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FORM FACTORS OF THE DECAY $D^+ \to \overline{K}^{*0} e^+ \nu_e$

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Recently, the study of the $D^+ \to \bar{K}^{*0}e^+\nu_e$ decay was in the focus of attention of many theoretical [1]- [6] and experimental papers [7]- [9]. In most of the theoretical papers [1]- [4] only the vector form factor of this decay is satisfactorily described whereas the estimates of the axial form factors deviate strongly enough from the experimental values (see Table 1).

The present paper is an attempt to find additional intermediate processes which would allow to improve this situation. For estimation we will use a simple phenomenological model providing only a qualitative description of the given decay. This model is similar to the known Nambu-Jona-Lasinio model (the NJL model) which has successfully been applied in the low-energy meson physics [10]-[13]. It has also been used to describe charmed mesons [14]. Taking into account the fact that the chiral symmetry, which underlies the NJL model, is strongly violated in the region of charmed mesons, we will use only some specific phenomenological methods of that model for estimating form factors of the $D^+ \to \bar{K}^{*0}e^+\nu_e$ decay.

The amplitude of the $D^+ \to \bar{K}^{*0}e^+\nu_e$ decay has the form

$$A_{D^+\to\bar{K}^{*0}e^+\nu_e} = i\sqrt{2}H^\mu l_\mu \ , \tag{1}$$

$$l_\mu = \frac{G_F}{\sqrt{2}} V_{cs}\bar{u}_e\gamma_\mu(1-\gamma_5)v_\nu \ , \tag{2}$$

$$\begin{aligned} H^\mu &= \ < K^{*0}(q,\epsilon^{(K)})|(J^\mu_{had} = V^\mu - A^\mu)|D^+(p) >= \\ &= (M_D + M_{K^*})A_1(t)\epsilon^\mu_{(K)} - \frac{2A_2(t)}{M_D+M_{K^*}}(\epsilon_{(K)}p)\cdot p^\mu - \\ &- i\frac{2V(t)}{M_D+M_{K^*}}\epsilon^{\mu\nu\rho\sigma}\epsilon^{(K)}_\nu p_\rho q_\sigma \ . \end{aligned} \tag{3}$$

Here, the fourth form factor in the zero-lepton-mass limit is omitted, l_μ and J^μ_{had} are lepton and hadron currents, G_F is the Fermi constant, V_{cs} is an element of the Kobayashi-Moskawa matrix which corresponds to the c-s quark transition, $t = (p-q)^2 = Q^2$ is the momentum of a lepton pair, M_D and M_{K^*} are masses of D^+ and $\bar{K}^{*0}$ mesons, $V(t)$ and $A_1(t), A_2(t)$ are the vector and axial form factors, respectively.

The largest difficulties arise in describing the $A_1(t)$ form factor. Many quark models lead to the values of A_2 which do not differ much from the values of V (see refs.[2]-[4]) whereas the experiment gives $A_2(0) = 0$ and

$V(0) = 0.9$. This resembles the situation that took place 5-10 years ago in describing in quark models vector and axial form factors h_A and h_V of the $\pi^- \to e\bar{\nu}\gamma$ decay [15]. Quark models led to equal values of these form factors, which was in contradiction with the experimental data ($\gamma^{exp} = h_A/h_V = 0.52 \pm 0.06$ [16]). [1] In our paper [18] we have shown the way of solving this problem. Since in the present paper we propose an analogous way to describe form factors of the $D^+ \to \bar{K}^{*0}e^+\nu$ decay, let us briefly remind the method used in paper [18].

The structure part of the decay amplitude $\pi^- \to e\bar{\nu}\gamma$ has the form similar to (3)

$$A_{\pi^-\to e\bar{\nu}\gamma} = i\sqrt{2}e[h_A(g^{\mu\nu}pq - p^\mu q^\nu) - ih_V\epsilon^{\mu\nu\rho\sigma}p_\rho q_\sigma]\epsilon_\mu^{(\gamma)}\bar{l}_\nu \,. \qquad (4)$$

Here p and q are pion and photon momenta, $\bar{l}_\nu = \frac{G_F}{\sqrt{2}}cos\theta_c\bar{v}_\nu\gamma_\nu(1-\gamma_5)u_e$, θ_c is the Cabibbo angle, e is the electric charge, $\epsilon_\mu^{(\gamma)}$ is the photon polarization, h_V and h_A are the vector and axial form factors.

