investigated in the $U(3)$-version of the superconducting quark model with allowance for $\phi A$-mixing. The total and differential widths of four $\tau$-lepton decays were obtained. The number of these decays per year at the c-т-factory was calculated.

The investigation has been performed at the Laboratory of Nuclear Problems, JINR.

The high luminosity of planned Charm- $\tau$ factories [1] will open good opportunities for investigations of Cabibbo suppressed $\tau$-lepton decays. It is worth mentioning that systematic investigation of $\tau$-lepton decays into open strange final states has only begun [2]. Observation and exploration of these decays will yield information about the structure of the strangeness-changing part of the weak hadron current in a new kinematic region. Here we bring a short review of our calculations of four $\tau$-decays induced only by the vector part of the weak hadron current. We consider semileptonic $\tau$-decays into strange and non-strange pseudoscalar mesons:

$$
\begin{align*}
& \tau^{-} \rightarrow \nu_{r}+\bar{K}^{0}+\pi^{-}  \tag{1}\\
& \tau^{-} \rightarrow \nu_{r}+K^{-}+\pi^{0}  \tag{2}\\
& \tau^{-} \rightarrow \nu_{\tau}+K^{-}+\eta  \tag{3}\\
& \tau^{-} \rightarrow \nu_{r}+K^{-}+\eta^{\prime} \tag{4}
\end{align*}
$$

In our calculations we used the phenomenological effective meson Lagrangian of the Superconductor Quark Model (QMST) [3], which stemmed from the well-known 4 -fermion Nambu-Jona-Lasinio theory and uniformly describes interactions of scalar, pseudoscalar, vector and axial vector meson nonets at low energy.

In the $\tau$-lepton rest frame the differential widths of the $\tau$-decays can be calculated by a standard formula

$$
\begin{equation*}
d \Gamma\left(\tau \rightarrow \nu_{\tau} \phi^{-} \phi^{0}\right)=\frac{\delta^{4}\left(p_{\tau}-p_{\nu}-p_{0}-p_{-}\right) \frac{\frac{1}{2} \Sigma|T|^{2}}{2 m_{\tau}} \prod_{i=1}^{3} \frac{d p_{i}^{3}}{2 E_{i}}, \frac{1}{5}}{2} \tag{5}
\end{equation*}
$$

where $i=(\nu, 0,-)$ corresponds to the neutrino, neutral and charged final state mesons respectively.

The amplitudes of all decays can be expressed in the form

$$
\begin{equation*}
T(\tau \rightarrow \nu 2 \phi)=-\frac{G_{F}}{2} \sin \theta_{c} \bar{\nu}_{r}\left(1+\gamma_{s}\right) \gamma_{\mu} \tau\left\{f_{+}\left(q^{2}\right) p^{\mu}+f-\left(q^{2}\right) q^{\mu}\right\} \tag{6}
\end{equation*}
$$

where $p=p_{-}-p_{0}, q=p_{-}+p_{0}$ and $p_{-}, p_{0}$ are the momenta of the charged and neutral final state mesons. Note that $m_{0}^{2}=p_{0}^{2}$ and $m_{-}^{2}=p_{-}^{2}$.



Fig. 16
Figure 1: Diagrams contributing to $\tau$-decays
For example, in the case of the $\tau \rightarrow \nu_{\tau} \bar{K}^{0} \pi^{-}$reaction we have $p_{-}=p_{\pi^{-}}$, $p_{0}=p_{\bar{K}^{0}}$. Two formfactors $f_{+}\left(q^{2}\right), f_{-}\left(q^{2}\right)$ are determined by dynamics of strong interaction. We calculated them in QMST using two weak vertices depicted in Fig. 1 (the filled dot in Fig.1b denotes a $V \sigma$-transition).

The first lepton weak vertex (Fig. 1a) generates an intermediate vector (V) strange $K^{*}(892)$-meson with mass $m_{K^{*}}=891.83 \pm 0.24 \mathrm{MeV}$ and width $\Gamma_{K^{*}}=49.8 \pm 0.8 \mathrm{MeV}$ [4].

