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MESONS AND DIQUARKS IN A NJL-MODEL AT FINITE TEMPERATURE AND CHEMICAL POTENTIAL

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1 Introduction

It is now generally believed that with increasing temperature and baryon number density the hadronic matter undergoes a phase transition to the quark- gluon plasma which is expected to appear in ultra- relativistic heavy ion collisions. Of particular interest is here the question how the quark condensate $< \bar{q}q >$ as a chiral order parameter changes in a hot and dense nuclear medium, and at which temperature and/or baryonic chemical potential the chiral symmetry is restored. Related interesting questions are the behaviour of the relevant quark and meson quantities (masses of constituent quarks and mesons, coupling and decay constants) as functions of the temperature and chemical potential.

In lattice QCD Monte Carlo simulations useful numerical results have been obtained in the region of the hadron and quark- gluon phase at vanishing baryon number density [1]. The quantitative reliability of lattice calculations is however yet somewhat limited technically (in particular, there are difficulties with taking into account the chemical potential dependence due to the complex fermion determinant); so they need to be supplemented by alternative analytical approaches applicable within "simpler" QCD- motivated models. Corresponding supplementary approaches to the question of the chiral phase transition have recently been offered by the study of effective Lagrangian models by using chiral perturbation theory [2] and of NJL models [3], which for zero temperature and density successfully reproduce the low energy meson physics [4]- [5]. (Clearly, the NJL models are incomplete since they do not contain a reliable confinement mechanism. The quantitative results should therefore be considered as only approximate ones.)

In addition to mesons, there exists an increasing interest in diquarks. Diquarks play an interesting role in the hadronization of the QCD action within the functional integral approach where they are required to form baryons as quark- diquark bound states [6].

In a recent letter [7], we have discussed some bulk properties of composite mesons $(\sigma, \vec{\pi}, \vec{\rho}, \omega, \vec{a_1})$ and diquarks (D_S, D_P, D_V, D_A) arising in an extended $SU(2) \times SU(2)$ NJL- model, restricting us to the case of zero temperature and density (see also ref. [8]).

In the present paper, we investigate the meson- diquark system again for the case of nonvanishing temperature and baryon number density (chemical potential μ) and study the implications of a hot and dense nuclear medium for the relevant meson and diquark quantities.

The paper is organized as follows. We follow closely the work [7], but to keep this paper self- contained, we review the bosonization formalism in some detail in section 2. This part is also needed to fit the model parameters (quark mass m, four- quark coupling constants $G_i, \tilde{G}_i$ and momentum cut-off Λ) from the properties of the vacuum ($< \bar{q}q >$) and the relevant meson and diquark characteristics at $T = \mu = 0$.

In section 3 we derive the chiral meson- diquark Lagrangian by functional integral bosonization techniques generalized to finite temperature and chemical potential. To calculate quark loop integrals arising from the loop expansion of the quark determinant, we find it convenient to use quark propagators of the "real time" formalism [9]. Then, we derive formulae for meson and diquark masses as well as coupling constants as functions of T and μ.

Section 4 contains discussions of meson and diquark properties at finite temperature and density.

Finally, in section 5 we present a short summary of the results.

2 The effective meson- diquark Lagrangian at zero temperature and density

Let us consider a $SU(2)_L \otimes SU(2)_R \otimes SU(3)_c$ NJL model defined by the Lagrangian

$$\mathcal{L}_{\rm NJL} = \bar{q}(i\hat{\partial} - m_0)q + \mathcal{L}^{(4)}_{\rm int} \ , \qquad (1)$$

where $q(x)$ are quark fields with three colours and two flavours, and m_0 is the mass of light current quarks.

The four- quark interaction term $\mathcal{L}^{(4)}_{\rm int}$ in (1) describes the interaction of $q\bar{q}$ and qq- pairs leading to the formation of meson and diquark bound states and is given by [7]

$$\begin{aligned} \mathcal{L}^{(4)}_{\rm int} &= \sum_{a=1}^{4}\Big\{G^a(\bar{q}\mathcal{M}^{\tau}_{M}q)(\bar{q}\mathcal{M}^{\tau}_{M}q) + \\ &+ \ \tilde{G}_a(\bar{q}\mathcal{M}^{\theta}_{D}C^T\bar{q}^T)(q^T C^T \mathcal{M}^{\theta}_{D}q)\Big\} \ . \end{aligned} \qquad (2)$$

(In the following we will omit summation symbols having in mind summation over repeated indices.)

In eq. (2) $\mathcal{M}_H$, and $\mathcal{M}_D$ are meson and diquark projection matrices, respectively,

$$\mathcal{M}^{\tau}_{M} \equiv \mathcal{M}^{al}_{M} = \frac{1}{\sqrt{3}}\mathcal{K}^a 1_c \mathcal{F}^l \ , \ \mathcal{M}^{\theta}_{D} \equiv \mathcal{M}^{a\rho g}_{D} = \frac{i}{\sqrt{6}}\mathcal{K}^a \epsilon^{\rho} \mathcal{H}^g \ , \qquad (3)$$

and C^T is the transposed matrix of the charge conjugation ($C = \gamma^2\gamma^4$). The quantities $\mathcal{K}^a$ are Dirac matrices

$$\{\mathcal{K}^a, a = 1,2,3,4\} \equiv \{1, i\gamma^5, \gamma^\mu, \gamma^\mu\gamma^5\} \ ,$$

$(\epsilon^\rho)_{\alpha\beta}$ is the Levi- Civita tensor in the color $SU(3)_c$ space and $\mathcal{F}^l$ and $\mathcal{H}^g$ are generators in the flavor $SU(2)_f$ space

$$\begin{aligned} \left\{\mathcal{F}^l, l = 0,1,2,3\right\} &\equiv \left\{\frac{1}{\sqrt{2}}, \frac{\sigma_1}{\sqrt{2}}, \frac{\sigma_2}{\sqrt{2}}, \frac{\sigma_3}{\sqrt{2}}\right\} , \\ \left\{\mathcal{H}^g, g = 0,1,2,3\right\} &\equiv \left\{\mathcal{F}^l, \underbrace{l = 2}_{1_A}, \underbrace{0,1,3}_{3_S}\right\} \end{aligned}$$

with σ_i being Pauli matrices.

