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SEMILEPTONIC HEAVY-TO-LIGHT DECAYS OF BARYONS

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1. Introduction

The spin-flavour symmetry of QCD [1, 2] discovered by Isgur and Wise opened the new era for theoretical investigations of weak decays of heavy hadrons. Specifically, s.l. decays of mesons and baryons with a single heavy quark in initial and final states were considered in detail in [5, 6, 20]. It is known, in this case the symmetry of Isgur and Wise takes place. The heavy baryon decays with heavy-to-light quark transition ($Q \to q$ transition) are very intresting, as well.

The research of s.l. decays with $Q \to q$ transition were carried out in the Free Quark Decay (FQD) [10, 11] which became the traditional tool for considering the s.l. decays of heavy quarks. However, we have to note that since a baryon in the final state contains only light degrees of freedom, we cannot identify the $Q \to q$ transition with weak decay of free quark into free quark. Thus, this approach does not give absolutely accurate predictions for heavy-to-light quark transitions. The obtained results in [10, 11] can be considered as preliminary estimates of these processes.

Quite a full analysis of weak decays of baryons containing only heavy quarks has been done in the spectator quark model (SQM) [4]- [9]. Specifically, the baryon decays corresponding to the $c \to s$ transition have been considered in the works [4, 8, 9]. However, the momentum dependence of form factors has not been calculated in the SQM. So, as "input" the SQM use the multipole q^2-dependence of weak form factors of heavy hadrons [5].

In this paper, we shall consider the s.l. decays of $\frac{1}{2}^+$ heavy baryon to $\frac{1}{2}^+$ light baryon final states in the framework of the Quark Confinement Model (QCM) [12]- [21]. The explicit expressions for form factors of these decays will be calculated. Also, we will analyse the limit $\frac{m_{B_q}}{m_{B_Q}} \ll 1$.

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Using the obtained form factors we will compute basic characteristics of s.l. decays $\Lambda_Q \to \Lambda_q e\nu$ and $\Sigma_Q \to \Sigma_q e\nu$: decay rates Γ, differential q^2-distributions $d\Gamma/dq^2$ and lepton E_l-spectra $d\Gamma/E_l$.

2. The main notions of the QCM

The Quark Confinement Model (QCM), a relativistic quark model, is based on some assumptions about the hadronisation and quark confinement. A more detailed description of the QCM is given in [12]- [21].

A dynamical description of hadron processes in the QCM is based on the interaction Lagrangians of hadrons with quarks. Specifically, the interaction Lagrangians of baryons with quarks have the following form

$$L_B = g_B \bar{B} J_B + h.c., \tag{1}$$

where B is the baryon field, g_B is the coupling constant. The J_B is the three-quark current with the quantum numbers of baryon B [14, 19]. The three-quark current with quantum numbers $J^P = \frac{1}{2}^+$ has two independent representaitions: tensor and vector variants. It was shown [14] that the tensor current gives a better description of the experimental data in the QCM. So, in this work we will use only T-variant of the three-quark current as in ref.[14, 20]. In this case, three-quark currents corresponding to $\Lambda_\mathrm{Q}, \Sigma_\mathrm{Q}$-baryons and nucleon $N(p,n)$ have the following form

$$\begin{aligned} J_{\Lambda_\mathrm{Q}} &= \varepsilon^{abc}\{\mathrm{Q}^a(u^b C\gamma^5 d^c) + \gamma^5 \mathrm{Q}^a(u^b C d^c)\} \\ J_{\Sigma_\mathrm{Q}} &= \varepsilon^{abc}\sigma^{\mu\nu}\gamma^5 \mathrm{Q}^a(u^b C\sigma^{\mu\nu}u^c) \\ J_p &= \varepsilon^{abc}\sigma^{\mu\nu}\gamma^5 d^a(u^b C\sigma^{\mu\nu}u^c) \\ J_n &= \varepsilon^{abc}\sigma^{\mu\nu}\gamma^5 u^a(d^b C\sigma^{\mu\nu}d^c) \end{aligned} \tag{2}$$

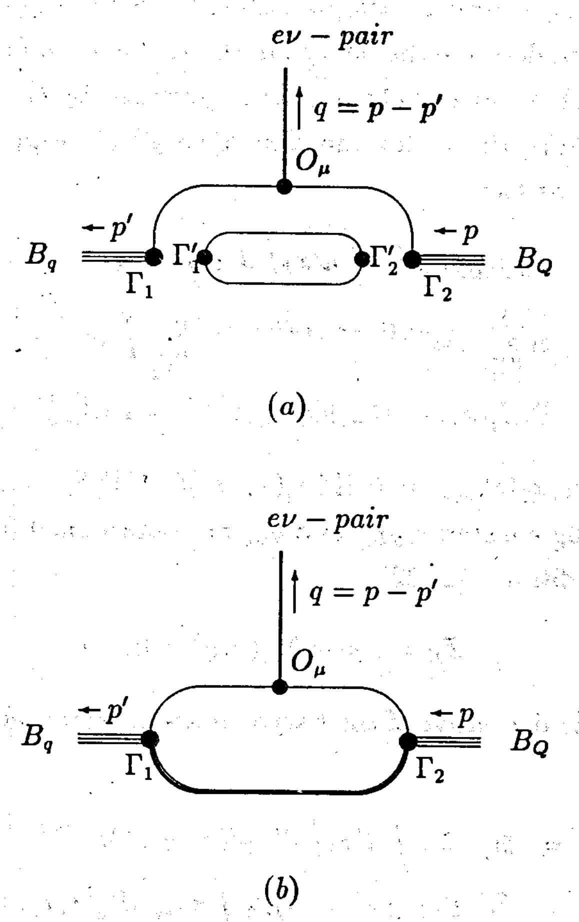

Fig.1. Heavy-Baryon Weak Form Factors in the QCM.

where a,b,c are colour indices, $C = \gamma^0\gamma^2$ is the matrix of the charge conjugation and $\mathrm{Q} = s, c$ or b quark.