The other weak vertex (Fig.1b) appears in QMST due to $V \sigma$-mixing and generates exchange with a scalar strange meson - $K_{0}^{*}(1340)$. The full weak Lagrangian is:

$$
\begin{equation*}
\mathcal{L}_{w}=G_{F} \sin \theta_{c} \frac{m_{K^{*}}^{2}}{g_{K^{*}}} \bar{\nu}_{\tau} \gamma_{\mu}\left(1-\gamma_{s}\right) \tau\left[K_{\mu}^{*+}+i \kappa Z_{K_{0}^{*}}^{1 / 2} \partial_{\mu} K_{0}^{*+}\right]+\text { h.c. } \tag{7}
\end{equation*}
$$

Here $\kappa$ and $Z_{K_{0}^{*}}$ denote $V \sigma$-mixing and scalar field renormalization parameters and for $K_{0}^{*}$-meson are equal to

$$
\begin{align*}
\kappa & =\sqrt{\frac{3}{2}\left(\frac{\left.m_{1}-m_{u}\right)}{m_{K^{*}}^{2}}\right.}=0.27 \mathrm{GeV}^{-1} \\
Z_{K_{0}^{*}} & =\left(1-\frac{3\left(m_{1}-m\right)^{2}}{2 m_{K^{*}}^{2}}\right)^{-1}=1.06 \tag{8}
\end{align*}
$$

In calculations we used $g_{\rho}=6.15$ and QMST-determined quark masses $m \equiv m_{u}=m_{d}=280 \mathrm{MeV}$ and $m_{s}=450 \mathrm{MeV}$.

So for formfactors we obtain expressions:

$$
\begin{equation*}
f_{+}=\frac{m_{V}^{2}}{q^{2}-m_{V}^{2}} \frac{g\left(\phi^{+} \phi^{0}\right)-g\left(\phi^{0} \phi^{+}\right)}{g_{V}} \tag{9}
\end{equation*}
$$

$$
\begin{gather*}
f_{-}=-\frac{m_{-}^{2}-m_{0}^{2}}{m_{V}^{2}} f_{+}-\frac{g\left(\phi^{+} \phi^{0}\right)+g\left(\phi^{0} \phi^{+}\right)}{g_{V}}  \tag{10}\\
-2 \kappa Z_{K_{0}^{*}}^{1 / 2} m_{V}^{2} q^{2}-m_{\sigma}^{2}
\end{gather*}
$$

Here squared masses of intermediate mesons are complex values $m_{V}^{2}=$ $m_{K^{*}}^{2}-i m_{K^{*}} \Gamma_{K^{*}}, m_{\sigma}^{2}=m_{K_{0}^{*}}^{2}-i m_{K_{0}^{*}} \Gamma_{K_{0}^{*}}$ and $m_{K^{*}}^{2}, \Gamma_{K^{*}}, m_{K_{0}^{*}}^{2}, \Gamma_{K_{0}^{*}}$ are experimentally determined values [4].

Hadron vertex constants $g\left(\phi^{+} \phi^{0}\right), g\left(\phi^{0} \phi^{+}\right)$and $g_{s}\left(\phi^{0} \phi^{+}\right)$are determined in the model after removing $\phi A$-mixing and pseudoscalar field renormalization. The constante can be obtained from hadron Lagrangians:

$$
\begin{align*}
& \mathcal{L}_{K \cdot-\phi^{2}}=K_{\mu}^{*}\left[g(K \phi) \phi \partial_{\mu} K+g(\phi K) K \partial_{\mu} \phi\right]  \tag{11}\\
& \mathcal{L}_{K_{0}^{*}-\phi^{2}}=K_{0}^{*-} g_{s}(K \phi) \phi K . \tag{12}
\end{align*}
$$