Finally, the constants G^a and $\tilde{G}_a$ are chosen to be equal in pairs

$$\begin{aligned} G^1 = G^2 \equiv G \ , \quad G^3 = G^4 \equiv -G' \ , \\ \tilde{G}_1 = \tilde{G}_2 \equiv \tilde{G} \ , \quad \tilde{G}_3 = \tilde{G}_4 \equiv -\tilde{G}'. \end{aligned} \tag{4}$$

Next, it is convenient to introduce the generating functional of the NJL model

$$Z[\mu_0] = \int \mathcal{D}q\,\mathcal{D}\bar{q}\exp\left\{i\int d^4x(\mathcal{L}_{\rm NJL} + \mu_0(\bar{q}\gamma_0 q)\right\} \ , \tag{5}$$

where μ_0 has the meaning of a (bare) chemical potential.

Using a standard procedure for introducing collective meson (Φ) and diquark (ω^+, ω) fields (for brevity Lorentz indices are omitted) we derive the expression [7]

$$Z = \int \mathcal{D}\Phi\,\mathcal{D}\omega^+\,\mathcal{D}\omega\,\exp\left\{i\int d^4x\mathcal{L}_{\rm eff}[\Phi, \omega^+, \omega]\right\} \tag{6}$$

with the effective meson- diquark Lagrangian

$$\begin{aligned} \mathcal{L}_{\rm eff} &= -\left[\frac{(\Phi^{al})^2}{4G_a} + \frac{1}{\tilde{G}_a}\omega^{+a\rho g}\omega^{a\rho g}\right] - \\ &- i{\rm Trln}(S_\Phi^{-1}) - \frac{i}{2}{\rm Trln}(1 + 4S_\Phi^T\Omega^+ S_\Phi\Omega) \ . \end{aligned} \tag{7}$$

In this formula the symbol Tr acts on internal as well as on spinor indices, and also includes integration over space- time variables. The expression S_Φ^T is the transpose of the quark propagator containing meson fields. Its inverse is given by

$$S_\Phi^{-1} = S_0^{-1} + \mathcal{M}_M^\tau \Phi^\tau \ , \tag{8}$$

$$S_0^{-1} = i\hat{\partial} - m_0 + \mu_0\gamma_0 \ ,$$

and

$$\Omega \equiv \mathcal{M}_D^\theta C^T\omega^\theta \ , \quad \Omega^+ \equiv \omega^{+\theta}C^T\mathcal{M}_D^\theta \ . \tag{9}$$

Let us remark that the expressions Trln$(\ldots)$ entering into (7) contain diverging integrals which should be regularized. Since this model aims to describe spin-1 mesons a gauge- invariant regularization is chosen [4, 5, 7].

For the description of the physical quantities we determine the quark- condensate $<\bar{q}q>_0 = (\sqrt{6}/2G) <\sigma>_0$ and the vacuum excitation value of the quark numbers $<\bar{q}\gamma^0 q>_0 = -(\sqrt{6}/2G') <\omega_0>_0$. The minimum conditions for $\mathcal{L}_{\rm eff}$ lead to equations for the scalar field $\Phi^{a=1,l=0} = \sigma$ and the time component of the ω- meson field

$$\begin{aligned} \frac{\delta\mathcal{L}_{\rm eff}}{\delta\sigma}\Big|_{\sigma=<\sigma>} &= -\frac{1}{2G} <\sigma>_0 - i{\rm Tr}\left[(\mathcal{M}_M^{10})S_{\Phi_0}\right] = 0 \ , \\ \frac{\delta\mathcal{L}_{\rm eff}}{\delta\omega_0}\Big|_{\omega_0=<\omega_0>} &= \frac{1}{2G'} <\omega_0>_0 - i{\rm Tr}\left[(\mathcal{M}_M^{30})_0 S_{\Phi_0}\right] = 0 \ , \end{aligned} \tag{10}$$

where the inverse of the modified "free" quark propagator S_{Φ_0} is now given by

$$S_{\Phi_0}^{-1} = i\hat{\partial} - m + \mu\gamma_0 \ , \tag{11}$$

with

$$m = m_0 - \frac{1}{\sqrt{6}} < \sigma >_0 = m_0 - \frac{G}{3} < \bar{q}q >_0 \ ,$$
$$\mu = \mu_0 + \frac{1}{\sqrt{6}} < \omega_0 >_0 = m_0 - \frac{G'}{3} < \bar{q}\gamma_0 q >_0 \ . \quad (12)$$

All other fields have $< \Phi^\tau >_0 = 0$. Equations (10) and (12) are the gap equations for the constituent quark mass m and the renormalized chemical potential μ.

In order to introduce physical fields σ and ω with vanishing vacuum expectation values we have to perform the change of variables

$$\sigma = < \sigma >_0 + \sigma' \ , \ \omega_0 = < \omega_0 >_0 + \omega_0' \ , \ \omega_i = \omega_i' \ , \ (i = 1, 2, 3) \quad (13)$$

Then, the inverse quark propagator has the form

$$S_\Phi^{-1} = [\gamma^0(i\frac{\partial}{\partial t} + \mu) + \gamma\nabla - m] + \mathcal{M}_M^\tau \Phi'^\tau \equiv$$
$$\equiv S_{\Phi_0}^{-1} + \mathcal{M}_M^\tau \Phi'^\tau \ , \quad (14)$$

(In the following we use again the notation $S_{\Phi_0} \to S_0$).

Let us now derive the expressions for meson and diquark masses as well as for coupling constants. To this we expand the Lagrangian $\mathcal{L}_{\text{eff}}$ to second order in meson and diquark fields. (Linear terms disappear as a consequence of the equation of motion). We obtain

$$\mathcal{L}^{(2)}_{\text{eff}} = -[\frac{(\Phi^\tau)^2}{4G_a} + \frac{1}{\tilde{G}_a}\omega^{+\theta}\omega^\theta] +$$
$$+ \frac{i}{2}\text{Tr}[S_0\mathcal{M}_M^\tau S_0 \mathcal{M}_M^{\tau'}]\Phi^\tau\Phi^{\tau'} + 2i\text{Tr}[S_0\mathcal{M}_D^\theta S_0\mathcal{M}_D^{\theta'}]\omega^{+\theta}\omega^{\theta'} \ , \quad (15)$$

where use has been made of the relation

$$C^T S_0^T(x,y) C^T = -S_0(x,y) \ .$$

i). Meson masses and coupling constants.