The s.l. decays of the heavy to light baryons are described within the QCM by the quark diagrams (Fig.1). So far as the heavy quarks weakly interact with the external background fields, in the works [18, 20] it was suggested to describe the heavy quarks as the usual Fermi particles whereas the behaviour of light quarks is governed by the confinement mechanism. Thus, the vertex function $\Lambda_\mu(p,p')$ corresponding to diagram Fig.1a is written as

$$\begin{aligned}\Lambda_\mu(p,p') &= 6 l_f g_{B_Q} g_{B_q} 3^4 i^2 \int d^4x_1 \int d^4x_2 \int d^4x_3 \cdot \\ &\cdot \delta(\sum_{i=1}^{3} x_i) e^{-ip(x_1-x_3)+p'(x_3-x_2)} \sum_{\Gamma_1\Gamma_2} \int d\sigma_{vac} \cdot \qquad (3)\\ &\cdot \Gamma_1 S_q(x_1,x_3|B_{vac}) O_\mu S_Q(x_3-x_2)\Gamma_2 \Pi^{\Gamma_1\Gamma_2}_{vac}(x_1,x_2|B_{vac}),\end{aligned}$$

where $\Pi^{\Gamma_1\Gamma_2}_{vac}(x_1,x_2|B_{vac}) = tr\,[\Gamma_1' S_q(x_1,x_2|B_{vac})\Gamma_2' S_q(x_1,x_2|B_{vac})]$. In (3) the coupling constants g_{B_Q} and g_{B_q} are determined from the compositeness condition [12, 22]

$$Z_B = 1 + g_B^2 \Sigma_B'(m_B) = 0, \qquad (4)$$

where Σ_B' is the derivative of the baryon mass operator which is defined as

$$\begin{aligned}\Sigma_{B_Q}(p) &= 6 l_f \cdot 2^4 i \int d^4x_1 \int d^4x_2 \delta(x_1+x_2) e^{-ip(x_1-x_2)} \qquad (5)\\ &\cdot \sum_{\Gamma_1\Gamma_2} \Gamma_1 S_Q(x_1-x_2)\Gamma_2 \int d\sigma_{vac} \Pi^{\Gamma_1\Gamma_2}_{vac}(x_1,x_2|B_{vac}).\end{aligned}$$

for the heavy baryon and

$$\begin{aligned}\Sigma_{B_q}(p) &= 6 l_f \cdot 2^4 i \int d^4x_1 \int d^4x_2 \delta(x_1+x_2) e^{-ip(x_1-x_2)} \qquad (6)\\ &\cdot \sum_{\Gamma_1\Gamma_2} \int d\sigma_{vac} \Pi^{\Gamma_1\Gamma_2}_{vac}(x_1,x_2|B_{vac}) \Gamma_1 S_q(x_1,x_2|B_{vac})\Gamma_2\end{aligned}$$

for the light baryon.

Here

$$S_Q(x_1-x_2) = <0|T(Q(x_1)\bar{Q}(x_2))|0> = i(\not p - m_Q)^{-1}\delta(x_1-x_2), \qquad (7)$$

is the heavy quark propagator with mass m_Q, and

$$S_q(x_1,x_2|B_{vac}) = <0|T(q(x_1)\bar{q}(x_2))|0> = i(\not p + \not B_{vac})^{-1}\delta(x_1-x_2), \qquad (8)$$

is the light quark propagator in the external background field.

The confinement hypothesis concerns the definition of an action of the vacuum gluon field $\not B_{vac}(x)$ on the quark fields. Our assumption consists in that light quarks (u,d and s) do not appear in the observable hadron spectrum and do not have definite value of mass. They turn into some quasiparticles with quantum numbers of quarks and do not exist as usual particles.

The analytical structure of measure $d\sigma_{vac}$ is defined by the following integral [13, 14]

$$\int \frac{d\sigma_v}{v-z} = G(z) = a(-z^2) + z b(-z^2), \qquad (9)$$

where the confinement functions are chosen to be

$$\begin{aligned}a(u) &= \int d\sigma_v \frac{v}{v^2+u} = 2\exp\{-u^2-u\}, \qquad (10)\\ b(u) &= \int d\sigma_v \frac{1}{v^2+u} = 2\exp\{-u^2+0.4u\}. \qquad (11)\end{aligned}$$

The Dirac matrices $\Gamma_{1(2)}$, $\Gamma'_{1(2)}$ and the flavour coefficients l_f come from the interaction Lagrangians of heavy baryons with quarks (see, formula (1)-(2)) and l_f are equal to

$$l_f = \begin{cases} 1, & \text{for } \Lambda_Q\text{-decay} \\ 2, & \text{for } \Sigma_Q\text{-decay} \end{cases}$$

In ref. [14] it was proposed to use the *Quark-Diquark Approximation of the Three-Quark Structure of Baryons* under calculation of the integral (3):

$$\int d\sigma_{vac}\Gamma_1 S_q(x_1,x_3|B_{vac})O_\mu S_Q(x_3-x_2)\Gamma_2\Pi^{\Gamma_1\Gamma_2}_{vac}(x_1,x_2|B_{vac}) \to$$
$$\int d\sigma_v\Gamma_1 S_v(x_1-x_3)O_\mu S_Q(x_3-x_2)\Gamma_2 D_v(x_1-x_2), \qquad (12)$$

where $D_v(x_1-x_2)$ is considered to be the diquark propogator

$$D_v(x_1-x_2)=\int\frac{d^4k}{(2\pi)^4}e^{-ik(x_1-x_2)}\frac{d^{\Gamma_1\Gamma_2}}{v^2\Lambda_D^2-k^2}, \qquad (13)$$

where $\Lambda_D=827.7$ MeV and

$$d^{\Gamma_1\Gamma_2}=\begin{cases}1, & \Gamma_1=\Gamma_2=S,P\\ 2\cdot g^{\mu\alpha}g^{\nu\beta}, & \Gamma_1=\Gamma_2=T\,.\end{cases}$$

This assumption essentially simplifies calculations because it allows us to replace two-loop quark diagrams (Fig.1a) by the one-loop quark-diquark diagrams shown in Fig.1b. From the physical point of view, this prescription can be justified by that the baryons containing the only heavy quark can be considered as two-particle systems Qd (d is a light diquark).

3. The heavy-to-light quark transition

In the work [20] we considered the decays of heavy baryons containing only heavy quark in the initial and final states. We showed that spin-flavour symmetry proposed by Isgur and Wise [1, 2] describes the decays of this kind quite accurate. In this connection, it is interesting to consider the decay of the baryon containing the single heavy quark into light baryon.