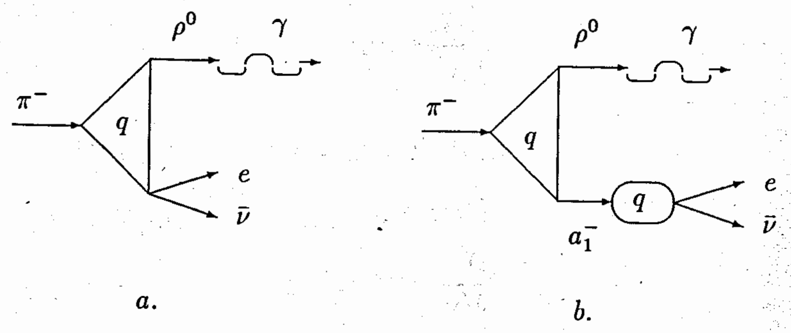

Fig.1. Diagrams describing the $\pi^- \to e\bar{\nu}\gamma$ decay.

[1] We should like to note that 11 years before the publication of this experimental value we obtained the estimate γ=0.57 [17] in the model of nonlinear chiral Lagrangian with baryon loops.

The diagram 1a leads to equal values of the form factors h_A and h_V [18]

$$h_A = h_V = \frac{1}{8\pi^2 F_\pi}. \qquad (5)$$

Here $F_\pi = 93MeV$ is the decay constant $\pi^- \to e\bar{\nu}$. This is the well-known result that has been obtained in various quark models [15]. However, the inclusion of diagram 1b with the intermediate axial-vector meson a_1 decreases the value of h_A in agreement with experimental data. Indeed, in the NJL model [10]-[12] for the amplitude of the decay $a_1^- \to e\bar{\nu}$ proceeding through the diverging quark loop the following expression is derived:

$$A_{a_1^-\to e\bar{\nu}} = \frac{g_\rho}{3\sqrt{2}}(\frac{(-i3)}{4\pi^4}\int^\Lambda \frac{dk}{(m_u^2-k^2)^2} = \frac{6}{g_\rho^2})[Q^\mu Q^\nu - g^{\mu\nu}Q^2 + 6m_u^2 g^{\mu\nu}]\epsilon_\mu^{(a_1)}\bar{l}_\nu. \qquad (6)$$

Here $Q^2 = (p-q)^2, g_\rho = 6$ is the decay constant $\rho \to 2\pi, m_u = 0.3GeV$ is the mass of the u or d constituent quark. The regularised diverging integral determines the constant g_ρ^{-2} [10]-[12].

Using the amplitude (6) one can easily express the contribution of diagram 1b through the corresponding contribution of diagram 1a with the factor $(-\frac{6m_u^2}{M_{a_1}^2})$, so that the total contribution to the axial form factor h_A becomes equal to

$$h_A^{(a+b)} = h_A^{(a)}(1 - \frac{6m_u^2}{M_{a_1}^2}) = h_A^{(a)}Z^{-1} = 0.6h_V \,, \qquad (7)$$

which approximates the experimental value. In the vector channel one could take into account the diagram with intermediate vector mesons but the amplitude , analogous to (6), would not contain a large constant term of the form $6m_u^2 g^{\mu\nu}$ and its contribution would be small. [2]

[2] The same result can easily be obtained in terms of the axial-vector dominance of weak interactions [19] (as it was made with a photon, see fig.1). The Lagrangian describing the transitions $W^- \to \rho^-$ and $W^- \to a_1^-$ has the form

$$L^W = \frac{\kappa}{2g_\rho}[M_\rho^2\rho_\mu^- + \frac{M_{a_1}^2}{Z}a_{1\mu}^-]W^{\mu+} + h.c. \,.$$

Here κ is the weak interaction constant, $W^{\mu+}$ is the vector W meson field, M_ρ and M_{a_1} are masses of ρ and a_1 mesons.