In two equations above we assume summation over $\phi$ where $\phi$ denotes a physical $\pi^{+}$field for $K \equiv K^{0}$ and physical fields $\pi^{0}, \eta, \eta^{\prime}$ for $K=$ $K^{+}$. Vertex constants are collected in Table 1. Here for pseudoscalar field renormalization constants $Z$ with physical masses of appropriate axial-vector mesons $m_{a_{1}}=1260 \mathrm{MeV}, m_{K_{1}}=1270 \mathrm{MeV}$ and $m_{f_{1}}=$ 1425 MeV one obtains

$$
\begin{aligned}
& Z_{\pi}=\left(1-\frac{6 m^{2}}{m_{1}^{2}}\right)^{-1}=1.43 \\
& Z_{K}=\left(1-\frac{3\left(m+m_{2}\right)^{2}}{2 m_{K_{1}}^{2}}\right)^{-1}=2.04 \\
& Z_{\eta^{\prime}}=\quad\left(1-\frac{6 m^{2}}{m_{f_{1}}^{2}}\right)^{-1}=2.70
\end{aligned}
$$

Other vertex constants are expressed through those given above:

$$
\begin{align*}
& g\left(K^{+} \pi^{0}\right)=g\left(K^{+} \eta\right)=\frac{1}{\sqrt{2}} g\left(K^{0} \pi^{+}\right), \\
& g\left(\pi^{0} K^{+}\right)=g\left(\eta K^{+}\right)=\frac{1}{\sqrt{2}} g\left(\pi^{+} K^{0}\right),  \tag{13}\\
& g_{s}\left(K^{+} \pi^{0}\right)=g_{s}\left(K^{+} \eta\right)=\frac{1}{\sqrt{2}} g\left(K^{0} \pi^{+}\right) .
\end{align*}
$$

Table 1: Hadron vertex constants (see also (13))

| Hadrons | $K^{+} \eta^{\prime}$ | $K^{0} \pi^{+}$ |
| :---: | :---: | :---: |
| $g(K \phi)$ | $i g_{\rho} \sqrt{\frac{z_{K} Z_{n 1}}{2}}\left[1-\frac{2 m}{m+m_{\rho}}\left(\frac{z_{K}-1}{Z_{K}}\right)\right]$ | $-i g_{\rho} \sqrt{\frac{z_{K} Z_{x}}{2}}\left[1-\frac{2 m_{1}}{m+m_{s}}\left(\frac{z_{K}-1}{Z_{K}}\right)\right]$ |
| $g(\phi K)$ | $-i g_{\rho} \sqrt{\frac{z_{K} Z_{n^{\prime}}}{2}}\left[1-\frac{\left.3 m_{0}-m\right)}{2 m_{0}}\left(\frac{z_{n^{\prime}}-1}{Z_{n^{\prime}}}\right)\right]$ | $i g_{\rho} \sqrt{\frac{z_{K} Z_{X}}{2}}\left[1-\frac{3 m-m_{r}}{2 m}\left(\frac{Z_{N}-1}{Z_{\pi}}\right)\right]$ |
| $g_{*}(K \phi)$ | $g_{\rho} 2 m \sqrt{\frac{z_{K_{Q}^{*}}}{3}}$ | $g_{\rho} 2 m, \sqrt{\frac{z_{K_{0}}}{g}}$ |

With above formulae we calculated the total $\Gamma\left(\tau \rightarrow \nu \phi^{-} \phi^{0}\right)$ and differential width $d \Gamma / d q^{2}$ of decays (1)-(4). We used an $S U(3)$ relation $g_{k^{*}}=g_{\rho}$. The relation yields good value for $B r\left(\tau \rightarrow \nu K^{*}\right)$.