The masses of all mesons and diquarks as well as the corresponding kinetic terms are expressed in the one-loop approximation by divergent integrals of the type

$$L^{aa'}(q) = \text{tr}_\gamma \int \frac{d^4k}{(2\pi)^4} \frac{1}{(\hat{k} - m)} \mathcal{K}^a \frac{1}{(\hat{k} - \hat{q} - m)} \mathcal{K}^{a'} \ , \quad (16)$$

which in a low- momentum expansion may be expressed by logarithmically and quadratically divergent parts

$$I_1 = -i \int^\Lambda \frac{d^4k}{(2\pi)^4} \frac{1}{(m^2 - k^2)}$$
$$= \frac{1}{8\pi^2}[\Lambda_3\sqrt{\Lambda_3^2 + m^2} - m^2 \ln(\frac{\Lambda_3}{m} + \sqrt{1 + \frac{\Lambda_3^2}{m^2}})] \ ,$$
$$I_2 = -i \int^\Lambda \frac{d^4k}{(2\pi)^4} \frac{1}{(m^2 - k^2)^2}$$
$$= \frac{1}{8\pi^2}[\ln(\frac{\Lambda_3}{m} + \sqrt{1 + \frac{\Lambda_3^2}{m^2}}) - (1 + \frac{m^2}{\Lambda_3^2})^{-1/2}] \ . \quad (17)$$

Here the quantity Λ_3 denotes a non- covariant cut- off in the three-momentum space introduced after integration over k_0 [2]. As a result, we get the following expression for the effective meson Lagrangian (15):

$$\mathcal{L}^{(2)}_{\text{eff}} = S\{-\frac{1}{2G} + 4[I_1 + (\frac{q^2}{2} - 2m^2)I_2]\}S +$$
$$+ P\{-\frac{1}{2G} + 4[I_1 + \frac{q^2}{2}I_2]\}P +$$

[2] This formulation with a non- covariant cut- off is convenient for the subsequent applications to the case of nonzero temperature.

$$
\begin{aligned}
&+ \; V'_\mu\{\frac{g^{\mu\nu}}{2G'} - \frac{4}{3}(q^2 g^{\mu\nu} - q^\mu q^\nu)I_2\}V_\nu + \\
&+ \; A_\mu\{\frac{g^{\mu\nu}}{2G'} - \frac{4}{3}(q^2 g^{\mu\nu} - q^\mu q^\nu - 6m^2 g^{\mu\nu})I_2\}A_\nu + \\
&+ \; A_\mu(-4imq^\mu I_2)P + P(4imq^\mu I_2)A_\mu .
\end{aligned} \tag{18}
$$

From this formula we obtain, taking into account field renormalizations, the following expressions for the coupling constants:

$$
\begin{aligned}
&S \to Z_S^{1/2} S \ , \ P \to Z_S^{1/2} P \ , \ Z_S \equiv 3g^2_{Sq\bar q} = \frac{1}{4I_2} \ , \\
&V_\mu \to Z_V^{1/2} V_\mu \ , \ A_\mu \to Z_V^{1/2} A_\mu \ , \ Z_V \equiv 3(\frac{g_\rho}{2})^2 = \frac{3}{8I_2} \ ,
\end{aligned} \tag{19}
$$

and meson masses

$$
\begin{aligned}
M_P^2 &= (\frac{1}{4GI_2} - 2\frac{I_1}{I_2}) \ , \ M_S^2 = M_P^2 + 4m^2 \ , \\
M_V^2 &= \frac{3}{8G'I_2} \ , \ M_A^2 = M_V^2 + 6m^2 \ .
\end{aligned} \tag{20}
$$

An additional rescaling of pseudoscalar fields appears when taking into account nondiagonal transitions of the type $\pi \to A_\mu$. In fact, diagonalizing the Lagrangian (18) by introducing physical fields $A'_\mu = A_\mu + \lambda \partial P$ ($\lambda = -\sqrt{6}m/M^2_{a_1}$), where M_{a_1} is the mass of the axial- vector meson a_1, one must rescale the pseudoscalar fields as follows:

$$
P_i = Z^{-1/2} P'_i \ , \ Z = 1 - 6\frac{m^2}{M^2_{a_1}} \ . \tag{21}
$$

As a result, we get for the masses of pseudoscalar and scalar mesons

$$
\begin{aligned}
M^2_{P'} &\equiv M^2_\pi = \frac{1}{4I_2}(\frac{1}{G} - 8I_1)Z^{-1} = \frac{1}{4I_2}\frac{m_0}{mG} \ Z^{-1} \ , \\
M^2_S &= M^2_\pi Z + 4m^2 \ ,
\end{aligned} \tag{22}
$$

and the coupling constant $g_{P'q\bar q}$ is equal to

$$
g_{P'q\bar q} = g_{\pi q\bar q} = g_{Sq\bar q} Z^{-1/2} \ . \tag{23}
$$

Now we determine the parameters of the NJL model in the case of a three momentum cut-off Λ_3.

The four principal parameters of the meson sector of our model m (or m_0), Λ_3, G, G' may be fixed by four physical quantities:

1. the pion decay constant $F_\pi \approx 93$Mev obeying the Goldberger-Treiman relation

$$
F_\pi = \frac{m}{g_{Sq\bar q} Z^{-1/2}} = \frac{m}{g_{\pi q\bar q}} \ ; \tag{24}
$$

2. the constant g_ρ of ρ- decay $\rho \to 2\pi$ ($g_\rho^2/4\pi \approx 3$) (see eq.(19));
3. the pion mass $M_\pi = 140$MeV (see eq.(22));
4. the ρ meson mass $M_\rho = 770$MeV (see eq.(20));

Note also that the gap equation (12)

$$
m = m_0 - \frac{1}{\sqrt{6}} < \sigma >_0 = m_0 + 8GmI_1 \ ,
$$

connects the constituent quark mass m and the current quark mass m_0.