If the (axial)-vector dominance of weak interactions is not used, the $D^+ \to \bar K^{*0}\bar e\nu$ decay is described by the three diagrams depicted in fig.2.Let us write down the Lagrangian that is necessary to calculate the amplitudes corresponding to these diagrams

$$L = i\sqrt{2}g_D\bar c\gamma_5 D^+ d + \frac{g_V(d,s)}{2}\bar d\gamma_\nu K^{*0\nu}s +$$

$$\frac{g_V(s,c)}{\sqrt{2}}\bar c\gamma_\nu(D_s^{*+\nu} + \gamma_5 D_{s_1}^{+\nu})s + \bar s\gamma_\nu(1-\gamma_5)cl^\nu \ . \qquad (8)$$

Here c, s and d are the fields of the charmed, strange and d quarks; D^+, K^{*0}, D_s^{*+} and $D_{s_1}^+$ are the fields of the pseudoscalar meson D^+, vector mesons K^{*0} and D_s^{*+} and the axial-vector meson $D_{s_1}^+$

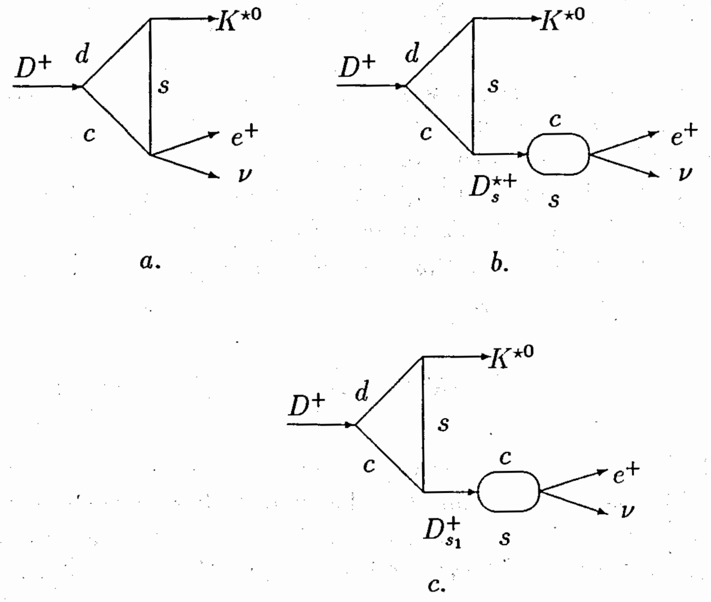

Fig.2. Diagrams describing the $D^+ \to \bar K^{*0}e^+\nu_e$ decay.

We will use relations arising in the NJL model between the coupling constants of pseudoscalar and vector (axial) mesons [10]-[14]

$$g_V(s,c) = \sqrt{6}g_{D_S}Z_{sc}^{-\frac{1}{2}}, \quad Z_{sc}^{-1} = 1 - \frac{3(m_s+m_c)^2}{2M^2_{D_{S_1}}}, \quad g_V(d,s) = g_\rho \ . \qquad (9)$$

Here m_s and m_c are masses of the s and c constituent quarks

$$m_s = 0.5GeV, \quad m_c = \frac{M_{J/\Psi}}{2} = 1.55GeV \ . \qquad (10)$$

$M_{D_{S_1}}$ is the mass of the axial-vector meson D_{s_1} . With the help of the Goldberger- Treiman identity the constants g_D and g_{D_S}.can be expressed through the decay constants F_D and F_{D_S}.

$$g_D = \frac{m_d + m_c}{2F_D}, \quad g_{D_s} = \frac{m_s + m_c}{2F_{D_S}} \ , \qquad (11)$$

where $F_D = 1.9F_\pi$ and $F_{D_S} = 2F_\pi$ (see ref.[21]). And the last we are going to take from the NJL model is the relation of the constants $g_V(i,j)$ with the regularised diverging quark loops [10]-[14]

$$g_V^{-2}(i,j) = \frac{(-i)}{8\pi^4}\int^\Lambda \frac{dk}{(m_i^2 - k^2)(m_j^2 - k^2)} \ , \qquad (12)$$

where Λ means some regularisation of this integral the concrete form of which is out of our interest.