There are antisymmetrical ($Q\{qq\}_A$) and symmetrical ($Q\{qq\}_S$) spin combinations of light quarks (see, formula (2)). We shall consider typical processes $\Lambda_Q\to\Lambda\{p,n\}e\nu$ and $\Sigma_Q\to\Sigma\{p,n\}e\nu$ as examples of the heavy baryon decays with the spin of light quarks $S=0$ and $S=1$, respectively. The corresponding matrix element has the standard form

$$M(Q\to q)=\frac{G_F}{\sqrt{2}}V_{Qq}\bar{B}_q(p')\Lambda_\mu(p,p')B_Q(p)\ell_\mu(q) \qquad (14)$$

where $B_Q(p)$ and $B_q(p')$ are the fields of heavy and light baryons, respectively.

$G_F=1.1664\cdot10^{-5}\text{GeV}^{-2}$ is the Fermi constant,

$\ell_\mu=\bar{u}_\nu\gamma_\mu(1-\gamma_5)u_\ell$ is a lepton current,

V_{Qq} is the Kobayashi-Mascawa matrix element.

It is known, the vertex function $\Lambda_\mu(p,p')$ can be decomposed into a combination of six relativistic form factors. Usually, one use the following combination

$$\begin{aligned}\Lambda_\mu(p,p') &= f_1(t)\gamma_\mu+f_2(t)i\sigma_{\mu\nu}q^\nu+f_3(t)q_\mu\\ &+ f_4(t)\gamma_\mu\gamma_5+f_5(t)i\sigma_{\mu\nu}q^\nu\gamma_5+f_6(t)q_\mu\gamma_5 \end{aligned}\qquad (15)$$

where $t=q^2$

For the calculation of the vertex function $\Lambda_\mu(p,p')$ in the framework of the QCM we use the *Quark-Diquark Approximation of the Three-Quark Structure of Baryons.* According to formulae (12) and (13) we have the following expression for $\Lambda_\mu(p,p')$

$$\begin{aligned}\Lambda_\mu(p,p') &= 6l_f g_{B_Q}g_{B_q}\sum_{\Gamma_1\Gamma_2}d^{\Gamma_1\Gamma_2}\int\frac{d^4k}{\pi^2 i}\int d\sigma_v\,\cdot \qquad (16)\\ &\Gamma_1\frac{1}{v\Lambda_q-(\not p'-\not k)}O_\mu\frac{1}{m_Q-(\not p-\not k)}\Gamma_2\frac{d^{\Gamma_1\Gamma_2}}{v^2\Lambda_D^2-k^2}\,\cdot\end{aligned}$$

By analogy we get the expression for the mass operators of heavy and light baryons

$$\Sigma_{B_Q}(p) = 6l_f \sum_{\Gamma_1\Gamma_2} d^{\Gamma_1\Gamma_2} \cdot \int \frac{d^4k}{\pi^2 i} \int d\sigma_v \Gamma_1 \frac{1}{m_Q - \not{p} - \not{k}} \Gamma_2 \frac{d^{\Gamma_1\Gamma_2}}{v^2\Lambda_D^2 - k^2} \quad (17)$$

$$\Sigma_{B_q}(p) = 6l_f \sum_{\Gamma_1\Gamma_2} d^{\Gamma_1\Gamma_2} \cdot \int \frac{d^4k}{\pi^2 i} \int d\sigma_v \Gamma_1 \frac{1}{v\Lambda_q - \not{p} - \not{k}} \Gamma_2 \frac{d^{\Gamma_1\Gamma_2}}{v^2\Lambda_D^2 - k^2} \cdot (18)$$

As in the work [20] we shall not distinguish between mass of the baryon containing a single heavy quark and mass of the corresponding heavy quark, i.e $m_Q = m_{B_Q}$. The hadron masses are taken from the experiment [23].

$$\begin{aligned} m_{\Lambda_b^0} &= 5.60\ GeV, \quad m_{\Sigma_b^+} = 5.73\ GeV, \quad m_{\Lambda_c^+} = 2.28\ GeV, \\ m_{\Sigma_c^{++}} &= 2.45\ GeV, \quad m_{\Lambda^0} = 1.12\ GeV, \quad m_{\Sigma^+} = 1.18\ GeV. \end{aligned}$$

Using the compositeness condition (4) we get the following expressions for the coupling constants of heavy (g_{B_Q}) and light (g_{B_q}) baryons with quarks, respectively (see, Appendix).

$$\begin{aligned} g_{B_Q}^{-2} &= \frac{3}{8\pi^2} l_B m_{B_Q}^2 R_0(m_{B_Q}^2) \cdot \frac{1}{1+z}, \quad (19) \\ g_{B_q}^{-2} &= \frac{3}{8\pi^2} l_B m_{B_q}^2 \int_0^1 d\alpha r(\alpha) a(-r(\alpha) m_{B_q}^2), \end{aligned}$$

where

$$r(\alpha) = \frac{\alpha(1-\alpha)}{1+(1-\alpha)z}$$

$$C(x,y,z) = \frac{1}{2y}\{\sqrt{(x-y+z)^2+4xy} - (x-y+z)\}.$$

$$C_0(u,x) = C(u,x,x) = \frac{\sqrt{u^2+4ux}-u}{2x}.$$

$$R_0(x) = \int_0^\infty du b(u) \frac{C_0(u,x)[1-C_0(u,x)]}{\sqrt{u^2+4ux}},$$

The parameter z denotes the ratio between Λ_D and Λ_q: $\Lambda_D^2 = \Lambda_q^2(1+z)$. The factor l_B being related to the flavour and T-product combinations is equal to

$$l_B = \begin{cases} 4, & \text{for } B = \Lambda_Q \\ 96, & \text{for } B = \Sigma_Q\ . \end{cases}$$