Then, from (19) and (24) we get the value

$$
m = 280\text{MeV} \ .
$$

From equations (19) we get the following expression for the integral $I_2(m)$:

$$
I_2 = \frac{1}{2g_\rho{}^2} = \frac{1}{8\pi}\left(\frac{g_\rho{}^2}{4\pi}\right)^{-1} = \frac{1}{24\pi} \ . \tag{25}
$$

which yields the following estimate for the cut-off parameter Λ_3

$$\Lambda_3 = 1.03\text{GeV} \ . \tag{26}$$

Furthermore, from (20) we can determine the constants G

$$G = 10.45(\text{GeV})^{-2} \ , \tag{27}$$

and G'

$$G' = \frac{9\pi}{M_\rho{}^2} = 48(\text{GeV})^{-2} \ . \tag{28}$$

Using these relations we obtain for the current quark mass the estimate

$$m_0 = 4mM_\pi{}^2 GZI_2 \approx 2\text{Mev} \ . \tag{29}$$

ii). Diquark masses

The diquark masses can be calculated from meson masses with the help of a simple substitution of the constants G and G' in (20) by $\tilde{G}/3$ and $\tilde{G}'/3$, respectively

$$M_{D_P} = M_S(G \to \frac{\tilde{G}}{3}) \ , \ M_{D_A} = M_V(G' \to \frac{\tilde{G}'}{3}) \ ,$$
$$M_{D_S} = M_P(G \to \frac{\tilde{G}}{3}) \ , \ M_{D_V} = M_A(G' \to \frac{\tilde{G}'}{3}) \ . \tag{30}$$

Futher, taking into account scalar- vector diquark mixing we get for the physical diquarks the mass formulae [7]

$$M^2_{D'_S} = (\frac{3}{4\tilde{G}I_2} - 2\frac{I_1}{I_2})\tilde{Z}^{-1} \ , \ M^2_{D_P} = M^2_{D'_S}\tilde{Z} + 4m^2 \ ,$$
$$M^2_{D_A} = \frac{9}{8\tilde{G}'I_2} \ , \ M^2_{D_V} = M^2_{D_A} + 6m^2 \ , \tag{31}$$

where the diquark mixing constant $\tilde{Z}$ is given by

$$\tilde{Z} = 1 - 6\frac{m^2}{M^2_{D_V}} \ . \tag{32}$$

3 NJL model at finite temperature and baryon number density

For the description of the properties of constituent quarks, mesons and diquarks at finite temperature and baryon number density we consider shortly the thermodynamical properties of the particle system. In relativistic theory, particle number is not conserved. Therefore, when discussing the thermodynamics of quantum field theory one uses the grand canonical formalism with two Lagrange multipliers: $\beta = 1/T$ (T is the temperature) and μ_0 is the chemical potential.

The thermal average of an operator $\mathcal{O}$ over the grand canonical ensemble associated with the NJL model (1) is

$$< \mathcal{O} >= \mathcal{Z}^{-1}\text{Tr}\Big(\mathcal{O}e^{-\beta(H-\mu_0 N)}\Big) \ , \tag{33}$$

where $\mathcal{Z}$ is the grand partition function defined by

$$\mathcal{Z} = \text{Tr}\Big(e^{-\beta(H-\mu_0 N)}\Big) = \sum_i < i|e^{-\beta(H-\mu_0 N)}|i > \ . \tag{34}$$

Here $N = \int d^3x\bar{q}\gamma_0 q$ is the number operator for u and d valence quark and $H = \int d^3x\mathcal{H}$ is the Hamiltonian derived from $\mathcal{L}_{\text{NJL}}$.

All the thermodynamical parameters of the system (entropy S, pressure P, baryon number $N/3$) can then be calculated from $\mathcal{Z}$.

By using the standard methods (see [10]) $\mathcal{Z}$ can be rewritten in terms of the following path integral:

$$Z = \mathcal{N} \int \mathcal{D}q\mathcal{D}\bar{q} \exp\Big\{\int_0^\beta d\tau \int d\mathbf{x}\big(\mathcal{L}_{\rm NJL}(\tau,\mathbf{x}) + \mu_0 N\big)\Big\} \; . \tag{35}$$

which is very similar to the zero temperature expression (5). Here $\tau = ix_0$, and the integration has to be performed over antiperiodic Grassman fields, $q(0) = -q(\beta)$.

In the "imaginary time" formalism one gets the following expression for the quark propagator in the momentum space

$$S_0^\beta(p) = \frac{1}{\not p - m_0} \; ,$$

where now $p^\mu = (i\omega_n + \mu_0, \mathbf{p})$, and $\omega_n = (2n+1)\pi/\beta$ are Matsubara frequencies. For a free field this looks like a zero- temperature propagator expect that p^0 is replaced by $(i\omega_n + \mu_0)$. In particular, the Feynman rules are similar to the zero- temperature case, with the following modifications

$$p^0 \longrightarrow (i\omega_n + \mu_0) \tag{36}$$

$$\int \frac{dp}{(2\pi)^4} \longrightarrow \int \frac{d\mathbf{p}}{(2\pi)^3}\frac{i}{\beta}\sum_n \; . \tag{37}$$

For subsequent applications it is more convenient to use instead of $S_0^\beta(p)$ an equivalent representation for the quark propagator derived in the "real time" formalism [9]

$$S(p,T,\mu_0) = (\hat{p} + m_0)\Big\{\frac{1}{p^2 - m_0^2 + i\epsilon} + i\,2\pi\delta(p^2 - m_0^2)\big[\theta(p_0)n(\mathbf{p},\mu_0) + \theta(-p_0)\bar{n}(\mathbf{p},\mu_0)\big]\Big\} \; , \tag{38}$$

where

$$n(\mathbf{p},\mu_0) = (1 + \exp(E - \mu_0)\beta)^{-1} \; , \tag{39}$$

$$\bar{n}(\mathbf{p},\mu_0) = (1 + \exp(E + \mu_0)\beta)^{-1} \; , \tag{40}$$

are the Fermi- Dirac functions for quarks and antiquarks, respectively, and $\beta = T^{-1}$,$E = \sqrt{\mathbf{p}^2 + m_0^2}$.

After introducing collective fields as in the zero- temperature case, the path- integral can now be rewritten in a form analogous to (7), where in S_Φ^{-1} the inverse propagator S_0^{-1} has to be replaced by the inverse of $S(p,T,\mu_0)$. Finally, due to the nonvanishing termal expectation values $<\sigma>$,$<\omega_0>$ the bare parameters in (38) have to be shifted according to $m_0 \to m, \mu_0 \to \mu$ (see eqs.(12), (13)).