Let us start our calculations with the vector part of diagram 2a. This anomalous- type diagram is similar to those describing the decays $(\pi^0,\eta,\eta') \to 2\gamma, \eta' \to \gamma(\rho,\omega), \rho \to \gamma(\pi,\eta), \omega \to \pi\gamma, \phi \to \gamma(\pi,\eta)$ and others (see ref.[11]). It is to be noted that in describing the above decays one should use only the first step of the momentum expansion of quark loops. Then, theoretical estimates will satisfactorily agree with the experimental data [11]. [3]

The contribution of diagram 2a to the form factor $V(0)$ in the above-mentioned approximation equals [4]

$$V^{(a)}(0) = g_\rho\frac{(m_c+m_d)(m_c+m_s+m_d)}{(4\pi)^2F_D}(M_D + M_{K^*})I(m_c,m_s,m_d) = 2, \qquad (13)$$

[3]The use of this method of calculation allows one to approximately conserve the group $U(3)$, which results in reasonable relations between the amplitudes of the above processes.The allowance for the influence of momenta of legs makes the results worse as it strongly violates this group.

[4]In the case $m_c = m_s = m_d$ and $F_D = F_\pi$ the form factor (13) easily results in expression (5) for the vector form factor of the $\pi^- \to e\bar\nu\gamma$ decay

where

$$I(m_c,m_s,m_d) = \frac{1}{i\pi^2}\int\frac{dk}{(m_c^2-k^2)(m_s^2-k^2)(m_d^2-k^2)} = \tag{14}$$

$$= \frac{2}{(m_c^2-m_s^2)}\left[\frac{ln(m_c/m_d)}{1-m_d^2/m_c^2}-\frac{ln(m_s/m_d)}{1-m_d^2/m_s^2}\right].$$

Now let us estimate the contribution of diagram 2b with the intermediate vector meson D_s . The amplitude of the $D_s^{*+}\to\bar{e}\nu$ decay proceeding through a quark loop has the form

$$A_{D_s^{*+}\to\bar{e}\nu} = \frac{g_V(s,c)}{3\sqrt{2}}\{\frac{6}{g_V^2(s,c)}\}[Q^\mu Q^\nu - g^{\mu\nu}Q^2 + \frac{3(m_c-m_s)^2}{2}g^{\mu\nu}]\epsilon_\mu^{(D_S^*)}l_\nu. \tag{15}$$

(The expression in the braces brackets corresponds to the diverging quark loop, see(12)). Using expression (15) one can easily estimate the contribution of diagram 2b to the vector part of the matrix element H^μ

$$\Delta^{(b)}H_V^\mu = -i\frac{2V^{(a)}(0)}{M_D+M_{K^*}}[Q^2-\frac{3}{2}(m_c-m_s)^2]\frac{1}{M_{D^*}^2}\epsilon^{\mu\nu\rho\sigma}\epsilon_\nu^{(K^*)}p_\rho q_\sigma \ . \tag{16}$$

Finally, this leads to the following expression for the vector form factor

$$V^{(a+b)}(0) = V^{(a)}(0)[1-\frac{3(m_c-m_s)^2}{2M_{D^*}^2}] = 1.3 \ . \tag{17}$$

This estimate satisfies the experimental data within the errors [8]

$$V^{exp}(0) = 0.9\pm0.3\pm0.1 \ . \tag{18}$$

Now let us calculate the axial form factor A_2 . The contribution of diagram 2a to the corresponding part of the amplitude H^μ has the form

$$\Delta^{(a)}H_A^\mu = ig_\rho g_D\frac{3\epsilon_\nu^{(K^*)}}{4\pi^4}\cdot$$

$$\int dk\frac{m_dp^\mu p^\nu-(m_c+m_s)p^\mu k^\nu+(m_s-m_d)p^\nu k^\mu+2(m_c-m_d)k^\mu k^\nu}{[m_d^2-k^2][m_s^2-(k-q)^2][m_c^2-(k-p)^2]} . \tag{19}$$