The vertex function $\Lambda_\mu(p,p')$ (16) is computed by analogy. In conformity with the definition $\Lambda_\mu(p,p')$ (15), we get the following expression for the form factors $f_i(t)$ $(i = 1, ..., 6)$

$$f_i(t) = \frac{3}{16\pi^2} l_B g_{B_Q} g_{B_q} \int_0^\infty du \int_0^1 d\alpha \tilde{f}_i(u,\tilde{Q}_\alpha) D(u,\tilde{Q}_\alpha) \quad (20)$$

where

$$\begin{aligned} D(u,\tilde{Q}_\alpha) &= \frac{1-C(u,\tilde{Q}_\alpha^2,m_Q^2)}{\sqrt{(u+\tilde{m}_Q^2-\tilde{Q}_\alpha^2)^2+4u\tilde{Q}_\alpha^2}} \cdot \frac{1}{1+(1-\alpha)z}, \\ Q_\alpha^2 &= (1-\alpha)(m_Q^2 - \alpha m_{B_q}^2) + \alpha t; \\ \tilde{Q}_\alpha^2 &= \frac{Q_\alpha^2}{1+(1-\alpha)z}, \qquad \tilde{m}_Q^2 = \frac{m_Q^2}{1+(1-\alpha)z}\ . \end{aligned}$$

The functions $\tilde{f}_i(u,\tilde{Q}^2\alpha)$ have the following form:

a. Decay $\Lambda_Q \to \Lambda\{p,n\}$

$$\begin{aligned} \tilde{f}_1 &= m_Q(1-\alpha)[1-C(u,\tilde{m}_Q^2,\tilde{Q}_\alpha^2)] \cdot [a(w) + m_{B_q} b(w)] \\ \tilde{f}_2 &= -\alpha \cdot a(w) + C(u,\tilde{m}_Q^2,\tilde{Q}_\alpha^2) \cdot [\alpha \cdot a(w) - m_Q b(w)] \\ \tilde{f}_3 &= \alpha \cdot a(w) - C(u,\tilde{m}_Q^2,\tilde{Q}_\alpha^2) \cdot [\alpha \cdot a(w) + m_Q b(w)] \quad (21) \\ \tilde{f}_4 &= -\tilde{f}_1, \qquad \tilde{f}_5 = -\tilde{f}_3, \qquad \tilde{f}_6 = -\tilde{f}_2. \end{aligned}$$

b. Decay $\Sigma_Q \to \Sigma\{p,n\}$

$$\begin{aligned}
\tilde{f}_1 &= \frac{1}{3}(1-\alpha)[1-C(u,\tilde{m}_Q^2,\tilde{Q}_\alpha^2)] \cdot \\
&\cdot \ [a(w)(m_Q+2m_{B_q})+m_Q b(w)\cdot(2m_Q+m_{B_q})] \\
\tilde{f}_2 &= \frac{1}{3}[a(w)(2-\alpha)+2m_Q(1-\alpha)b(w)-C(u,\tilde{m}_Q^2,\tilde{Q}_\alpha^2)\cdot \\
&\cdot \ \{a(w)(2-\alpha)+m_Q b(w)(1-2\alpha)\}] \\
\tilde{f}_3 &= \frac{1}{3}[a(w)(2+\alpha)-2m_Q(1-\alpha)b(w)-C(u,\tilde{m}_Q^2,\tilde{Q}_\alpha^2)\cdot \\
&\cdot \ \{a(w)(2+\alpha)+m_Q b(w)\cdot(1+2\alpha)\}] \\
\tilde{f}_4 &= -\frac{1}{3}(1-\alpha)[1-C(u,\tilde{m}_Q^2,\tilde{Q}_\alpha^2)]\cdot \qquad (22) \\
&\cdot \ [a(w)(m_Q-2m_{B_q})-m_Q b(w)\cdot(2m_Q-m_{B_q})] \\
\tilde{f}_5 &= \frac{1}{3}[a(w)(2-\alpha)-2m_Q(1-\alpha)b(w)-C(u,\tilde{m}_Q^2,\tilde{Q}_\alpha^2)\cdot \\
&\cdot \ \{a(w)(2-\alpha)-m_Q b(w)(1-2\alpha)\}] \\
\tilde{f}_6 &= \frac{1}{3}[a(w)(2+\alpha)+2m_Q(1-\alpha)b(w)-C(u,\tilde{m}_Q^2,\tilde{Q}_\alpha^2)\cdot \\
&\cdot \ \{a(w)(2+\alpha)-m_Q b(w)(1+2\alpha)\}],
\end{aligned}$$

where $w = u - \alpha(1-\alpha)\tilde{m}_{B_q}^2$.

There are graphics of differential distributions $d\Gamma/dq^2$ and E_l-spectra $d\Gamma/dE_l$ in Fig.(3-14). For comparison the analogous graphics for the decay $\Lambda_c^+ \to \Lambda^0 e\nu$ obtained in the SQM [4, 8, 9] are given in Fig.(5,11). One can see, both results are similarly quality. One can see our graphics go in the few times higher than one in the SQM. Probably, this difference in the results can be explained by a more complex momentum dependence of the QCM form factors than dipole form factors, which are used in the SQM. The calculated values of the widths of s.l. decays of Λ_Q and Σ_Q into light baryons are presented in Table 1. For comparison the results obtained in the SQM [8, 9] and FQD [10, 11] are given.

Table 1. Decay Rates of $B_Q \to B_q e\nu$ Decays

Flavour exchange	Process	Decay Rate, $\Gamma/\lvert V_{Qq}\rvert$ $10^{12} sec^{-1}$			
		QCM		SQM [8]	FQM [6]
		asymptotic results	explicit results		
$b \to u$	$\Lambda_b^0 \to pe\nu$	32.65	148.98		118.
	$\Sigma_b^- \to ne\nu$	608.16	395.92		
$c \to s$	$\Lambda_c^+ \to \Lambda^0 e\nu$	1.56	0.59	0.23	0.35
	$\Sigma_c^{++} \to \Sigma^+ e\nu$	4.26	0.45		
$c \to d$	$\Lambda_c^+ \to ne\nu$	1.33	0.90		0.54
	$\Sigma_c^{++} \to pe\nu$	3.82	0.90		

We also considered the limit $\frac{m_{B_q}}{m_{B_Q}} \ll 1$ $(m_Q \to \infty)$. To demonstrate the calculation technique of weak baryon form factors we compute the asymptotics of the typical integral:

$$I(p,p') = \int \frac{d^4k}{\pi^2 i} \int d\sigma_v \frac{1}{v^2\Lambda_q^2-(k-p')^2} \cdot \frac{1}{m_Q^2-(k-p)^2} \cdot \frac{1}{v^2\Lambda_D^2-k^2} \cdot \qquad (23)$$