Let us first look at the analogue of eqs. (10) for the quark condensate and averaged quark number. Using the Green function $S(p,T,\mu)$ and performing a countour integration in the complex (p_0)- plane we obtain [3]

$$\begin{aligned} <\bar{q}q> &\equiv -i6\int \frac{d^4p}{(2\pi)^4}\mathrm{tr}_\gamma\Big[S(p,T,\mu)\Big] \\ &= -\frac{6m}{\pi^2}\int_0^{\Lambda_3} dp\frac{p^2}{E}\Big(1 - n(\mathbf{p},\mu) - \bar{n}(\mathbf{p},\mu)\Big) \; , \end{aligned} \tag{41}$$

[3] In contrast with ref. [11] there appear independent constants G, G' in eq.(12). Since our model contains a vector interaction, we do not need any Fierz transformations to generate $< q^+q > \neq 0$.

$$< \bar{q}\gamma_0 q > \equiv -i6\int\frac{d^4p}{(2\pi)^4}\mathrm{tr}_\gamma\Big[\gamma_0 S(p,T,\mu)\Big]$$
$$= \frac{6}{\pi^2}\int_0^{\Lambda_3}p^2dp\Big(n(\mathbf{p},\mu)-\bar{n}(\mathbf{p},\mu)\Big)\ , \tag{42}$$

where Λ_3 is a noncovariant three- momentum cut- off which is adjusted to reproduce various meson properties at $T=\mu=0$.

Then, we may represent the thermal gap equation for the constituent quark mass in the form

$$m(T,\mu)=m_0+8Gm(T,\mu)I_1(m,T,\mu)\ , \tag{43}$$

where

$$I_1(m,T,\mu)=\frac{1}{(2\pi)^2}\int_0^{\Lambda_3}dp\frac{p^2}{E}\Big(1-n(\mathbf{p},\mu)-\bar{n}(\mathbf{p},\mu)\Big) \tag{44}$$

denotes the T- and μ- dependent generalization of the function $I_1(m)$, introduced in (17). Analogously, the expression $I_2(m)$ becomes now a function of the temperature T and the chemical potential μ is given by

$$I_2(m,T,\mu)=\frac{1}{2(2\pi)^2}\int_0^{\Lambda_3}dp\frac{p^2}{E^3}\Big(1-n(\mathbf{p},\mu)-\bar{n}(\mathbf{p},\mu)\Big)\ . \tag{45}$$

To calculate meson masses we have to evaluate the loop integrals (16) by the substitution $S(p)\rightarrow S(p,T,\mu)$

$$L^{aa'}(q)=\mathrm{tr}_\gamma\int\frac{d^4p}{(2\pi)^4}S_\beta(p,T,\mu)\mathcal{K}^a S_\beta(p-q,T,\mu)\mathcal{K}^{a'}\ . \tag{46}$$

In our treatment of the NJL model the meson and diquark masses get contributions from the constant parts integral (46) in the limit $\mathbf{p}\rightarrow 0$. Clearly, the meson and diquark masses are now functions of T and μ.

Let us now consider the behaviour of meson and diquark characteristics in a hot and dense medium. Obviously the coupling and renormalization constants depend on the temperature and chemical potential through the integral $I_2(m,T,\mu)$

$$g^2_{Sq\bar{q}}(T,\mu)=\frac{1}{12I_2(m,T,\mu)}\ , \tag{47}$$

and also

$$g_{Vq\bar{q}}(T,\mu)=g_{Aq\bar{q}}(T,\mu)=\sqrt{6}g_{Sq\bar{q}}(T,\mu)\ . \tag{48}$$

Additionally, taking into account the $\pi-A$-mixing effects we obtain

$$g_{\pi q\bar{q}}(T,\mu)=\Big[1-\frac{6m^2(T,\mu)}{M_A{}^2(T,\mu)}\Big]^{-\frac{1}{2}}g_{Sq\bar{q}}(T,\mu)\ ; \tag{49}$$

and using the Goldberger- Treiman relation (24),

$$F_\pi(T,\mu)=\frac{m(T,\mu)}{g_{\pi q\bar{q}}(T,\mu)}\ . \tag{50}$$

After having determined $m(T,\mu)$ and $F_\pi(T,\mu)$ by (43) and (50) we can calculate all meson masses at finite temperature and chemical potential (see eqs.(20), (22))

$$\begin{aligned}
M^2_\pi(T,\mu) &= 3\frac{m_0}{G}\frac{m(T,\mu)}{F^2_\pi(T,\mu)}=\frac{m_0}{4G}\frac{Z^{-1}(T,\mu)}{m(T,\mu)I_2(T,\mu)},\\
M^2_S(T,\mu) &= M^2_\pi(T,\mu)\cdot Z(T,\mu)+4m^2(T,\mu),\\
M^2_\rho(T,\mu) &= \frac{3g^2_{Vq\bar{q}}(T,\mu)}{4G'}=\frac{3}{8G'I_2(T,\mu)},\\
M^2_{a_1}(T,\mu) &= M^2_\rho(T,\mu)+6m^2(T,\mu)\ .
\end{aligned} \tag{51}$$

In a similar manner it is possible to evaluate diquark masses from the corresponding loop integrals $L^{DD}(q)$. As a result, we obtain (see eqs.(31), (32))

$$M^2_{D'_s}(T,\mu) = \Big(\frac{3}{4\tilde{G}I_2(T,\mu)} - 2\frac{I_1(T,\mu)}{I_2(T,\mu)}\Big)\tilde{Z}^{-1}(T,\mu)$$
$$M^2_{D_P}(T,\mu) = M^2_{D'_s}(T,\mu)\cdot\tilde{Z}(T,\mu) + 4m^2(T,\mu), \quad (52)$$

$$M^2_{D_A}(T,\mu) = \frac{9g^2_{Vqq}(T,\mu)}{4\tilde{G}'} = \frac{9}{8\tilde{G}'I_2(T,\mu)},$$
$$M^2_{D_V}(T,\mu) = M^2_{D_A}(T,\mu) + 6m^2(T,\mu) \ ,$$

$$\tilde{Z}(T,\mu) = = 1 - 6\frac{m^2(T,\mu)}{M^2_{D_V}(T,\mu)} .$$

Note that there exists an important difference between M^2_π and $M^2_{D'_s}$ despite their formal similarity. In fact, because of $\tilde{G} \neq G$ the first factor in $M^2_{D'_s}$ (see eq.(52)) cannot be rewritten to be proportional to m_0 by using the gap equation (43). Thus, for $m_0 \to 0$, $M^2_{D'_s} \neq 0$, in contrast with $M^2_\pi \to 0$. This simply reflects the fact that diquarks cannot be Goldstone bosons.