Hence, for the form factor $A_2^{(a)}$ we have

$$A_2^{(a)}(0) = 3 \ . \tag{20}$$

Let us consider now diagram 2c with the intermediate axial-vector meson $D_{s_1}^+$. For the decay amplitude $D_{s_1}^+\to\bar{e}\nu$ we derive the expression analogous to (6) [18]

$$A^\mu_{D_{S_1}^+\to\bar{e}\nu} = \frac{g_V(s,c)}{3\sqrt{2}}(\frac{6}{g_V^2(s,c)})[Q^\mu Q^\nu - g^{\mu\nu}Q^2+\frac{3}{2}(m_c+m_s)^2g^{\mu\nu}]\epsilon_\mu^{(D_{S_1})}l_\nu \ . \tag{21}$$

Using (21) we arrive at the following expression for the total contribution of diagrams 2a and 2c to the axial part of the amplitude H^μ

$$\Delta^{(a+c)}H_A^\mu = \Delta^{(a)}H_A^\mu[1-\frac{3(m_c+m_s)^2}{2M_{D_{S_1}}^2}+\frac{Q^2}{M_{D_{S_1}}^2}] \ . \tag{22}$$

As a result, for the form factor $A_2(0)$ we finally get

$$A_2^{(a+c)}(0) = 3[1-\frac{3(m_c+m_s)^2}{2M_{D_{S_1}}^2}] = 0.06 \ . \tag{23}$$

which is in complete agreement with the experimental value [8]

$$A_2^{exp}(0) = 0.0\pm0.2\pm0.1 \ . \tag{24}$$

Finally, let us estimate the axial form factor $A_1(0)$. The decisive contribution to this form factor comes from the diverging parts of diagrams 2a and 2c. Let us demonstrate this.

The diverging part of diagram 2a has the form

$$\Delta_{div}^{(a)}H_A^\mu = 2g_\rho g_D(m_c+2m_s-m_d)\frac{(-i3)}{(2\pi)^4}\int^\Lambda\frac{dk}{(m_c^2-k^2)(m_s^2-k^2)}g^{\mu\nu}\epsilon_\nu^{(K^*)} = \tag{25}$$

$$= (m_c+2m_s-m_d)[\frac{3g_\rho g_D}{g_V^2(s,c)} = g_\rho\frac{F_{D_S}^2}{F_D}\frac{(m_c+m_d)}{(m_c+m_s)^2}Z_{sc}]g^{\mu\nu}\epsilon_\nu^{(K^*)} \ .$$

After the inclusion of diagram 2c the factor Z_{sc} disappears and the contribution of diagrams 2a and 2c to the form factor A_1 becomes equal to

$$A_1(0) = g_\rho\frac{F_{D_S}^2}{F_D}\frac{(m_c+m_d)(m_c+2m_s-m_d)}{(m_c+m_s)^2(M_D+M_{K^*})} = 0.42 \ . \tag{26}$$

The experimental value is

$$A_1^{exp}(0) = 0.46 \pm 0.05 \pm 0.05 \ . \tag{27}$$

The contributions of the finite parts of diagrams 2a and 2c cannot essentially change the result (26) as they are multiplied by the small factor analogously to the form factor $A_2^{(a)}$ (see (22)).

Summing up the study of the form factors of the $D^+ \to \bar{K}^{*0}e^+\nu$ decay we should like to emphasize again that the author does not pretend to a good quantitative description of the above-mentioned quantities as he uses very rough phenomenological approximations. However, we think that a qualitative physical picture of this process is explained. It is very similar to the situation existing in the $\pi^- \to e\bar{\nu}\gamma$ decay . Indeed, in the vector form factor the decisive role is played by diagram 2a. Therefore, all the models give more or less the same, satisfactory enough estimates. A more complicated situation occurs with the axial form factor A_2 . Here arises an additional diagram 2c with an intermediate heavy axial meson whose contribution almost completely cancels the contribution of the basic diagram 2a. Therefore, quark models disregarding this effect give very overestimated results. Finally, in the form factor A_1 in a part of the amplitude corresponding to diagram 2a there arises a large factor Z_{sc} compensating the effect of subtraction of two almost equal quantities. This again leads to a considerable deviation from zero of the form factor A_1 .