After using the Feynman α-parametrization expression (23) has the following form

$$I(p,p') = \int \frac{d^4k}{\pi^2 i} \int d\sigma_v \int_0^1 d\alpha \frac{1}{m_Q^2-(k-Q_\alpha)^2} \cdot \frac{1}{[v^2\Lambda^2(\alpha)-k^2-\alpha(1-\alpha)p'^2]^2}, \qquad (24)$$

where

$$\not{Q}_\alpha = \not{p} - \alpha \not{p}',$$
$$\Lambda^2(\alpha) = \alpha\Lambda_q^2 + (1-\alpha)\Lambda_D^2 .$$

Performing the Wick rotation $k_0 \to e^{i\frac{\pi}{2}k_4}$, using the spherical coordinate system and recalling the definition of the confinement function we obtain

$$\begin{aligned} I(p,p') &= \frac{2}{\pi}\int_0^\infty du \cdot u \int_0^\pi d\theta \sin^2\theta \cdot \\ &\cdot \int_0^1 d\alpha \frac{m_Q^2 + u - Q_\alpha^2}{(m_Q^2+u-Q_\alpha^2)^2 + 4uQ_\alpha^2\cos^2\theta} b(z) = \\ &= \frac{2}{\pi}\int_0^\infty du \cdot u \int_0^1 d\tau \sqrt{\frac{1-\tau}{\tau}} \cdot \\ &\cdot \int_0^1 d\alpha \frac{m_Q^2+u-Q_\alpha^2}{(m_Q^2+u-Q_\alpha^2)^2+4uQ_\alpha^2\tau} b(z) . \end{aligned} \tag{25}$$

Here $z = (u - \alpha(1-\alpha)p'^2)/\Lambda(\alpha)$; $\tau = \cos^2\theta$.

As $m_{B_Q} = m_Q \to \infty$ we assume $m_q, \Lambda_q, \Lambda_D \ll m_Q$. This allows us to neglect the difference between Λ_q and Λ_D and to consider that $\Lambda_q = \Lambda_D$, which essentianly simplifies our calculations.

$$\begin{aligned} I(p,p') &= \frac{2}{\pi m_Q^2}\int_0^\infty du \cdot ub(u) \int_0^1 d\tau \sqrt{\frac{1-\tau}{\tau}} \cdot \\ &\int_0^1 d\alpha \frac{k^2+\alpha y}{(k^2-2k^2\tau+\alpha y)^2 + 4k^2\tau(1+k^2-k^2\tau)}, \end{aligned} \tag{26}$$

where $y = 1 - q^2/m_Q^2$, $k^2 = u/m_Q^2$.

Integrating over the α variable gives us the result

$$\begin{aligned} I(p,p') &= \frac{2}{\pi m_Q^2}\int_0^\infty du \cdot ub(u) \int_0^1 d\tau \sqrt{\frac{1-\tau}{\tau}} \cdot \\ &\cdot \left[\frac{1}{2y} Ln(k^2,y,\tau) + \frac{4k^2\tau}{\sqrt{\Delta}} Ar(k^2,y,\tau)\right], \end{aligned} \tag{27}$$

where

$$\begin{aligned} Ln(k^2,y,\tau) &= \ln\frac{(k^2+y)^2 + 4\tau k^2(1-y)}{k^2(k^2+4\tau)}, \\ Ar(k^2,y,\tau) &= \text{arctg}\frac{\sqrt{\Delta}}{2k^2[k^2+4\tau+y(1-2\tau)]}, \\ \Delta &= 16\tau k^2 y^2[1+k^2(1-\tau)]. \end{aligned} \tag{28}$$

The functions $Ln(k^2,y,\tau)$ and $Ar(k^2,y,\tau)$ in the limit $m_Q \to \infty$ look as

$$Ln(k^2,y,\tau) = 2\ln m_Q$$
$$Ar(k^2,y,\tau) = \frac{\pi}{2} .$$

Thus, we have

$$I(p,p') = \frac{2}{\pi m_Q^2}\int_0^\infty du \cdot ub(u) \int_0^1 d\tau \sqrt{\frac{1-\tau}{\tau}} \left[\frac{\ln m_Q}{y} + \frac{\pi}{2}k\right]. \tag{29}$$

Finally, integrating over the τ variable we obtain the resulting expression for $I(p,p')$ in the Isgur-Wise limit

$$I(p,p') = \frac{\ln m_Q}{m_Q^2 y}\int_0^\infty du \cdot ub(u) + \frac{2}{3}\frac{1}{m_Q^3 y}\int_0^\infty du \cdot u^{3/2} b(u) . \tag{30}$$

By anology we obtain expressions for the form factors $f_i(t)$ in (20). They have the following form

a. Decay $\Lambda_Q \to \Lambda\{p,n\}$

$$\begin{aligned} f_1 &= \frac{\ln m_Q}{m_Q y}\frac{\sqrt{6}A_0}{\sqrt{m_{B_q}B_0R(m_{B_q})}} \\ f_2 &= -\frac{\sqrt{6}}{m_Q^2 y}\frac{A_0+B_{1/2}}{\sqrt{m_{B_q}B_0R(m_{B_q})}} \\ f_3 &= \frac{\sqrt{6}}{m_Q^2 y}\frac{A_0-B_{1/2}}{\sqrt{m_{B_q}B_0R(m_{B_q})}} \\ f_4 &= -f_1, \qquad f_5=-f_3, \qquad f_6=-f_2 \ . \end{aligned} \tag{31}$$

b. Decay $\Sigma_Q \to \Sigma\{p,n\}$

$$\begin{aligned} f_1 &= m_Q\Phi(q^2) \\ f_2 &= \Phi(q^2) \\ f_3 &= -f_2 = f_5 = -f_6, \qquad f_4 = f_1 \ . \end{aligned} \tag{32}$$

Here

$$\begin{aligned} R(m_{B_q}) &= \int\limits_0^1 du \frac{u}{\sqrt{1-u}} a\left(-\frac{u m_{B_q}^2}{4}\right) \\ \Phi(q^2) &= \frac{4\sqrt{2}}{3}\frac{\ln m_Q^2}{m_Q y}\sqrt{\frac{B_0}{m_{B_q}R(m_{B_q})}} \\ A_0 &= \int\limits_0^1 du\, a(u), \qquad B_{1/2} = \int\limits_0^1 du\sqrt{u}\, b(u) \ . \end{aligned}$$