4 Discussions of constituent quark masses and meson and diquark properties at finite T and μ

Note that the dependence on temperature and chemical potential of masses of constituent quarks, mesons and diquarks as well as of coupling constants is determined in our model by two characteristic integrals I_1 and I_2 (see (44), (45)). The first interesting question is the

behaviour of the constituent quark mass $m(T,\mu)$ as a function of T and μ. The answer is found by a self- consistent solution of the termal gap equations (41), (43) with inclusion of eq.(12). The behaviour of $m(T,\mu)$ is shown in Fig.1a and Fig.1b. For $m_0 = 0$ the restoration of chiral symmetry is indicated by the vanishing of the order parameter $< \bar{q}q > \to 0$ at critical values of the temperature and chemical potential, and consequently by $m(T,\mu) \to 0$. When $m_0 \neq 0$ the sharp phase boundary disappears. From Figures 1 we read off the values of the critical temperature and density $T_c = 210$Mev, $\mu_c = 350$Mev which are in agreement with other works [3]. Figures 2 exhibit the behaviour of the meson masses M_σ, M_π, M_ρ and M_{A_1} as functions of T and μ. As T,μ increases the mass of the σ meson decreases sharply as a result of the decrease of the constituent quark mass. On the other hand, the mass of the (would be) Goldstone pion will persistently stay constant until the critical conditions for chiral restoration are reached, beyond which it ceases to exist. Furtherfore, m_ρ is merely independent of T (a weak T- dependence for $T \geq 0$ Mev arises from $g_\rho(T,\mu)$, whereas M_A shows a sharp decrease similar to that of M_σ. Finally, above the critical temperature one obtains $M_{A_1} = M_\rho$, as is expected for a chiral symmetric phase. The behaviour of the meson coupling constants $g_{\sigma q\bar{q}}, g_{Vq\bar{q}} = g_{Aq\bar{q}}$ and of the $\pi - A$ mixing factor Z at finite temperature and chemical potential is shown in Figures 3. The coupling constants remain approximatively constant until $T \approx 100$Mev, $\mu \approx 150$Mev and then slightly decrease whereas Z slightly increases for $T > 100$Mev. Next, the pion decay constant is an important physical quantity which determines the scale of low- energy chiral meson theories. Its T- dependence is shown in Fig.4. Clearly, the sharp decrease in T near $T \approx T_c = 210$Mev is associated to the constituent quark mass, as follows from the Goldberger-Treiman relation. Recall that in our model F_π includes the effects of $\pi - A$ mixing which were not taken into account in [12] (for the inclusion of mixing effects in a density- dependent theory see also [3]). From Fig.1- Fig.4 we can conclude that our results derived for the meson sector of the NJL- model (1) within the "real time" formalism are in agreement with other works based on nonlinear chiral Lagrangians [2] as well as variants NJL models [3]. Finally, let us consider the diquark sector. As has already been mentioned, despite of formal similarities in mass formulae (between the pion and the scalar diquark), scalar diquarks are not Goldstone boson, so that $M^2_{D_s} \neq 0$ for $m_0 \to 0$. Nevereless, taking into account the formal similarities of mass formulae for mesons and diquarks under substitutions $G \to \tilde{G}/3, G' \to \tilde{G}'/3$, (see eq.(30)), i.e. $M_\sigma \sim M_{D_P}, M_V \sim M_{D_A}, M_A \sim M_{D_V}$ one expects to find the (rescaled) behaviour shown in Figures 5. Again the mass of the vector (pseudoscalar) diquark is larger than the mass of the axial (scalar) diquark due to the additional contribution of the constituent quark mass (see eq.(52)). Note that the diquark masses approach each other already at a lower temperature $T \approx 100$Mev and chemical potential.

5 Summary and outlook

In this paper we have extended the investigation of the NJL model with both meson and diquark sectors considered in [7] for $T = 0, \mu = 0$ to the case of nonvanishing temperature and chemical potential. A convenient tool for studying the behaviour of the quark system together with

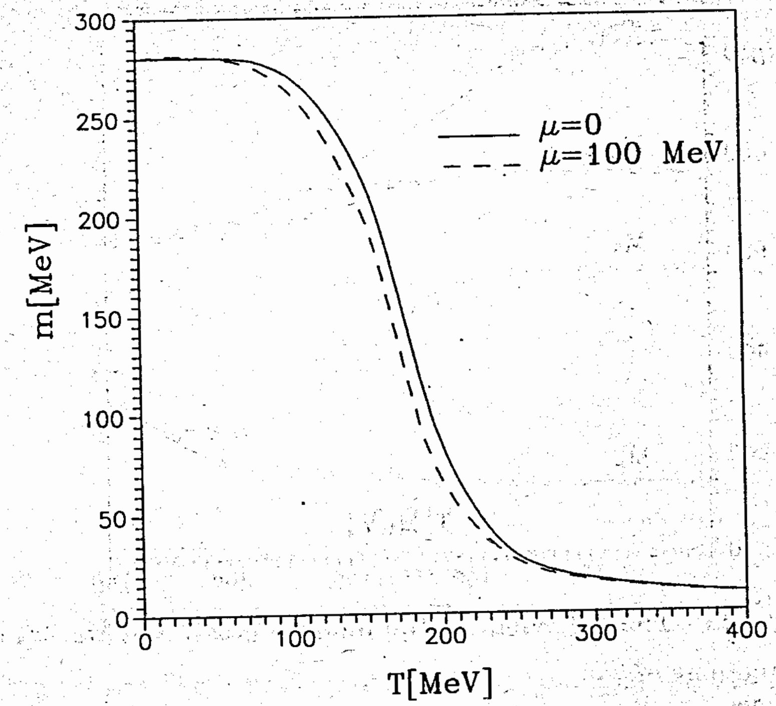

Figure 1a. The T- dependence of quark mass m. μ is the chemical potential.

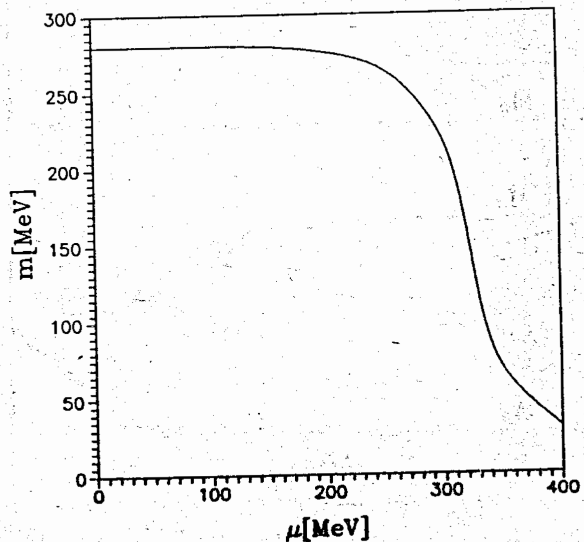

Figure 1b. The μ- dependence of quark mass m.