In our subsequent papers we are going to study the momentum dependence of these form factors using one of the versions of potential quark models to estimate vertices described in the present paper by simple quark loops. Then, the ratio between the limiting and transverse polarisations in the decay $D^+ \to \bar{K}^{*0}e^+\nu_e$ as well as form factors of the B meson decay could be estimated.

Table 1. Experimental values for form factors of the decay $D^+ \to \bar{K}^{*0}e^+\nu_e$ from [8] compared to prediction of various models ([1]- [6] and this paper).

	E 691 [8]	[1]	[2]	[3]	[4]	[5]	[6]	This paper
$A_1(0)$	$0.46 \pm 0.05 \pm 0.05$	0.8	0.8	0.9	1.0	0.52 ± 0.07	0.43	0.42
$A_2(0)$	$0.0 \pm 0.2 \pm 0.1$	0.6	0.8	1.2	1.0	0.05 ± 0.35	0.29	0.06
$V(0)$	$0.9 \pm 0.3 \pm 0.1$	1.5	1.1	1.3	1.0	0.85 ± 0.08	0.50	1.3

References

[1] F.J.Gilman, R.L.Singleton (jr.), Phys.Rev.D41 (1990) 142

[2] N.Isgur, D.Scora, Phys.Rev.D4O (1989) 1491

[3] M.Bauer, M.Wirbel, Z.Phys. C42 (1989) 671

[4] J.G.Korner, G.A.Shuler, Z.Phys. C38 (1988)511

[5] V.Lubicz, G.Martinelli, C.T.Sachrajda, Nucl.Phys. B356 (1991),301

[6] R.N.Faustov, V.O.Galkin, A.Yu.Mishurov, Preprint JINR E2-91-451, Dubna, 1991

[7] J.C.Anjos et al., Phys.Rev.Lett.62 (1989) 722

[8] J.C.Anjos et al., Preprint Fermilab.Pub.-90/124-E (E-691); Phys.Rev.Lett.67 (1991) 1507

[9] M.Adamovich et al., Preprint CERN/PPE 91-99, June 1991

[10] M.K.Volkov, D.Ebert, Yad.Fiz. 36 (1982) 1265 (in Russian); Z.Phys.C 16 (1983) 205

[11] M.K.Volkov, Ann.Phys. 157 (1989) 282; Particles and Nuclei 17 (1986) 433

[12] D.Ebert, R.Reinhardt, Nucl.Phys.B271 (1986) 188

[13] U.Vogl, W.Weise, Preprint TPR-91-6, Regensburg,1991

[14] D.Ebert, Z.Phys.C 28 (1985) 433

[15] D.A.Bryman, P.Dopommier, C.Lecroy, Phys.Rep.88 (1982) 151; B.R.Holstein, Phys.Rev.D33 (1986) 3316; J.Gasser, H.Leutwyler, Ann.Phys.158 (1984) 142; E.Z.Avakian et al., Yad.Fiz.46 (1987) 576; N.Power, M.D.Scadron, Nuovo Cim. A78 (1983) 159

[16] A.Bay et al., Phys.Lett. B174 (1986) 445

[17] V.N.Pervushin, M.K.Volkov, Phys.Lett.B58 (1975) 74

[18] A.N.Ivanov, M.Nagy, M.K.Volkov, Phys.Lett.B200 (1988) 171

[19] M.K.Volkov, M.Nagy, A.A.Osipov, Teor.Mat.Fiz 87 (1991) 57 (in Russian)

[20] Particle Data Group, Phys.Lett B239 (1990) 1

[21] H.Albrecht et al., Preprint DESY 91-121, October, 1991

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