Note, in the limit $\frac{m_{B_q}}{m_{B_Q}} \ll 1$ all form factors are inversely proportional to y, i.e. they have a monopole dependence. The q^2-dependence of f_1 form factors for $\Lambda_b^0 \to pe\nu$ decay in Fig.(2) is drawn. However, it is to be remarked that explicit form of $f_1(q^2)$ is strong different from

asymptotic one. One can see from the Table 1 and graphics of differential q^2-distribution $d\Gamma/dq^2$ given in Fig.3,4 the approximation $\frac{m_{B_q}}{m_{B_Q}} \ll 1$ does not give the right description of the processes with heavy-to-light quark transition. So the Isgur-Wise limit in the heavy-to-light decays of baryons is not correct. Analogous conclusion has been done by N.Isgur in the work [3].

However, one has to note that in the semileptonic decays of heavy baryons with a single b-quark in the initial state and a single c-quark in the final state the Isgur-Wise spin-flavour symmetry takes place. Particularly, in our recent pâper [20] we have shown that the behaviour of weak form factors arising in these decays practically coincides with the behaviour of their mathematical asymptotics ($m_Q \to \infty$).

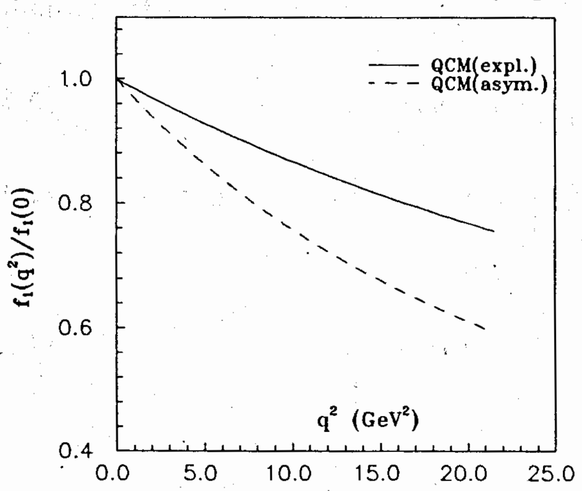

Fig.2

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Appendix

The heavy baryon mass operator is written as

$$\Sigma^{\Gamma_1\Gamma_2}(p) = \int \frac{d^4k}{\pi^2 i}\int d\sigma_v \Gamma_1 \frac{1}{m_Q-(\not p-\not k)}\Gamma_2 \frac{d^{\Gamma_1\Gamma_2}}{v^2\Lambda_D^2-k^2} =$$

$$= \Lambda_q m_Q \Gamma_1 \Gamma_2 d^{\Gamma_1\Gamma_2} \int \frac{d^4k}{\pi^2 i} \int d\sigma_v \cdot \frac{1}{[v^2(1+z) - k^2][m_Q^2 - (\not p - \not k)^2]} \quad \text{(A.1)}$$

The typical four-dimensional integral in the expression (A.1)

$$I(p^2) = \int \frac{d^4k}{\pi^2 i} \frac{b(-k^2)}{m_Q^2 - (k-p)^2} \quad \text{(A.2)}$$

is calculated in a standard manner (see, [18, 21]):
(i) The transition to the Euclidean region is performed for the internal momentum $k^0 \to ik_4$, $k^2 \to -k_E^2$ and external ones $p^0 \to ip_4$, $p^2 \to -p_E^2$;
(ii) The integration over sphere angles is carried out using the formula:

$$\int_0^\pi d\theta \frac{\sin^2\theta}{r + \cos\theta} = \pi[r - \sqrt{r^2-1}] \qquad (r \geq 1); \quad \text{(A.3)}$$

(iii) The analytical continuation to the physical region over external momentum is fulfilled.
Finally we have

$$I(p^2) = \int_0^\infty du\, b(u) C(u, p^2, m_Q^2). \quad \text{(A.4)}$$

Then we have

$$\Sigma^{\Gamma_1\Gamma_2} = \Lambda_q m_Q \Gamma_1 \Gamma_2 d^{\Gamma_1'\Gamma_2'} \int d\sigma_v \int du \frac{C(u,p^2,m_Q^2)}{v^2(1+z) - u} =$$
$$= \Lambda_q m_Q \Gamma_1 \Gamma_2 d^{\Gamma_1'\Gamma_2'} \int du\, b(u) C(u, \tilde p^2, \tilde m_Q^2), \quad \text{(A.5)}$$