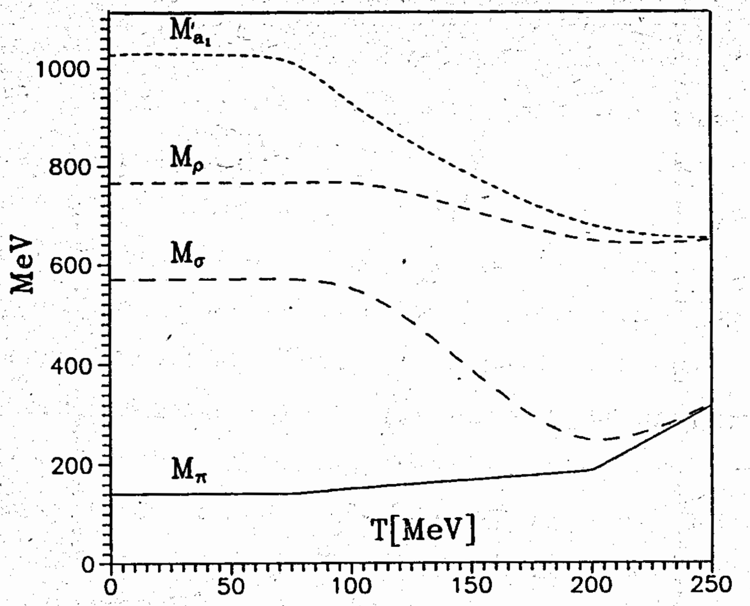

Figure 2a. The behaviour of the meson masses M_σ, M_π, M_ρ and M_{A_1} as functions of T.

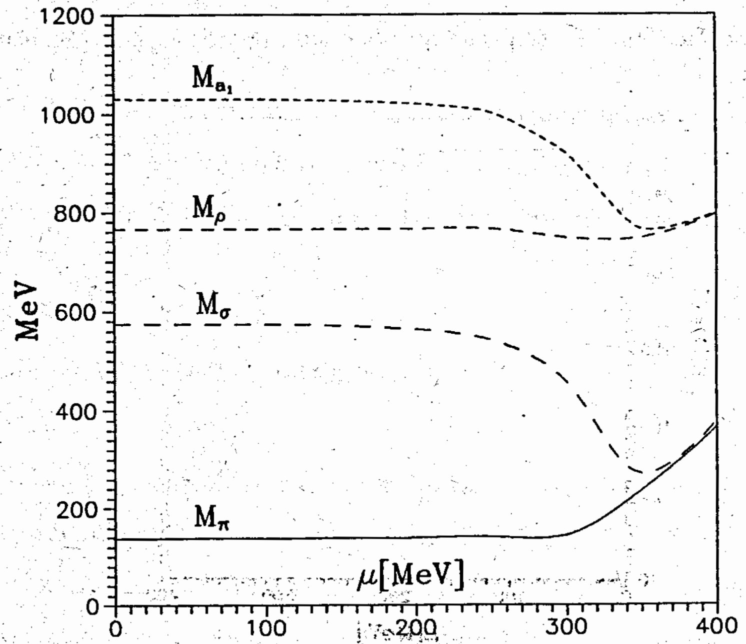

Figure 2b. The behaviour of the meson masses M_σ, M_π, M_ρ and M_{A_1} as functions of μ.

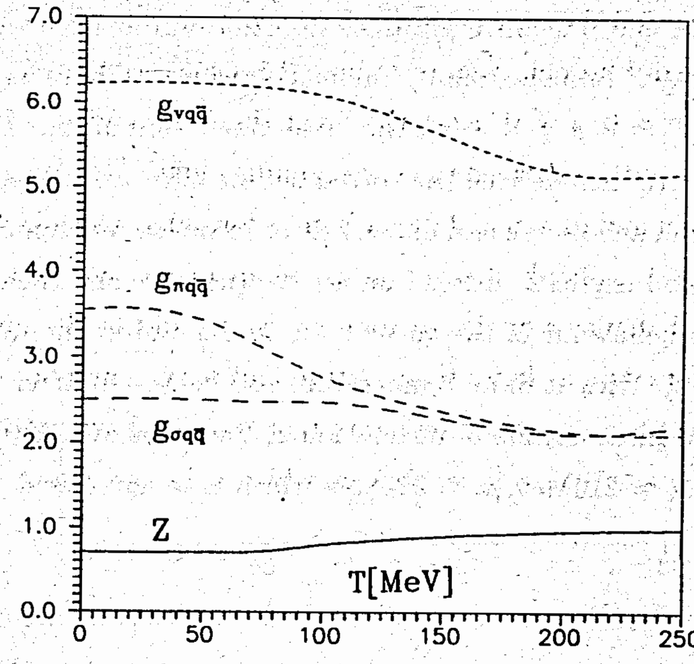

Figure 3a. The behaviour of the meson coupling constants $g_{\sigma q\bar{q}}, g_{Vq\bar{q}} = g_{Aq\bar{q}}$ and of the $\pi - A$ mixing factor Z as functions of temperature T.