The total widths (in $G e V$ ), $\Gamma(\tau)$, branching ratios, $B r$, contributions from $f_{+}$-proportional term, $\Gamma\left(f_{+}\right)$, and $f_{-}$-proportional term, $\Gamma\left(f_{-}\right)$, contributions from $K^{*}, \Gamma\left(K^{*}\right)$, and $K_{0}^{*}, \Gamma\left(K_{0}^{*}\right)$, intermediate states and the expected number of useful events, $N$, per year for all decays in question are collected in Table 2. We used the input value of planned $10^{7} \tau$-pairs per year and the total $\tau$-decay width equal to $0.2 \cdot 10^{-11} \mathrm{GeV}$ [4]. The first two decays with "light" final state (1),(2) have a distinct resonance structure at $q^{2}$ in the vicinity of the $K^{*}$ mass. This region yields general contribution to the total decay width. Accurate $q^{2}$-scanning of the resonance shape would bring a good chance for precise extraction of the mass and width of the intermediate strange vector meson. It is not a priori obvious that these parameters will coincide with the $K^{*}$ mass and width extracted from pure hadronic reactions. The vector strange meson $K^{*}$ dominates due to a big contribution to the formfactor $f_{+}$. In $\tau$-decays the weak formfactor can be studied in another kinematic region which


Figure 2: Differential width of decay $\tau^{-} \rightarrow \nu_{\tau} \bar{K}^{0} \pi^{-}$
differs from the low $q^{2}$-region of the $K_{13}$-decay. The scalar strange meson $K_{0}^{*}$ contributes only to formfactor $f_{-}$.

Due to a big $\tau$-lepton mass the $f_{-}$-formfactor contribution is sufficiently large as compared with that from $K_{i 3}$ decay. In all kinematic domains the $f_{+}$-contribution disguises a near-three-time lower contribution from the formfactor $f_{\text {- }}$. But their destructive interference seems to bring good opportunities for experimental extraction of $q^{2}$-dependence of $f$ - for large momentum transferred. In accordance with the AdemolloGatto theorem [6], the $\phi A$-mixing practically doesn't contribute to formfactor $f_{+}$. Unfortunately, their contribution to $f$ - is not big enough to be noticeable in the decay width.

For example, differential widths $d \Gamma / d q^{2}$ of decay (1), contributions from $f_{+}$- and $f_{-}$-proportional terms are depicted in Fig. 2.

Those "heavy" $\tau$-decays into $\eta$ and $\eta^{\prime}(3),(4)$ do not have a distinct resonance structure due to a sufficiently great invariant mass of the final state, therefore the total widths are considerably smaller. But here the role of the intermediate scalar $K_{0}^{*}$-meson and the $f_{-}$-proportional term becomes visible and important. Here we neglect $\eta-\eta^{\prime}$ mixing.


Figure 3: Differential width of decay $\tau^{-} \rightarrow \nu_{\tau} K^{-} \eta$

The most interesting case is the decay $\tau^{-} \rightarrow \nu_{\tau} K^{-} \eta$.
There is no significant $\phi A$-transition contribution to $f_{+}$, but the decay formfactor $f$, is determined practically totally by the $\phi A$-transition and the $f_{-}$-contribution becomes equal to $f_{+}$-ones (see Fig. 3 and Table 2). As a result, due to the $\phi A$-transition, the total width is doubled.

As the detailed analysis shows, $\phi A$-mixing and $K_{0}^{*}$-meson destructively interfere, changing however $f-q^{2}$-dependence and the total differential width too (see Fig.3). Study the decay shape one might chance to obtain $\phi A$-mixing manifestations. Another way for that is to study the ordinary Dalitz-plot. The manifestations of $\phi A$-mixing are clearly seen from comparison of two plots in Fig. 4.

The Cabibbo suppressed $\tau$-lepton decays into a non-strange pseudoscalar meson and a kaon were investigated. Our calculations ware performed in the superconducting quark model on the basis of $U(3)$-violated effective Lagrangian from [3]. Two-particle vertices (A $\phi$ and $V \sigma$ ) in the Lagrangian were eliminated, which resulted in a new effective hadron Lagrangians (11), (12).