where

$$\tilde p^2 = \frac{p^2}{1+z} \cdot$$

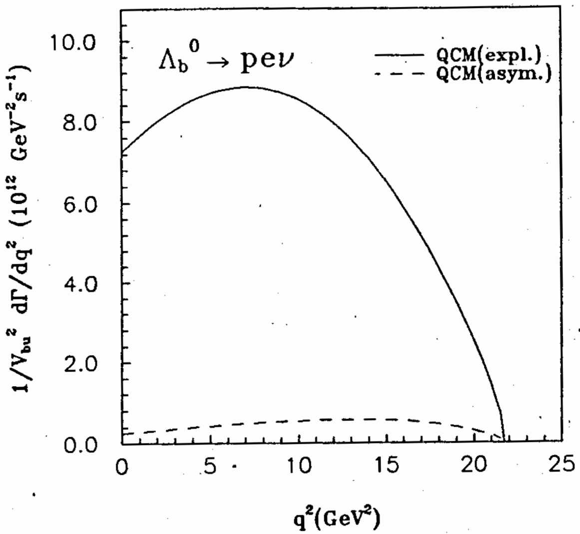

Fig.3

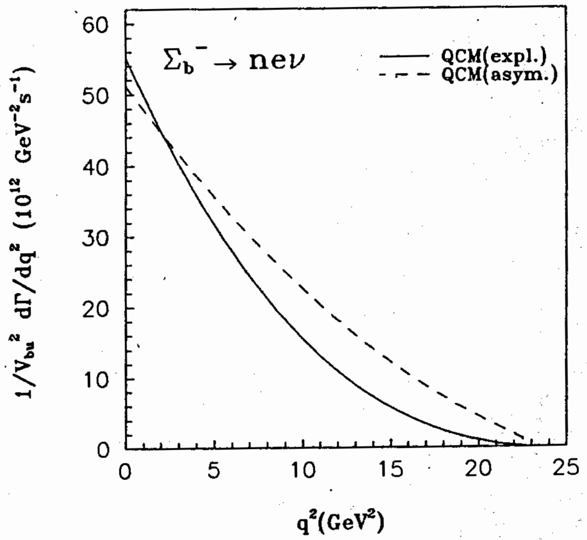

Fig.4

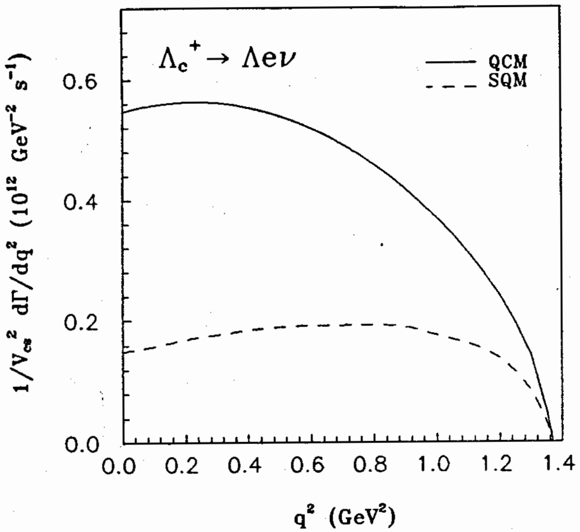

Fig.5

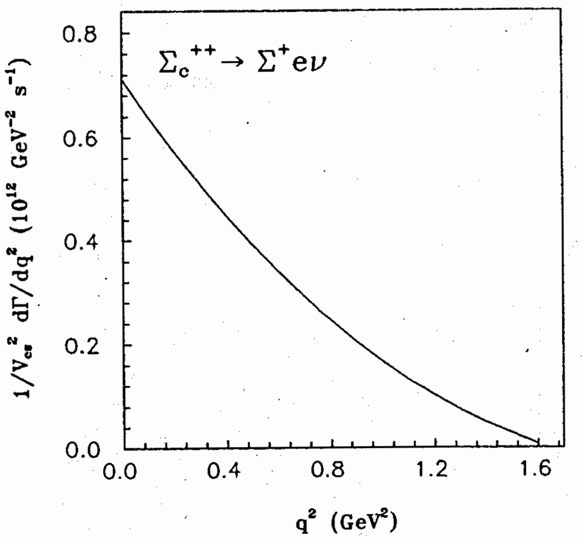

Fig.6

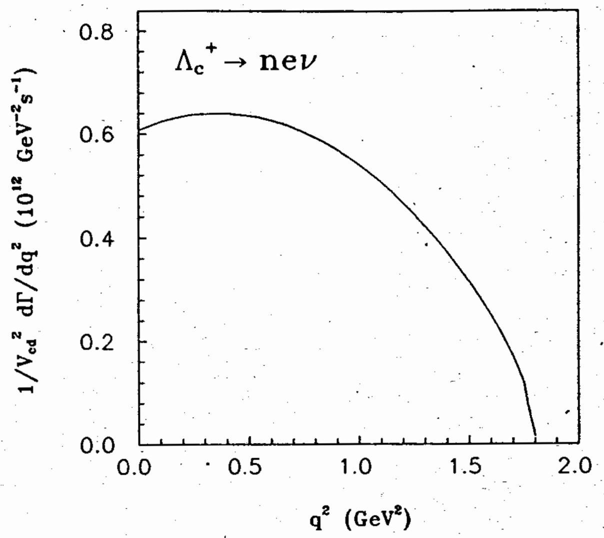

Fig.7

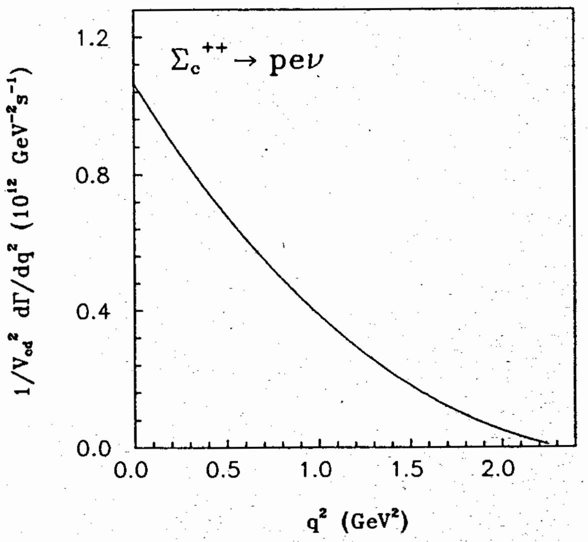

Fig.8

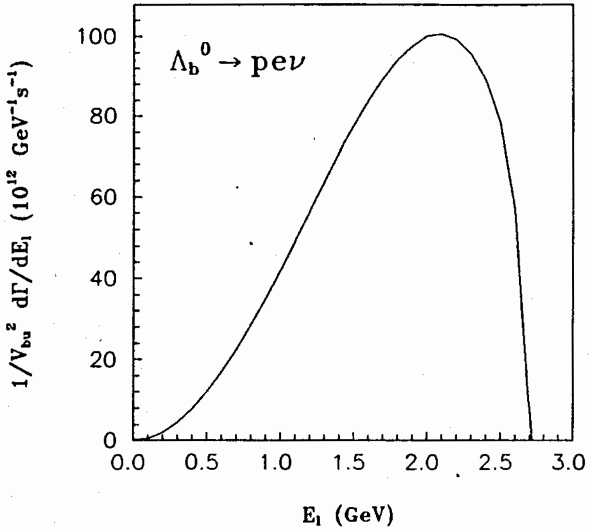

Fig.9

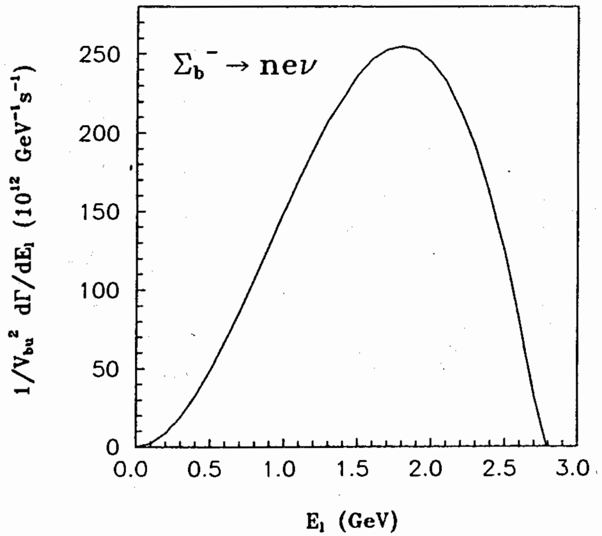

Fig.10

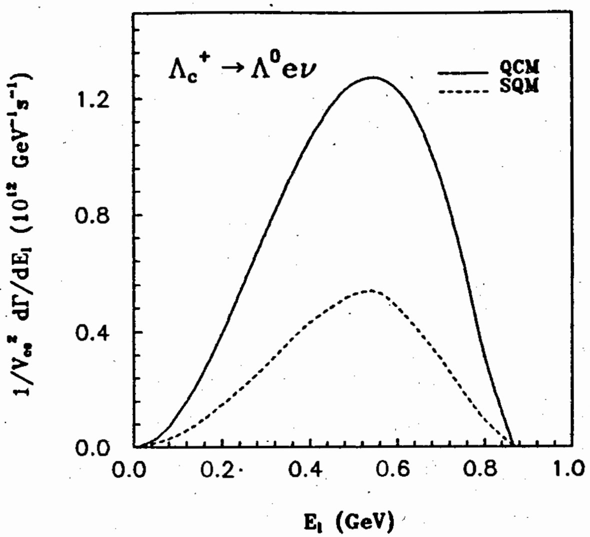

Fig.11

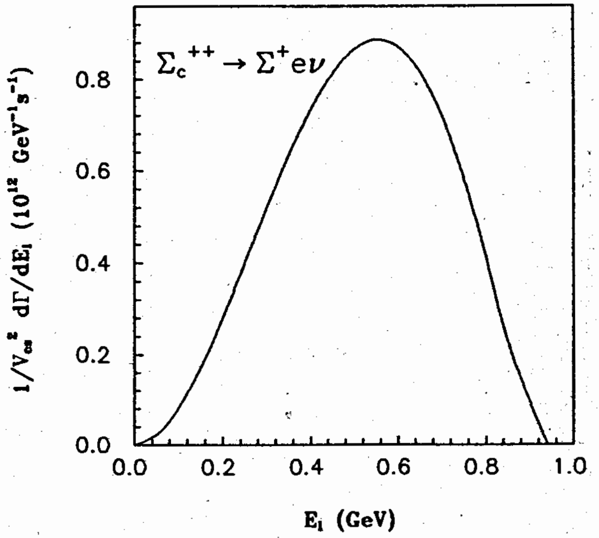

Fig.12

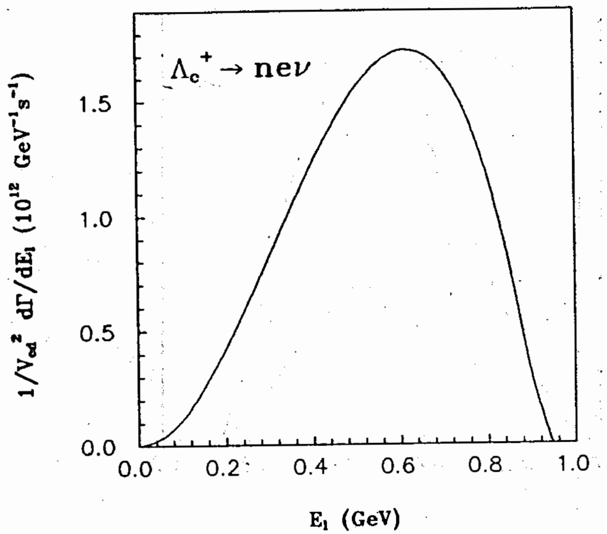

Fig.13

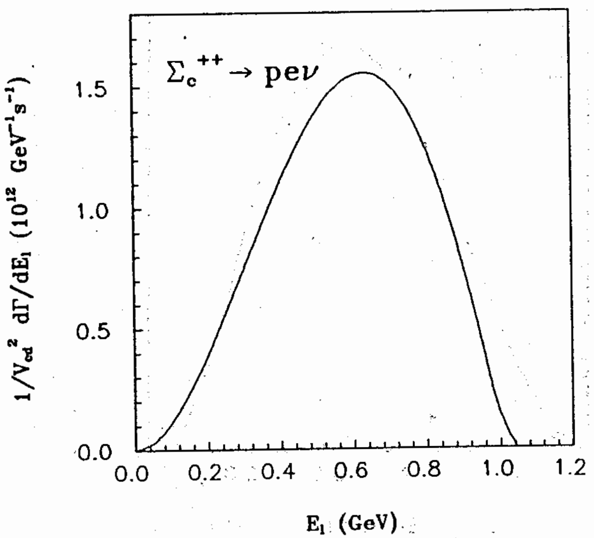

Fig.14

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Ефимов Г.В. и др. E2-92-106
Полулептонные распады
тяжелых барионов в легкие барионы

Дано детальное описание полулептонных распадов барионов, содержащих один тяжелый кварк, в легкие барионы. Данные распады рассмотрены в рамках модели конфайнмированных кварков. Вычислены слабые формфакторы, ширины и дифференциальные распределения полулептонных распадов тяжелых барионов в легкие. В работе также рассмотрен предел $m_Q \to \infty$ (предел Изгура-Вайзе).

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1992

Efimov G.V. et al. E2-92-106
Semileptonic Heavy-to-Light
Decays of Baryons

We report our results about semileptonic decays of baryons with only heavy quark into light baryons. These processes are considered in the framework of the Quark Confinement Model. Weak form factors, decay rates and differential distributions of semileptonic heavy-to-light baryon decays are calculated. The limit $m_Q \to \infty$ is examined.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1992