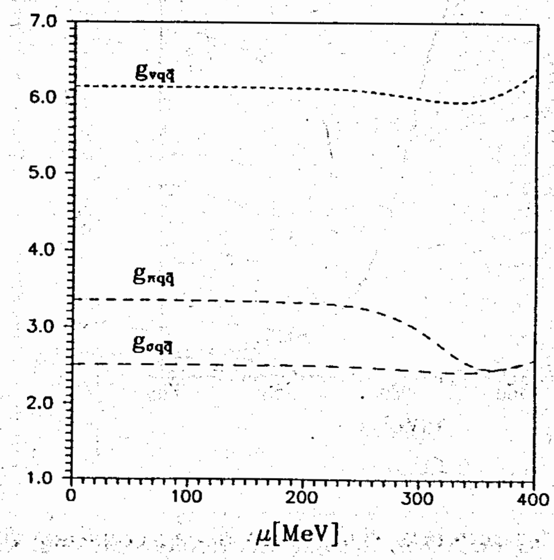

Figure 3b. The behaviour of the meson coupling constants $g_{\sigma q\bar{q}}, g_{Vq\bar{q}} = g_{Aq\bar{q}}$ as functions of chemical potential μ.

collective meson and diquark excitations in a hot and dense medium is the path- integral bosonization technique [12] which we have formulated here for $T \neq 0, \mu \neq 0$ using the "real time" formalism. From our model we have then derived the corresponding effective meson- diquark Lagrangian and meson and diquark mass formulae which include mixing effects and explicitly depend on the temperature and chemical potential. The behaviour of the vacuum ($< \bar{q}q >$) and of the meson and diquark properties at finite temperature and baryon number density exhibits the phenomenon of restoration of the chiral symmetry at critical values $T_c \approx 210$Mev, $\mu_c \approx 350$Mev which is in agreement with other works.

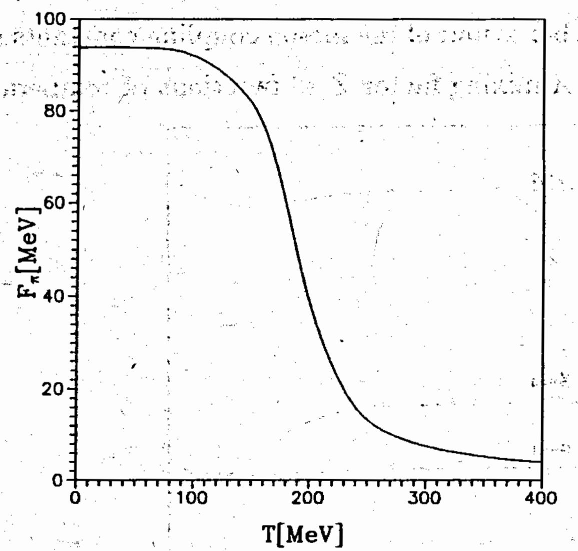

Figure 4 The T- dependence of the pion decay constant F_π.

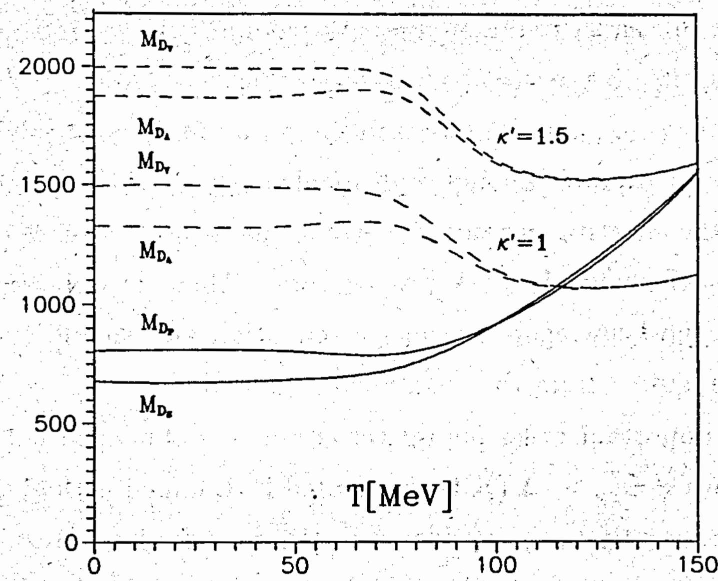

Figure 5a. The behaviour of the diquark masses $M_{D_S}, M_{D_P}, M_{D_A}, M_{D_V}$ as functions of T. κ' is a parameter $\tilde{G} = \kappa' \tilde{G}'$.

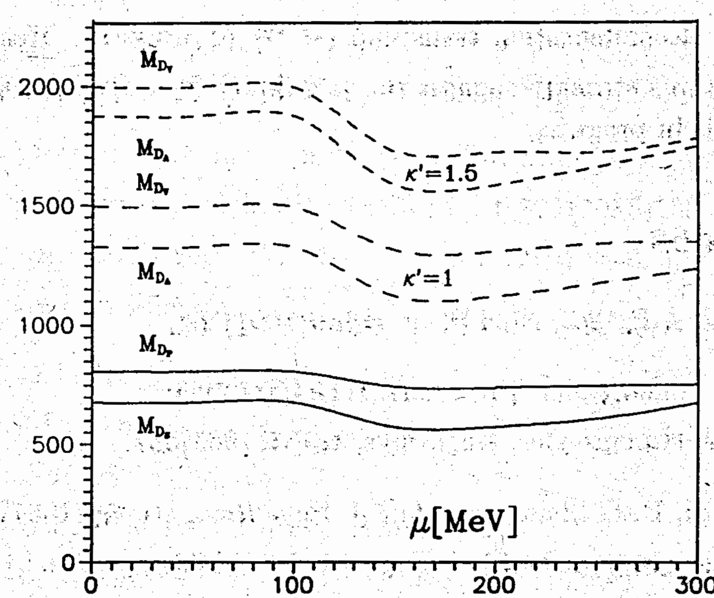

Figure 5b. The behaviour of the diquark masses $M_{D_S}, M_{D_P}, M_{D_A}, M_{D_V}$ as functions of μ. κ' is a parameter $\tilde{G} = \kappa' \tilde{G}'$.

We consider this effective meson- diquark theory as an intermediate step towards an effective meson- baryon theory [6].

An important question to be solved in the future for such a type of models is how to incorporate enough additional non- perturbative gluon dynamics into the effective current (x) current quark interaction of (1) in order to model confined quark propagators. These are obtained as solutions of the Schwinger- Dyson gap equation and should keep the properties required from the spontaneous breakdown of the chiral symmetry. An important order parameter of gluon confinement is the gluon condensate $< G^2_{\mu\nu} >$. A QCD- motivated NJL model with gluon condensate has recently been considered in [13] for zero temperature. Clearly, it is interesting to extend this kind of models to the case $T \neq 0, \mu \neq 0$ including a temperature dependent gluon condensate and to study the relation between the chiral phase transition ($< \bar{q}q(T_c) > \approx 0$) and the gluon decondensation transition ($< G^2_{\mu\nu}(T_d) > \approx 0$). Recent lattice calculations strongly suggest the possibility $T_c = T_d$. Work in this direction is in progress.

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