We used a tree-level approximation and, only for simplicity, U(3) protected relation $g_{K^{*}}=g_{\rho}$. This value for $g_{K^{*}}$ doesn't contradict the experiment and can be made more precise due to extraction from the data if it is necessary.


Figure 4: The role of $\phi A$-mixing in decay $\tau^{-} \rightarrow \nu_{\tau} K^{-} \eta$
We suppose that taking into account $\mathrm{U}(3)$ violation in determination of the $g_{K}$. and consideration of loop contributions in our exploration of decays in question as well as a very important problem of background processes acquire real significance only in seting up a new special experiment or in processing data obtained.

The mosi interesting results are depicted in figures 2-4 and table 2. The open question is the role of axial-pseudoscalar and vector-scalar mixing in extension of the QMST to U(3) violated case. Experimental verification of these results is very important.

It is shown that the planned number of $\tau$-interactions at the Charm$r$ factory is practically enough for detailed investigation of all above mentioned decays without probably the $\tau^{-} \rightarrow \nu_{r} K^{-} \eta^{\prime}$ - one.

Up to now the structure of the strange component of the hadron weak vector current has been studied in detail only in ordinary K-meson decays. Due to smallness of the final state lepton mass only $f_{+}$-formfactor can be reliably delermined. For $\tau$-decays this limitation is lifted and one can try to investigate the $f_{-}$formfactor too.

Table 2. Calculated widths (in GeV )

|  | $\tau^{-} \rightarrow \nu_{\tau} \bar{K}^{0} \pi^{-}$ | $r^{-} \rightarrow \nu_{\tau} K^{-} \pi^{0}$ | $r^{-} \rightarrow \nu_{r} K^{-} \eta$ | $r^{-} \rightarrow \nu_{\tau} K^{-} \eta^{\prime}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\Gamma\left(f_{+}\right)$ | $2.013 \cdot 10^{-14}$ | $1.044 \cdot 10^{-14}$ | $0.669 \cdot 10^{-16}$ | $0.240 \cdot 10^{-17}$ |
| $\Gamma\left(f_{-}\right)$ | $0.566 \cdot 10^{-14}$ | $0.284 \cdot 10^{-14}$ | $0.700 \cdot 10^{-16}$ | $2.222 \cdot 10^{-17}$ |
| $\Gamma\left(K^{*}\right)$ | $1.437 \cdot 10^{-14}$ | $0.740 \cdot 10^{-14}$ | $1.270 \cdot 10^{-16}$ | $0.248 \cdot 10^{-17}$ |
| $\Gamma\left(K_{0}^{*}\right)$ | $0.012 \cdot 10^{-14}$ | $0.006 \cdot 10^{-14}$ | $0.363 \cdot 10^{-16}$ | $2.127 \cdot 10^{-17}$ |
| $\Gamma(r)$ | $1.442 \cdot 10^{-14}$ | $0.756 \cdot 10^{-14}$ | $1.286 \cdot 10^{-16}$ | $0.635 \cdot 10^{-17}$ |
| $B r$ | $7.2 \cdot 10^{-3}$ | $3.8 \cdot 10^{-3}$ | $0.6 \cdot 10^{-4}$ | $0.3 \cdot 10^{-5}$ |
| $N$ | 7200 | 3800 | 600 |  |
| $N$ |  |  | 30 |  |

## References

[1] J.Kirkby, CERN-EP/87-210 (1987) C $\tau$-factory proposals
[2] Goldberg M. et al., Phys.Lett.B 251 (1990) p.223-228
[3] M.K. Volkov, Annals of Physics v. 157 (1984) p. 282
[4] Particle Data Group, Phys.Lett.B239 (1990) p. 1
[5] D.Ebert,H.Reinhardt, Núcl.Phys. B271 (1986) p. 188
[6] M.Ademollo, R.Gatto, Phys.Rev.Lett. 13 (1964) p. 264
Received by Publishing Department on January 15, 1992.

